PS#27

Nueva Multivariable Calculus

1. Suppose you have a plate of metal, described by the plane $z = 3x + 2y + 7$, whose projection onto the xy plane is the rectangle with corners at the origin and $(7, 11)$. The plate has some sort of electrical charge on it, such that the amount of electrical charge at any point on it is proportional to its distance from the xy-plane. What's the total amount of electrical charge on the plate?

2. Suppose an infinitely-thin piece of wire is lying in the shape of the upper-right quadrant of a circle with radius 7, centered at the origin. Suppose that piece of wire has a density at any point (x, y) of xy grams per unit length. How much does the piece of wire weigh?

3. We've done a couple problems with what I've been calling **infinite volcanoes**. (This is very much an Andrewism!) For instance, we've thought about the 1D infinite volcano:

   $$f(x) = \frac{1}{x^{2k}}$$

   And we've found the areas underneath it, between 0 and 1 and between 1 and ∞ (as a function of k).

   Likewise, we've found the volume under a 2D infinite volcano, above a circle of radius 1:

   $$f(x) = \frac{1}{(x^2 + y^2)^k}$$

   Now suppose you have not just a 2D volcano or a 3D volcano, but an nD volcano!!!!

   $$f : \mathbb{R}^n \to \mathbb{R}^1$$

   $$f(x_1, x_2, x_3, \ldots, x_n) = \frac{1}{\left((x_1)^2 + (x_2)^2 + (x_3)^2 + \cdots + (x_n)^2\right)^k}$$

   Suppose we're adding up the total n-dimensional hypervolume of this shape, inside a n-dimensional unit sphere, same as in the previous two versions. Same questions! For what values of k is that volume finite? For what values is it infinite?

   (Do go back and look at your answers to the previous problems in this series! And feel free to do research to help yourself solve this one, too! Feel free to try to look stuff up.)