#ret #hw

1 | High frequency arbitrage

1.1 | The Problem

Optimize the payoff for both players in a system with two markets and three unique items. Selling two distinct items on the same market yields a payoff of m for both parties, whereas selling the same item on the same market yields a payoff of $-M$ for both parties, where $0 < m < M$. Selling on distinct markets always yields a payoff of 0.

Each play can sell on their local market instantly, and on the foreign market after 40ms. Communication between players also takes 40ms, one way.

Every 0.5ms each play receives a randomly chosen item to sell, and decides where to sell it on the same tick.

1.2 | The Solution

Players A and B could utilize the following deterministic scheme based on what items they received to determine what market to sell on.

Item	A	B
x	m_1	m_2
y	m_2	m_1
z	m_1	m_2

In a system with only two items, x and y, this strategy yields the theoretical perfect payoff:

A Item	B Item	A Market	B Market	Reward
x	y	m_1	m_1	$2m$
y	x	m_2	m_2	$2m$
x	x	m_1	m_2	0
y	y	m_2	m_1	0

In scenarios where players receive distinct items, a maximal reward is always received. When players receive the same item, $-M$ is always avoided, yielding again the maximal reward. Moving to three items, while $-M$ is always avoided, $\frac{1}{3}$ of the time an opportunity to receive $2m$ will be missed.

1.2.1 | Generalizing

Using the same strategy scheme, the probability of receiving the maximal payoff is $\frac{m}{n}$ where m is the number of markets and n is the number of items.

$$P(2m) = \frac{m}{n}$$

$$P(-2M) = 0$$

$$P(O) = P(2m^c) = 1 - \frac{m}{n}$$

Thus, as the $n - m$ increases, our expected value decreases significantly, eventually rendering the strategy unviable.