1 | Dot product:

- Name: dot product
- Result: Scalar
- Interpretation (what it measures): parallelity
  - the more parallel the larger the dot product
- Magnitude (with sign): $|\vec{a}||\vec{b}|cos(\theta)$
- Geometric magnitude: $|\vec{a}||\vec{b}_{||\vec{a}}|$
- Direction: no direction
- Algebraic form: $a_xb_x + a_yb_y + a_zb_z$
- Algebraic properties:
  - commutative
  - associative
  - distributive across addition

2 | Cross product:

- Name: Cross product
- Result: Vector
- Interpretation (what it measures): Orthgonality
  - the more orthogonal the longer the cross product
- Magnitude (with sign): $|\vec{a}||\vec{b}|\,\text{sin}(\theta)$
- Geometric Magnitude: $|\vec{a}||\vec{b}_{\perp\vec{a}}|$
- Direction: perpendicular to the two vectors
  - by the right hand rule by rotating the first vector into the second vector
- Albraic form: $\langle a_yb_z - a_zb_y, a_xb_z - a_zb_x, a_xb_y - a_yb_x\rangle$
- Algebraic properties:
  - Anticommutative
  - $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
  - $(\vec{A} \times \vec{B}) \perp \vec{A}$
  - $(\vec{A} \times \vec{B}) \perp \vec{B}$
  - Antiassociative

3 | Application of cross product:

- In physics there is something called torque, notated τ
  - Torqe is the net force of things that rotate, so:
    - $F_{net} = ma$
    - $\tau_{net} = I\omega$
- Somethings to note about τ:
  - It increases with a longer lever
  - It increases with a greater force
    - that is perpendicular to the lever
- Given these requirements we can make a formula:
  - $|\tau| = |\vec{r}||\vec{F}_{\perp \vec{r}}|$, where $\vec{F}$ is the force applied to the door, and $\vec{r}$ is the radius of the lever.
  - this, the right side of the equation, can be described using the dot product: $|\tau| = \vec{r} \times \vec{F}$

4 | Derivation of cross product algebraic form:

To start, we can define:

$$\vec{A} = (A_x, A_y, A_z) = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

$$\vec{B} = (B_x, B_y, B_z) = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$$

Next we have to assume that the dot product is distributive across addition:
$\vec{A} \times \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_Z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$
$= A_xB_x\hat{\imath} \times \hat{\imath} + A_xB_y\hat{\imath} \times \hat{\jmath} + A_xB_z\hat{\imath} \times \hat{k}$
$+ A_yB_x\hat{\jmath} \times \hat{\imath} + A_yB_y\hat{\jmath} \times \hat{\jmath} + A_yB_z\hat{\jmath} \times \hat{k}$
$+ A_zB_x\hat{k} \times \hat{\imath} + A_zB_y\hat{k} \times \hat{\jmath} + A_zB_z\hat{k} \times \hat{k}$

From the definition of a cross product, we know that the cross product between any two vectors that are parallel is zero, thus:
$= A_xB_y\hat{\imath} \times \hat{\jmath} + A_xB_z\hat{\imath} \times \hat{k}$
$+ A_yB_x\hat{\jmath} \times \hat{\imath} + A_yB_z\hat{\jmath} \times \hat{k}$
$+ A_zB_x\hat{k} \times \hat{\imath} + A_zB_y\hat{k} \times \hat{\jmath}$

$\hat{\imath} \times \hat{\jmath}$ would yield a vector length one in the direction of a vector that is perpendicular to both $\hat{\imath}$ and $\hat{\jmath}$, which would be $\hat{k}$. Conversly, $\hat{\imath} \times \hat{\jmath} = -\hat{k}$. Therefore:
$= A_xB_y\hat{k} - A_xB_z\hat{\jmath}$
$- A_yB_x\hat{k} + A_yB_z\hat{\imath}$
$+ A_zB_x\hat{\jmath} - A_zB_y\hat{\imath}$
$= A_xB_y\hat{k} - A_yB_x\hat{k}$
$+ A_yB_z\hat{\imath} - A_zB_y\hat{\imath}$
$- A_xB_z\hat{\jmath} + A_zB_x\hat{\jmath}$
$= (A_xB_y - A_yB_x)\hat{k} + (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath}$
$= (A_yB_z - A_zB_y, A_zB_x - A_xB_z, A_xB_y - A_yB_x)$

Now we need to show that the cross product is distributive across addition:

We can start with:

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

due to the definition of the cross product, and the fact that all of the terms are the cross prouduct with A and some other vector, we know that all of the terms are coplaner vectors, in which the plane is perpendicular to $\vec{A}$. We also know that the term $\vec{A} \times \vec{B}$ is perpendicular to $\vec{B}$ and that $\vec{A} \times \vec{C}$ is perpendicular to $\vec{C}$.

We can then choose a coordinate system so that $\vec{A}$ is in the z direction. This means that A would be on the z axis, and that the vectors $\vec{A} \times \vec{B}$, $\vec{A} \times \vec{C}$, $\vec{A} \times (\vec{B} + \vec{C})$ are all in the xy plane.

In the xy plane we know that $\vec{A} \times \vec{B} \perp \vec{B}$, we can claim that $\vec{A} \times \vec{B} \perp \vec{B}_{\perp \vec{A}}$ (Note: $\vec{B}_{\perp \vec{A}} = p\vec{roj}_{xy}\vec{B}$). To prove this claim we need to prove that $\vec{B}_{\perp A}$ is in the plane that has the vectors $\vec{A}$ and $\vec{B}$. This is by the definition of the cross product the $\vec{A}\vec{B}$ plane is perpendicular to $\vec{A} \times \vec{B}$ and so if $\vec{B}_{\perp \vec{A}}$ is in the plane then it is prependicular to $\vec{A} \times \vec{B}$. We can prove this with the linear algebra definition of a plane.

5 | Determinate form of cross product:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Evaluating determinates: `https://www.youtube.com/watch?v=CcbyMH3Noow`

It is not actually a determinate because it has vectors, it is just a good way to remember what the cross product is.