#flo #hw

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1 | Dimension

- how should we define dimension?
  - should make the dimension of $F^n = n$
  - nice to define as the length of a basis
    - all basis of a given vector space have the same length

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basis len does not on basis
-> any two bases of a finite-dimension vector space have the same length
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cool proof!

- V is finite dimensional
- B_1 and B_2 both bases of V.
- Thus, B_1 is linearly independent in V and B_2 spans V
  - by @axler-2.23 (len of lin indepedent list $\leq$ len of spanning list)
  - len(B_1) $\leq$ len(B_2)
  - swap B_1 and B_2
    - len(B_2) $\leq$ len(B_1)
    - therefore, B_1 and B_2 need to be of equal len.

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title: dimension
the *dimension* of a finite-dimensional vector space is the len of any basis of the vector space.
denoted as dim $V$ if V is finite-dimensional
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- which are all the same!

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title: dimension of a subspace @2.38
if $V$ is finite-dimensional and $U$ is a subspace of $V$, then dim $U\leq$ dim $V$.
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- linearly independent list of the right length is a basis

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title: linearly independant list of the right length is a basis @2.39
suppose $V$ if finite-dimensional. Then very linearly indepedent list of vectors in $V$ with length dim
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- makes sense, because linearly independent list with the len of the dimension has to include all the info, and thus it spans, and thus it is a basis.
  - we don't need to check that it spans!
    - instead, we just need to check that it is linearly independent and that it has the same len as the dim

- why does the list being linearly independent mean that its dim is >= 3? #question @axler-2.41
- what??? #question what is happening at the end of @axler-2.41

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title: spanning list of the right length is a basis @2.42
suppose $V$ is finite-dimensional. Then every spanning list of vectors in $V$ with length dim $V$ is a
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- every list of vecs which both spans and is the same len as the dim is a basis.
  - because in order to span, you need all the info
  - and to be the len of the dim, you can't have repeat info
  - so it's a basis.

1.0.1 | **formula for dimension of sum of two subspaces**

in a finite-dimensional vector space.

like counting formula: - the num of elems in the union of two finite sets is the num of elems in the first set + the number of elems in the second set - the overlap (intersection)

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title: dimension of a sum @2.43
if $U_1$ and $U_2$ are subspaces of a finite-dimensional vector space, then
$$dim(U_1+U_2) = dim U_1 +dim U_2 - dim(U_1\cap U_2)$$
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#review the ending proof.