1 | Problem 1

1.1 | a)

Inorder for the boat to be going in the right direction we know that $\vec{C}+\vec{S}=\alpha\vec{D}$, where $\vec{C}$ is the current of the river, $\vec{S}$ is the speed of the boat, α is some scalar and $\vec{D}$ is the vector that goes from the boatman's starting point to their desired endpoint.

We can set the boatman's start point as $(0,0)$, and thus $\vec{D}=\langle 3,2\rangle$. We also know that $\vec{C}=\langle 0,-3.5\rangle$. Lastly, $\vec{S}=\langle 13\sin(\theta),13\cos(\theta)\rangle$, where θ is the angle between the side of the river and $\vec{S}$.

We can then plug in these values into the equation written above:

$\vec{C}+\vec{S}=\alpha\vec{D}$
$\Rightarrow \langle 0,-3.5\rangle+\langle 13\sin(\theta),13\cos(\theta)\rangle=\alpha\langle 3,2\rangle$
$\Rightarrow \langle 13\sin(\theta),-3.5+13\cos(\theta)\rangle=\langle \alpha 3,\alpha 2\rangle$
$\Rightarrow 13\sin(\theta)=\alpha 3, -3.5+13\cos(\theta)=\alpha 2$
$\Rightarrow 6\alpha=26\sin(\theta), 6\alpha=-10.5+39\cos(\theta)$
$\Rightarrow 26\sin(\theta)=-10.5+39\cos(\theta)$

plug it into wolfram alpha:

$\theta\approx 0.75686$ radians or $\approx 43.36°$

1.2 | b)

The net velocity of the boat is $\vec{S}+\vec{C}=\langle 13\sin(\theta),13\cos(\theta)-3.5\rangle$, where θ is the answer to part a. To get the speed of the boat we find the magnitude of this vector:

$|\vec{S}+\vec{C}|=\sqrt{(13\sin(\theta))^2+(13\cos(\theta)-3.5)}\approx 10.7282$ km/h

Now we need to find the distance traveled by the boat, which should be the magnitude of $\vec{D}$:

$|\vec{D}|=\sqrt{2^2+3^2}=\sqrt{4+9}=\sqrt{13}\approx 3.60555$ km

To get the time it took to take the trip we divide the distace by the speed:

$\frac{3.60555}{10.7282}=0.336$ hours, which is 20.2 minutes

2 | Problem 2

2.1 | a)

$\vec{r}(t)=(R\cos(\omega_o t),R\sin(\omega_o t))$

This is because the x coordinate is defined as $r\cos(\theta)$ and the y coordinate is defined as $r\sin(\theta)$. In this case r or the radius is R and θ is $\omega_o t$, because $\omega_o=\frac{\theta}{t}$ (definition of angular velocity.

2.2 | b)

$\vec{v}(t)=\vec{r}'(t)=(\frac{d}{dt}R\cos(\omega_o t),\frac{d}{dt}R\sin(\omega_o t))=(-R\omega_o\sin(\omega_o t),R\omega_o\cos(\omega_o t))$

Answer: $\vec{v}(t)=(-R\omega_o\sin(\omega_o t),R\omega_o\cos(\omega_o t))$

2.3 | **c)**

$\vec{a}(t) = \vec{r}''(t) = (\frac{d}{dt}(-R\omega_o \sin(\omega_o t)), \frac{d}{dt}R\omega_o \cos(\omega_o t)) = (-R\omega_o^2 \cos(\omega_o t), -R\omega_o^2 \sin(\omega_o t))$
Answer: $\vec{a}(t) = (-R\omega_o^2 \cos(\omega_o t), -R\omega_o^2 \sin(\omega_o t))$

2.4 | **d)**

The tangent:

Because $\vec{v}(t)$ is a vector it can be placed anywhere on the plane, so the only requierment for $\vec{v}(t)$ is that it has to be perpendicular to $\vec{r}(t)$, which is the radius of the circle. This means that the slope of $\vec{v}(t)$ has to be the opposite reciprocol of $\vec{r}(t)$:

Slope of $\vec{r}(t) = \frac{\Delta y}{\Delta x} = \frac{R\sin(\omega_o t)-0}{R\cos(\omega_o t)-0} = \frac{\sin(\omega_o t)}{\cos(\omega_o t)}$

Slope of $\vec{v}(t) = \frac{\Delta y}{\Delta x} = \frac{R\omega_o\cos(\omega_o t)-0}{-R\omega_o\sin(\omega_o t)-0} = -\frac{\cos(\omega_o t)}{\sin(\omega_o t)}$

The slopes are opposite reciprocols, thus $\vec{v}(t)$ is perpendicular to $\vec{r}(t)$, thus $\vec{v}(t)$ is tangent to the circle.

The magnitude:

$|\vec{v}(t)| = \sqrt{(-R\omega_o\cos(\omega_o t))^2 + (R\omega_o\sin(\omega_o t))^2} = \sqrt{R^2\omega_o^2\cos^2(\omega_o t + R^2\omega_o^2\sin^2(\omega_o t)} = \sqrt{R^2\omega_o^2(\cos^2(\omega_o t) + \sin^2(\omega_o t)} = \sqrt{R^2\omega_o^2(1)} = \sqrt{R^2\omega_o^2} = R\omega_o$

2.5 | **e)**

Point towards the center of the circle:

$\vec{a}(t)$ is a scalar multiple of $\vec{r}(t)$: $\vec{a}(t) = -\omega_o^2 \cdot \vec{r}(t) = -\omega_o^2(R\cos(\omega_o t), R\sin(\omega_o t)) = (-R\omega^2\cos(\omega_o t), -R\omega_o^2\sin(\omega_o t)) = \vec{a}(t)$

because the scalar multiple is negative $\vec{v}(t)$ points in the opposite direction of $\vec{r}(t)$, which is towards the center of the circle because $\vec{r}(t)$ points from the center of the circle outwards.

The magnitude:

$|\vec{a}(t)| = \sqrt{(-R\omega^2\cos(\omega_o t))^2 + (-R\omega_o^2\sin(\omega_o t))^2} = \sqrt{R^2\omega_o^4\cos^2(\omega_o t) + R^2\omega_o^4\sin^2(\omega_o t)} = \sqrt{R^2\omega_o^4(\cos^2(\omega_o t) + \sin^2(\omega_o t))} = \sqrt{R^2\omega_o^4(1)} = \sqrt{R^2\omega_o^4} = R\omega_o^2$

$\frac{|\vec{v}(t)|^2}{R} = \frac{(R\omega_o)^2}{R} = \frac{R^2\omega_o^2}{R} = R\omega_o^2$

2.6 | **f)**

$\theta'(t) = \int \theta''(t)dt = \int \alpha_o dt = \alpha_o t + c$, where c is the constant of integration

Answer: $\alpha_o t + c$

2.7 | **g)**

$\theta(t) = \int \theta'(t)dt = \int(\alpha_o t + c)dt = \int \alpha_o t dt + \int c dt = \frac{\alpha_o t^2}{2} + ct + c'$ where c' is another constant of integration.

Answer: $\frac{\alpha_o t^2}{2} + ct + c'$

2.8 | h)

For simplicity's sake, I am going to assume that $c = 0$ and $c' = 0$:
$\vec{r}(t) = (R\cos(\frac{\alpha_o t^2}{2}), R\sin(\frac{\alpha_o t^2}{2}))$

2.9 | i)

$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = (\frac{d}{dt}R\cos(\frac{\alpha_o t^2}{2}, \frac{d}{dt}R\sin(\frac{\alpha_o t^2}{2})) = (-R\alpha_o t\sin(\frac{\alpha_o t^2}{2}), R\alpha_o t\cos(\frac{\alpha_o t^2}{2}))$
Answer: $\vec{v}(t) = (-R\alpha_o t\sin(\frac{\alpha_o t^2}{2}), R\alpha_o t\cos(\frac{\alpha_o t^2}{2}))$

2.10 | j)