1 | circumfrence

Start with the arc length formula

$$2\int_0^a \sqrt{1+f'^2(x)}dx$$

$$f(x)=\sqrt{a^2-x^2} \text{ so } f'(x)=\frac{1}{2\sqrt{a^2-x^2}}(-2x)=-\frac{x}{\sqrt{a^2-x^2}}$$

$$2\int_0^a \sqrt{1+\frac{x^2}{a^2-x^2}}dx=2\int_0^a \sqrt{\frac{a^2}{a^2-x^2}}dx$$

Now we need to use trig substitution. Lets use $x=a\sin\theta$, with the limit assumptions $-\frac{\pi}{2}\leq\theta\leq\frac{\pi}{2}$. Then, $dx=a\cos\theta d\theta$

$$= 2 \int \sqrt{\frac{a^2}{a^2 - a^2 \sin^2 \theta}} d\theta$$

$$= 2 \int \sqrt{\frac{1}{1 - \sin^2 \theta}} d\theta$$

$$= 2 \int \sqrt{\frac{1}{\cos^2 \theta}} d\theta$$

$$= 2 \int \sqrt{\sec^2 \theta} d\theta$$

$$= 2 \int \sec \theta d\theta$$

$$= 2 \ln|\sec \theta + \tan \theta|$$

Now for the triangle part! $\sec\theta=\frac{1}{\cos\theta}=\frac{\sqrt{a^2-x^2}}{a}, \tan\theta=\frac{x}{\sqrt{a^2-x^2}}$ $=2\ln\left|\frac{a}{\sqrt{a^2-x^2}}+\frac{x}{\sqrt{a^2-x^2}}\right|$

Seems like a lot of dividing by zero

2 | surface area

I don't really understand how we use arclength in the calculation of surface area. I tried just using the circumfrence, but that didn't work with the slicing assumption because the surface curves.