0 | lets find a general formula

The problem is to find the slope of f in the θ direction at the point $P_0 \in \mathbb{R}^2$.

We want to turn this 2d problem into a 1d problem, so that we can find the derivative generally, then just apply it. For each version of the problem, we are given a function $f: \mathbb{R}^2 \to \mathbb{R}$ (with a 2d input), but we would rather have a function $h: \mathbb{R} \to \mathbb{R}$ that encodes the planar slice that we care about.

We can do this by composing f with some other $g: \mathbb{R} \to \mathbb{R}^2$ s.t. $f \circ g(t)$ is that planar slice. This way, g just has to spit out xy-coordinates in the domain of f.

For convienence, lets also ensure that $g(0) = P_0$, In summary, we want a function g(t) that parameterizes a line that passes through P_0 and has slope θ , ie. goes in the direction $\langle \cos \theta, \sin \theta \rangle$.

$$g(t) = \vec{P_0} + t \langle \cos \theta, \sin \theta \rangle$$

Now, to solve any of the following problems, we can take

$$\frac{d}{dt}\Big|_{0} f \circ g$$

1 |
$$f(x,y) = x^2 + y^2$$

Before we do anything, lets find $\frac{d}{dt}f \circ g$ generally for any P_0 and θ :

$$\begin{split} \frac{d}{dt}f \circ g &= \frac{d}{dt} \bigg(\left(P_{0x} + t \cos \theta \right)^2 + \left(P_{0y} + t \sin \theta \right)^2 \bigg) \\ &= \frac{d}{dt} \Big(P_{0x}^2 + t^2 \cos^2 \theta + 2 P_{0x} t \cos \theta + P_{0y}^2 t^2 \sin^2 \theta + 2 t P_{0y} \sin \theta \Big) \\ &= \frac{d}{dt} \Big(P_{0x}^2 + P_{0y}^2 + t^2 \left(\cos^2 \theta + \sin^2 \theta \right) + 2 t \left(P_{0x} \cos \theta + P_{0y} \sin \theta \right) \\ &= 2 t \Big(\cos^2 \theta + \sin^2 \theta \Big) + 2 \left(P_{0x} \cos \theta + P_{0y} \sin \theta \right) \\ &= 2 \left(P_{0x} \cos \theta + P_{0y} \sin \theta \right) \end{split}$$

Because this tells us the slope, we can find the steepest slope through optimization.

$$\frac{d}{d\theta}2\left(P_{0x}\cos\theta + P_{0y}\sin\theta\right) = 2\left(-P_{0x}\sin\theta + P_{0y}\cos\theta\right) = 0$$

Likewise, the slope is zero for all \$ θ \$s s.t.

$$P_{0x}\cos\theta + P_{0y}\sin\theta = 0$$

(I don't actually know how to solve these equations, but Wolfram Alpha does! Is there a trig identity for sin + cos?)

Now, we just plug values in.

(d)
$$14\sqrt{2}$$
 (e) 20 (f) 19.32

(g) (above) (h)
$$\frac{4\pi}{3}, \frac{\pi}{3}$$
 (i) (above)

Taproot • 2021-2022 Page 1

$$2 \mid f(x,y) = x^2 - y^2$$

We can follow the same process as above

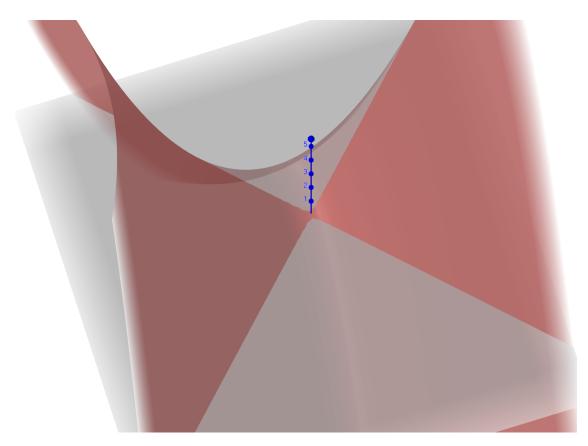
$$\begin{split} f \circ g &= (P_{0x} + t \cos \theta)^2 - (P_{0y} + t \sin \theta)^2 \\ &= P_{0x}^2 + t^2 \cos^2 \theta + 2t P_{0x} \cos \theta - P_{0y}^2 - t^2 \sin^2 \theta - 2t P_{0y} \sin \theta \\ &= P_{0x}^2 - P_{0y}^2 + t^2 \left(\cos^2 \theta - \sin^2 \theta\right) + 2t \left(P_{0x} \cos \theta - P_{0y} \sin \theta\right) \\ h(t) &= \frac{d}{dt} \Big|_0 f \circ g = 2 \left(P_{0x} \cos \theta - P_{0y} \sin \theta\right) \\ &\frac{d}{d\theta} h = -2 \left(P_{0x} \sin \theta + P_{0y} \cos \theta\right) = 0 \\ &\theta = n\pi - \tan^- \left(\frac{P_{0y}}{P_{0x}}\right) \end{split}$$

- **(a)** 0

(c) 6

- **(d)** 0
- (b) -24

- (f) -5.17
- (g) (above) (h) $\frac{2\pi}{3}$ steepest, add ninties for others



Taproot • 2021-2022 Page 2