

1 | Types of Proofs

- Proof by Example/Counterexample
- Proof by Cases
- Proof by Contradiction
- Proof by Induction
- Proof by Strong Induction

Proof: The number of primes is infinite.

- This will be an example of proof by contradiction.

Assume for the purposes of contradiction that the number of primes is finite.

Then we can list them:

- $\{P_1, P_2, \dots\}$

Consider the number $S = P_1 \times P_2 \times P_3 \times \dots + 1$

Case 1: S is prime

- Contradiction as S is not in the set.

Case 2: S is not prime

- If S is not prime it must be divisible by at least two prime > numbers.
- However, all primes are in that list, so if S is divided by any > number in the list the remainder will be one.
- Contradiction!

All cases have contradictions so our assumptions must be false.

Q.E.D.

Proof by induction

- Prove something is true for a smaller number and show that doing so > implies it is true for larger numbers.
- There are 5 steps.
 - Declare proof by induction
 - Declare inductive hypothesis
 - * Inductive hypothesis is typically whatever you are trying to > prove.
 - Prove the base case
 - * Like dominos where a proof of one number leads to the proof > of the next number
 - Show that $P(n) \rightarrow P(n+1)$
 - Invoke induction.

Proof: $1+2+3+\dots+n = n(n+1)/2$

- Proof is by induction.

$P(n)$ is the hypothesis that $1+2+3+\dots+n = n(n+1)/2$

$P(1)$:

$$1 = 1(1+1)/2$$

$$1 = 2/2$$

$$1 = 1$$

$P(n+1)$:

We need to show $1+2+3+4+\dots+n+n+1 = ((n+1)(n+2))/2$

Assume that $P(n)$ is true.

Left side simplifies to $(n(n+1)/2) + (n+1)$

Algebraic manipulation leaves you with $((n+2)(n+1))/2$

If $P(n)$ is true then $P(n+1)$ is true and $P(1)$ is true therefore $P(n)$ is true for all numbers.

Proof by Strong Induction

- Use $P(1), P(2) \dots P(n)$ to prove $P(n+1)$
- Same list but instead of proving $P(n) \rightarrow P(n+1)$ it will be $P(1), > P(2) \dots P(n) \rightarrow P(n+1)$

Proof: Any group of students ≥ 12 can be divided into some combination of groups of 4 and groups of 5.

- This will be a proof by strong induction.

$$P(12) = 4+4+4$$

$$P(13) = 4+4+5$$

$$P(14) = 4+5+5$$

$$P(15) = 5+5+5$$

$$P(n) \rightarrow P(n+4)$$

If $P(n-3)$ is true, then there exists a combination of groups of 4 and 5 to make up $(n-3)$ students. Add a group of 4 and that gives us out combination for $(n+1)$ students.

$$P(n-3) \rightarrow P(n+1)$$

By induction this is true for all $n \geq 12$

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See Induction for more examples.