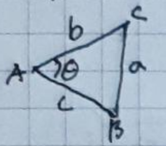
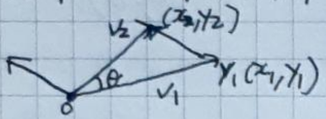
제3강 Dot product의 여러가지 성질

* 제 2코사인 법칙



$\cos\theta = \frac{b^2+c^2-a^2}{2bc}$

① Dot product의 기하학적 의미 $V_1=(x_1,y_1)$, $V_2=(x_2,y_2)$



$V_2-V_1=(x_2-x_1,\, y_2-y_1)$

$\cos\theta = \frac{\|V_2\|^2+\|V_1\|^2-\|V_2-V_1\|^2}{2\|V_2\|\|V_1\|}$

$= \frac{*}{2\|V_1\|\|V_2\|} = \frac{2\,V_1 V_2}{2\|V_1\|\|V_2\|}$

$* = \|V_2\|^2+\|V_1\|^2-\|V_2-V_1\|^2 \quad \|V\|=\sqrt{V\cdot V}$

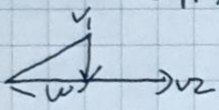
$= (\sqrt{V_2\circ V_2})^2+(\sqrt{V_1\circ V_1})^2-(\sqrt{(V_2-V_1)\circ(V_2-V_1)})^2$

$= 2V_1\circ V_2$

$\Rightarrow V_1\circ V_2 = \|V_1\|\|V_2\|\cos\theta$ $(0\sim 90°)$

$(90°\sim180°$ 사이$)$ 음수

② 벡터의 사영 projection 투영.



$proj_{V_2}V_1 = W$

$proj_{V_2}V_1 = \frac{V_1\circ V_2}{V_2\circ V_2}V_2$

$V_1\circ V_2 = \|W\|\|V_2\|$

$\|W\| = \frac{V_1\circ V_2}{\|V_2\|}$

$\frac{V_2}{\|V_2\|} \rightsquigarrow W = \frac{V_1\circ V_2}{\|V_2\|}\circ\frac{V_2}{\|V_2\|} = \frac{V_1\circ V_2}{\|V_2\|^2}V_2 = \frac{V_1\circ V_2}{V_2\circ V_2}V_2$

③ 코시슈바르츠 부등식 : $(ax+by)^2 \le (a^2+b^2)(x^2+y^2)$

$U=(a,b)\ V=(x,y)\quad U\circ V = ax+by$

$(U\circ V)^2 \le \|U\|^2\|V\|^2$

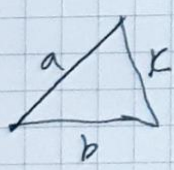
$|U\circ V| \le \|U\|\|V\|$

proof) $U\circ V = \|U\|\|V\|\cos\theta \ (-1\le\cos\theta\le1)$

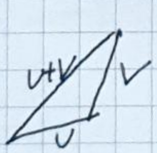
$-\|U\|\|V\| \le U\circ V \le \|U\|\|V\|$

$|U\circ V| \le \|U\|\|V\|$

④ 삼각부등식



$c \le a+b$

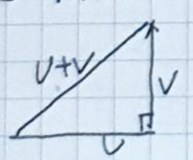


$\|U+V\|^2 \le (\|U\|+\|V\|)^2$

$(U+V)(U+V) = U\circ U + 2U\circ V + V\circ V$

$= \|U\|^2+2U\circ V+\|V\|^2 \le \|U\|^2+2|U\circ V|+\|V\|^2$

⑤ 피타고라스



$\|U+V\|^2 = (U+V)\circ(U+V)$

$= U\circ U + 2U\circ V + V\circ V$

$= \|U\|^2 + \cancel{2U\circ V} + \|V\|^2$

$= \|U\|^2+\|V\|^2$

$U\circ V = \|U\|\|V\|\cos\theta$

6 수직

$U\perp V \Leftrightarrow U\circ V = 0$