PS#14: Slopes!!!

Nueva Multivariable Calculus

Don't forget to type this all up in LaTeX! Submit a PDF to Canvas, plus a link (in the comments) to your Overleaf source. I've also linked to Matthew Gill's AMAZING Nueva math homework template—it's so, so great, and you should definitely use it instead of mine.

0. Read the notes (linked on Canvas) on "Functions in Higher Dimensions" and (unredacted!) on "Differentiation in Higher Dimensions."

1. Find all the first, second, and third partial derivatives of this function (including the mixed ones):

$$f(x, y) = 4x^2y^5 + 3x^3y^2$$

2. Find the full derivative matrix for each of the following functions:

(a) $f : \mathbb{R}^2 \to \mathbb{R}^1$; $\quad f(x, y) = \dfrac{xy}{x^2 + y^2}$

(b) $f : \mathbb{R}^3 \to \mathbb{R}^2$; $\quad f(x, y, z) = \langle xy + 2yz,\, 2xy^2x \rangle$

3. Suppose you have a function from $\mathbb{R}^2$ to $\mathbb{R}^1$. How many total first partial derivatives does it have? What about second partial derivatives (including the mixed ones, but counting the mixed ones taken in a different order as equivalent)? What about third partial derivatives (again, including the mixed ones)? What about k'th partial derivatives?

4. Consider the function:

$$f(x, y) = 7x + 2x^2y^3 + 10y^2$$

You took a bunch of partial derivatives of this last time! It's a two-dimensional cubic—note how the highest power is a cube, in the y.

Anyway, graph this, using the 3D graphing software of your choice! (Include a picture in your writeup.) If you can figure out how to make a top-down **contour plot** (easy to do in Sage, my preferred software), include that, too!

Then: suppose you're standing on this function, looking in the direction of the vector $\left\langle -\frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$. How steep is the function? Give your answer both in normal slope units ("change in y over change in x") and also as an angle up from the horizontal xy-plane.

Suppose you want to turn such that you're facing in the direction such that the function is steepest. What direction do you need to be facing? What if you want to turn such that the function is steepest *downhill*?

Suppose you want to turn such that you're facing in the direction such that the function is flat. What direction do you need to turn?

Note that I'm not giving numbers to each of the individual questions here, so make sure your writeup is clearly and coherently organized!

If you can include pictures on your plot of where you're standing and the various directions you need to face, that'd be even better!