0 | **meta**

This homework took ~3h whilst discussing with peers… I really need to practice this type of algebra.

1 | $\int \frac{\sqrt{x-1}}{x} dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}} dx$

$$\begin{aligned}
\int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\
&= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\
&= 2 \int \frac{\cancel{u^2+1}}{\cancel{u^2+1}} + \frac{-1}{u^2+1} du \\
&= 2 \int 1 du - \frac{1}{u^2+1} + C \\
&= 2 \int 1 du - \tan^{-} u + C \\
&= 2u - \tan^{-} u + C \\
&= \boxed{2\sqrt{x-1} - \tan^{-}\left(\sqrt{x-1}\right) + C}
\end{aligned}$$

Polynomial long division?

When you have a square root with a sum/difference inside, there's not much you can do. So, your best bet is to substitute either the stuff inside the root as u or the entire radical as u.

2 | $\int \frac{x^2}{x^2+1} dx$

Let $u = x^2 + 1$, $du = 2x dx$

$$\begin{aligned}
\int \frac{x^3}{x^2+1} dx &= \frac{1}{2} \int \frac{u-1}{u} du \\
&= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C \\
&= \frac{1}{2} (u - \ln u) + C \\
&= \boxed{\frac{1}{2} \left(x^2 + 1 - \ln(x^2+1)\right) + C}
\end{aligned}$$

JUST SPLIT THE FRACTION

AND LOOK FOR TANsr X

3 | $\int \frac{x-4}{x^2}dx$

$$\begin{aligned}\int \frac{x-4}{x^2}dx &= \int \frac{x}{x^2}\frac{4}{x^2}dx \\ &= \int \frac{1}{x}dx + 4\int \frac{1}{x^2}dx \\ &= \boxed{\ln x - \frac{4}{x} + C}\end{aligned}$$

4 | $\int (x+1)e^{x^2+2x}dx$

Let $u = x^2 + 2x$, $du = x + 1dx$

$$\begin{aligned}\int (x+1)e^{x^2+2x}dx &= \frac{1}{2}\int e^u du \\ &= \frac{1}{2}e^u \\ &= \boxed{\frac{1}{2}e^{x^2+2x} + C}\end{aligned}$$

5 | $\int \tan^2 x + 1dx$

$$\begin{aligned}\int \tan^2 x + 1dx &= \int \sec^2 x - 1 + 1dx \\ &= \int \sec^2 xdx \\ \text{Let } u = x, du = dx & \\ &= \int \sec^2 udu \\ &= \tan u + C \\ &= \boxed{\tan x + C}\end{aligned}$$

6 | $\int \frac{6x^2-4}{x}dx$

$$\begin{aligned}\int \frac{6x^2-4}{x}dx &= \int \frac{6x^2}{x}dx - 4\int \frac{1}{x}dx \\ &= \int 3xdx - 4\ln|x| + C \\ &= \boxed{3x^2 - 4\ln|x| + C}\end{aligned}$$

7 | $\int \frac{e^x-1}{e^x} dx$

$$\begin{aligned}\int \frac{e^x-1}{e^x} dx &= \int 1 - \frac{1}{e^x} dx \\ &= \int 1 - e^{-x} dx \\ &= x + e^{-x} + C \\ &= \boxed{e^{-x} + x + C}\end{aligned}$$

8 | $\int \frac{\sec^2 x}{\csc x} \sin x dx$

$$\begin{aligned}\int \frac{\sec^2 x}{\csc x} \sin x dx &= \int \tan^2 x dx \\ &= \int \sec^2 x - 1 dx \\ &= \int \sec^2 x dx - \int 1 dx \\ &= \boxed{\tan x - x + C}\end{aligned}$$

9 | $\int \sin x \cos x dx$

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{aligned}\int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x + C}\end{aligned}$$

Product of $sin $ and $cos $, so we can use a double angle formula

$$\begin{aligned}\int \sin x \cos x dx &= \int \frac{1}{2} \sin 2x dx \\ &= \frac{1}{2} \int \sin 2x dx \\ &= -\frac{1}{4} \cos 2x\end{aligned}$$

10 | $\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx$

$$\begin{aligned}
\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx &= \int \frac{\sin^2 x+\cos^2 x}{\tan^2 x+1}dx \\
&= \int \frac{1}{\tan^2 x+1}dx \\
&= \int \frac{1}{\sec^2 x}dx \\
&= \int \cos^2 x dx \\
&= \int \frac{1}{2}(\cos 2x+1)dx \\
&= \frac{1}{2}\int \cos 2x+1dx \\
&= \frac{1}{2}\left(\int \cos 2xdx+\int 1dx\right)
\end{aligned}$$

Let $u=2x$, $du=2dx$

$$\begin{aligned}
\frac{1}{2}\int \cos 2xdx+\frac{x}{2}+C &= \frac{1}{4}\int \cos udu+\frac{x}{2}+C \\
&= \frac{1}{4}\sin u+\frac{x}{2}+C \\
&= \boxed{\frac{1}{4}\sin 2x+\frac{x}{2}+C}
\end{aligned}$$

11 | $\int \frac{\sec x\tan x}{1+\sec^2 x}dx$

Let $u=\sec x$, $du=\sec x\tan xdx$

$$\begin{aligned}
\int \frac{\sec x\tan x}{1+\sec^2 x}dx &= \int \frac{du}{1+u^2}dx \\
&= \int \frac{1}{1+u^2}du \\
&= \tan^{-} u+C \\
&= \boxed{\tan^{-}\sec x+C}
\end{aligned}$$

12 | $\int x^2e^{x^3}dx$

Let $u=x^3$, $du=3x^2dx$

$$\begin{aligned}\int x^2 e^{x^3} dx &= \frac{1}{3}\int e^u du \\ &= \frac{1}{3}e^u + C \\ &= \boxed{\frac{1}{3}e^{x^3} + C}\end{aligned}$$

13 | $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$

$$\begin{aligned}\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx &= 2\int e^u du \\ &= 2e^u + C \\ &= \boxed{2e^{\sqrt{x}} + C}\end{aligned}$$