1 | A Problem

Normal or "easy" limits are rather simple, as examples like $\lim_{x\to 4} \frac{x+3}{x^2+1}$ just need some plugging in. Derivatives are usually harder as $\lim_{x\to 0} \frac{f(x_0+\Delta x)-f(x_0)}{x-x_0}$ always evaluates to $\frac{0}{0}$, and needs some cancellation.

2 | Some Notation

DEFINITION Right hand limit or $lim_{x\to x_0^+} f(x)$ indicates that x is greater than x_0 (or that x begins on the right side of the number line).

DEFINITION Left hand limit or $lim_{x\to x_0^-} f(x)$ indicates that x is less than x_0 (or that x begins on the left side of the number line). These notations will make dealing with limits of these functions more convienient.

EXAMPLE Take the following example of a conditional function:

if $x > 0$, $f(x) = x + 1$

if $x < 0$, $f(x) = -x + 2$

$lim_{x\to x_0^+} f(x) = lim_{x\to x_0} x + 1 = 1$

$lim_{x\to x_0^-} f(x) = lim_{x\to x_0} - x + 2 = 2$

We did not need a $x = 0$ value to compute these limits!

3 | Nested Limits

A checklist for what to do before dealing with nested limits.

EXAMPLE: sin $\sqrt{x}$

- ☐ Check domain + range of inner function (in this case $[0, \infty)$, $[0, \infty)$).
- ☐ Check domain + range of outer function as well as what it takes in. (takes in $[0, \infty)$, range is $[-1, 1]$)
- ☐ Restrict domain based on requirements of inner + outer functions

EXAMPLE: ln sin x

- ☒ Domain of sin x is $(-\infty, \infty), range is $[-1, 1]$.
- ☒ Domain of ln x is $[0, \infty)$, range is $(-\infty, \infty)$.
- ☒ As ln x takes only positive values, the restricted domain for the composite function is $[0, \pi]$, $[2\pi, 3\pi]$, etc. The range of the composite function would be $(-\infty, 0]$.
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4 | Links

Adjacent to this: Continuity

Building upon this: Calculating Derivatives

Further reference can be found at Limits and Continuity Practice.