

```
a,b,c,d,e,f = var("a b c d e f")
u,v = var("u v")
w,x = var("w x")
y,z = var("y z")
```

```
s0 = vector([a, i*a])
s1 = vector([u,v])
s2 = vector([w,x])
s3 = vector([y,z])
```

```
# these are equivalent
s1p = (s0*b) + (s1*c)
s2p = (s1*d) + (s3*e)
s3p = (s2*f)
```

```
eqns = [
    s1[0] == s1p[0],
    s1[1] == s1p[1],
    s2[0] == s2p[0],
    s2[1] == s2p[1],
    s3[0] == s3p[0],
    s3[1] == s3p[1],
]
```

```
solve(eqns, (u,v,w,x,y,z))
```

```
valuify = lambda x:x.subs(a==sqrt(3/6), b=sqrt(4/6), c=sqrt(2/6), d=sqrt(5/6), e=sqrt(1/6), f=sqrt(6/6))
```

```
res = -a*b/(c - 1)
float(simplify(expand(valuify(res*res.conjugate()))))
```

```
float(-1/2/(3*sqrt(1/3) - 2))
```

$$\left[\left[u = -\frac{ab}{c-1}, v = -\frac{iab}{c-1}, w = \frac{abd}{(ef-1)c-ef+1}, x = \frac{iabd}{(ef-1)c-ef+1}, y = \frac{abdf}{(ef-1)c-ef+1}, z = \frac{iabdf}{(ef-1)c-ef+1} \right] \right. \quad (1)$$

$$u = \frac{1}{4}\sqrt{3} + \frac{1}{4}i, v = \frac{1}{4}i\sqrt{3} - \frac{1}{4} \quad (2)$$

$$w = \frac{1}{28}\sqrt{6}\sqrt{5}\sqrt{3} + \frac{1}{28}i\sqrt{6}\sqrt{5} + \frac{1}{28}i\sqrt{5}\sqrt{3} - \frac{1}{28}\sqrt{5}, x = \frac{1}{28}i\sqrt{6}\sqrt{5}\sqrt{3} - \frac{1}{28}\sqrt{6}\sqrt{5} - \frac{1}{28}\sqrt{5}\sqrt{3} - \frac{1}{28}i\sqrt{5} \quad (3)$$

$$y = \frac{1}{28}\sqrt{6}\sqrt{5}\sqrt{3} + \frac{1}{28}i\sqrt{6}\sqrt{5} + \frac{1}{28}i\sqrt{5}\sqrt{3} - \frac{1}{28}\sqrt{5}, z = \frac{1}{28}i\sqrt{6}\sqrt{5}\sqrt{3} - \frac{1}{28}\sqrt{6}\sqrt{5} - \frac{1}{28}\sqrt{5}\sqrt{3} - \frac{1}{28}i\sqrt{5} \quad (4)$$

```
x = -1/308*(13*sqrt(6)*sqrt(5) - 5*sqrt(6)*sqrt(3) + 8*I*sqrt(5) - 86*I*sqrt(3))
y = 1/308*(13*I*sqrt(6)*sqrt(5) - 5*I*sqrt(6)*sqrt(3) - 8*sqrt(5) + 86*sqrt(3))
```

```
x*x.conjugate()
```

$$\left[\left[s_0 = \frac{1}{2}i\sqrt{2}d_0 + \frac{1}{2}\sqrt{2}u_0, s_1 = -\frac{1}{308} \left(13\sqrt{6}\sqrt{5} - 5\sqrt{6}\sqrt{3} + 8i\sqrt{5} - 86i\sqrt{3} \right) d_0 + \frac{1}{308} \left(13i\sqrt{6}\sqrt{5} - 5i\sqrt{6}\sqrt{3} - 8\sqrt{5} + 86\sqrt{3} \right) u_0 \right] \right. \\ \left. \right] \quad (5)$$