1 | Complex Number Review

Notes taken on (12/7/21)

Complex numbers were invented so that we can represent $\sqrt{-1}$.

A complex number is an ordered pair of numbers (a, b), and is represented as $a + bi$. The set of all complex numbers is $C = \{a + bi : a, b \in \mathbb{R}\}$.

Addition and subtraction works pretty standardly; $(a + bi) + (c + di) = (a + c) + (b + d)i$.

There's also the powers of i, but this is trivial.

Complex number properties:

Commulative $\alpha + \beta = \beta + \alpha$

Associative $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$; $(\alpha\beta)\lambda = \alpha(\beta\lambda)$

Identities $\alpha + 0 = \alpha$; $\alpha \cdot 1 = \alpha$

Multiplicative Inverse $\forall\alpha\exists\beta : \alpha\beta = 1$

Distributive $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$

The book goes into proving this but I won't do that here. Also, in Axler, $\mathbb{F}$ will mean either $\mathbb{C}$ or $\mathbb{R}$. Theorems that work for $\mathbb{F}$ will work for both $\mathbb{C}$ and $\mathbb{R}$.

If $\alpha \in \mathbb{F}$, then α is a scalar. Definition of a scalar. Axler rambles about powers of numbers now, but it's pretty self-evident so I won't cover this here.

Then he talks about $\mathbb{R}^n$ and $\mathbb{C}^n$. The formal definition for a particular n (e.g. 2) is $\mathbb{R}^2 = \{(a, b) : a, b \in \mathbb{R}\}$. To abstract this for any n, we go over lists. The notation for lists is $(x_1, ..., x_n)$. Lists are always finite in length. We can have an empty list: $()$. Lists care about their order and repetitions. Using lists, we can define $\mathbb{F}^n$ as

$$\mathbb{F}^n = \{(x_1, ..., x_n) : x_j \in \mathbb{F} \text{ for } j = 1, ..., n\}$$

Most of the content following this is redundant review that doesn't introduce anything new so I will skip it.

Also, sometimes when we add 0, we actually mean a list full of zeroes.

2 | Vector Space Definition

Notes taken on (12/7/21)

A vector space is a *set* such that **addition** and **scalar multiplication** are defined like in $\mathbb{F}^n$. That is, for a vector space V

$$u + v \in V \text{ given } u, v \in V$$
$$\lambda v \in V \text{ given } \lambda \in \mathbb{F} \quad v \in V$$

Formally, a vector space is a set that follows the rules above, as well as holds the following properties:

- **Commulative**
- **Associative**
- **Identities**
- **Additive Inverse**

- **Distributive Property**

Elements of a vector space are called **Vectors** or **Points**. Also, when you need to be precise about what type of scalar you multiply by for scalar multiplication, you can say that V is a **vector space over** $\mathbb{F}$, for example. Usually it's implied in the vector space definition.

The notation $\mathbb{F}^S$ denotes the set of functions from S to $\mathbb{F}$.

- $f + g \in \mathbb{F}^S$ means that $(f + g)(x) = f(x) + g(x)$.
- $\lambda f \in \mathbb{F}^S$

Also, for the rest of the book, V will notate a vector space over $\mathbb{F}$.

3 | Subspaces

Notes taken on (12/7/21)