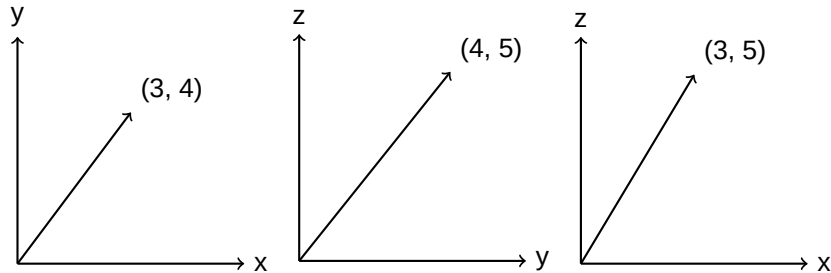


1 | projections or smt

1.1 | projections



1.2 | language of projections?

Let $\vec{c} = \vec{B} - \vec{A}$. Our goal is to find the magnitude $|\vec{c}|$.

Lets form a right triangle:

$$\vec{c} = \vec{c}_{x,y} + \vec{c}_z$$

and thus,

$$|\vec{c}| = \sqrt{|\vec{c}_{x,y}|^2 + |\vec{c}_z|^2} \text{ pro}$$

The length of the projection onto the xy-plane can be found using the Pythagorean theorem.

$$\begin{aligned} |\vec{c}| &= \sqrt{|\vec{c}_{x,y}|^2 + |\vec{c}_z|^2} \\ &= \sqrt{\sqrt{|\vec{c}_x|^2 + |\vec{c}_y|^2}^2 + |\vec{c}_z|^2} \\ &= \sqrt{|\vec{c}_x|^2 + |\vec{c}_y|^2 + |\vec{c}_z|^2} \end{aligned}$$

2 | vectors problems

2.1 | adding two vectors $\langle a_1, a_2, a_3 \rangle$ and $\langle b_1, b_2, b_3 \rangle$

2.1.1 | the coordinates of the sum

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

2.1.2 | adding vectors

Geometrically, it is putting the vectors tip to tail. Follow one, then follow the other. Algebraically, it is adding each of the components. See the previous part

2.1.3 | subtracting vectors

We want to define $\vec{c} = \vec{a} - \vec{b}$ such that $\vec{b} + \vec{c} = \vec{a}$.

Geometrically, that means following \vec{a} , and then following \vec{b} backwards (ie. we want to define a negative vector as the same vector backwards). Algebraically, we see that it inherits the properties from addition/subtraction.

2.2 | finding the vector between two points

Take the points as vectors, and subtract them.

$$\langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

2.3 | practice problems

2.3.1 | magnitude of a

$$|\langle 4, 0, 3 \rangle| = \sqrt{4^2 + 3^2} = 5$$

2.3.2 | magnitude of b

$$|\langle -2, 1, 5 \rangle| = \sqrt{(-2)^2 + 1^2 + 5^2} = \sqrt{30} = 5.47722557505$$

2.3.3 | $\vec{a} + \vec{b}$

$$\langle 2, 1, 8 \rangle$$

2.3.4 | $\vec{a} - \vec{b}$

$$\langle 6, -1, -2 \rangle$$

2.3.5 | $3\vec{b}$

$$\langle -6, 3, 15 \rangle$$

2.3.6 | $2\vec{a} + 5\vec{b}$

$$\langle -2, 5, 31 \rangle$$

2.3.7 | \hat{a}, \hat{b}

$$\left\langle \frac{4}{5}, 0, \frac{3}{5} \right\rangle$$

$$\left\langle \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right\rangle$$

2.3.8 | $\theta_{\vec{a}x}$

Lets make a right triangle in the plane that contains the tip and tail of the vector and the x-axis.

The height will be from the x-axis to the tail, so we'll take the diagonal in the yz plane

$$h = \sqrt{a_y^2 + a_z^2}$$

The base of the triangle will be along the x-axis. So, the base is just the x component a_x .

And so, we can find theta using the tangent

$$\tan \theta = \frac{\sqrt{a_y^2 + a_z^2}}{a_x}$$

You could also do it with the cosine, as in dot product:

$$\cos \theta = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

2.4 | triangle proof

Lets let \vec{a}, \vec{b} be the two sides and \vec{c} be the middle side. This is the small triangle. Then, let's double each of the side lengths:

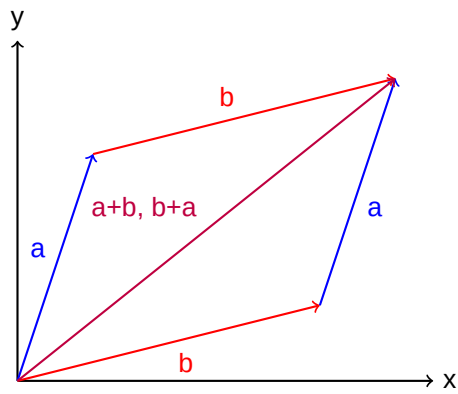
$$2\vec{a} + 2\vec{b} = 2(\vec{a} + \vec{b}) = 2\vec{c}$$

Thus, the middle line is half the magnitude of the longer third side. Because they are scalar multiples of eachother, the lines are parallel.

3 | proving vector properties

You are really stretching my \LaTeX abilities here

3.1 | $a + b = b + a$



3.2 | $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$