

## 1 | Formula

$$\int_a^b \sqrt{1 + f'^2(x)} dx$$

## 2 | Derivation

Let  $S$  equal the total length of the curve.

Start with the pythagorean theorem: for any differentiable function,

$$S = \sum_a^b dS$$

where  $dS$  can be calculated using the pythagorean theorem:

$$dS = \sqrt{\Delta y^2 + \Delta x^2}$$

We can simplify this by dividing  $dS$  by  $\Delta x$

$$\begin{aligned} \frac{dS}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \sqrt{\frac{\Delta y^2}{\Delta x^2} + \frac{\Delta x^2}{\Delta x^2}} \\ &= \lim_{\Delta x \rightarrow 0} \sqrt{\frac{\Delta y^2}{\Delta x^2} + 1} \\ &= \sqrt{f'^2(x) + 1} \end{aligned}$$

Now, to find  $dS$  again, we just have to multiply by  $dx$

$$dS = \frac{dS}{\Delta x} dx = \sqrt{1 + f'^2(x)} dx$$