- Vector spaces and fields are like groups
  - With 2 operations
- Vector
  - direction and magnitude
  - numbers with an order
    - list = ordered set
    - Nx1 matrix
  - A vector is not just an Nx1 matrix. **A vector exists in a vector space**
    - might be full of physics vectors, matrices, or polynomials
- Field
  - Addition and multiplication might be different
    - might be related to normal addition/multiplication
    - might by any binary operation
    - Addition is "primary" operation, multiplication is "secondary"
      - addition is really good (more group like)
      - multiplication needs to exclude the additive identity (because it can't have an inverse)
    - questions
      - multiplication is repeated addition?
      - not necessarily
      - binary expressions?
      - associative?
      - both yes
    - 1.3 demonstrates that the complex numbers are a field
      - commutativity
      - associativity
      - identities
      - additive inverse
      - multiplicative inverse except additive identity
      - distributive
  - usually means Reals or Complex
    - integers mod 3 are a field
      - #bonushw show integers mod 3 are a field
  - higher dimensions
    - R^2 is a Cartesian plane, R^4 is a space
  - operations
    - addition is really nice (element wise)
    - scalar multiplication is easy enough

        * vector vector multiplication is hard to find

- two square roots of i
    - fundamental theorem of algebra
        * (important)
    - So, i should have two square roots
    - Powers of i go in a circle (90 degrees rotation)
        * Complex number rotation gives a preferred direction
        * So that's why the quadrants are numbered in that direction
    - One can be found geometrically 20math530srcSquareRootI.png
    - We could also try it algebraically
        * $(a + bi)^2 = i = a^2 - b^2 + 2abi$
        * so $a^2 - b^2 = 0$ and $2ab = 1$
- dot product
    - How much of $\vec{A}$ is in the direction of $\vec{B}$ multiplied by the magnitude of $\vec{B}$
    - $\vec{A} \cdot \vec{B} = |A||B|cos\theta$
        * #bonushw prove that ^^
    - Identity: $\frac{\vec{A} \cdot \vec{B}}{|A||B|} = cos\theta$
- cross product
    - only works on 3x1 matrices
    - steps
        * determinant
        * i, j, k are the unit vectors
        * $$\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \left| \begin{bmatrix} i & j & k \\ 2 & 1 & 0 \\ 1 & 2 & -1 \end{bmatrix} \right| = i \left| \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} \right| - j \left| \begin{bmatrix} 2 & 0 \\ 1 & -1 \end{bmatrix} \right| + k \left| \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right| = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$$
- dropping zero: $0a = (0 + 0)a = 0a + 0a \Rightarrow 0a = 0$, so the additive identity can't have a multiplicative inverse (everything multiplied it will just be the additive identity)
    - 20math530srcFieldsMultiplyCannotBeGroup.png
- determinant
    - measures the "size" of a matrix, denoted absolute value (relevant to inverse of a matrix multiplication)
- #todo #exr0n #future prove identities are unique