

Take, for instance, problem e. From taking two partial derivatives in the x and y dimensions, we deduce that the partial derivative values are...

$$\frac{\partial f}{\partial x} = 10 \quad (1)$$

$$\frac{\partial f}{\partial y} = 10\sqrt{3} \quad (2)$$

We could, therefore, treat these terms as two separate vectors lying at the x and y directions. That is, we know that the multidimensional "slope" of the function could be represented by a combination of vectors...

$$\left\{ \begin{pmatrix} 10 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 10\sqrt{3} \end{pmatrix} \right\} \quad (3)$$

The "slope" created by the two slope values at a 60° angle is essentially the sums of the two partial derivative vectors projected at 60° . Hence, we have to project the two vectors' magnitudes to a shared 60° angle, and sum it up.

We first note that, to project the two *orthogonal* vectors to the same, shared "60-degrees" direction, we must project one vector to 60° and the other to $(90 - 60)^\circ$ to actually result in the projections' alignment.

Conventionally, we will project the x-direction vector to 60° . and the y-direction vector to $(90 - 60)^\circ$, but the 60 degree direction that we aim to share is actual arbitrary.

To perform the actual magnitude projection, we perform the follows.

$$\text{let } \vec{X} = \begin{pmatrix} 10 \\ 0 \end{pmatrix} \quad (4)$$

$$\vec{Y} = \begin{pmatrix} 0 \\ 10\sqrt{3} \end{pmatrix} \quad (5)$$

$$\vec{X}_p = \|\vec{X}\| \cos(60^\circ) \quad (6)$$

$$= 10 \times \frac{1}{2} \quad (7)$$

$$= 5 \quad (8)$$

$$\vec{Y}_p = \|\vec{Y}\| \cos((90 - 60)^\circ) \quad (9)$$

$$= \|\vec{Y}\| \sin(60^\circ) \quad (10)$$

$$= 10\sqrt{3} \times \frac{\sqrt{3}}{2} \quad (11)$$

$$= \frac{30}{2} = 15 \quad (12)$$

Finally, the sum of slopes in that shared direction would therefore be 20.