1 | **Evaluating a Cylindrical Integral**

Considering the function:

$$f(x, y, z) = \sqrt{x^2 + y^2} \tag{1}$$

To evaluate the integral, we will convert it to cylindrical coordinates. We note first that the integral is to be evaluated inside the cylinder of $x^2 + y^2 = 16$, which means that we wish to evaluate it in a circle with center at the origin with radius 4.

Furthermore, we understand that the bounds of the function are to be evaluated between $[-5, -4]$.

If we set up the integral, we will get:

$$\int_{-5}^{-4} \int_C \sqrt{x^2 + y^2}\, dx\ dy\ dz \tag{2}$$

This is convenient. We can evaluate the inner integral first like in $\mathbb{R}^2 \to \mathbb{R}^1$, then simply evaluate the other integral after.

Let's do so.

Note that the inner integral is a normal cylindrical coordinate setup. Therefore, we can take the following substitution:

$$\sqrt{x^2 + y^2} = r \tag{3}$$

Furthermore, that:

$$dx\ dy = dr\ d\theta \tag{4}$$

With the appropriate bounds, then:

$$\int_0^{2\pi} \int_0^4 r\ dr\ d\theta \tag{5}$$

$$\Rightarrow \int_0^{2\pi} \frac{r^2}{2}\Big|_0^4 d\theta \tag{6}$$

$$\Rightarrow \int_0^{2\pi} 8\ d\theta \tag{7}$$

$$\Rightarrow 16\pi \tag{8}$$

Finally, we will take the integral of this value dz:

$$\int_{-5}^{-4} 16\pi\ dz = 16\pi \tag{9}$$

Therefore, the value of the integral is 16π.

2 | **Uselessly Spherical Integral**

We first recall that the differential volume can be written as:

$$dV = \rho^2 \sin\phi \; d\rho \; d\phi \; d\theta \tag{10}$$

To take this integral, then, we have to figure the distance ρ to a rectangle for every point (ϕ, θ).