We begin by defining a system

θ is the angle by which the shooter is aimed, the shooter shoots at v_0, the projectile travels a distance of R.

So, define a function $R(\theta) = R$.

Hence, the goal of this project is to find local mix, min points (critical points that arn't inflection points), which means — at a minimum…

vs

$$\frac{dR}{d\theta} = 0 \tag{1}$$

which would therefore indicate a θ such that the distance would be the longest.

Hence, to get the longest distance, solve.

There was apparently my old notes on this. But not sure if its helpful.

$$y(t), y_0 = 0, y_f = 0 \tag{2}$$
$$x(t), x_0 = 0, y_f = R \tag{3}$$
$$y(t) = \frac{-1}{2} g t^2 + V_{0y} t + y_0, V_{0y} = V_0 \sin\theta \tag{4}$$
$$y(t) = \frac{-1}{2} g t^2 + V_0 \sin\theta t + y_0 \tag{5}$$
$$x(t) = 0 (g = 0) + V_{0x} t + x_0, V_{0x} = V_0 \cos\theta \tag{6}$$
$$x(f) = 0 (g = 0) + V_0 \cos\theta t + x_0 \tag{7}$$
$$0\ (end\ up\ on\ ground) = y_f = y(t_f) = -\frac{1}{2} g t_f^2 + (v_0 \sin\theta) t_f \tag{8}$$
$$R\ (want\ to\ travel\ R) = x_f = x(t_f) = (v_0 \cos\theta) t_f \tag{9}$$
$$\tag{10}$$