

$$1 \mid \int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$$

$$\begin{aligned} \int -e^u du &= -e^u + C \\ &= -e^{\frac{1}{2}} + e^{\frac{1}{1}} \\ &= e - e^{\frac{1}{2}} \end{aligned}$$

$$2 \mid \int_0^1 r e^{\frac{r}{2}} dr$$

$$\begin{aligned} \int_0^1 r e^{\frac{r}{2}} dx &\Rightarrow r 2 e^{\frac{r}{2}} - \int 2 e^{\frac{r}{2}} dr \\ &= 2 r e^{\frac{r}{2}} - \int 2 e^{\frac{r}{2}} dr \\ &= 2 r e^{\frac{r}{2}} - 4 e^{\frac{r}{2}} \\ &= 2 r e^{\frac{r}{2}} - 4 e^{\frac{r}{2}} \\ &\Rightarrow 2 e^{\frac{1}{2}} - 4 e^{\frac{1}{2}} - (-4) \\ &= 4 - 2 e^{\frac{1}{2}} \end{aligned}$$

$$3 \mid \int_4^9 \frac{\ln y}{\sqrt{y}} dy$$

$$\begin{aligned} \int \frac{\ln y}{\sqrt{y}} dy &= 2 \ln y \sqrt{y} - \int 2 \frac{1}{y} \sqrt{y} dy \\ &= 2 \ln y \sqrt{y} - \int 2 \frac{1}{\sqrt{y}} dy \\ &= 2 \ln y \sqrt{y} - 2 \int y^{-\frac{1}{2}} dy \\ &= 2 \ln y \sqrt{y} - 4 \sqrt{y} + C \\ &= 2 \sqrt{y} (\ln y - 2) + C \\ &\Rightarrow 6 (\ln 9 - 2) - 4 (\ln 4 - 2) \end{aligned}$$

4 | $\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$, $dx = 2u du$

$$\begin{aligned} \int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= 2 \int u \cos u du \\ &= 2u \sin u - 2 \int \sin u du \\ &= 2u \sin u + 2 \cos u \\ &= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} \\ \Rightarrow 2\pi^{\frac{1}{4}} \sin \pi^{\frac{1}{4}} + 2 \cos \pi^{\frac{1}{4}} - 2 \end{aligned}$$

5 | $\int_1^e \sin \ln x dx$

$$\begin{aligned} \int_1^e \sin \ln x dx &= x \sin \ln x - \int x \frac{1}{x} \cos \ln x dx \\ &= x \sin \ln x - \int \cos \ln x dx \\ &= x \sin \ln x - \left(x \cos \ln x + \int x \frac{1}{x} \sin \ln x dx \right) \\ &= x \sin \ln x - x \cos \ln x - \int \sin \ln x dx \\ 2 \int \sin \ln x dx &= x \sin \ln x - x \cos \ln x \\ \int \sin \ln x dx &= \frac{1}{2} x (\sin \ln x - \cos \ln x) \\ \Rightarrow \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0) \\ &= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2} \end{aligned}$$

6 | $\int_0^1 \frac{x^3}{\sqrt{4+x}} dx$

Let $u = 4 + x^2$, $du = 2x dx$

$$\begin{aligned} \int \frac{x^3}{\sqrt{4+x}} dx &= \frac{1}{2} \int \frac{(u-4)}{\sqrt{u}} du \\ &= \frac{1}{2} \int \sqrt{u} dx - 2 \int \frac{1}{\sqrt{u}} dx \end{aligned}$$

7 | (additional problems)

7.1 | $\int \sin^2 x dx$

$$\begin{aligned}
 \int \sin^2 x dx &= -\sin x \cos x + \int \cos^2 x dx \\
 &= -\sin x \cos x + \int 1 dx - \int \sin^2 x dx \\
 2 \int \sin^2 x dx &= -\sin x \cos x + x \\
 \int \sin^2 x dx &= \frac{1}{2}(x - \sin x \cos x)
 \end{aligned}$$

7.2 | $\int \cos^2 x dx$

$$\begin{aligned}
 \int \cos^2 x dx &= \cos x \sin x + \int \sin^2 x dx \\
 &= \cos x \sin x + \int 1 dx - \int \cos^2 x dx \\
 2 \int \cos^2 x dx &= \cos x \sin x + x \\
 \int \cos^2 x dx &= \frac{1}{2} \cos x \sin x + \frac{x}{2} + C
 \end{aligned}$$

7.3 | $\int \sin^2 x \cos^2 x dx$

$$\begin{aligned}
 \sin 2x &= 2 \sin x \cos x \\
 \int \sin^2 x \cos^2 x dx &= \frac{1}{4} \int \sin^2 2x dx \\
 \text{Let } u &= 2x, du = 2dx \\
 &= \frac{1}{8} \int \sin^2 u du \\
 &= \frac{1}{8} \frac{1}{2} (u - \sin u \cos u) \\
 &= \frac{1}{16} (2x - \sin 2x \cos 2x) + C
 \end{aligned}$$

7.4 | $\int \sin^3 x dx$

$$\begin{aligned}\int \sin^3 x dx &= \int \sin x(1 - \cos^2 x) dx \\&= \int \sin x dx - \int \cos^2 x \sin x dx \\ \text{Let } u &= \cos x, du = -\sin x dx \\&= \int \sin x dx + \int u^2 du \\&= -\cos x + \frac{1}{3}u^3 + C \\&= \frac{1}{3}\cos^3 x - \cos x + C\end{aligned}$$

7.5 | $\int \cos^3 x dx$

$$\begin{aligned}\int \cos^3 x dx &= \int \cos x(1 - \sin^2 x) dx \\&= \int \cos x - \sin^2 x \cos x dx \\&= \sin x - \int u^2 du \\&= \sin x - \frac{1}{3}u^3 + C \\&= \sin x - \frac{1}{3}\sin^3 x + C\end{aligned}$$