

## 1 | Precessional Velocity

Taking the setup, we can figure the sum of the angular momentums and leverage it to figure the spin angular momentum.

Let's first define a system:  $\hat{i}$  is "right" on the figure,  $\hat{j}$  "in" the page,  $\hat{k}$  "up" the figure.

We note that the normal spin of the flywheel gives us:

$$\vec{L}_s = I\vec{\omega}_s \hat{i} \quad (1)$$

As the flywheel is rotating at a constant speed, we have actually no torque that this contributes to the net system — that is  $\frac{d\vec{L}_s}{dt} = 0$ .

Furthermore, we can figure torque—and subsequent angular momentum contribution—of gravity as follows:

$$\vec{\tau}_g = lmg\hat{j} \quad (2)$$

The total net torque on the system, then:

$$\vec{\tau}_{net} = \vec{\tau}_g + 0 \quad (3)$$

$$= \vec{\tau}_g \quad (4)$$

We also have that:

$$\vec{\tau}_{net} = \frac{d\vec{L}_{net}}{dt} = \Delta\vec{L}_s = lmg \quad (5)$$

We see that, because of small-angle approximation,  $\Delta\vec{L}_s = L_s\Omega$

Therefore, we can replace the values determined above and solve for  $\Omega$ :

$$\Delta\vec{L}_s = L_s\Omega \quad (6)$$

$$\Rightarrow lmg = I\vec{\omega}_s\Omega \quad (7)$$

$$\Rightarrow \Omega = \frac{lmg}{I\vec{\omega}_s} \blacksquare \quad (8)$$

## 2 | Discussion Questions

### 2.1 | Gyro in the Opposite Direction

If  $\omega_s$  was in the opposite direction,  $\vec{L}_s = -\vec{L}_{s\text{old}}$  — by the right hand rule, it would be in the other direction.

The direction of precession would be in the same direction, "into" the page, by the  $\hat{j}$  direction.

Therefore, the direction of  $\vec{L}_s$  would be inching up and to the right—resulting in precession in the opposite ("clockwise") direction.

## 2.2 | Tilted Gyro

In our expressions above, the thing that will change is the fact that the value of  $r \times F$  for  $\vec{\tau}_g$  would change to account for the angle  $\phi$ :  $\vec{\tau}_g = l \sin \tau mg$ .

Therefore, the final expression  $\Omega$  becomes:

$$\Omega = \frac{lmg \sin \tau}{I\vec{\omega}_s} \quad (9)$$

## 2.3 | Biker

If a bicyclist leans towards the right, they are adding a small "forward" change to the angular momentum originally pointing strictly towards the left. This would orient the axis of rotation a little bit more towards the forward direction from being completely towards the left, which points the axis a little more towards the front.

The bike then responds to this axis change by turning to an angle orthogonal (per the right hand rule) to the location a little "more forward", which is a little "more right."

## 2.4 | Forces on the COM

### 2.4.1 | Normal Force and Torque

When the gyro is precessing, it performs uniform circular motion by a radius  $l$ . We see that, as determined above, it is precessing at a constant speed of  $\frac{lmg}{I\vec{\omega}_s}$ .