# 1 | **Dot product:**

- Name: dot product

- Result: Scalar

- Interpretation (what it measures): parallellity

    - the more parallel the larger the dot product

- Magnitude (with sign): $|\vec{a}||\vec{b}|cos(\theta)$

- Geometric magnitude: $|\vec{a}||\vec{b}_{\parallel \vec{a}}|$

- Direction: no direction

- Algebraic form: $a_x b_x + a_y b_y + a_z b_z$

- Algebraic properties:

    - commutative

    - associative

    - distributive across addition

# 2 | **Cross product:**

- Name: Cross product

- Result: Vector

- Interpretation (what it measures): Orthgonality

    - the more orthogonal the longer the cross product

- Magnitude (with sign): $|\vec{a}||\vec{b}| \sin(\theta)$

- Geometric Magnitude: $|\vec{a}||\vec{b}_{\perp \vec{a}}|$

- Direction: perpendicular to the two vectors

    - by the right hand rule by rotating the first vector into the second vector

- Albraic form: $\langle a_y b_z - a_z b_y, a_x b_z - a_z b_x, a_x b_y - a_y b_x \rangle$

- Algebraic properties:

    - Anticommutative

    - $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

    - $(\vec{A} \times \vec{B}) \perp \vec{A}$

    - $(\vec{A} \times \vec{B}) \perp \vec{B}$

    - Antiassociative

## 3 | **Application of cross product:**

- In physics there is something called torque, notated $\tau$
  - Torqe is the net force of things that rotate, so:
    * $F_{net} = ma$
    * $\tau_{net} = I\omega$
- Somethings to note about $\tau$:
  - It increases with a longer lever
  - It increases with a greater force
    * that is perpendicular to the lever
- Given these requirements we can make a formula:
  - $|\tau| = |\vec{r}||\vec{F}_{\perp\vec{r}}|$, where $\vec{F}$ is the force applied to the door, and $\vec{r}$ is the radius of the lever.
  - this, the right side of the equation, can be described using the dot product: $|\tau| = \vec{r} \times \vec{F}$

## 4 | **Derivation of cross product algebraic form:**

To start, we can define:

$$\vec{A} = (A_x, A_y, A_z) = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = (B_x, B_y, B_z) = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Next we have to assume that the dot product is distributive across addition:

$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_Z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$
$= A_xB_x\hat{i} \times \hat{i} + A_xB_y\hat{i} \times \hat{j} + A_xB_z\hat{i} \times \hat{k}$
$+ A_yB_x\hat{j} \times \hat{i} + A_yB_y\hat{j} \times \hat{j} + A_yB_z\hat{j} \times \hat{k}$
$+ A_zB_x\hat{k} \times \hat{i} + A_zB_y\hat{k} \times \hat{j} + A_zB_z\hat{k} \times \hat{k}$

From the definition of a cross product, we know that the cross product between any two vectors that are parallel is zero, thus:

$= A_xB_y\hat{i} \times \hat{j} + A_xB_z\hat{i} \times \hat{k}$
$+ A_yB_x\hat{j} \times \hat{i} + A_yB_z\hat{j} \times \hat{k}$
$+ A_zB_z\hat{k} \times \hat{i} + A_zB_z\hat{k} \times \hat{j}$