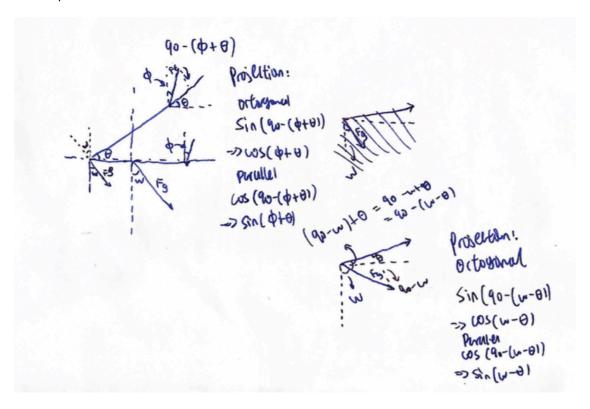
Let's draw a picture of this situation!



We first set up the basic assumptions and variables.

```
GRAV <- 9.8 \# gravity (m/s^2)

MASS <- 3.11*10^(-5) \# mass (kg)

I_CM <- 9.85*10^(-5) \# roational inertia at the centre of gravity (kg m^2)

L1 <- 0.0017 \# distance from rotation point to CoM (m)

L2 <- 0.0034 \# distance from rotation point to tension (m)

PHI <- pi/6 \# angle of Ft relative to floor (orthogonal) (rad)

FT <- 5*10^(-4) \# tension force (N)

OMEGA <- 0 \# angle of line orthogonal to floor relative to gravity (rad) (because shifted axis)
```

Additionally, we set the time interval and seed values for time and theta (distance from flat):

```
dt <- 0.0001
t_max <- 1

vx <- 0
vy <- 0

x <- 0
y <- 0

theta <- 0
thetadot <- 0
time <- 0</pre>
```

Great. Let's start generating the table! We essentially write a for loop to appends to a few different vectors. Variables appended with c reflect the column vectors that we will put together.

```
cTime = NULL
cTheta = NULL
cDDTheta = NULL
cDTheta = NULL
cTorqueNet = NULL
cAccelX = NULL
cAccelY = NULL
cVelX = NULL
cVelY = NULL
cPosX = NULL
cPosY = NULL
cPosPX = NULL
cPosPY = NULL
cFFriction = NULL
cFNormal = NULL
# debugging values
cFNetY = NULL
cFTensionPhiComponent = NULL
cFGravityPhiComponent = NULL
cMuStatic = NULL
cKERot = NULL
cKETrans = NULL
Awesome. Let's now run a lovely little for loop to actually populate the values recursively.
for (i in 0:(t_max/dt)) {
    # We first populate the time column with the time, theta column with theta
    cTime[i] = time
    # Given the theta value, we calculate the net torque and set that
    I_ROT <- I_CM + MASS * L1^2 # we calculate I_ROT using</pre>
# the Parallel axis theorem
    torque <- L2 * FT * cos(theta + PHI) - L1 * MASS * GRAV * cos(theta - OMEGA)
    cTorqueNet[i] = torque
    # Now that we know the net torque, we could know how much the angular
    # acceleration is by just dividing out the rotational inertia
    thetadotdot <- torque/I_ROT
    cDDTheta[i] = thetadotdot
    # We could also multiply the theta acceleration by time to get the
    # velocity at that point
    thetadot <- dt*thetadotdot + thetadot</pre>
    cDTheta[i] = thetadot
    # we then tally the theta value
    theta <- dt*thetadot + theta
    cTheta[i] = theta
    # We could therefore component-ize the acceleration in theta, times
```

```
# of the com
    ax <- -1 * L1 * sin(theta) * thetadotdot
    cAccelX[i] = ax
    ay <- L1 * cos(theta) * thetadotdot
    cAccelY[i] = ay # @mark isn't sin and cos backwards?
    # "position prime": calculated positino
    cPosPX[i] = cos(theta)*L1
    cPosPY[i] = sin(theta)*L1
    # We also tally the components seperately for velocity
    vx \leftarrow ax*dt + vx
    vy \leftarrow ay*dt + vy
    # We finally tally the positions as well
    x \leftarrow vx*dt + x
    y \leftarrow vy*dt + y
    cPosX[i] = x
    cPosY[i] = y
    # Based on these accelerations, we therefore could calculate the relative
    # force of friction and normal force by subtracting the force in that direction
    # out of net
    ffriction <- FT*sin(PHI) + MASS*GRAV*sin(OMEGA)-MASS*ax
    fnormal <- MASS*ay-FT*cos(PHI)+MASS*GRAV*cos(OMEGA)</pre>
    cFNetY[i] = MASS*ay
    cFTensionPhiComponent[i] = FT*cos(PHI)
    cFGravityPhiComponent[i] = -MASS*GRAV*cos(OMEGA)
    cFFriction[i] = ffriction
    cFNormal[i] = fnormal
    # Then, we calculate the energies
    cKERot[i] = 0.5 * I ROT * thetadot^2
    cKETrans[i] = 0.5 * MASS * (vx^2+vy^2)
    # Dividing the friction force by the normal force, of course, will result in
    # the (min?) friction coeff
    cMuStatic[i] = ffriction/fnormal
    # We incriment the time and also increment theta by multiplying the velocity
    # by dt to get change in the next increment
    time <- dt + time
}
We now put all of this together in a dataframe.
rotating_link <- data.frame(cTime,</pre>
    cTheta,
    cDTheta,
    cDDTheta,
```

the length of the object until com, to figure the acceleratinos

cTorqueNet, cAccelX, cAccelY, cPosX, cPosY,

```
cPosPX,
            cPosPY.
            cFFriction,
            cFNormal,
            cMuStatic,
            cKERot,
            cKETrans)
names(rotating_link) <- c("time",</pre>
      "theta",
      "d.theta"
      "dd.theta"
      "net.torque",
      "accel.x",
      "accel.y",
      "pos.x",
      "pos.y",
      "pos.p.x";
      "pos.p.y",
      "friction.force",
      "normal.force",
      "friction.coeff",
      "ke.rot",
      "ke.trans")
Let's import some visualization tools, etc.
library(tidyverse)
Let's first see the head of this table:
head(rotating_link)
1e-04\ 2.9059380176551e-10\ 1.9372920117422e-06\ 0.00968646005829307\ 9.54117186351211e-07\ -4.7852029317815e-100
1.64669820990982e-05 -7.97533822000296e-23 4.94009463001366e-13 0.0017 4.94009463001367e-13
2e-04 5.81187603505943e-10 2.90593801740433e-06 0.0096864600566213 9.54117186186541e-07 -9.570405861498
1.64669820962562e-05 -2.39260146573936e-22 9.88018925960103e-13 0.0017 9.88018925960103e-13
3e-04 9.68646005787513e-10 3.8745840228157e-06 0.00968646005411363 9.54117185939536e-07 -1.595067643078
1.64669820919932e-05 -5.5827367525568e-22 1.64669820983877e-12 0.0017 1.64669820983877e-12
4e-04 1.45296900857678e-09 4.8432300278927e-06 0.00968646005077009 9.54117185610197e-07 -2.392601463619
1.64669820863092e-05 -1.11654735029939e-21 2.47004731458053e-12 0.0017 2.47004731458053e-12
0.00025000000000001 - 0.000128232189769076 - 1.94958848047598 \ 1.1552522514671 \\ e^{-15} \ 1.05414032857393 \\ e^{-21} \ 1.05414032857393 \\ e^{-1} \ 1.054140328573 \\ e^{-1} \ 1.054140328573 \\ e^{-1} \ 1.05414032857 \\ e^{-1} \ 1.0541403285
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1.64669820792041e-05 -2.00978523007641e-21 3.45806624011433e-12 0.0017 3.45806624011433e-12
0.00025000000000001 - 0.000128232189769077 - 1.94958848047597 \ 1.66356324158625 = -15 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.5179620726616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.51796207266616 = -21 \ 1.517962072666616 = -21 \ 1.517962072666616 = -21 \ 1.517962072666616 = -21 \ 1.517962072666616 = -21 \ 1.
```

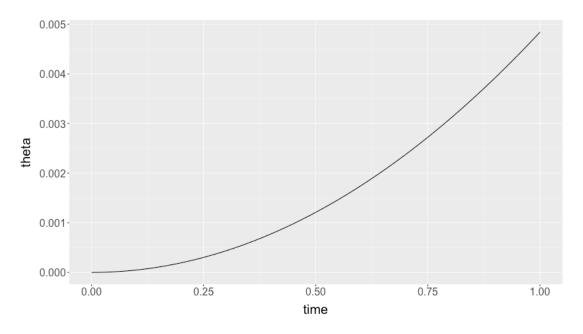
6e-04 2.71220881550289e-09 6.7805220367093e-06 0.00968646004157534 9.54117184704513e-07 -4.466189393682 1.64669820706781e-05 -3.34964204922164e-21 4.61075498635491e-12 0.0017 4.61075498635491e-12 0.00025000000000001 -0.000128232189769077 -1.94958848047597 2.26429441131234e-15 2.06611504257856e-21

Before we start graphing, let's set a common graph theme.

 $\texttt{default.theme} \leftarrow \texttt{theme}(\texttt{text} = \texttt{element_text}(\texttt{size=20}), \ \texttt{axis.title.y} = \texttt{element_text}(\texttt{margin} = \texttt{margin}(\texttt{t} = \texttt{0}, \texttt{margin}))$

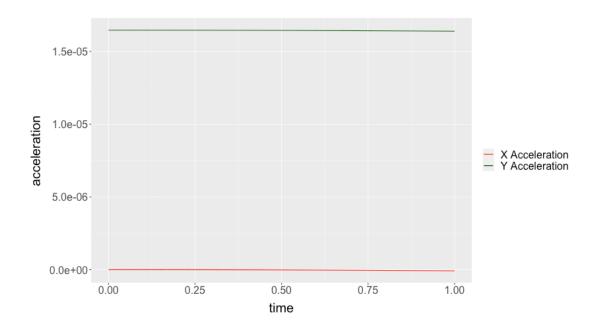
Cool! We could first graph a function for theta over time.

rotating_link %% ggplot() + geom_line(aes(x=time, y=theta)) + default.theme



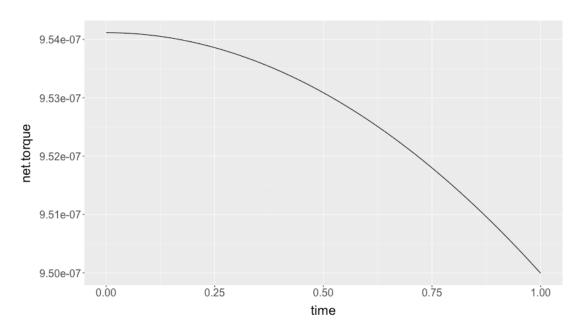
And, similarly, we will graph ax and ay on top of each other:

rotating_link %>% ggplot() + geom_line(aes(x=time, y=accel.x, colour="X Acceleration")) + geom_line(aes



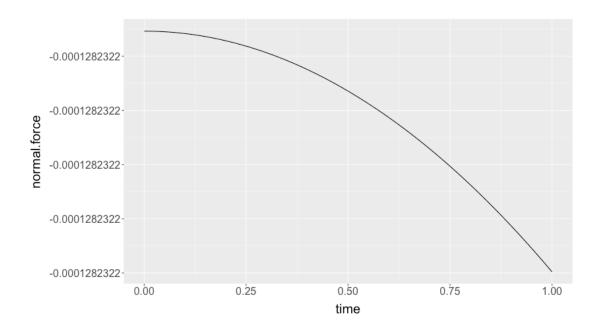
Let's also plot torque as well.

rotating_link %>% ggplot() + geom_line(aes(x=time, y=net.torque)) + default.theme



And. Most importantly! Let's plot the normal force.

 $\verb|rotating_link \%>\%| ggplot() + geom_line(aes(x=time, y=normal.force)) + default.theme|$

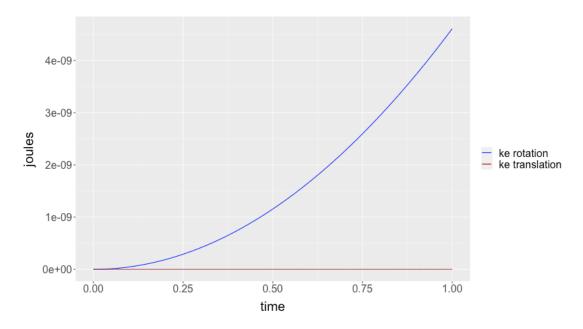


Obviously, after the normal force becomes negative, this graph stops being useful.

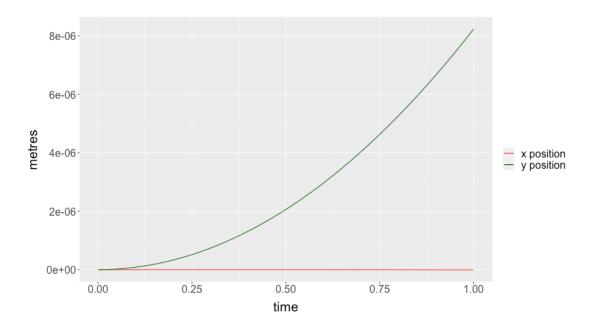
Theta dot atop theta:

We finally, plot KE rotation and translation

 $\verb|rotating_link| \%>\% | \texttt{ggplot()} + \texttt{geom_line(aes(x=time, y=ke.rot, colour="ke rotation"))} + \texttt{geom_line(aes(x=time, y=ke.rot, colour="ke rotation")} + \texttt{geom_line(aes(x=time, y=ke.rot, colour="ke rotation")} + \texttt{geom_line(aes(x=time, y=ke.$

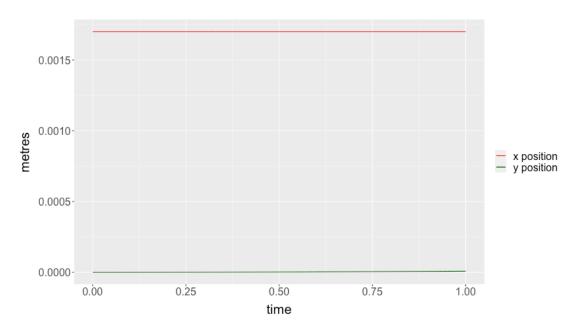


rotating_link %>% ggplot() + geom_line(aes(x=time, y=pos.x, colour="x position")) + geom_line(aes(x=time, y=pos.x, colour

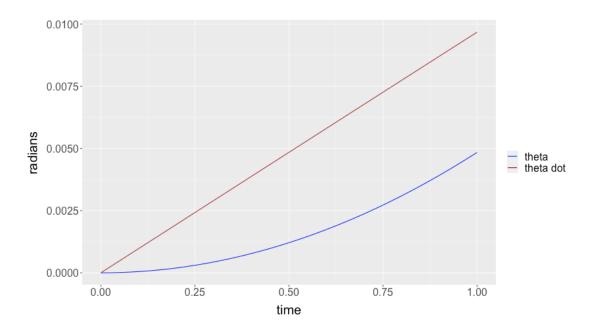


floor

rotating_link %>% ggplot() + geom_line(aes(x=time, y=pos.p.x, colour="x position")) + geom_line(aes(x=t



rotating_link %>% ggplot() + geom_line(aes(x=time, y=theta, colour="theta")) + geom_line(aes(x=time, y=theta, theta))) + geom_line(aes(x=time, y=theta, theta)) + geom_line(aes(x=time, y=theta))) + geom_line(aes(x=time, y=theta)) + geom_line(aes(x=time, y=theta))) + geom_



 $\verb|rotating_link \%>\% | ggplot() + geom_line(aes(x=time, y=dd.theta, colour="thetadd")) + scale_colour_manual| | for the property of the prope$