

Here are four easy integrals.

1 | Single Value Function

$$f_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (1)$$

$$f_1(x, y) = 0 \quad (2)$$

What's the area of this function?

We can take the area of the shape, essentially by taking the volume by height 1: that is, for a rectangle of l, w, h , its top-area is simply $l \cdot w$, also known as $lw \cdot 1$. Therefore:

$$\int_0^7 \int_0^5 1 dx dy = 35 \quad (3)$$

The area of the shape is therefore 35.

2 | Area of the Plane

We want to first figure the correction per every given slice $dA = n dV$ to setup a surface integral. By pythagoras (i.e. projecting the changes to the parallelity of the surface), we have that:

$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dV \quad (4)$$

What's the area of the following function by (5, 7)?

$$f_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (5)$$

$$f_2(x, y) = 2x + 3y + 10 \quad (6)$$

What's the area of this function?

$$dA = \sqrt{1 + 4 + 9} dV = \sqrt{14} dV \quad (7)$$