1 | **Tangent to equation**

We get an equation of which we need to find the tangent plane at $(1, 1, 3)$. The equation is as follows:

$$x^2 - 3y^2 + z^2 = 7 \tag{1}$$

Based on this, we can isolate z^2 and then take the square root (keeping in mind that $(1, 1, 3)$ is a solution to the equation) in order to create a function $z(x, y)$:

$$\begin{aligned} z^2 &= -x^2 + 3y^2 + 7 \\ z &= \pm\sqrt{-x^2 + 3y^2 + 7} \\ 3 &= \sqrt{-(1) + 3(1) + 7} \\ z(x, y) &= \sqrt{-x^2 + 3y^2 + 7} \end{aligned}$$

It turns out that we didn't really need to think about the sign so much because we know that 3 is positive. We can now take the gradient of $z(x, y)$.

$$\nabla z(x, y) = \begin{bmatrix} -\frac{x}{\sqrt{-x^2+3y^2+7}} \\ \frac{3y}{\sqrt{-x^2+3y^2+7}} \end{bmatrix} \tag{2}$$

We can plug in $(1, 1)$ to get the derivative "vector":

$$\begin{aligned} \nabla z(1, 1) &= \begin{bmatrix} -\frac{(1)}{\sqrt{-(1)^2+3(1)^2+7}} \\ \frac{3(1)}{\sqrt{-(1)^2+3(1)^2+7}} \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{3} \\ 1 \end{bmatrix} \end{aligned}$$

Given these vectors in the directions $\hat{\imath}$ and $\hat{\jmath}$, we can find the plane that is spanned by these two points (and through the point $(1, 1, 3)$.

The equation for the plane is given by

$$-\frac{\partial z}{\partial x}x - \frac{\partial z}{\partial y}y + z + \frac{\partial z}{\partial x}P_x + \frac{\partial z}{\partial y}P_y - P_z = 0 \tag{3}$$

Where the point "anchoring" the plane is given by (P_x, P_y, P_z). This gives us the following equation:

$$\begin{aligned} -\left(-\frac{1}{3}\right)x - (1)y + z + \left(-\frac{1}{3}\right)(1) + (1)(1) - (3) &= 0 \\ \frac{1}{3}x - y + z - \frac{7}{3} &= 0 \end{aligned}$$

2 | **1D**

We are given a function $f(x) = x^4 - 9x^2$. Taking the derivative, we get the following:

$$f'(x) = 4x^3 - 18x \tag{4}$$

We can factor this to get the following: