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MultilayerGraphs.jl: A Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs

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Summary

MultilayerGraphs.jl is a Julia package for the creation, manipulation and analysis of the structure, dynamics and functions of multilayer graphs extending Graphs.jl (Fairbanks et al., 2021) and fully integrating with the JuliaGraphs ecosystem.

A multilayer graph is a graph consisting of multiple standard subgraphs called *layers* which can be interconnected through bipartite graphs called *interlayers* composed of the vertex sets of two different layers and the edges between them. The vertices in each layer represent a single set of nodes, although not all nodes have to be represented in every layer.

Formally, a multilayer graph can be defined as a triple $G = (V, E, L)$, where:

- V is the set of vertices;
- E is the set of edges, pairs of nodes (u, v) representing a connection, relationship or interaction between the nodes u and v;
- L is a set of layers, which are subsets of V and E encoding the nodes and edges within each layer.

Each layer ℓ in L is a tuple (V_ℓ, E_ℓ), where V_ℓ is a subset of V that represents the vertices within that layer, and E_ℓ is a subset of E that represents the edges within that layer.

[A FEW WORDS ABOUT THE MAIN FEATURES, POSSIBLY EXTRACTED FROM TUTORIAL / README]

Statement of Need

Multiple theoretical frameworks have been proposed to formally integrate all instances of multilayer graphs (Aleta & Moreno, 2019; Artime et al., 2022; Bianconi, 2018; Boccaletti et al., 2014; Cozzo et al., 2018; M. D. Domenico et al., 2013; M. D. Domenico, 2022; Kivela et al., 2014; Lee et al., 2015).

Multilayer graphs have been adopted to model the structure and dynamics of a wide spectrum of high-dimensional, non-linear, multi-scale, time-dependent complex systems including physical, chemical, biological, neuronal, socio-technical, epidemiological, ecological and economic networks (Amato et al., 2017; Arruda et al., 2017; Azimi-Tafreshi, 2016; Baggio et al., 2016; Buldú & Porter, 2018; Cozzo et al., 2013; Dickison et al., 2016; M. D. Domenico, 2017; M. D. Domenico et al., 2016; Estrada & Gómez-Gardeñes, 2014; Gosak et al., 2018; Granell et al., 2013; Lazega & Snijders, 2016; Lim et al., 2019; Mangioni et al., 2020; Massaro & Bagnoli, 2014; Pilosof et al., 2017; Soriano-Paños et al., 2018; Timóteo et al., 2018).

The Journal of Open Source Software

We have chosen the Julia language for this software package because it is a modern, open-source, high-level, high-performance dynamic language for technical computing (Bezanson et al., 2017). At the best of our knowledge there are currently no software packages dedicated to the creation, manipulation and analysis of multilayer graphs implemented in the Julia language apart from MultilayerGraphs.jl itself (Moroni & Monticone, 2022).

Main Features

- Main structs
- Different formalisms
- Main methods and metrics
- Extension of Graphs.jl (Fairbanks et al., 2021), fully integrated within the JuliaGraphs ecosystem
- Integrated within the JuliaDynamics ecosystem: in particular `Multilayer(Di)Graphs` can be utilised as an argument to `GraphSpace` in Agents.jl (Datseris et al., 2022).

Installation and Usage

To install MultilayerGraphs.jl it is sufficient to activate the `pkg` mode by pressing `]` in the Julia REPL and then run the following command:

```
pkg> add MultilayerGraphs
```

In the following code chunks we synthetically illustrate how to define, handle and analyse a `MultilayerGraph` in order to showcase some of the main features outlined in the previous section.

First of all we need to import the necessary dependencies and set a few constants:

```
using Revise
using StatsBase, Distributions
using Graphs, SimpleWeightedGraphs, MetaGraphs, SimpleValueGraphs
using MultilayerGraphs

const vertextype   = Int64
const _weighttype  = Float64
const min_vertices = 5
const max_vertices = 7
const min_edges    = 1
const max_edges    = max_vertices*(max_vertices-1)
const n_nodes      = max_vertices
```

[...]

Then we define a multilayer graph by specifying its layers and interlayers:

```
multilayergraph = MultilayerGraph(  layers,
                                    interlayers;
                                    default_interlayers_null_graph = SimpleGraph{vertext
                                    default_interlayers_structure = "multiplex"
)
```

[...]

For a more comprehensive exploration of the package features and functionalities we strongly recommend consulting the tutorial included in the package documentation.

Related Packages

R

Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the R language:

- `muxViz` implements functions to perform multilayer correlation analysis, multilayer centrality analysis, multilayer community structure detection, multilayer structural reducibility, multilayer motifs analysis and utilities to statically and dynamically visualise multilayer graphs (D. Domenico et al., 2014);
- `multinet` implements functions to import, export, create and manipulate multilayer graphs, several state-of-the-art multiplex graph analysis algorithms for centrality measures, layer comparison, community detection and visualization (Magnani et al., 2021);
- `mully` implements functions to import, export, create, manipulate and merge multilayer graphs and utilities to visualise multilayer graphs in 2D and 3D (Hammoud & Kramer, 2018);
- `multinets` implements functions to import, export, create, manipulate multilayer graphs and utilities to visualise multilayer graphs (Lazega et al., 2008).

Python

Here is a list of software packages for the creation, manipulation, analysis and visualisation of multilayer graphs implemented in the Python language:

- `MultiNetX` implements methods to create undirected networks with weighted or unweighted links, to analyse the spectral properties of adjacency or Laplacian matrices and to visualise multilayer graphs and dynamical processes by coloring the nodes and links accordingly;
- `PyMNet` implements data structures for multilayer graphs and multiplex graphs, methods to import, export, create, manipulate multilayer graphs and for the rule-based generation and lazy-evaluation of coupling edges and utilities to visualise multilayer graphs (Kivela et al., 2014).

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