Hello, fellow person that comes across this. I have had one brief exposure with Linear Algebra following MATH 21-1 at UCSC. However, Axler is just so cool, so I am trying to learn a bit of linalg on the side to supplement my much more traditional linalg experience at the UC.

A few things of note. This whole thing is very "partial": in the sense that its contents contain many a parts of things omitted which I feel like I have a very good grasp on from 21-1 such that I don't need to be reminded again; I only include things that maybe useful to me later either b/c I don't know it or I want to be reminded of it. As such, I don't think this will be helpful for most people.

1 | **1.A**

1.1 | **Things of Note**

- $\lambda \in \mathbb{F}$ is called a "scalar". I mean duh but still.

1.1.1 | **Defining a list**

A list of length n is a collection of n elements (any mathematical object?) separated by commas.

"Identical" lists are established when lists have:

- the same length
- same elements
- in the same order.

Its also called a n-tuple.

n must be a finite non-negative value. Therefore, an "infinitely long list" is not a list.

1.1.2 | **Sets vs Lists**

Lists have order and repetition. In sets, order and repetitions don't matter.

1.1.3 | $\mathbb{F}$

- A set
- Containing 2 elements $0, 1$
- Operators of "addition" and "multiplication" that satisfy the following properties

1. Properties of $\mathbb{F}$ That, with $\alpha, \beta, \lambda \in \mathbb{F}$:
    - **Commutativity** $\alpha + \beta = \beta + \alpha$ and $\alpha\beta = \beta\alpha$
    - **Associativity** $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$ and $(\alpha\beta)\lambda = \alpha(\beta\lambda)$
    - **Existence of Identities** $\lambda + 0 = \lambda$ and $\lambda 1 = \lambda$
    - **Additive Inverse** for every α, $\exists \beta$ s.t. $\alpha + \beta = 0$
    - **Multiplicative Inverse** for every $\alpha \neq 0$, $\exists \beta$ s.t. $\alpha\beta = 1$
    - **Distribution** $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$

1.1.4 | $\mathbb{F}^n$

$$\mathbb{F}^n = \{(x_1, \ldots, x_n) : x_j \in \mathbb{F} \; for \; j = 1, \ldots, n\} \tag{1}$$

We say x_j is the j^{th} coordinate of $(x_1, \ldots, x_n)$.
In $\mathbb{F}^n$...

1. Addition
$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n) \tag{2}$$
2. Zero
$$0 = (0, \ldots, 0) \tag{3}$$
3. Additive Inverse ...of $x \in \mathbb{F}^n$:
$$x + (-x) = 0 \tag{4}$$
That:
$$x = (x_1, \ldots, x_n), -x = (-x_1, \ldots, -x_n) \tag{5}$$
4. Scalar Multiplication
$$\lambda(x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n) \tag{6}$$

1.2 | In-Text Exercises

1.2.1 | Verify that $i^2 = -1$

$(0 + 1i)(0 + 1i) = (0 + 0 + 0 + ii) = -1$

1.2.2 | Defining subtraction and division

$\alpha, \beta \in \mathbb{C}$
Subtraction could be defined in that:

- Let $-\alpha$ be defined as the additive inverse of α
- Subtraction, therefore, is defined $\beta - \alpha = \beta + (-\alpha)$

Division could be defined in that:

- Let $1/\alpha$ be defined as the multiplicative inverse of α
- Subtraction, therefore, is defined $\beta/\alpha = \beta(1/\alpha)$

1.3 | **Actual Exercises**

1: Suppose $a, b \in \mathbb{R}$, $a, b \neq 0$, find $c, d \in \mathbb{R}$ s.t. $\frac{1}{(a+bi)} = c + di$

$$\frac{1}{(a+bi)} = \frac{(a-bi)}{(a+bi)(a-bi)} = \tag{7}$$

$$\Rightarrow \frac{a-bi}{a^2-(bi)^2} = c + di \tag{8}$$

$$\Rightarrow \frac{a-bi}{a^2+b^2} = c + di \tag{9}$$

$$\Rightarrow \frac{a}{a^2+b^2} - \frac{bi}{a^2+b^2} = c + di \tag{10}$$

Therefore:

$$c = \frac{a}{a^2+b^2} \tag{11}$$

$$d = \frac{-b}{a^2+b^2} \tag{12}$$

2: Show that $\frac{-1+\sqrt{3}i}{2}$ is the cube root of 1.

$$(\frac{-1+\sqrt{3}i}{2})^3 \tag{13}$$

$$\Rightarrow (\frac{-1+\sqrt{3}i}{2})(\frac{-1+\sqrt{3}i}{2})(\frac{-1+\sqrt{3}i}{2}) \tag{14}$$

$$\Rightarrow \frac{(-1+\sqrt{3}i)(-1+\sqrt{3}i)(-1+\sqrt{3}i)}{8} \tag{15}$$

$$\Rightarrow \frac{(1-2\sqrt{3}i-3)(-1+\sqrt{3}i)}{8} \tag{16}$$

$$\Rightarrow \frac{(1-2\sqrt{3}i-3)(-1+\sqrt{3}i)}{8} \tag{17}$$

$$\Rightarrow \frac{8}{8} = 1 \tag{18}$$

3: Find two distinct square roots of i

?

4: Show that $\alpha + \beta = \beta + \alpha, \forall \alpha, \beta \in C$

Let:

$\forall a, b, c, d \in \mathbb{R}$

- $\alpha = (a + bi)$
- $\beta = (c + di)$

$$\begin{aligned}
\alpha + \beta &= (a+bi) + (c+di) && (19)\\
&= (a+c) + (b+d)i && (20)\\
&= (c+a) + (d+b)i && (21)\\
&= (c+di) + (a+bi) && (22)\\
&= \beta + \alpha \blacksquare && (23)
\end{aligned}$$

5: Show that $(\alpha+\beta)+\lambda = \alpha + (\beta+\lambda), \forall \alpha, \beta, \lambda \in \mathbb{C}$

Let:

$\forall a, b, c, d, e, f \in \mathbb{R}$

- $\alpha = (a+bi)$
- $\beta = (c+di)$
- $\lambda = (e+fi)$

$$\begin{aligned}
(\alpha+\beta)+\lambda &= ((a+bi)+(c+di)) + (e+fi) && (24)\\
&= ((a+c)+(b+d)i) + (e+fi) && (25)\\
&= (a+c+e) + (b+d+f)i && (26)\\
&= (a+(c+e)) + (b+(d+f))i && (27)\\
&= (a+bi) + (c+e) + (d+f)i && (28)\\
&= (a+bi) + ((c+di)+(e+fi)) && (29)\\
&= \alpha + (\beta+\lambda) \blacksquare && (30)
\end{aligned}$$