True or false (and give a proof of your answer): The exists $T \in \mathcal{L}(\mathbb{R}^3)$ s.t. T is not self-adjoint but there is a basis of \mathbb{R}^3 consisting of eigenvectors of T.

The real spectral theorem suggests that if a map is self-adjoint, it must have an orthogonal basis of eigenvectors. Thus, we can choose a map which does not have an orthogonal basis of eigenvectors. For instance, choose the non-orthogonal eigenpairs

$$T(1,0,0) = (1,0,0)$$

$$T(0,1,0) = (0,2,0)$$

$$T(1,1,1) = (3,3,3)$$

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