1 | ~

Given functions $f(n)$ and $g(n)$, if:

$$\lim_{n\to\infty}\left(\frac{f(n)}{g(n)}\right) = 1 \tag{1}$$

we say that $f \sim g$.
That – the relationship between f and g grows in a similar fashion as n increases. For instance:

- $f(n) = n + 1$
- $g(n) = n + 2$

Therefore:

$$f \sim g = \lim_{n\to\infty}\frac{f(n)}{g(n)} = \lim_{n\to\infty}\frac{n+1}{n+2} = 1 \tag{2}$$

The $\sim$ operator is *commutative* ($f \sim g \Rightarrow g \sim f$) and *transitive* ($f \sim g, g \sim h \Rightarrow f \sim h$).

2 | o(n)

Given two functions $f(n)$, $g(n)$, if their relationship shows:

$$\lim_{n\to\infty}\frac{f(n)}{g(n)} = 0 \tag{3}$$

we can write it as

$$f = o(g) \tag{4}$$

This tells us that if n becomes very large, g becomes much larger than f. f does not grow nearly as fast as g.
The operation is *not* commutative, but is *transitive* ($f = o(g), g = o(h) \Rightarrow f = o(h)$)

3 | O(n)

Given two functions $f(n)$, $g(n)$.

$$\lim_{n\to\infty}\frac{f(n)}{g(n)} < \infty \tag{5}$$

that the relationship between $f(n)$ and $g(n)$ is countable as n trends to infinity.
We can also say that, given n, n_0, and some c which $\forall n, n > n_0$, there is:

$$|f(n)| < |cg(n)| \tag{6}$$

This tells us that $f(n)$ does not grow much much faster than $g(n)$.
Therefore:

- If $f \sim g$, $f = O(g)$ (as they grow together, f is not much faster)
- If $f = o(g)$, $f = O(g)$ (as f does not grow at all, f is not faster)

4 | θ(n)

$f = \theta(g)$ IFF $f = O(g)$ and $g = O(f)$, its essentially $\sim$ but without the strict requirement of a 1:1 ratio.

5 | ω(n) and Ω(n)

The inverses of O and o:

- $f(n) = O(g(n)) \Rightarrow g(n) = \omega(f(n))$
- $f(n) = o(g(n)) \Rightarrow g(n) = \Omega(f(n))$