




MultilayerGraphs.jl: A Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs

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DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

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Submitted: 01 January 1970

Published: unpublished

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Summary

MultilayerGraphs.jl is a Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs.

A multilayer graph is a graph consisting of multiple standard subgraphs called *layers* which can be interconnected through [bipartite graphs](#) called *interlayers* composed of the vertex sets of two different layers and the edges between them. The vertices in each layer represent a single set of nodes, although not all nodes have to be represented in every layer.

Formally, a multilayer graph can be defined as a triple $G = (V, E, L)$, where:

- V is the set of vertices;
- E is the set of edges, pairs of nodes (u, v) representing a connection, relationship or interaction between the nodes u and v ;
- L is a set of layers, which are subsets of V and E encoding the nodes and edges within each layer.

Each layer ℓ in L is a tuple (V_ℓ, E_ℓ) , where V_ℓ is a subset of V that represents the vertices within that layer, and E_ℓ is a subset of E that represents the edges within that layer.

MultilayerGraphs.jl is an integral part of the [JuliaGraphs](#) ecosystem extending Graphs.jl ([Fairbanks et al., 2021](#)) so all the methods and metrics exported by Graphs.jl work for multilayer graphs, but due to the special nature of multilayer graphs the package features a peculiar implementation that maps a standard integer-labelled vertex representation to a more user-friendly framework exporting all the objects an experienced practitioner would expect such as nodes (Node), vertices (MultilayerVertex), layers (Layer), interlayer (Interlayer), etc.

MultilayerGraphs.jl features multilayer-specific methods and metrics including the global clustering coefficient, the overlay clustering coefficient, the multilayer eigenvector centrality, the multilayer modularity and the Von Neumann entropy.

Finally, MultilayerGraphs.jl has been integrated within the [JuliaDynamics](#) ecosystem so that any Multilayer(Di)Graph can be utilised as an argument to the GraphSpace constructor in Agents.jl ([Datseris et al., 2022](#)).

Statement of Need

Multiple theoretical frameworks have been proposed to formally subsume all instances of multilayer graphs ([Aleta & Moreno, 2019](#); [Artime et al., 2022](#); [Bianconi, 2018](#); [Boccaletti et al., 2014](#); [Cozzo et al., 2018](#); [M. D. Domenico et al., 2013](#); [M. D. Domenico, 2022](#); [Kivela et al., 2014](#); [Lee et al., 2015](#)).

38 Multilayer graphs have been adopted to model the structure and dynamics of a wide spectrum
39 of high-dimensional, non-linear, multi-scale, time-dependent complex systems including physi-
40 cal, chemical, biological, neuronal, socio-technical, epidemiological, ecological and economic
41 networks ([Aleta et al., 2022, 2020](#); [Amato et al., 2017](#); [Arruda et al., 2017](#); [Azimi-Tafreshi,](#)
42 [2016](#); [Baggio et al., 2016](#); [Buldú & Porter, 2018](#); [Cozzo et al., 2013](#); [M. D. Domenico, 2017](#);
43 [M. D. Domenico et al., 2016](#); [Estrada & Gómez-Gardeñes, 2014](#); [Gosak et al., 2018](#); [Granell](#)
44 [et al., 2013](#); [Lim et al., 2019](#); [Mangioni et al., 2020](#); [Massaro & Bagnoli, 2014](#); [Pilosof et al.,](#)
45 [2017](#); [Soriano-Paños et al., 2018](#); [Timóteo et al., 2018](#)).

46 We have chosen the [Julia language](#) for this software package because it is a modern, open-
47 source, high-level, high-performance dynamic language for technical computing ([Bezanson et](#)
48 [al., 2017](#)). At the best of our knowledge there are currently no software packages dedicated to
49 the creation, manipulation and analysis of multilayer graphs implemented in the Julia language
50 apart from MultilayerGraphs.jl itself ([Moroni & Monticone, 2022](#)).

51 Main Features

52 The two main data structures are MultilayerGraph and MultilayerDiGraph: collections of
53 layers connected through interlayers.

54 The **vertices** of a multilayer graph are representations of one set of distinct objects called
55 Nodes. Each layer may represent all the node set or just a subset of it. The vertices
56 of Multilayer(Di)Graph are implemented via the MultilayerVertex custom type. Each
57 MultilayerVertex encodes information about the node it represents, the layer it belongs to
58 and its metadata.

59 Both the **intra-layer** and ****inter-layer* *edges**** are embedded in the MultilayerEdge struct,
60 whose arguments are the two connected multilayer vertices, the edge weight and its metadata.
61 It's important to highlight that Multilayer(Di)Graphs are weighted and able to store metadata
62 by default (i.e. they have been assigned the IsWeighted and IsMeta traits from [SimpleTraits.jl](#)).

63 The **layers** are implemented via the Layer struct composed of an underlying graph and a
64 mapping from its integer-labelled vertices to the collection of MultilayerVertices the layer
65 represents. **Interlayers** are similarly implemented via the Interlayer mutable struct, and they
66 are generally constructed by providing the two connected layers, the (multilayer) edge list
67 between them and a graph. This usage of underlying graphs allows for an easier debugging
68 procedure during construction and a more intuitive analysis afterwards allowing the package to
69 leverage all the features of the JuliaGraphs ecosystem so that it can be effectively considered
70 as a real proving ground of its internal consistency.

71 The Multilayer(Di)Graph structs are weighted and endowed with the functionality to store
72 both vertex-level and edge-level metadata by default so that at any moment the user may
73 add or remove a Layer or specify an Interlayer and since different layers and interlayers
74 could be better represented by graphs that are weighted or unweighted and with or without
75 metadata, it was crucial for us to provide the most general and adaptable structure. A
76 Multilayer(Di)Graph is instantiated by providing the ordered list of layers and the list of
77 interlayers to the constructor. The latter are automatically specified, so there is no need to
78 instantiate all of them.

79 Alternatively, it is possible to construct a Multilayer(Di)Graph making use of a graph
80 generator-like signature allowing the user to set the degree distribution or the degree sequence
81 and employs graph realisation methods such as the Havel-Hakimi algorithm for undirected
82 graphs ([Hakimi, 1962](#)) and the Kleitman-Wang algorithm for directed ones ([Kleitman & Wang,](#)
83 [1973](#)).

84 Multilayer(Di)Graphs structure may be represented via dedicated WeightTensor,
85 MetadataTensor and SupraWeightMatrix structs, all of which support indexing with

86 MultilayerVertexs. Once a Multilayer(Di)Graph has been instantiated, its layers and
87 interlayers can be accessed as their properties.

88 Installation and Usage

89 To install MultilayerGraphs.jl it is sufficient to activate the pkg mode by pressing] in the Julia
90 REPL and then run the following command:

```
pkg> add MultilayerGraphs
```

91 In the following code chunks we synthetically illustrate how to define, handle and analyse a
92 MultilayerGraph in order to showcase some of the main features outlined in the previous
93 section.

94 First of all we need to import the necessary dependencies and set a few relevant constants:

```
using Revise  
using StatsBase, Distributions  
using Graphs, SimpleWeightedGraphs, MetaGraphs, SimpleValueGraphs  
using MultilayerGraphs
```

```
const vertextype = Int64  
const _weighttype = Float64  
const min_vertices = 5  
const max_vertices = 7  
const min_edges = 1  
const max_edges = max_vertices*(max_vertices-1)  
const n_nodes = max_vertices
```

95 [...]

96 Then we define a multilayer graph by specifying its layers and interlayers:

```
multilayergraph = MultilayerGraph( layers,  
                                   interlayers;  
                                   default_interlayers_null_graph = SimpleGraph{vertextype}(),  
                                   default_interlayers_structure = "multiplex"  
                                   )
```

97 [...]

98 For a more comprehensive exploration of the package features and functionalities we strongly
99 recommend consulting the [tutorial](#) included in the package documentation.

100 Related Packages

101 R

102 Here is a list of software packages for the creation, manipulation, analysis and visualisation of
103 multilayer graphs implemented in the [R language](#):

- 104 ■ [muxViz](#) implements functions to perform multilayer correlation analysis, multilayer central-
105 ity analysis, multilayer community structure detection, multilayer structural reducibility,
106 multilayer motifs analysis and utilities to statically and dynamically visualise multilayer
107 graphs ([D. Domenico et al., 2014](#));
- 108 ■ [multinet](#) implements functions to import, export, create and manipulate multilayer
109 graphs, several state-of-the-art multiplex graph analysis algorithms for centrality measures,
110 layer comparison, community detection and visualization ([Magnani et al., 2021](#));

- `mully` implements functions to import, export, create, manipulate and merge multilayer graphs and utilities to visualise multilayer graphs in 2D and 3D (Hammoud & Kramer, 2018);
- `multinets` implements functions to import, export, create, manipulate multilayer graphs and utilities to visualise multilayer graphs (Lazega et al., 2008).

Python

Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the `Python` language:

- `MultiNetX` implements methods to create undirected networks with weighted or unweighted links, to analyse the spectral properties of adjacency or Laplacian matrices and to visualise multilayer graphs and dynamical processes by coloring the nodes and links accordingly;
- `PyMNet` implements data structures for multilayer graphs and multiplex graphs, methods to import, export, create, manipulate multilayer graphs and for the rule-based generation and lazy-evaluation of coupling edges and utilities to visualise multilayer graphs (Kivela et al., 2014).

Acknowledgements

This open-source research software project received no financial support.

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