Suppose V is a complex inner product space and  $T\in\mathcal{L}(V)$  is a normal operator such that  $T^9=T^8$ . Prove that T is self-adjoint and  $T^2=T$ .

If T=0, then  $0^2=0$  and 0 is self-adjoint. Thus, let  $T\neq 0$ .

In 7.1, Axler asserts that V is finite-dimensional.

By the complex spectral theorem, T has a diagonal matrix w.r.t. an orthonormal basis of V.

Let these entries equal  $d_1,\ldots,d_n$ .  $T^k$  will have on it's diagonal  $d_1^k,\ldots,d_n^k$ . For each  $d_i$ ,  $d_i^8=d_i^9$ . The only values in  $\mathbb F$  that satisfy this are zero and one; thus every  $d_i$  must be a zero or a one.

Thus,  $T = T^2$  and T is self-adjoint.

## 1 | :noexport:

$$TT^*=T^*T$$

First, we will show that  $T^2 = T$ . Suppose T is invertible. Then,

$$T^{9} = T^{8}$$
 $T^{9}T^{-7} = T^{8}T^{-7}$ 
 $T^{2} = T$ 

Suppose T is not invertible and not equal to zero. Then, T has some zero entries on it's diagonal and some non-zero entries on it's diagonal.

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