1 | A surface integral

We are defining a function:

$$f(x, y, z) = y^2 \tag{1}$$

and slicing out a vertical organ pipe shape with a sliced edge. That is:

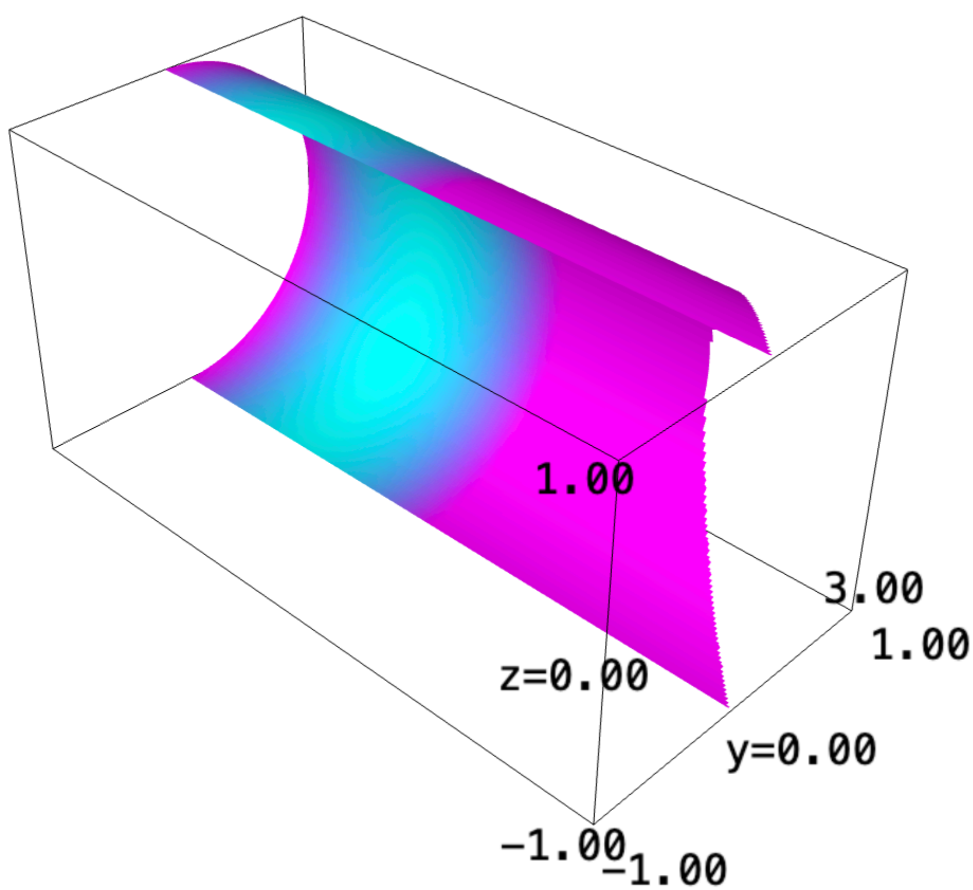
$$x^2 + z^2 = 1 \tag{2}$$

bounded by $y > 0$ and $y < 3 - x$.

Let's plot this:

```
var('x,y,z')
f = y^2

implicit_plot3d(x^2+z^2 == 1, (y,-1,3), (x,-1,1), (z, -1,1), region=(lambda x,y,z: y > 0 and y<3-x), co
```



that's honestly pretty cool!

Great, now let's take the actual surface integral. Note that, because the "pipe" does not have a defining ending point, we will set its end at the xy plane, that it ends at $x = 0$ at the other end in addition to $y = 3 - x$.

Looking at the actual function for which we are taking the integral, we have:

$$x^2 + z^2 = 1 \tag{3}$$

We will rearrange this expression in terms of z:

$$z = \sqrt{1 - x^2} \tag{4}$$

Fortunately, we see already that the function's derivative w.r.t. y is 0; indeed, it doesn't change along the y direction (the cylinder is centered around it after all.)

Taking the derivative in the x direction:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\sqrt{1 - x^2} \tag{5}$$
$$= \frac{-2x}{2\sqrt{-x^2 + 1}} \tag{6}$$
$$= \frac{-x}{\sqrt{-x^2 + 1}} \tag{7}$$

Squaring the expression below:

$$\frac{x^2}{-x^2 + 1} \tag{8}$$

And finally, we have the correction factor:

$$dA = \sqrt{\frac{x^2}{-x^2 + 1} + 1}\ dV \tag{9}$$
$$= \sqrt{\frac{1}{-x^2 + 1}}\ dV \tag{10}$$

Lastly, we can multiply the actual value function to this to this expression to get the expression for the integral:

$$\iint_V y^2 \sqrt{\frac{1}{-x^2 + 1}}\ dx\ dy \tag{11}$$

Furthermore, our bounds are also a little complicated:

$$\int_0^3 \int_0^{3-x} y^2 \sqrt{\frac{1}{-x^2 + 1}}\ dx\ dy \tag{12}$$

Great, we will now ask Sage to take the actual integral for us:

2 | Jacobian Matrix

We will attempt to take the derivative matrix for this expression methodically.

Let's define the function as:

$$f(x, y, z) = (z^2 - \sin(y))\hat{h} + (x + y + z)\hat{\imath} + (e^y + 7x)\hat{\jmath} + (ln(x + y - 2z))\hat{k} \tag{13}$$

2.1 | f_x

- $f_x \cdot \hat{h} = 0$
- $f_x \cdot \hat{i} = 1$
- $f_x \cdot \hat{j} = 7$
- $f_x \cdot \hat{k} = \frac{1}{x+y-2z}$

2.2 | f_y

- $f_y \cdot \hat{h} = -cos(y)$
- $f_y \cdot \hat{i} = 1$
- $f_y \cdot \hat{j} = e^y$
- $f_y \cdot \hat{k} = \frac{1}{x+y-2z}$

2.3 | f_z

- $f_z \cdot \hat{h} = 0$
- $f_z \cdot \hat{i} = 1$
- $f_z \cdot \hat{j} = 0$
- $f_z \cdot \hat{k} = \frac{-2}{x+y-2z}$

2.4 | **Jacobian**

Finally, we can assemble the Jacobian matrix

$$\nabla f = \begin{bmatrix} 0 & -\cos(y) & 0 \\ 1 & 1 & 1 \\ 7 & e^y & 0 \\ \frac{1}{x+y-2z} & \frac{1}{x+y-2z} & \frac{-2}{x+y-2z} \end{bmatrix} \tag{14}$$