

1 | Axler 7.18 normal

def

Not to be confused with normal vectors, which have norm 1.

- An operator on an inner product space is called *normal* if it commutes with its adjoint.
- aka: $T \in \mathcal{L}(V)$ is *normal* if

$$TT^* = T^*T$$

Every self adjoint operator is normal, because $TT = TT$

2 | results

2.1 | **Axler 7.20** T is normal iff $\|Tv\| = \|T^*v\|$ for all v

This implies that $T = T^*$ for all normal operators T .

2.2 | **Axler 7.21** For T normal, T and T^* have the same eigenvectors

And the corresponding eigenvalues are conjugates of one another.

2.3 | **Axler 7.22** Normal operators have orthogonal eigenvectors

Suppose $T \in \mathcal{L}(V)$ is normal. Then eigenvectors of T corresponding to distinct eigenvalues are orthogonal.