

- MultilayerGraphs.jl: A Julia package for the creation,
- <sup>2</sup> manipulation and analysis of the structure, dynamics
- 3 and functions of multilayer graphs
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#### Software

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# Summary

MultilayerGraphs.jl is a Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs extending Graphs.jl (Fairbanks et al., 2021) and fully integrating with the JuliaGraphs ecosystem.

A multilayer graph is a graph consisting of multiple standard subgraphs called *layers* which can be interconnected through bipartite graphs called *interlayers* composed of the vertex sets of two different layers and the edges between them. The vertices in each layer represent a single set of nodes, although not all nodes have to be represented in every layer.

Formally, a multilayer graph can be defined as a triple G = (V, E, L), where:

- V is the set of vertices;
- E is the set of edges, pairs of nodes (u, v) representing a connection, relationship or interaction between the nodes u and v;
- lacksquare L is a set of layers, which are subsets of V and E encoding the nodes and edges within each layer.

Each layer  $\ell$  in L is a tuple  $(V_\ell, E_\ell)$ , where  $V_\ell$  is a subset of V that represents the vertices within that layer, and  $E_\ell$  is a subset of E that represents the edges within that layer.

[A FEW WORDS ABOUT THE MAIN FEATURES, POSSIBLY EXTRACTED FROM TUTO-23 RIAL / README]

## Statement of Need

Multiple theoretical frameworks have been proposed to formally integrate all instances of multilayer graphs (Aleta & Moreno, 2019; Artime et al., 2022; Bianconi, 2018; Boccaletti et al., 2014; Cozzo et al., 2018; M. D. Domenico et al., 2013; M. D. Domenico, 2022; Kivela et al., 2014; Lee et al., 2015).

Multilayer graphs have been adopted to model the structure and dynamics of a wide spectrum of high-dimensional, non-linear, multi-scale, time-dependent complex systems including physical, chemical, biological, neuronal, socio-technical, epidemiological, ecological and economic networks (Amato et al., 2017; Arruda et al., 2017; Azimi-Tafreshi, 2016; Baggio et al., 2016; Buldú & Porter, 2018; Cozzo et al., 2013; Dickison et al., 2016; M. D. Domenico, 2017; M. D. Domenico et al., 2016; Estrada & Gómez-Gardeñes, 2014; Gosak et al., 2018; Granell et al., 2013; Lazega & Snijders, 2016; Lim et al., 2019; Mangioni et al., 2020; Massaro & Bagnoli, 2014; Pilosof et al., 2017; Soriano-Paños et al., 2018; Timóteo et al., 2018).



We have chosen the Julia language for this software package because it is a modern, opensource, high-level, high-performance dynamic language for technical computing (Bezanson et al., 2017). At the best of our knowledge there are currently no software packages dedicated to the creation, manipulation and analysis of multilayer graphs implemented in the Julia language apart from MultilayerGraphs.jl itself (Moroni & Monticone, 2022).

#### Main Features

- Main structs
  - Different formalisms
- Main methods and metrics
  - Extension of Graphs.jl (Fairbanks et al., 2021), fully integrated within the JuliaGraphs ecosystem
- Integration with Agents.jl (Datseris et al., 2022), fully integrated within the Julia Dynamics ecosystem

# Installation and Usage

- To install MultilayerGraphs.jl it is sufficient to activate the pkg mode by pressing ] in the Julia REPL and then run the following command:
  - THE E and then run the following community
  - pkg> add MultilayerGraphs
    [HERE WE SHOULD INSERT A FEW LINES OF CODE SHOWACASING THE MAIN
- 54 FEATURES WRITTEN ABOVE
- $_{55}$  In the package documentation you can find a comprehensive tutorial that illustrates all its
- 56 main features and functionalities.

## Related Packages

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- Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the R language:
  - muxViz implements functions to perform multilayer correlation analysis, multilayer centrality analysis, multilayer community structure detection, multilayer structural reducibility, multilayer motifs analysis and utilities to statically and dynamically visualise multilayer graphs (D. Domenico et al., 2014);
  - multinet implements functions to import, export, create and manipulate multilayer graphs, several state-of-the-art multiplex graph analysis algorithms for centrality measures, layer comparison, community detection and visualization (Magnani et al., 2021);
  - mully implements functions to import, export, create, manipulate and merge multilayer graphs and utilities to visualise multilayer graphs in 2D and 3D (Hammoud & Kramer, 2018):
  - multinets implements functions to import, export, create, manipulate multilayer graphs and utilities to visualise multilayer graphs (Lazega et al., 2008).

## **Python**

- Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the Python language:
  - MultiNetX implements methods to create undirected networks with weighted or unweighted links, to analyse the spectral properties of adjacency or Laplacian matrices and



- to visualise multilayer graphs and dynamical processes by coloring the nodes and links accordingly:
- PyMNet implements data structures for multilayer graphs and multiplex graphs, methods to import, export, create, manipulate multilayer graphs and for the rule-based generation and lazy-evaluation of coupling edges and utilities to visualise multilayer graphs (Kivela et al., 2014).

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