1 | Problem 1

1.1 | KE

Given a distance from axis l_i, mass m_i, relative position $\vec{r_i}'$ and angular velocity ω, you can get rotational KE via $\frac{1}{2}\sum m_i(l_i\omega)$. $l_i\theta$ would give arclength of the path of the mass, its so time derivative $l_i\omega$ would give its velocity. Additionally, this checks out in terms of dimensional analysis: radians are dimensionless so it's 1/s times meters to get m/s.

$$
\begin{aligned}
KE_r =& \frac{1}{2}\sum m_i(v_i')\\
=& \frac{1}{2}\sum m_i(l_i\omega)\\
=& \frac{1}{2}\sum m_i l_i^2\omega^2\\
=& \frac{1}{2}\omega^2\underbrace{\sum m_i l_i^2}_{I}\\
=& \frac{1}{2}I\omega^2
\end{aligned}
$$

1.2 | Rotational Inertia of Ring

$$I = \sum m_i l_i^2$$

l_i is constant on a ring and equal to R.

$$I = \sum m_i R^2$$

$$I = R^2 \sum m_I$$

$\sum m_i$ is defined to be M in previous problems.

$$I = MR^2$$

1.3 | Rotational Inertia of Sphere

I would expect it to be less than I_{disk} because more of the mass is spread out across the volume of the sphere, and more of it has a smaller l_i, meaning that $\sum m_i l_i^2$ will be smaller and therefore I_{sphere} is smaller.

2 | Problem 2

2.1 | $v(t)$ and $y(t)$

$$a(t) = a_0$$
$$\int a(t)dt = \int a_0$$
$$v(t) = a_0 t + C$$
We know that $v(0) = v_0 = C$
$$\boxed{v(t) = a_0 t + v_0}$$

$$v(t) = a_0 t + v_0$$
$$\int v(t)dt = \int a_0 t + v_0 dt$$
$$y(t) = \frac{a_0}{2} t^2 + v_0 t + C$$
We know that $y(0) = y_0 = C$
$$\boxed{y(t) = \frac{a_0}{2} t^2 + v_0 t + y_0}$$

2.2 | Equation for $v^2(t)$

$$v(t) = a_0 t + v_0$$
$$v^2(t) = (a_0 t + v_0)^2$$
$$v^2(t) = a_0^2 t^2 + 2a_0 t v_0 + v_0^2$$
$$v^2(t) = v_0^2 + 2a_0(\frac{1}{2} a_0 t^2 + v_0 t)$$
$$v^2(t) = v_0^2 + 2a_0(\frac{1}{2} a_0 t^2 + v_0 t + y_0 - y_0)$$
We know from the previous problem that $\frac{1}{2} a_0 t^2 + v_0 t + y_0$ is equal to $y(t)$.
$$\boxed{v^2(t) = v_0^2 + 2a_0(y(t) - y_0)}$$

2.3 | Equation for Δy

$$\begin{aligned}
v(t) &= a_0 t + v_0 \\
\int_{t_1}^{t_2} v(t) &= \int_{t_1}^{t_2} a_0 t + v_0 \\
y(t_2) - y(t_1) &= \left(\frac{a_0}{2} t_2^2 + v_0 t_2\right) - \left(\frac{a_0}{2} t_1^2 + v_0 t_1\right) \\
\Delta y &= \frac{(a_0 t_2^2 + 2v_0 t_2) - (a_0 t_1^2 + 2v_0 t_1)}{2} \\
\Delta y &= \frac{(a_0 t_2 + v_0) + (a_0 t_1 + v_0)}{2}(t_2 - t_1) \\
\text{We know from earlier that } v(t) &= a_0 t + v_0. \\
\Delta y &= \frac{v(t_2) - v(t_1)}{2}(t_2 - t_1) \\
&\boxed{\Delta y = \frac{v(t_2) - v(t_1)}{2}\Delta t}
\end{aligned}$$

3 | Problem 3

The constants that are flipped are a_0, v_0, and y_0. The equations for v(t) and y(t) become flipped as a result:

$$\begin{aligned}
a(t) &= -a_0 \\
\int a(t) dt &= \int -a_0 \\
v(t) &= -a_0 t + C \\
\text{We know that } v(0) &= -v_0 = C \\
&\boxed{v(t) = -a_0 t - v_0}
\end{aligned}$$

$$\begin{aligned}
v(t) &= -a_0 t - v_0 \\
\int v(t) dt &= \int -a_0 t + -v_0 dt \\
y(t) &= -\frac{a_0}{2} t^2 + -v_0 t + C \\
\text{We know that } y(0) &= -y_0 = C \\
&\boxed{y(t) = -\frac{a_0}{2} t^2 - v_0 t - y_0}
\end{aligned}$$

The sign change initially doesn't matter during the $v^2(t)$ derivation due to the square, but the substitution with $y(x)$ is flipped (see above) so part of the answer is flipped.

$$v(t) = -a_0 t - v_0$$
$$v^2(t) = (-a_0 t - v_0)^2$$
$$v^2(t) = a_0^2 t^2 + 2a_0 t v_0 + v_0^2$$
$$v^2(t) = v_0^2 + 2a_0(\frac{1}{2}a_0 t^2 + v_0 t)$$
$$v^2(t) = v_0^2 + 2a_0(\frac{1}{2}a_0 t^2 + v_0 t + y_0 - y_0)$$

We know from the previous problem that $-\frac{1}{2}a_0 t^2 - v_0 t - y_0$ is equal to $y(t)$.

$$\boxed{v^2(t) = v_0^2 + 2a_0(-y(t) - y_0)}$$

Finally, the final equation in problem 2 would stay the same because it is for the *change* in position and addition is commutative.