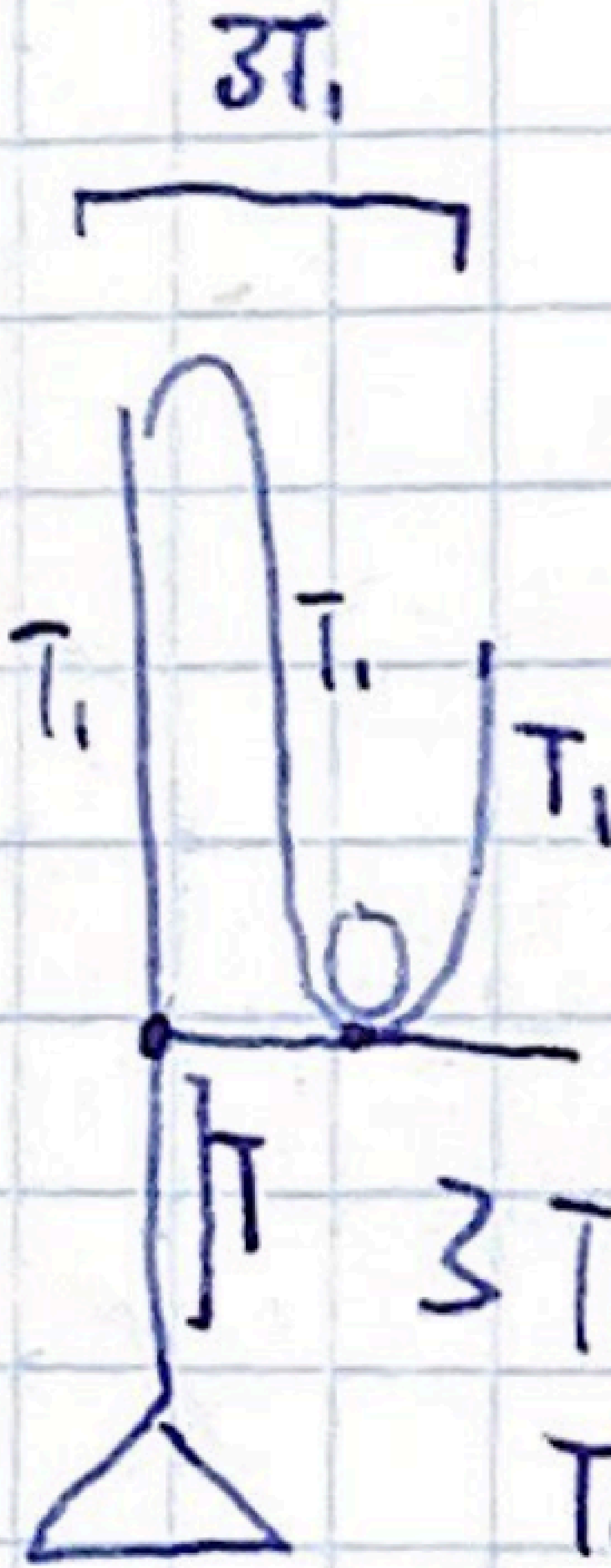
1 | Pulley Problem

1.1 | Number 1

Three tie points, tied onto one point.

$$3T_1 = T \tag{1}$$

$$\Rightarrow T_1 = \frac{1}{3}T \tag{2}$$



$$3T_1 = T$$

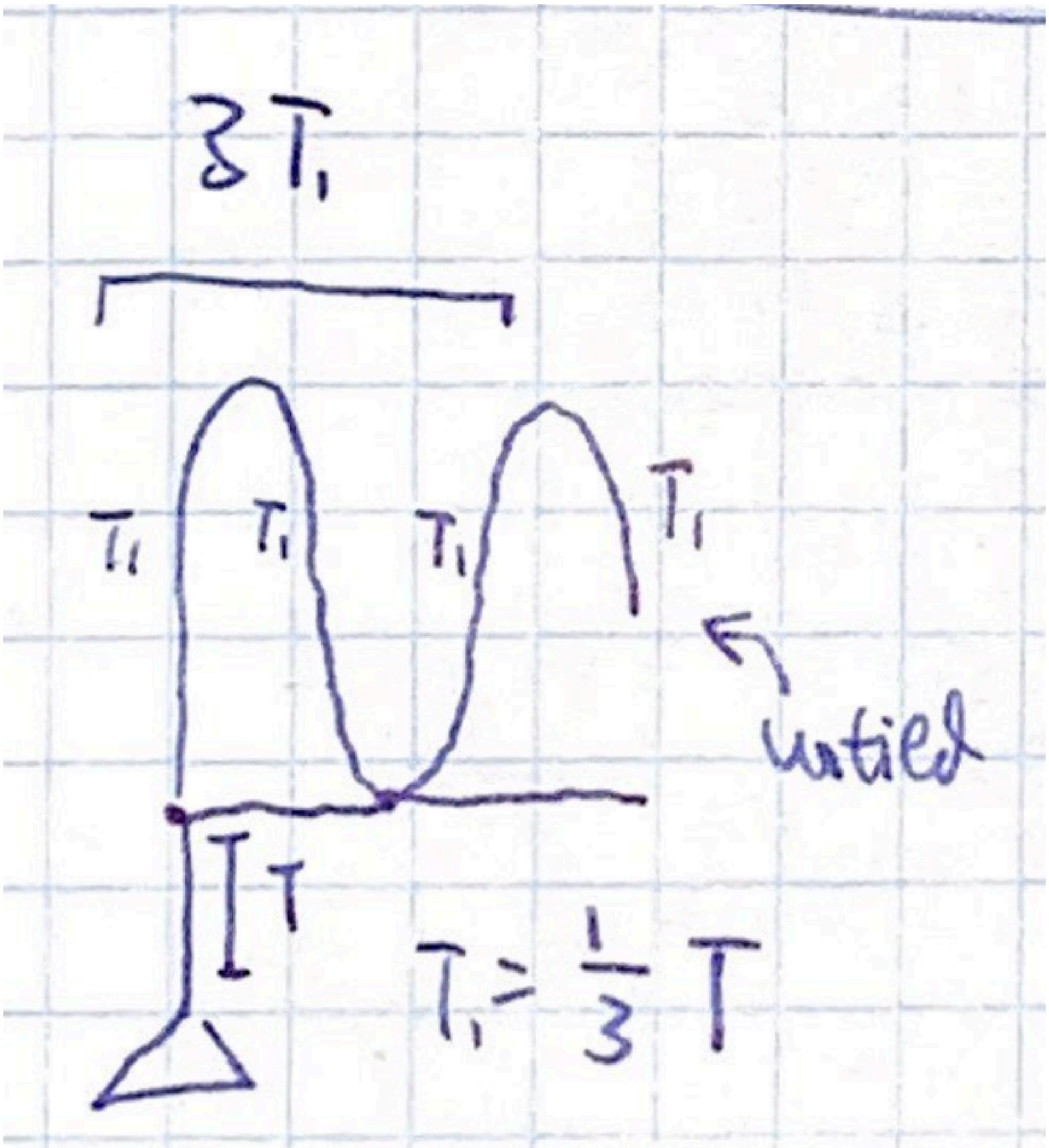
$$T_1 = \frac{1}{3}T$$

1.2 | Number 2

Three tie points, tied onto one point. The extra additional pull, given its not tied, simply transfers tension.

$$3T_1 = T \tag{3}$$

$$\Rightarrow T_1 = \frac{1}{3}T \tag{4}$$

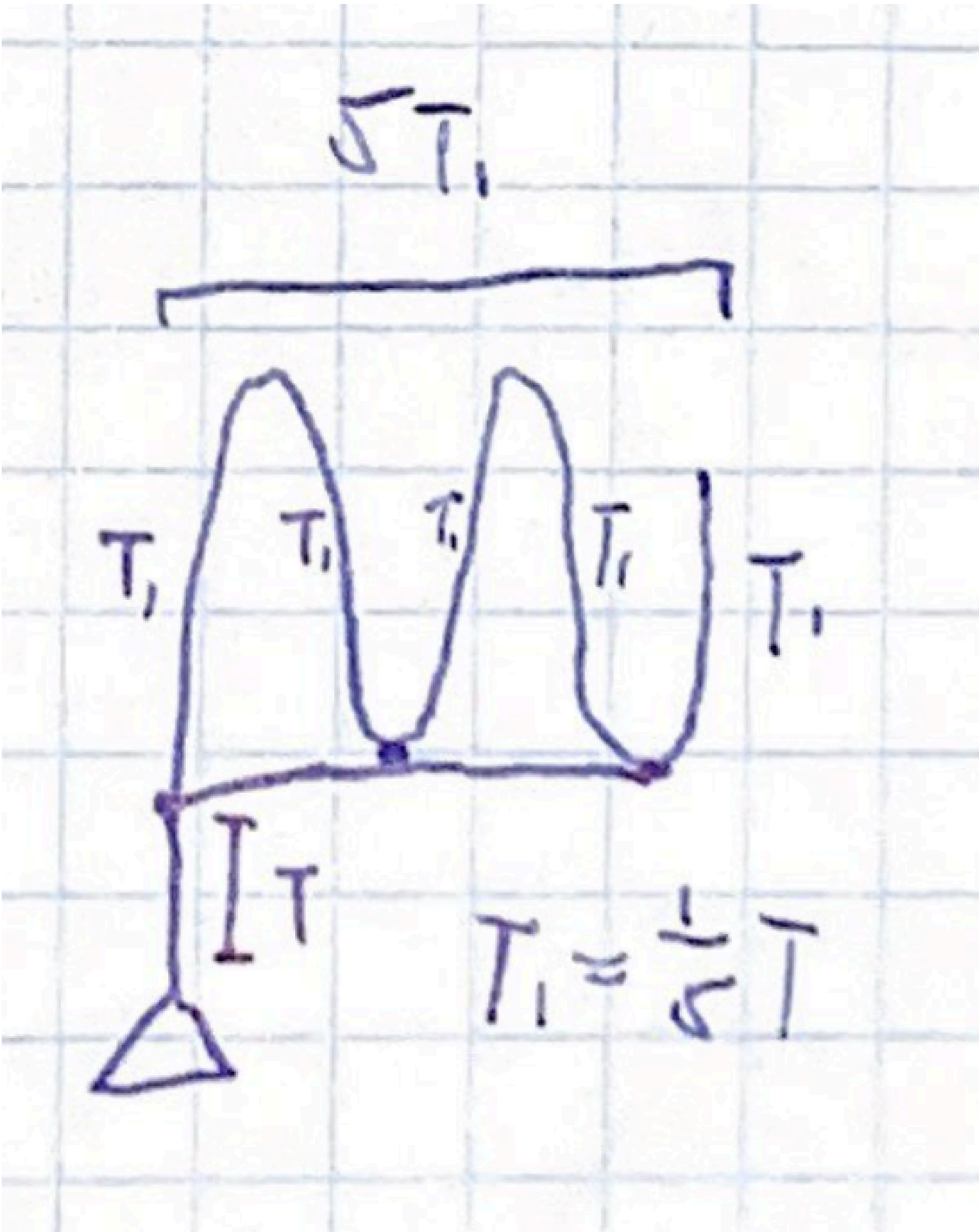


1.3 | Number 3

Five points, tied onto one point.

$$5T_1 = T \tag{5}$$

$$\Rightarrow T_1 = \frac{1}{5}T \tag{6}$$

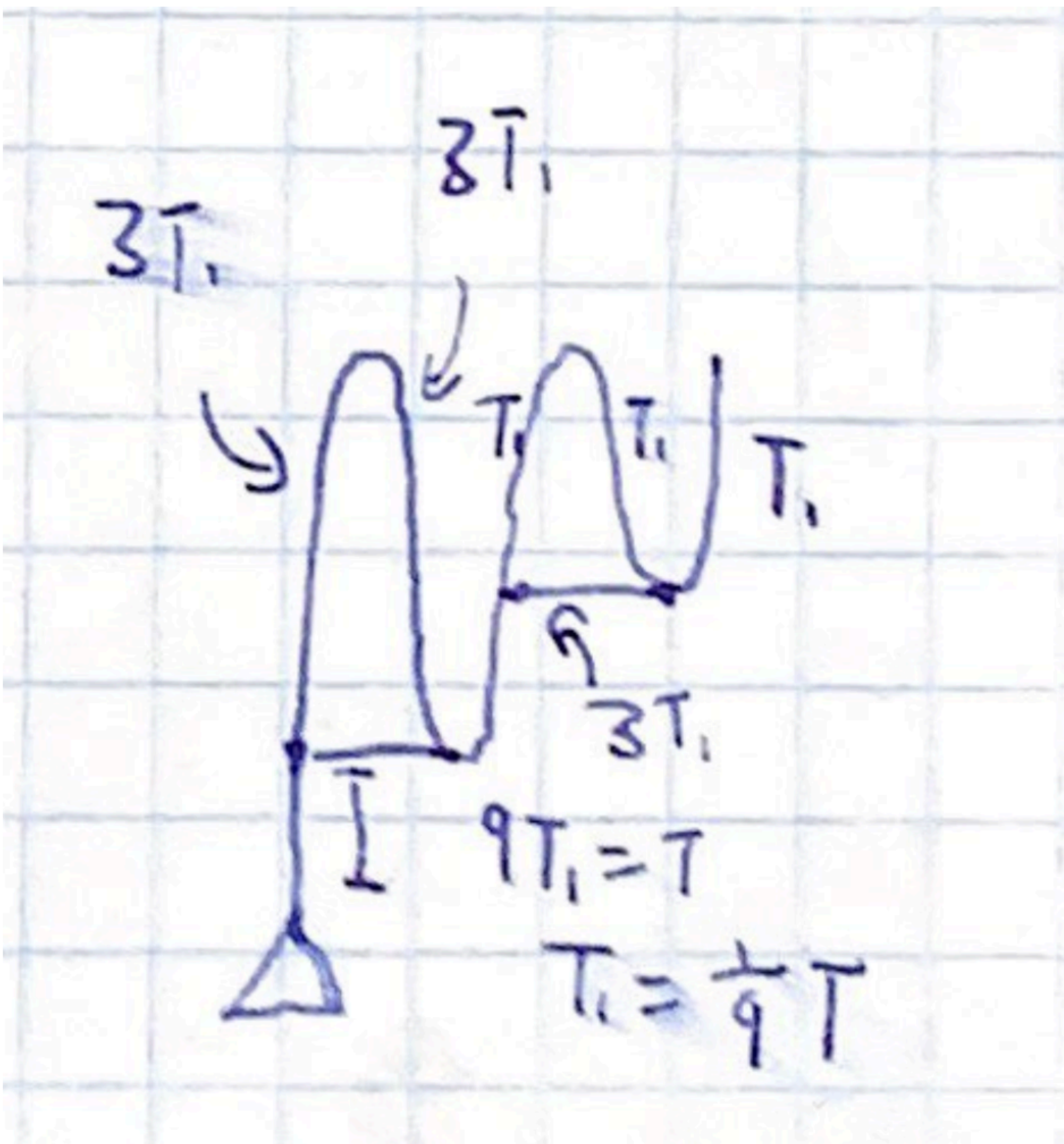


1.4 | Number 4

Two tie points on one, which is added to another set of tie points. The higher (smaller) set has a total tension of $3T_1$, which is tied again onto the bottom tie point.

$$9T_1 = T \tag{7}$$

$$\Rightarrow T_1 = \frac{1}{9}T \tag{8}$$

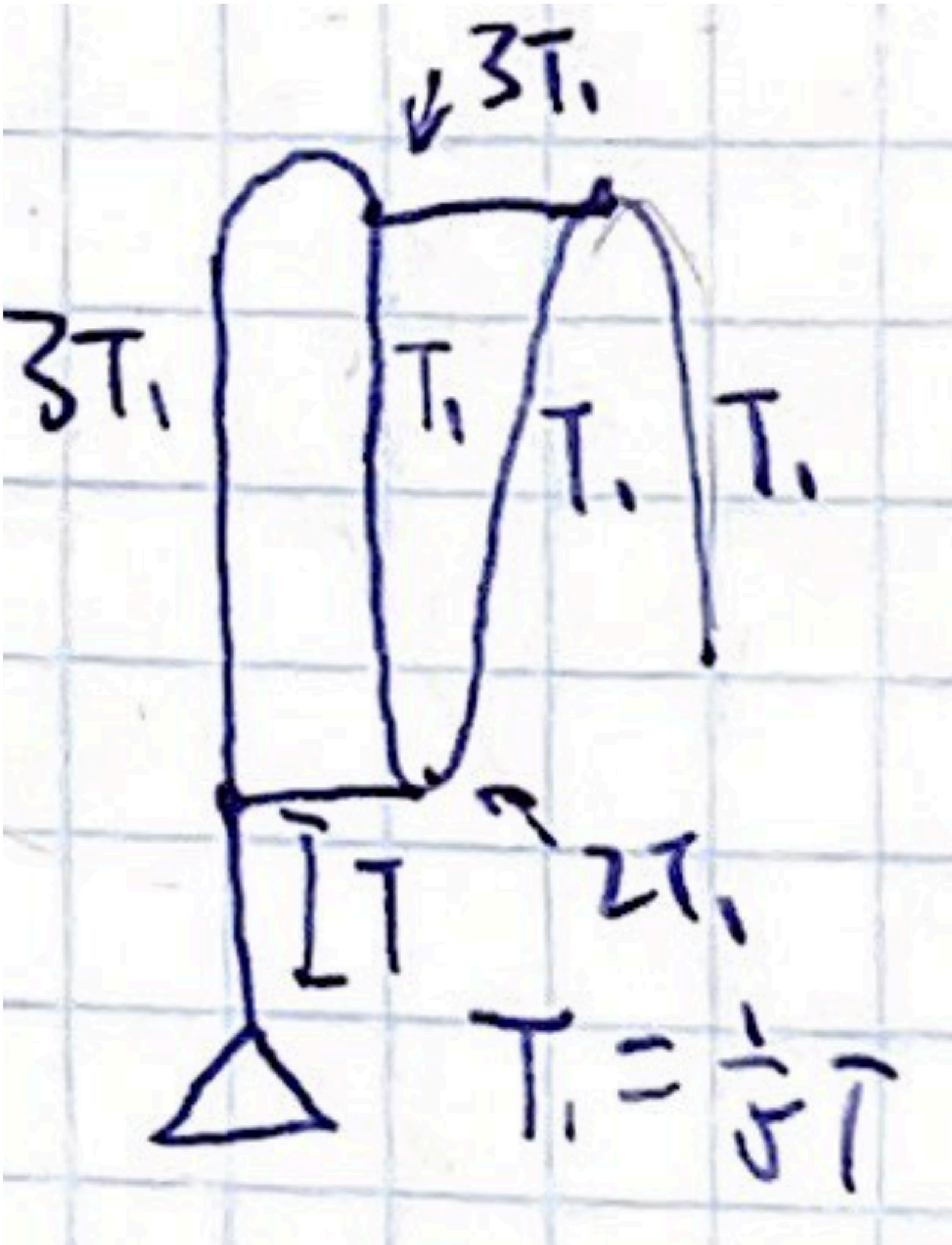


1.5 | Number 5

The top tie point has $3T_1$ of tension. The bottom has $2T_1$, and they are finally all added together.

$$3T_1 + 2T_1 = T \tag{9}$$

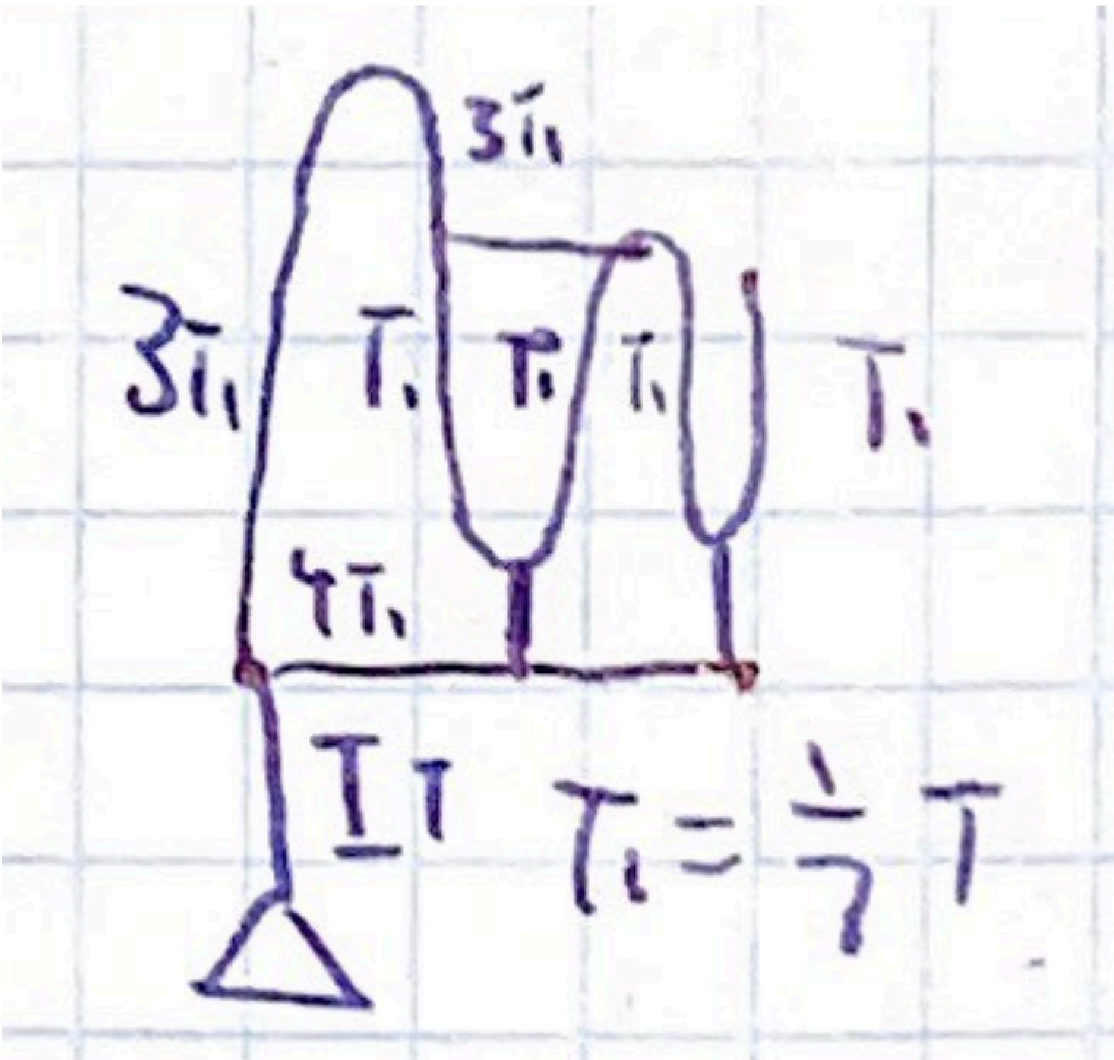
$$\Rightarrow T_1 = \frac{1}{5}T \tag{10}$$



1.6 | Number 6

The top tie point has $3T_1$ of tensions, and the bottom has four tie points and tied together towards the bottom.

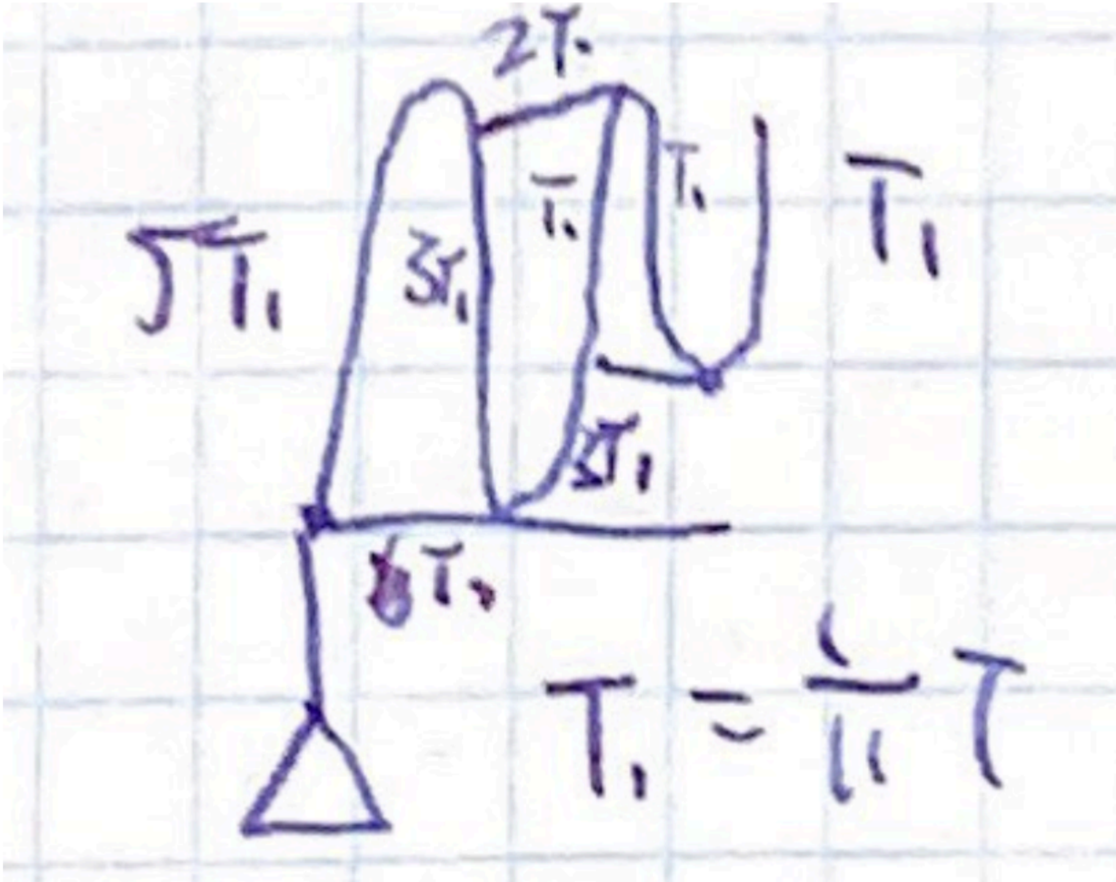
$$3T_1 + 4T_1 = T \tag{11}$$
$$\Rightarrow T_1 = \frac{1}{7}T \tag{12}$$



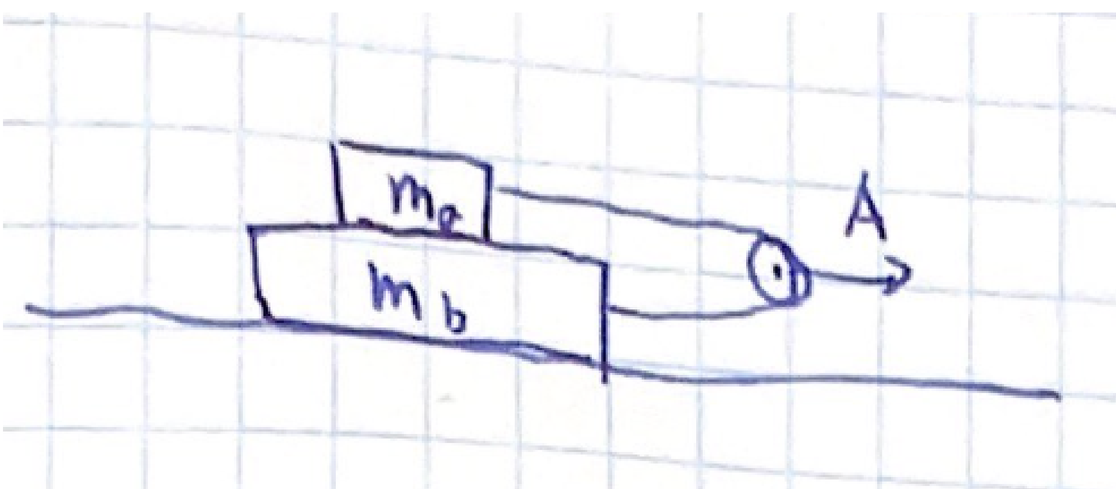
1.7 | Number 7

The top tie point has $2T_1$ of ties, the bottom has $3T_1$, which is finally tied to another propagated to another $6T_1$ tie point. In total, therefore:

$$2T_1 + 3T_1 + 6T_1 = T \tag{13}$$
$$\Rightarrow T_1 = \frac{1}{11}T \tag{14}$$

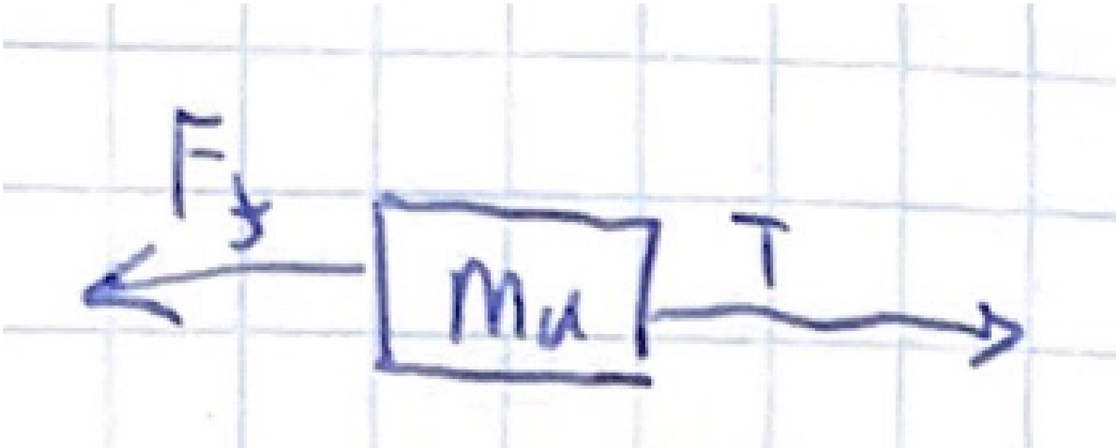


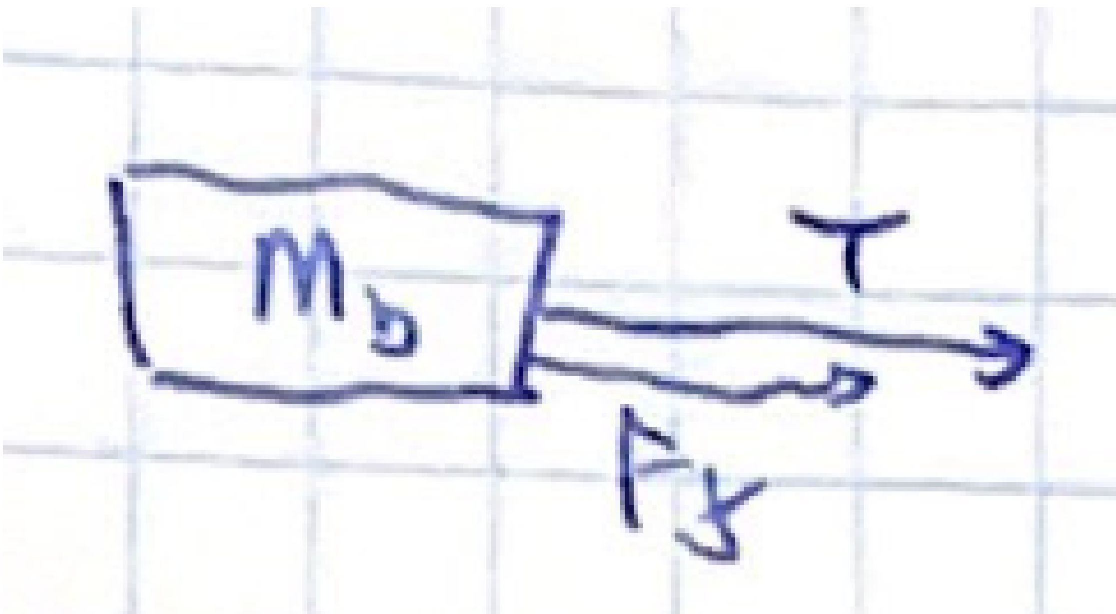
2 | Sliding Blocks



Given the masses M_a, M_b, F_f between them, and A the acceleration of the pully pulling them, we are to figure the tension T on the rope.

We will first draw free-body diagrams of each of the objects, like so:





We will note that, given $M_b > M_a$, M_a will slide to the "right" (+x) direction. As such, in the reference frame of M_a, the force of friction is in the opposite direction as T, whereas this is the opposite case for M_b.

According to these free-body diagrams, then, we can conclude that:

$$\begin{cases} M_a a_a = T - F_f \\ M_b a_b = T + F_f \end{cases} \tag{15}$$

As we have three unknowns and two expressions, we have one remaining degree of freedom to address. This can be analysed by looking at the rope connecting M_a and M_b.

We will define the position of M_a as x_a, M_b as x_b, and finally the position of the pulley as x_p. We will further introduce a temporary constant value L representing the length of the perfect rope connecting the two blocks.

We can, therefore, claim that:

$$(x_p - x_a) + (x_p - x_b) = L \tag{16}$$

That the two distances from block to pulley added up is the length of the rope.

Performing slight algebraic simplifications:

$$
\begin{aligned}
&(x_p - x_a) + (x_p - x_b) = L && (17)\\
\Rightarrow &2x_p - (x_a + x_b) = L && (18)\\
\Rightarrow &2x_p = L + (x_a + x_b) && (19)\\
\Rightarrow &x_p = \frac{L + (x_a + x_b)}{2} && (20)
\end{aligned}
$$

We can now take two derivatives w.r.t. time:

$$\frac{d^2 x_p}{dt^2} = \frac{\ddot{x_a} + \ddot{x_b}}{2} \tag{21}$$

And, therefore:

$$A = \frac{a_a + a_b}{2} \tag{22}$$

We now have three unknowns and three equations, which therefore allows for solving:

$$\begin{cases} M_a a_a = T - F_f \\ M_b a_b = T + F_f \\ A = \frac{a_a + a_b}{2} \end{cases} \tag{23}$$

Solving for the equations above symbolically, we arrive at the following expressions:

$$\left[a_a = \frac{2\left(AM_b - F_f\right)}{M_a + M_b}, a_b = \frac{2\left(AM_a + F_f\right)}{M_a + M_b}, T = \frac{(2\,AM_b - F_f)M_a + F_f M_b}{M_a + M_b}\right] \tag{24}$$

The value for tension, therefore, is the last expression highlighted above.