

## 1 | A surface integral

We are defining a function:

$$f(x, y, z) = y^2 \quad (1)$$

and slicing out a vertical organ pipe shape with a sliced edge. That is:

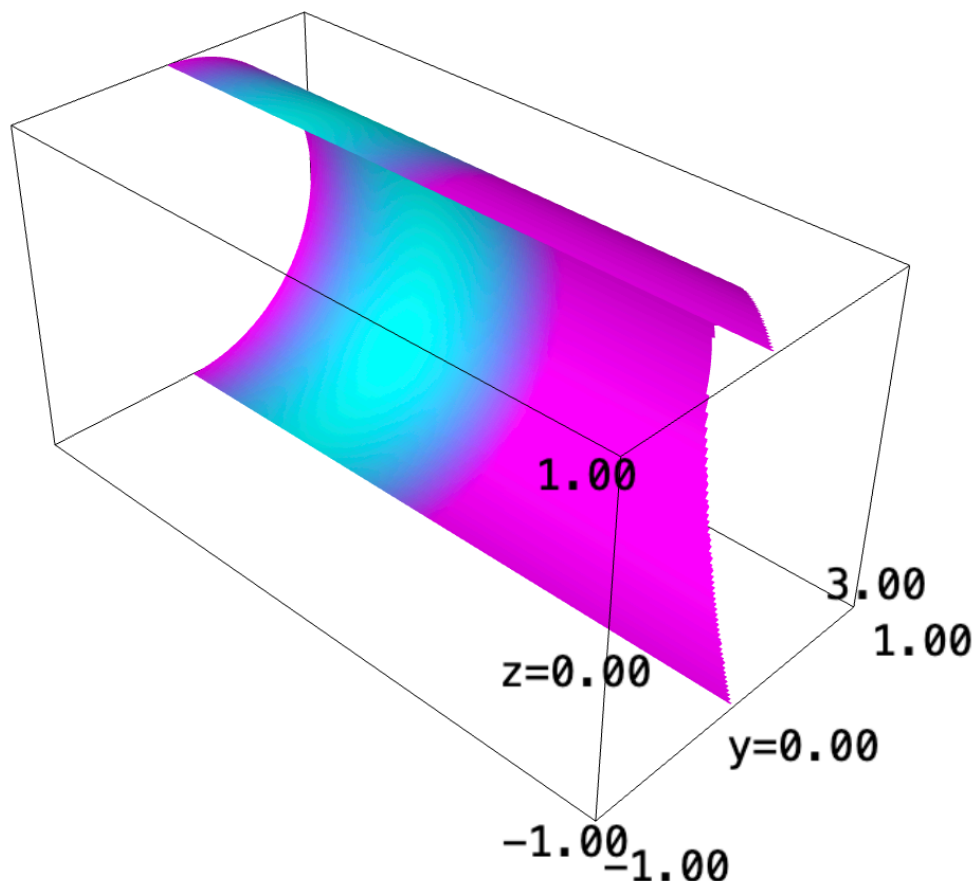
$$x^2 + z^2 = 1 \quad (2)$$

bounded by  $y > 0$  and  $y < 3 - x$ .

Let's plot this:

```
var('x,y,z')
f = y^2
```

```
implicit_plot3d(x^2+z^2 == 1, (y,-1,3), (x,-1,1), (z, -1,1), region=(lambda x,y,z: y > 0 and y<3-x), co
```



that's honestly pretty cool!

Great, now let's take the actual surface integral. Note that, because the "pipe" does not have a defining ending point, we will set its end at the  $xy$  plane, that it ends at  $x = 0$  at the other end in addition to  $y = 3 - x$ .

Looking at the actual function for which we are taking the integral, we have:

$$x^2 + z^2 = 1 \quad (3)$$

We will rearrange this expression in terms of  $z$ :

$$z = \sqrt{1 - x^2} \quad (4)$$

Fortunately, we see already that the function's derivative w.r.t.  $y$  is 0; indeed, it doesn't change along the  $y$  direction (the cylinder is centered around it after all.)

Taking the derivative in the  $x$  direction:

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \sqrt{1 - x^2} \quad (5)$$

$$= \frac{-2x}{2\sqrt{-x^2 + 1}} \quad (6)$$

$$= \frac{-x}{\sqrt{-x^2 + 1}} \quad (7)$$

Squaring the expression below:

$$\frac{x^2}{-x^2 + 1} \quad (8)$$

And finally, we have the correction factor:

$$dA = \sqrt{\frac{x^2}{-x^2 + 1} + 1} dV \quad (9)$$

$$= \sqrt{\frac{1}{-x^2 + 1}} dV \quad (10)$$

Lastly, we can multiply the actual value function to this to this expression to get the expression for the integral:

$$\iint_V y^2 \sqrt{\frac{1}{-x^2 + 1}} dx dy \quad (11)$$

Furthermore, our bounds are also a little complicated:

$$\int_0^3 \int_0^{3-x} y^2 \sqrt{\frac{1}{-x^2 + 1}} dx dy \quad (12)$$

Great, we will now ask Sage to take the actual integral for us:

```
f(x,y) = sqrt(1/-x^2+1)
f.integrate(x, 0, 3-x)
```