Ted's Slides

# 1 | Intro, and a Book!

Logicomix!

Logic and atheism as quintessentially modern.

> Reality does not need to be consistent, but our models of reality should be consistent.

# 2 | Category Theory

A specialty of mathematics that is somewhat new. Category theory aims to reshape mathematics to instead of studying the *objects*, we study the transformations.

Category theory has objects and morphism: "things" and "ways to go between things".

## 2.1 | Extensions of Category Theory

Category is a generalization of proofs

- Logic => Objects represent *propersitions*, morphisms represent *proofs*
- Programs => Objects represent datatype, morphisms represent programs

Category Theory	Physics	Topology	Logic	CS
Object	System	Manifold	Proposition	Data Type
Morphism	Process	Cobordism	Proof	Program

As you could see, category theory is a generalization.

#### 2.2 | Category

Define category C

- Collection of objects, where if X is an object of C, we write  $X \in C$ .
- For every pair (X,Y), a set hom(X,Y) of morphisms from X to Y. We call this set hom(X,Y) a homset. If  $f \in hom(X,Y)$ , then we write f: X > Y.

Objects and morphisms are independent, but an *morphism* of one category could be *objects* of another. Such that...

- Every object X has an identity morphism:  $1_x: X \to X$
- Morphisms are composable: given  $f:X\to Y$  and  $g:Y\to Z$  there is a composite morphism  $qf:X\to Z$ , sometimes also written  $g\circ f$
- An identity morphism is both a left and a right unit for composition: if  $f: X \to Y$ , then  $y1_x = f = 1_y f$
- Composition operation is associative, so  $(gf)a \cong g(fa)$ .

A category is a network of composable relationships.

### 2.3 | Funtors

Mapping operators between categories. It could act on both objects and morphisms.

Define cat C, cat D, functor F maps C to D

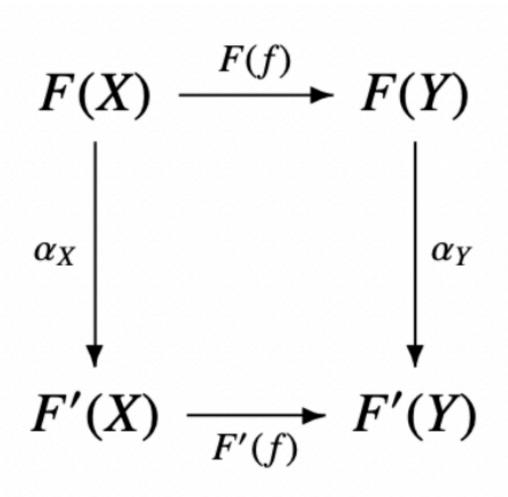
- Any object  $X \in C$  to an object  $F(X) \in D$  (maps objs to obj images)
- Any morphism  $f:X\to Y$  in C to a morphism  $F(f):F(X)\to F(Y)$  in D (morphs to morphs on the images)

in such a way that...

- F Identities are preserved:  $X \in C, F(1_x) = 1_{F(x)}$
- F Preserves composition: for pairs of morphisms  $f:X\to Y,g:Y\to Z$  both in C , F(gf)=F(g)F(f)

## 2.4 | Natural Transformations

Natural Transformation are operations that make a functor to another functor.



You could go two different ways around this square diagram, you get the same answer F'(Y), hence the name "commutative diagrams."

F and F' are secretly representations of G on Hilbert spaces