

Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V . Prove that U is invariant under T iff U^\perp is invariant under T^* .

For all pairs $u \in U$ and $w \in U^\perp$,

$$\begin{aligned}\langle Tu, w \rangle &= 0 \\ \langle u, T^*w \rangle &= 0\end{aligned}$$

This implies that the range of $T^*|_{U^\perp} \subseteq U^\perp$, aka that T^* is invariant under U^\perp

This implies both directions, since $U = U^{\perp\perp}$ and $T = (T^*)^*$.

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For all $u \in U$, $Tu = u' \in U$. Let $w \in U^\perp$. Then, $\langle T^*w, u \rangle = \langle w, Tu \rangle = \langle w, u' \rangle = 0$

$$\langle u, T^*w \rangle = \langle Tu, w \rangle = \langle u', w \rangle$$