

We are given that the object m_1 collides with the rod with velocity v_0 , and the rod is floating in free space. Given m_1 , v_0 , m_2 , I_0 , and r , we are to figure out the final velocity of m_1 after collision v_f , the velocity of m_2 after collision v_{CM} , and of course the rotation of the rod after collision ω .

We are assuming that this collision elastic.

We have, then, for conservation of linear momentum:

$$m_1 v_0 = m_1 v_f + m_2 v_{CM} \quad (1)$$

Furthermore, we understand that kinetic energy is also conserved here; therefore:

$$\frac{1}{2} m_1 v_0^2 = \left(\frac{1}{2} m_1 v_f^2 \right) + \left(\frac{1}{2} m_2 v_{CM}^2 \right) + \left(\frac{1}{2} I_0 \omega^2 \right) \quad (2)$$

$$\Rightarrow m_1 v_0^2 = (m_1 v_f^2) + (m_2 v_{CM}^2) + (I_0 \omega^2) \quad (3)$$

as the point mass does not have any rotational inertia, and the rod is not rotating at the start.

Lastly, we understand that the angular velocity is the same as that which the ball travels after the collision; that:

$$v_f = \omega R \quad (4)$$

We now have a system of three equations that can be combined to solve for three unknowns v_f , v_{CM} , and ω .

Performing the actual solution,

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var("I m1 v0 m2 r vf vcm w")
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expand(solve([m1*v0 == m1*vf+m2*vcm, m1*v0^2==m1*vf^2+m2*vcm^2+I*w^2, vf==w*r], vf, vcm, w, to_poly_sol
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