#ret #hw

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1 | **Proof Prez**

The problem:

Suppose $U_1, ..., U_m$ are finite-dimensional subspaces of V. Prove that $U_1 + ... + U_m$ is finite-dimensional

$$\dim(U_1 + ... + U_m) \leq \dim U_1 + ... + \dim U_m$$

- 1.39:
  - sum of subspaces is the smallest containing subspace.
    * Suppose $U_1, ..., U_m$ are subspaces of V. Then $U_1 + ... + U_m$ is the smallest subspace of V containing $U_1, ..., U_m$.
- 2.26
  - Finite-dimensional subspaces
    * every subspace of a finite-dimensional vector space is finite-dimensional
- 2.43
  - dimension of a sum
    * if U_1 and U_2 are subspaces of a finite-dimensional vector space, then

$$\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - dim(U_1 \cap U_2)$$

- 1.8
  - definiton list, length
  - supposed n is a non-negative integer
- 2.36
  - dimension
    * dimension of a finite dimensional vector space is the lenght of any basis of the vector space

1. proving finite dimensional By 1.39, we know that the sum of subspaces in V is a subspace in V. By 2.26, we know that every subspace of a finite dimensional vector space is finite-dimensional V is finite dimensional

   therefore, sum of subspaces in V is finite-dimensional

2. proving dim by 2.36, we know that the dimension is the lenght of the basis by 1.8, we know that a lenght cannot be negative **thus, dim U_1 + dim U_2 - dim (U_1 intsct U_2) will always be <= dim U_1 + dim U_2** – can't subtract and a positive number and get a larger number.

   by 2.43, we know that dim(U_1+U_2) = dim U_1 + dim U_2 - dim (U_1 intsct U_2) **therefore, dim(U_1+U_2) will always be <= dim U_1 + dim U_2**

   generalize to list: by 1.39, we know that the sum of subspaces in V is a subspace of V so, let U_1 = u_1, and U_2 = u_2 + … + u_m **thus, dim(U_1 + U_2) = dim(u_1 ... u_m)**

   therefore, dim(u_1 ... u_m) <= dim u_1 ... u_m

1.1 | Formalizing