> Suppose $T \in \mathcal{L}(V)$ and U is a subspace of V. Prove that U is invariant under T iff $U^\perp$ is invariant under T^*.

For all pairs $u \in U$ and $w \in U^\perp$,

$$\langle Tu, w\rangle = 0$$
$$\langle u, T^* w\rangle = 0$$

This implies that the range of $T^*|_{U^\perp} \subseteq U^\perp$, aka that T^* is invariant under $U^\perp$

This implies both directions, since $U = {U^\perp}^\perp$ and $T = (T^*)^*$.

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For all $u \in U$, $Tu = u' \in U$. Let $w = U^\perp$. Then, $T*w = $

$$\langle u, T^* w\rangle = \langle Tu, w\rangle = \langle u', w\rangle$$