#flo #ret #hw

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1 | Problem 21!

Suppose V is finite-dimensional and $T \in L(V, W)$. Prove that T is injective if and only if there exists $S \in L(W, V)$ such that ST is the identity map on V. NNOO!

3.16 injectivity is equivalent to null space equals {0} 3.15 injective: T: V -> W is injective if $Tu = Tv$ implied $u = v$ 3.8 product of linear maps:

identity map on v: Iv = v

product of linear maps:

if $T \in L(U, V)$ and $S \in L(V, W)$, then the *product* $ST \in L(U, W)$ is defined by

$$(ST)(u) = S(Tu)$$

for all $u \in U$.

2 | Problem 21, pt 2.

Suppose V is finite-dimensional and $T \in \mathcal{L}(V, W)$. Prove that T is surjective if and only if there exists $S \in \mathcal{L}(W, V)$ such that TS is the identity map on W.

3.20 :: def of surjective 3.17 :: def of range 3.2 :: def of linear map

- define a S that takes range of T back to V such that TS(v) = v
  - prove that this is linear, and we are done with the forwards direction
    - homogeneity
    - additivity
- additivity
  - $T(u + v) = Tu + Tv$ for all $u, v \in V$
  - $S(u + w) = Su + Sw$ for all $u, w \in W$

Since T is additive, and $TS(v) = v$, we can say that:

$$TS(v + u) = v + u = T(Sv) + T(Su) = T(Sv + Su)$$

Therefore, S is additive.

- homogeneity
  - $T(\lambda V) = \lambda(Tv)$ for all $\lambda \in F$ and all $v \in V$.
- homogeneity $T(S\lambda v) = \lambda v = \lambda T(Sv) = T(\lambda Sv)$

backwards direction:

assume TS is the identity map. prove that T is surjective.