# 1 | Problem 1

$$\text{KE}_{\text{total}} = \sum_{i=1}^{N} \frac{1}{2} m_i (v_i \cdot v_i)$$

$$\text{KE}_{\text{total}} = \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{V}_{\text{CM}} + \vec{v}_i')^2$$

$$\text{KE}_{\text{total}} = \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{V}_{\text{CM}}^2 + 2\vec{V}_{\text{CM}} v_i' + (\vec{v}_i')^2)$$

$$\text{KE}_{\text{total}} = \sum_{i=1}^{N} \left( \frac{1}{2} m_i \vec{V}_{\text{CM}}^2 + m_i \vec{V}_{\text{CM}} \vec{v}_i' + \frac{1}{2} m_i (\vec{v}_i')^2 \right)$$

$$\text{KE}_{\text{total}} = \sum_{i=1}^{N} \frac{1}{2} m_i \vec{V}_{\text{CM}}^2 + \sum_{i=1}^{N} m_i \vec{V}_{\text{CM}} \vec{v}_i' + \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{v}_i')^2$$

$$\text{KE}_{\text{total}} = \frac{1}{2} \vec{V}_{\text{CM}}^2 \sum_{i=1}^{N} m_i + \vec{V}_{\text{CM}} \sum_{i=1}^{N} m_i \vec{v}_i' + \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{v}_i')^2$$

Define $M = \sum_{i=1}^{N} m_i$.

$$\vec{r}_{\text{CM}}' = \frac{1}{M} \sum_i m_i \vec{r}_i$$

$\vec{r}_{\text{CM}}' = 0$ by definition (it is relative to itself).

$$0 = \frac{1}{M} \sum_i m_i \vec{r}_i$$

Differentiate with respect to time.

$$0 = \frac{1}{M} \sum_i m_i \vec{v}_i$$

$$0 = \sum_i m_i \vec{v}_i$$

Eliminate the middle term $\vec{V}_{\text{CM}} \sum_{i=1}^{N} m_i \vec{v}_i'$ as it is equal to 0.

$$\text{KE}_{\text{total}} = \frac{1}{2} \vec{V}_{\text{CM}}^2 \sum_{i=1}^{N} m_i + \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{v}_i')^2$$

$$\boxed{\text{KE}_{\text{total}} = \frac{1}{2} M \vec{V}_{\text{CM}}^2 + \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{v}_i')^2}$$