1 | A Double Integral

> Find the volume of the shape bounded on the top by the function $f(x, y) = xy$, and on the base/sides by the rectangle with corners $(x = 1, y = 0)$ and $(x = 4, y = 2)$.

Taking this integral in two ways is essentially just taking the double integral of the expression, in two different methods.

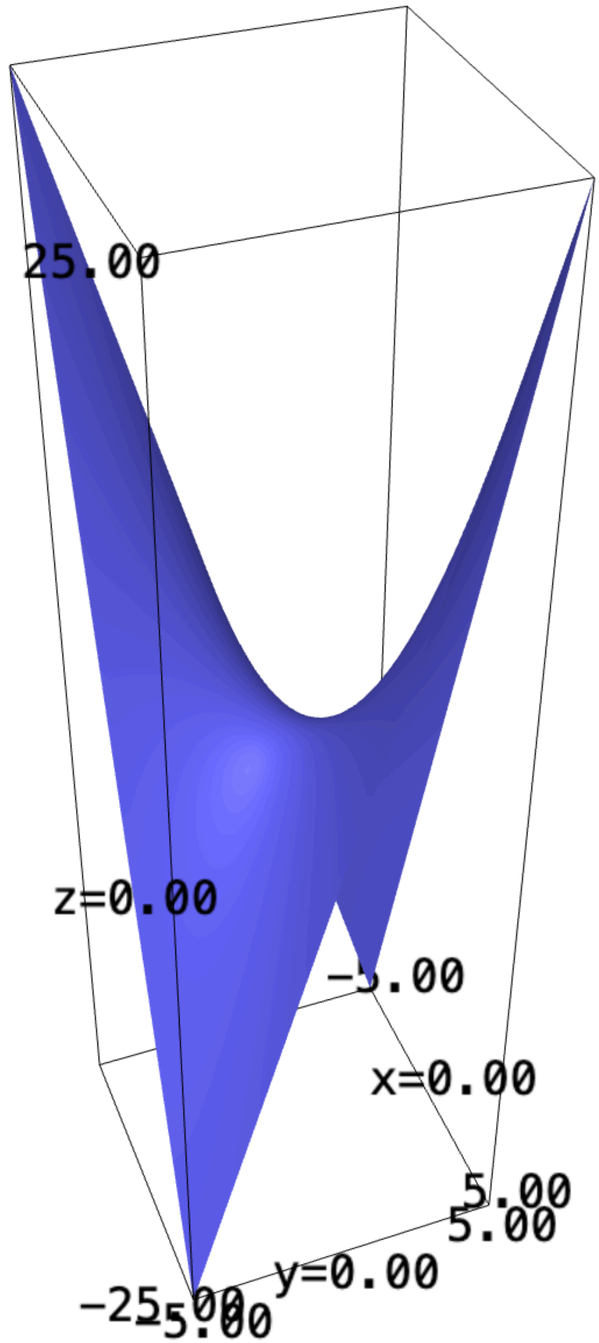
$$\int_0^2 \int_1^4 xy\ dx\ dy \tag{1}$$
$$\Rightarrow \int_0^2 \left.\frac{x^2y}{2}\right|_1^4 dy \tag{2}$$
$$\Rightarrow \int_0^2 \frac{16y-y}{2} dy \tag{3}$$
$$\Rightarrow \int_0^2 \frac{15y}{2} dy \tag{4}$$
$$\Rightarrow \left.\frac{15y^2}{4}\right|_0^2 \tag{5}$$
$$\Rightarrow \frac{60}{4} \tag{6}$$

We can now do that again.

$$\int_1^4 \int_0^2 xy\ dy\ dx \tag{7}$$
$$\Rightarrow \int_1^4 \left.\frac{xy^2}{2}\right|_0^2 dy \tag{8}$$
$$\Rightarrow \int_1^4 \frac{4x}{2} dx \tag{9}$$
$$\Rightarrow \left.\frac{4x^2}{4}\right|_1^4 \tag{10}$$
$$\Rightarrow \frac{64-4}{4} \tag{11}$$
$$\Rightarrow \frac{60}{4} \tag{12}$$

As you can see, both results in the same value.

```
f(x,y) = x*y
plot3d(f, (x,-5,5), (y,-5,5))
```



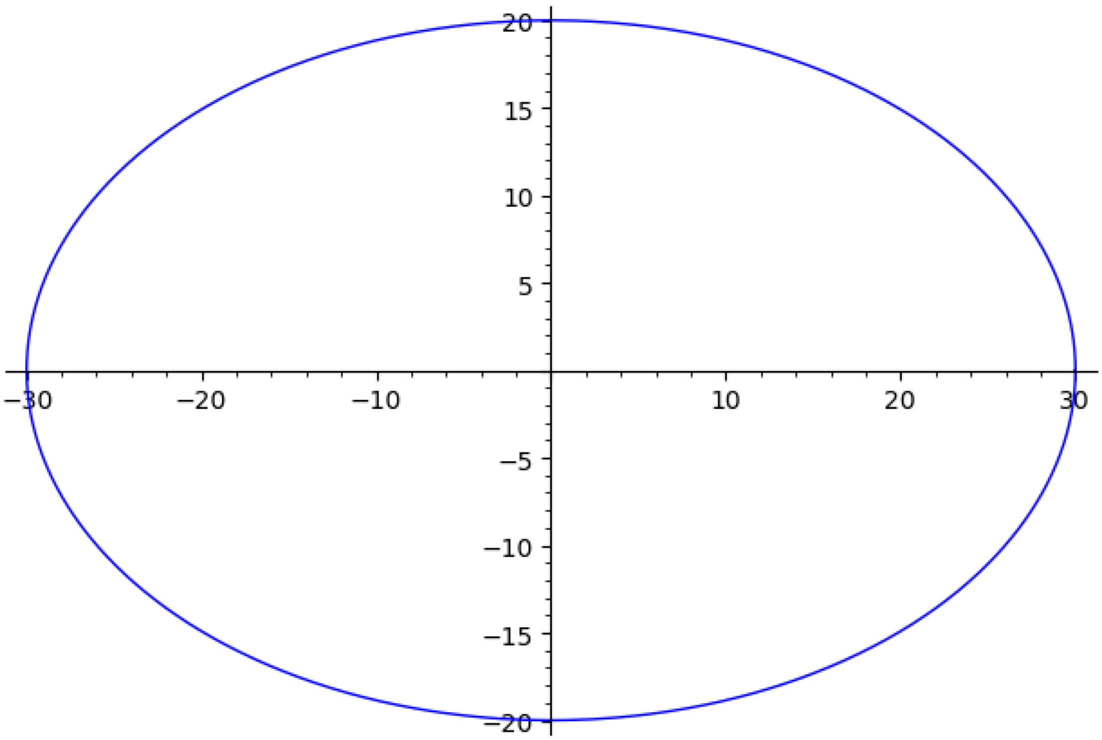
2 | **Pringles**

Building a Pringle's house.

We first begin by creating the elliptical projection of the shape downwards.

```
x(t) = 30*cos(t)
y(t) = 20*sin(t)

parametric_plot([x,y], (0, 2*pi))
```



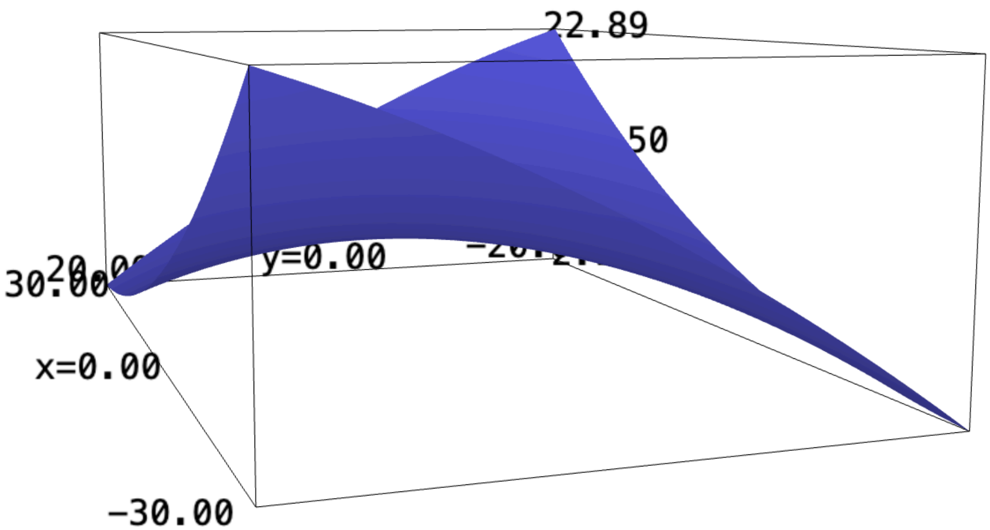
The function for which the projection is created is given by the problem as well, that:

$$r(x,y) = \frac{1}{400}\left(\sqrt{3}x - y\right)^2 - \frac{1}{400}\left(\sqrt{3}y - x\right)^2 + 10 \tag{13}$$

The plot of this function appears as:

```
r(x,y) = (1/400)*(sqrt(3)*x-y)^2 - (1/400)*(sqrt(3)*y+x)^2 + 10

plot3d(r, (x,-30,30), (y,-20,20), contours=True)
```



This question is simply a matter of parameterization. We have been given the parameterization:

$$\begin{cases} x(t) = 30\ cos(t) \\ y(t) = 20\ sin(t) \end{cases} \tag{14}$$

We first figure the derivative of each expression w.r.t. t:

$$\begin{cases} \frac{dx}{dt} = -30\ sin(t) \\ \frac{dy}{dt} = 20\ cos(t) \end{cases} \tag{15}$$

Lastly, we will square this expression to figure the value for $\frac{df}{dt} = \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2$.

$$\frac{df}{dt} = \sqrt{900\ sin^2(t) + 400\ cos^2(t)} \tag{16}$$

And finally, to figure the amount of paint that's needed to paint the sides, we will need to take the line integral of the function parameterized by $x(t)$ and $y(t)$.

$$\int_0^{2\pi} = \left(\frac{1}{400}\left(\sqrt{3}\ 30\ cos(t) - 20\ sin(t)\right)^2 - \frac{1}{400}\left(\sqrt{3}\ 20\ sin(t) + 30\ cos(t)\right)^2 + 10\right)\sqrt{900\ sin^2(t) + 400\ cos^2(t)}dt \tag{17}$$

We will take this integral digitally.

```
t = var("t")
f(x,y) = (1/400)*(sqrt(3)*x-y)^2 - (1/400)*(sqrt(3)*y+x)^2 + 10
dfdt = sqrt(900*(sin(t))^2 + 400*(cos(t))^2)

monte_carlo_integral(f(-30*sin(t), 20*cos(t))*dfdt, [0], [2*pi], 10000000)
```