

We first set up the basic assumptions and variables.

```
GRAV <- 9.8 # gravity (m/s^2)
MASS <- 1 # mass (kg)
I_CM <- 1/12 # rotational inertia at the centre of gravity (kg m^2)
L1 <- 0.5 # distance from rotation point to CoM (m)
L2 <- 1 # distance from rotation point to tension (m)
PHI <- 0 # angle of Ft relative to floor orthogonal (rad)
FT <- 11 # tension force (N)
OMEGA <- 0 # angle of line orthogonal to floor relative to gravity (rad) (because shifted axis)
```

Additionally, we set the time interval and seed values for time and theta (distance from flat):

```
dt <- 0.0001
t_max <- 5

vx <- 0
vy <- 0

theta <- 0
thetadot <- 0
time <- 0
```

First, let's create a function for torque in terms of theta (and the constants above):

```
net_torque <- function(theta) {
  return(L2 * FT * cos(theta + PHI) - L1 * MASS * GRAV * cos(theta - OMEGA))
}
```

Great. Let's start generating the table! We essentially write a for loop to append to a few different vectors. Variables appended with *c* reflect the column vectors that we will put together.

```
cTime = NULL
cTheta = NULL
cDDTheta = NULL
cDTheta = NULL
cTorqueNet = NULL
cAccelX = NULL
cAccelY = NULL
cVelX = NULL
cVelY = NULL
cFFriction = NULL
cFNormal = NULL

# debugging values
cFNetY = NULL
cFTensionPhiComponent = NULL
cFGravityPhiComponent = NULL

cMuStatic = NULL
cKERot = NULL
cKETrans = NULL
```

Awesome. Let's now run a lovely little for loop to actually populate the values recursively.

```

for (i in 0:(t_max/dt)) {
  # We first populate the time column with the time, theta column with theta
  cTime[i] = time
  cTheta[i] = theta

  torque <- net_torque(theta)
  # Given the theta value, we calculate the net torque and set that
  cTorqueNet[i] = torque
  # Now that we know the net torque, we could know how much the angular
  # acceleration is by just dividing out the rotational inertia
  thetadotdot <- torque/I_ROT
  cDDTheta[i] = thetadotdot
  # We could also multiply the theta acceleration by time to get the
  # velocity at that point
  thetadot <- dt*thetadotdot + thetadot
  cDTheta[i] = thetadot
  # We could therefore component-ize the acceleration in theta into
  # ax and ay
  ax <- -1 * L1 * sin(theta) * thetadotdot
  cAccelX[i] = ax
  ay <- L1 * cos(theta) * thetadotdot
  cAccelY[i] = ay # @mark isn't sin and cos backwards?
  # We also tally the components separately for velocity
  vx <- ax*dt + vx
  vy <- ay*dt + vy

  # Based on these accelerations, we therefore could calculate the relative
  # force of friction and normal force by subtracting the force in that direction
  # out of net
  ffriction <- FT*sin(PHI) + MASS*GRAV*sin(OMEGA)-MASS*ax
  fnormal <- MASS*ay-FT*cos(PHI)+MASS*GRAV*cos(OMEGA)

  cFNetY[i] = MASS*ay
  cFTensionPhiComponent[i] = FT*cos(PHI)
  cFGravityPhiComponent[i] = -MASS*GRAV*cos(OMEGA)

  cFFriction[i] = ffriction
  cFNormal[i] = fnormal

  # Then, we calculate the energies
  I_ROT <- I_CM + MASS * (L1*cos(theta))^2 # we calculate I_ROT using
  # the Parallel axis theorem
  cKERot[i] = 0.5 * I_ROT * thetadot^2
  cKETrans[i] = 0.5 * MASS * (vx^2+vy^2)

  # Dividing the friction force by the normal force, of course, will result in
  # the (min?) friction coeff
  cMuStatic[i] = ffriction/fnormal

  # We increment the time and also increment theta by multiplying the velocity
  # by dt to get change in the next increment
  time <- dt + time
  theta <- dt*thetadot + theta
}

```

We now put all of this together in a dataframe.

```
rotating_link <- data.frame(cTime,
  cTheta,
  cDTheta,
  cDDTheta,
  cTorqueNet,
  cAccelX,
  cAccelY,
  cFFriction,
  cFNormal,
  cMuStatic,
  cKERot,
  cKETrans)
```

```
names(rotating_link) <- c("time",
  "theta",
  "d.theta",
  "dd.theta",
  "net.torque",
  "accel.x",
  "accel.y",
  "friction.force",
  "normal.force",
  "friction.coeff",
  "ke.rot",
  "ke.trans")
```

Let's import some visualization tools, etc.

```
library(tidyverse)
```

Let's first see the head of this table:

```
head(rotating_link)
```

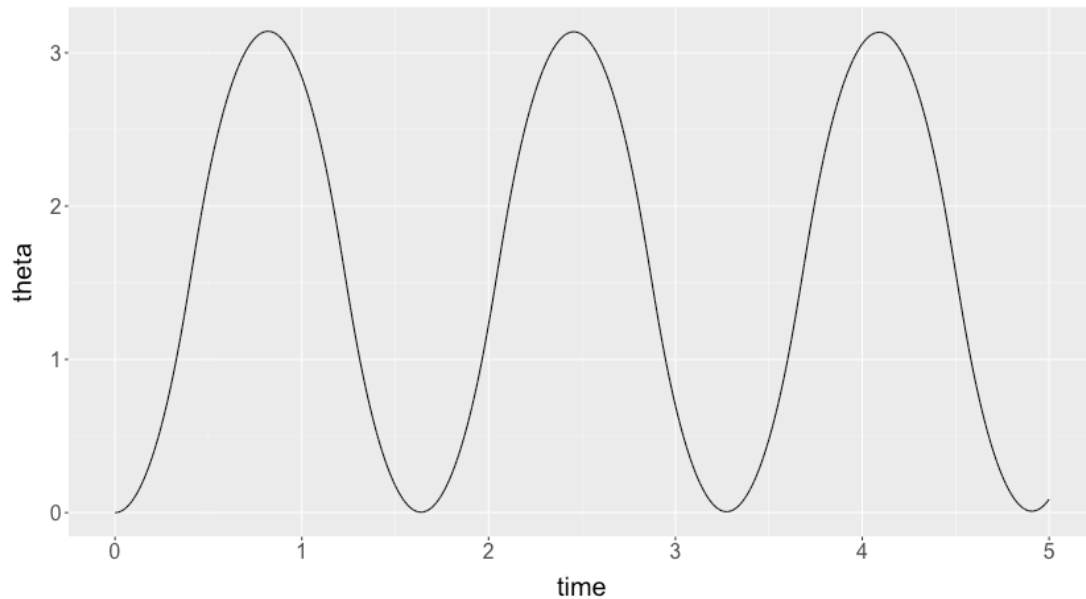
```
1e-04 6.1e-08 0.00244 18.3 6.099999999999999 -5.581499999999999e-07 9.149999999999999 5.581499999999999e-07
7.949999999999999 7.02075471698115e-08 9.922666666666666e-07 7.441999999999999e-07
2e-04 3.05e-07 0.004269999999999999 18.299999999999999 6.09999999999999972 -2.790749999999983e-06
9.149999999999999 2.790749999999983e-06 7.949999999999999 3.51037735849072e-07 3.038816666666634e-06
2.279112499999987e-06
3e-04 7.319999999999999e-07 0.006099999999999999 18.299999999999999 6.0999999999999837 -6.69779999999998e-06
9.149999999999999 6.69779999999998e-06 7.949999999999999 8.42490566037935e-07 6.201666666666327e-06
4.651249999999989e-06
4e-04 1.341999999999999e-06 0.007929999999999999 18.299999999999999 6.0999999999999451 -1.227929999999987e-05
9.149999999999987 1.227929999999987e-05 7.949999999999987 1.54456603773704e-06 1.048081666666489e-05
7.860612499999953e-06
5e-04 2.134999999999981e-06 0.009759999999999999 18.299999999999999 6.0999999999999861 -1.9535249999999653e-05
9.14999999999997065 1.9535249999999653e-05 7.94999999999997065 2.4572641509481e-06 1.587626666666024e-05
1.1907199999999857e-05
6e-04 3.110999999999995e-06 0.011589999999999999 18.299999999999999 6.09999999999997048 -2.8465649999999091e-05
9.14999999999994273 2.8465649999999091e-05 7.94999999999994273 3.58058490567474e-06 2.2388016666664823e-05
1.679101249999642e-05
```

Before we start graphing, let's set a common graph there.

```
default.theme <- theme(text = element_text(size=20), axis.title.y = element_text(margin = margin(t = 0,
```

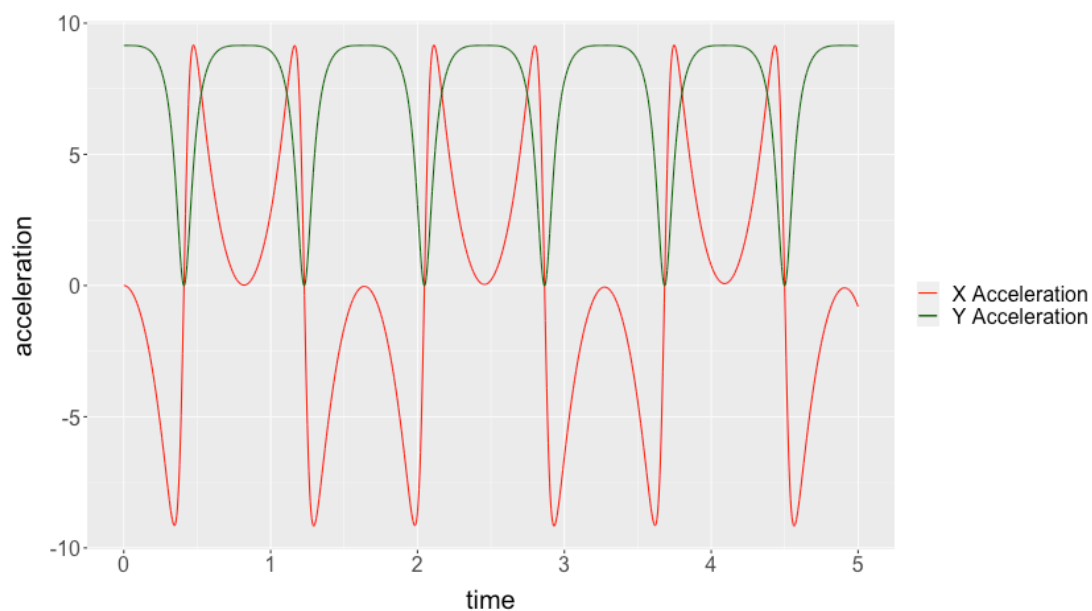
Cool! We could first graph a function for theta over time.

```
rotating_link %>% ggplot() + geom_line(aes(x=time, y=theta)) + default.theme
```



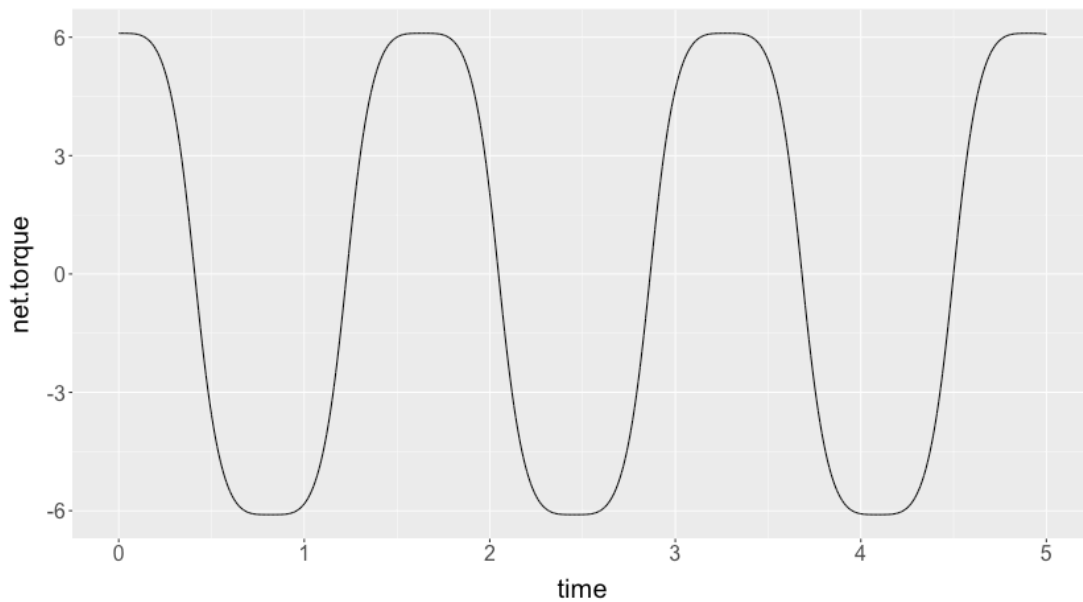
And, similarly, we will graph ax and ay on top of each other:

```
rotating_link %>% ggplot() + geom_line(aes(x=time, y=accel.x, colour="X Acceleration")) + geom_line(aes
```



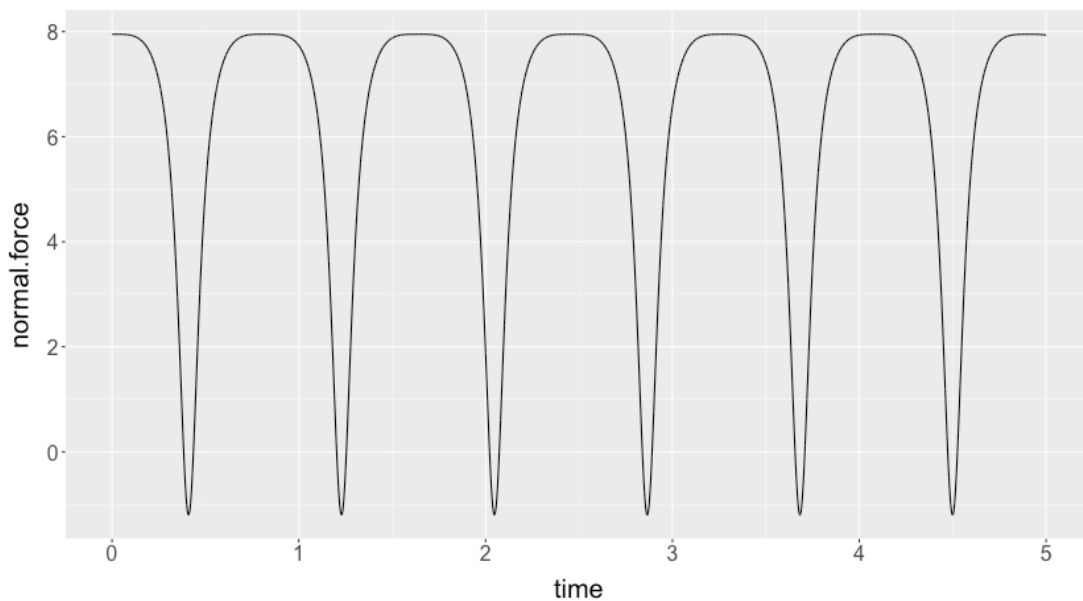
Let's also plot torque as well.

```
rotating_link %>% ggplot() + geom_line(aes(x=time, y=net.torque)) + default.theme
```



And. **Most importantly!** Let's plot the normal force.

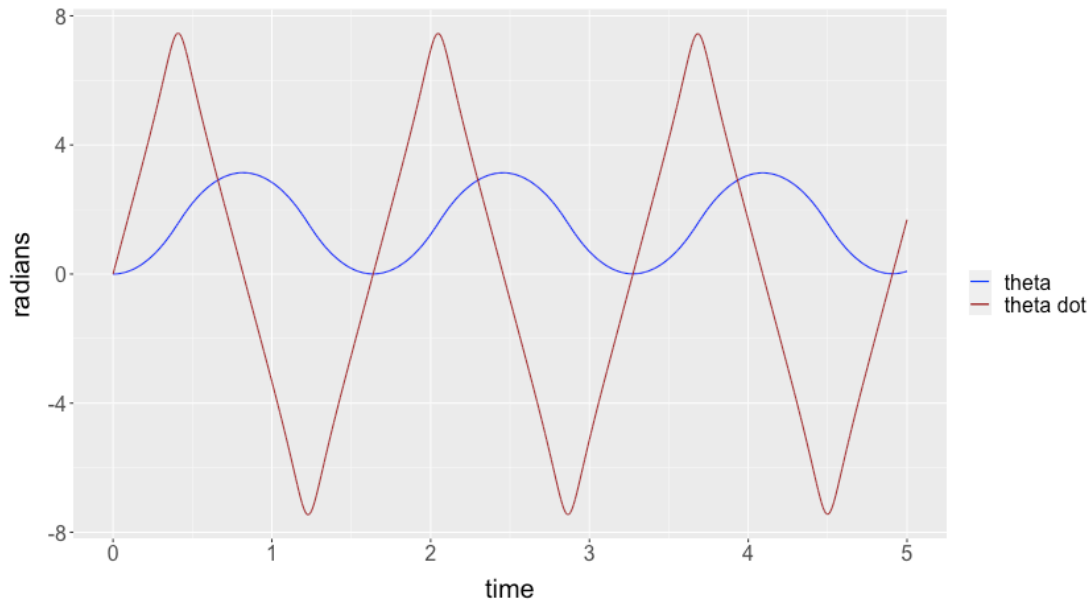
```
rotating_link %>% ggplot() + geom_line(aes(x=time, y=normal.force)) + default.theme
```



Obviously, after the normal force becomes negative, this graph stops being useful.

Theta dot atop theta:

```
rotating_link %>% ggplot() + geom_line(aes(x=time, y=theta, colour="theta")) + geom_line(aes(x=time, y=
```



We finally, plot KE rotation and translation

```
rotating_link %>% ggplot() + geom_line(aes(x=time, y=ke.rot, colour="ke rotation")) + geom_line(aes(x=t
```

