

## 1 | Openstax Calc vol 1 chap 2.4 ex 134

$$g(t) = \frac{1}{t} + 1$$

which is basically  $\frac{1}{x}$  shifted up by one, so there is an infinite discontinuity at  $x = 0$

## 2 | 136

There is a jump discontinuity at  $x = 2$ , because normally  $y = \frac{x}{x}$  simplifies to  $y = 1$ , but the sign flips at  $x = 2$ .

## 3 | 142

$$f(y) = \frac{\sin(\pi y)}{\tan(\pi y)} = \frac{\sin(\pi y) \cos(\pi y)}{\cancel{\sin(\pi y)}}$$

So there is a removable discontinuity at  $y = 1$ , because there is a discontinuity but it can be removed with algebra.

## 4 | 148

$$e^{4k} = 4 + 3$$

$$e^{4k} = 7$$

$$4k = \ln(7)$$

$$k = \frac{\ln(7)}{4}$$

## 5 | TODO 174

Prove  $f(x)$  is continuous everywhere, meaning show that  $\forall c \in \mathbb{R}$

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Because we can always evaluate  $f(x)$ , the limit always exists.

## 6 | Paul's online math notes Section 2-9: 23

The IVT states that when a function is continuous over a closed interval  $[a, b]$ , then for all  $\min\{f(a), f(b)\} \leq y \leq \max\{f(a), f(b)\}$  there exists some  $a \leq c \leq b$  s.t.  $f(c) = y$ . In this case, we have  $f(4) = 193$  and  $f(8) = -511$ .  $f(x)$  is a polynomial, so it is continuous over the range. Because our values straddle zero, there must be some value  $4 \leq c \leq 8$  s.t.  $f(c) = 0$ .

## 7 | Boundedness theorem

Given a function  $f(x)$  that is continuous on a closed interval  $[a, b]$ , there exists some  $M \in \mathbb{R}$  s.t.  $f(c) \leq M$  for all  $a \leq c \leq b$  aka  $M$  is an upper bound on  $f(x)$  over the interval  $[a, b]$ . There's also one that's less than all  $c$ . Doesn't work for open intervals.

7.1 |  $(0, 1]$ : **not continuous, not a closed interval**

7.2 |  $[0, 1)$ : **not a closed interval**

7.3 |  $(0, 1]$ : **not a closed interval**

7.4 |  $(0, 1]$ : **not continuous, not a closed interval**

7.5 |  $f(x) = \frac{1}{x}$ : **not continuous**

## 8 | Epilouge

Other than Problem 5, this took roughly 40 minutes. I still don't know how to do problem 5..