

## 1 | Deriving Rotational KE and Inertia

Given  $m_i$ , mass,  $\vec{r}_i'$ , location of the center of mass,  $l_i$ ,  $\omega$ , the angular velocity, figure a  $KE_{tot,rot}$ .

Because of the fact that the value  $\omega$  is in units  $\frac{d\theta}{dt}$ , the rate of radians change, and we know of a radius of the spin  $l_i$ , we could figure the velocity at which it is moving by simply scaling the change in radians up to a circle of radius  $l_i$ , that is:

$$V_i' = l_i \omega \quad (1)$$

(note that, to understand this, radians  $\frac{arclength}{radius}$ )

And so, substituting into the statement of  $\sum_{i=1}^N \frac{1}{2} m_i \vec{v}_i'^2$

$$KE_{rot} = \sum_{i=1}^N \frac{1}{2} m_i \vec{v}_i'^2 \quad (2)$$

$$= \sum_{i=1}^N \frac{1}{2} m_i (l_i \omega)^2 \quad (3)$$

$$= \sum_{i=1}^N \frac{1}{2} m_i l_i^2 \omega^2 \quad (4)$$

$$= \frac{1}{2} \omega^2 \sum_{i=1}^N (m_i l_i^2) \quad (5)$$

### 1.1 | Rotational Inertia

The right sum — the mass times the distance away from maxis of rotation ( $\sum_{i=1}^N (m_i l_i^2)$ ) — is defined as the rotational (moment) of inertia (spiny mass). That is,

$$I = \sum_{i=1}^N (m_i l_i^2) \quad (6)$$

Replacing that value in the prior statement, the statement of  $KE_{rot}$  is defined as:

$$KE_{rot} = \frac{1}{2} \omega^2 I \quad (7)$$

### 1.2 | Rotational Inertia for a Ring

For a ring (that's perfectly circular) rotating on an axis perpendicular to the plane of the ring, the  $l_i$  — distance from axis of rotation — is the same value: namely, the radius  $R$  as the radius of a circle is the same for all positions. Meaning,

$$l_i = R \quad (8)$$

regardless of which value  $i$ .

Hence, the value of  $KE_{rot}$  would be evaluated as...

$$KE_{rot} = \sum_{i=1}^N (m_i l_i^2) \quad (9)$$

$$= \sum_{i=1}^N (m_i R^2) \quad (10)$$

$$= R^2 \sum_{i=1}^N m_i \quad (11)$$

$$(12)$$

Substituting  $M$  as the sum of all masses in the ring ( $M = \sum_{i=1}^N m_i$ ), the statement is therefore:

$$KE_{rot} = MR^2 \quad (13)$$

### 1.3 | Rotational Inertia of a Solid Sphere

I believe that the rotational inertia of  $I_{sphere}$  to be less than  $I_{disk}$ . This is because, as the dimension of the object increases, it would be easier to change its velocity (a disk is easier to spin than a ring, etc.). Hence, my intuition states that  $I_{sphere}$  would be lower than  $I_{disk}$ .

Mathematically, as  $M$  is staying at the same value, in the disk case has more mass closer to the axis of rotation — meaning that the  $m_i R^2$  term would be smaller in more of the point masses than that of an object at a lower dimension. Hence, the sphere would have more points with lower  $m_i R^2$  terms than that of disk; hence,  $I_{sphere}$  would be less than  $I_{disk}$ .

## 2 | Kinematics Equations

Given  $a = a_0$ , initial velocity  $v_0$ , and position  $y_0$ , we derive the kinematics equations.

$$a(t) = a_0 \quad (14)$$

$$\int a(t) dt = \int a_0 dt \quad (15)$$

$$v(t) = a_0 t + C \quad (16)$$

We are given that  $v(0) = v_0$ .  $v(0) = C = v_0$ , hence,  $C = v_0$ . The velocity statement is therefore,

$$v(t) = a_0 t + v_0 \quad (17)$$

Continuing with integration:

$$v(t) = a_0 t + v_0 \quad (18)$$

$$\int v(t) = \int a_0 t + v_0 dt \quad (19)$$

$$y(t) = \frac{1}{2} a_0 t^2 + v_0 t + C \quad (20)$$

$$(21)$$

Again, substituting  $C = y_0$  by the same logic above —  $y(0) = C = y_0$ , we derive the statement for the position equation.

$$y(t) = \frac{1}{2}a_0t^2 + v_0t + y_0 \quad (22)$$

## 2.1 | Proving $v^2(t) = v_0^2 + 2a_0(y(t) - y_0)$

We start at the statement for  $v(t)$ , squaring it, and substituting the necessary statements.

$$v(t) = a_0t + v_0 \quad (23)$$

$$\Rightarrow v^2(t) = a_0^2t^2 + 2a_0v_0t + v_0^2 \quad (24)$$

$$v^2(t) = v_0^2 + 2a_0\left(\frac{1}{2}a_0t^2 + v_0t\right) \quad (25)$$

$$v^2(t) = v_0^2 + 2a_0\left(\frac{1}{2}a_0t^2 + v_0t + y_0 - y_0\right) \quad (26)$$

$$v^2(t) = v_0^2 + 2a_0(y(t) - y_0) \quad (27)$$

It is therefore shown that:

$$v^2(t) = v_0^2 + 2a_0(y(t) - y_0) \quad (28)$$

## 2.2 | Proving $\Delta y = \frac{v(t_1)+v(t_2)}{2}\Delta t$

Showing  $\Delta y = \frac{v(t_1)+v(t_2)}{2}\Delta t$ , defining  $\Delta y = y(t_2) - y(t_1)$  and  $\Delta t = t_2 - t_1$ . Substituting the appropriate values for  $v(t)$ ,  $\Delta y$ ,  $\Delta t$  and solving...

$$\Delta y = \frac{v(t_1) + v(t_2)}{2}\Delta t \quad (29)$$

$$y(t_2) - y(t_1) = \frac{v(t_1) + v(t_2)}{2}t_2 - t_1 \quad (30)$$

$$y(t_2) - y(t_1) = \frac{((a_0t_1 + v_0) + (a_0t_2 + v_0))}{2}t_2 - t_1 \quad (31)$$

$$y(t_2) - y(t_1) = \frac{((a_0t_1t_2 + v_0t_2) - (a_0t_1^2 + v_0t_1) + (a_0t_2^2 + v_0t_2) - (a_0t_1t_2 + v_0t_1))}{2} \quad (32)$$

$$y(t_2) - y(t_1) = \frac{((a_0t_1t_2 + v_0t_2) - (a_0t_1^2 + v_0t_1) + (a_0t_2^2 + v_0t_2) - (a_0t_1t_2 + v_0t_1))}{2} \quad (33)$$

$$(34)$$