

## 1 | Axler 5.22 matrix of an operator, $\mathcal{M}(T)$

def

Suppose  $T \in \mathcal{L}(V)$  and  $v_1, \dots, v_n$  is a basis of  $V$ . The *matrix of  $T$*  wrt this basis is the  $n$ -by- $n$  matrix

$$\mathcal{M}(T) = \begin{pmatrix} A_{1,1} & \cdots & A_{1,n} \\ \vdots & \ddots & \vdots \\ A_{n,1} & \cdots & A_{n,n} \end{pmatrix}$$

whose entries  $A_{j,k}$  are defined by

$$Tv_k = A_{1,k}v_1 + \cdots + A_{n,k}v_n$$

Specify a basis with  $\mathcal{M}(T, (v_1, \dots, v_n))$

### 1.1 | intuition

#### 1.1.1 | each column is where the map takes a basis vector

## 2 | Simplifying The Matrix Representation

2.1 | 'A central goal of linear algebra is to show that given an operator  $T \in \mathcal{L}(V)$ , there exists a basis of  $V$  wrt which  $T$  has a reasonably simple matrix'

2.2 | If by simple we mean "has many zeros" or RREF, then we know enough to ensure that there exists a basis s.t. the first column has zeros everywhere except the first row.

$\left[ \begin{pmatrix} \lambda & & \\ 0 & * & \\ \vdots & & \\ 0 & & \end{pmatrix} \right]$  Where  $*$  denotes all the other entries. Find  $\lambda$  by taking the lone eigenvalue and letting it's eigenvector be the first basis vector.