#ret

1. Suppose A = $\begin{pmatrix}1 & 3\\ 2 & -1\end{pmatrix}$ and B = $\begin{pmatrix}0 & -1\\ 2 & 1\end{pmatrix}$. Multiply AB and BA. What do you notice???

Nothing. Matrix multiplication is not commutative.

1. Use matrices to solve the system: $\begin{aligned}2x-y&=3\\ x+3y&=5\end{aligned}$

\[
$$\begin{aligned}
\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}3\\ 5\end{bmatrix}\\
\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix}\\
\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}2 & -1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}3 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}3\\ 5\end{bmatrix}\\
\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}6 & -3\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}9\\ 5\end{bmatrix}\\
\begin{bmatrix}7 & 0\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} &= \begin{bmatrix}14\\ 5\end{bmatrix}\\
\begin{bmatrix}7x\\ x+3y\end{bmatrix} &= \begin{bmatrix}14\\ 5\end{bmatrix}\\
\begin{bmatrix}\frac{1}{7} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}7x\\ x+3y\end{bmatrix} &= \begin{bmatrix}\frac{1}{7} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}14\\ 5\end{bmatrix}\\
\begin{bmatrix}x\\ x+3y\end{bmatrix} &= \begin{bmatrix}2\\ 5\end{bmatrix}
\end{aligned}$$
\] \[$\begin{aligned}x&=2\\ x+3y&=5\end{aligned}$ \] I'm not sure how to solve the rest of it with matrices, so I'll just do it normally: \[
$$\begin{aligned}x&=2\\ x+3y&=5\\ 2+3y&=5\\ 3y&=3\\ y&=1\end{aligned}$$
\] 3. > Do 2x2 matrices form a group under > a. addition? > b. multiplication?

See KBe2020math530refGroups I'll assume that our matrices have real numbers in them.

Operation Property	Closed	Identity	Inverse	Associative?	Final
Addition	Yes	$e=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$	$\begin{bmatrix}a & b\\ c & d\end{bmatrix}+\begin{bmatrix}-a & -b\\ -c & -d\end{bmatrix}=e$	"Inherits from addition"	Yes
Multiplication	Yes	$e=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$	Maybe?	Yes, see below	Undecided

Associativity of 2x2 matrices under multiplication: \[
$$\begin{aligned}
\left(\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}e & f\\ g & h\end{bmatrix}\right)\begin{bmatrix}i & j\\ k & l\end{bmatrix} &= \begin{bmatrix}ae+bg & af+bh\\ ce+dg & cf+dh\end{bmatrix}\begin{bmatrix}i & j\\ k & l\end{bmatrix}\\
&= \begin{bmatrix}aei+bgi+afk+bhk & aej+bgj+afl+bhl\\ cei+dgi+cfk+dhk & cej+dgj+cfl+dhl\end{bmatrix}\\
&= \begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}ei+fk & ej+fl\\ gi+hk & gj+hl\end{bmatrix} = \begin{bmatrix}a & b\\ c & d\end{bmatrix}\left(\begin{bmatrix}e & f\\ g & h\end{bmatrix}\begin{bmatrix}i & j\\ k & l\end{bmatrix}\right)
\end{aligned}$$
\]

I can't figure out if 2x2 matrices have multiplicative inverses… I tried to work it out using algebra but kept proving identities. Not sure what the right way to go about this is…

I spent far too long on this assignment (1.6h), so I probably won't spend as much time LaTeXing my homework in the future.