1 | Definitions of normal in math

- normal: perpendicular
- normalized: to turn something into a single unit
  - for example: a normalized vetor is the unit vector

2 | Defining a plane (that passes through the origin) with vectors:

- you can say that a plane is all of the vectors perpendicular to one vector, and pass through the origin.
- the "one vector" is called the normal vector
- the question is, how do you say this in math
- you can use the fact that if two vectors are perpendicular then their dot product is zero so:
- so then this should work: $\vec{r} = \{\vec{r} : \vec{r} \cdot \vec{n} = 0, \vec{n} \in \mathbb{R}^3\} = plane \perp \vec{n}$ passing through the origin
  - to show that this works we need to show that all of the vectors in $\vec{r}$ are in $plane \perp \vec{n}$ passing through the origin and that all vectors in $plane \perp \vec{n}$ passing through the origin are in $\vec{r}$
  - to show that this is true we can pick a generic vector in $\vec{r}$ and show thath that same vector is in $plane \perp \vec{n}$ passing through the origin and vice versa.

3 | Defining a plane (that might not pass thourgh the origin) with vectors:

- because you want to shift all of the vectors in the plane by $\vec{P}_o$, then you can subtract $\vec{r}$ by $\vec{P}_o$. This gives you:
  - $\{\vec{r} : (\vec{r} - \vec{P}_o) \cdot \vec{n} = 0, \vec{n} \in \mathbb{R}^3, \vec{P}_o \in \mathbb{R}^3\} = plane \perp \vec{n}$ passing through the point $\vec{P}_o$

4 | Converting this definition of a plane into a cartisian definition:

- First we can defined:
  - $\vec{n} = (n_x, n_y, n_z)$
  - $\vec{P}_o = (P_{ox}, P_{oy}, P_{oz})$
  - $\vec{r} = (x, y, z)$

- then we can evaluate: $(\vec{r} - \vec{P}_o) \cdot \vec{n} = 0$: $(\vec{r} - \vec{P}_o) \cdot \vec{n} = 0$
$\Rightarrow \vec{r} \cdot \vec{n} - \vec{P}_o \cdot \vec{n} = 0$
$\Rightarrow \vec{r} \cdot \vec{n} = \vec{P}_o \cdot \vec{n}$
$\Rightarrow xn_x + yn_y + zn_z = P_{ox}n_x + P_{oy}n_y + P_{oz}n_z$

so we see that the cartisian definition of a plane is: $xn_x + yn_y + zn_z = P_{ox}n_x + P_{oy}n_y + P_{oz}n_z$