Ted's Slides

1 | Intro, and a Book!

Logicomix!

Logic and atheism as quintessentially modern.

> Reality does not need to be consistent, but our models of reality should be consistent.

2 | Category Theory

A specialty of mathematics that is somewhat new. Category theory aims to reshape mathematics to instead of studying the *objects*, we study the transformations.

Category theory has *objects* and *morphism*: "things" and "ways to go between things".

2.1 | Extensions of Category Theory

Category is a generalization of proofs

- Logic => Objects represent *propersitions*, morphisms represent *proofs*
- Programs => Objects represent *datatype*, morphisms represent *programs*

Category Theory	Physics	Topology	Logic	CS
Object	System	Manifold	Proposition	Data Type
Morphism	Process	Cobordism	Proof	Program

As you could see, category theory is a generalization.

2.2 | Category

Define category C

- Collection of objects, where if X is an object of C, we write $X \in C$.
- For every pair (X, Y), a set $hom(X, Y)$ of morphisms from X to Y. We call this set $hom(X, Y)$ a homset. If $f \in hom(X, Y)$, then we write $f : X- > Y$.

Objects and morphisms are independent, but an *morphism* of one category could be *objects* of another.

Such that…

- Every object X has an identity morphism: $1_x : X \to X$
- Morphisms are composable: given $f : X \to Y$ and $g : Y \to Z$ there is a composite morphism $gf : X \to Z$, sometimes also written $g \circ f$
- An identity morphism is both a left and a right unit for composition: if $f : X \to Y$, then $y1_x = f = 1_y f$
- Composition operation is associative, so $(gf)a \cong g(fa)$.

A category is a network of composable relationships.

2.3 | Funtors

Mapping operators *between* categories. It could act on *both* objects and morphisms.

Define cat C, cat D, functor F maps C to D

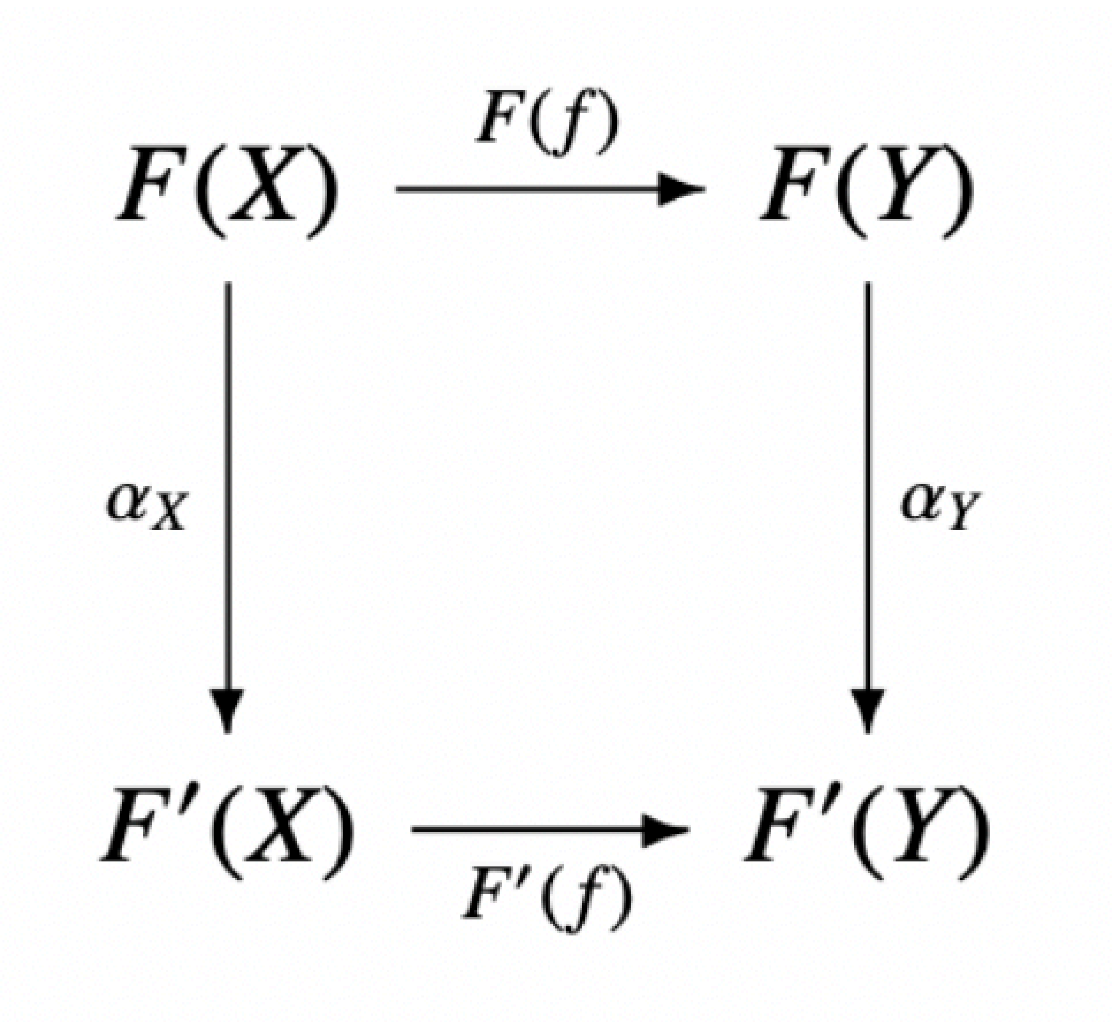
- Any object $X \in C$ to an object $F(X) \in D$ (maps objs to obj images)
- Any morphism $f : X \to Y$ in C to a morphism $F(f) : F(X) \to F(Y)$ in D (morphs to morphs on the images)

in such a way that…

- F **Identities are preserved**: $X \in C, F(1_x) = 1_{F(x)}$
- F **Preserves composition**: for pairs of morphisms $f : X \to Y, g : Y \to Z$ both in C , $F(gf) = F(g)F(f)$

2.4 | Natural Transformations

Natural Transformation are operations that make a functor to another functor.



You could go two different ways around this square diagram, you get the same answer $F'(Y)$, hence the name ”commutative diagrams.”

F and F' are secretly representations of G on Hilbert spaces