1 | orthogonal def

Two vectors $u, v \in V$ are called *orthogonal* if $\langle u, v \rangle = 0$

2 | results

2.1 | orthogonal ~= perpendicular

2.2 | Axler 6.12 orthogonality and zero

2.2.1 | 0 is orthogonal to every vector in V

2.2.2 | 0 is the only vector in V that is orthogonal to itself

2.3 | Axler 6.13 Pythagorean Theorem

Suppose u and v are orthogonal vectors in V. Then

$$\|u + v\|^2 = \|u\|^2 + \|v\|^2$$

2.3.1 | proof with more algebra written out

$$\begin{aligned} \|u + v\|^2 &= \langle u + v, u + v \rangle \\ &= \langle u, u + v \rangle + \langle v, u + v \rangle \\ &= \langle u, u \rangle + \cancelto{0}{\langle u, v \rangle} + \cancelto{0}{\langle v, u \rangle} + \langle v, v \rangle \\ &= \|u\|^2 + \|v\|^2 \end{aligned}$$