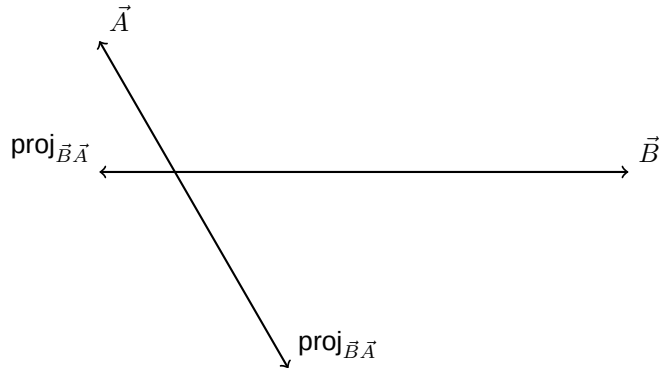


## 1 | vectors at an angle

### 1.1 | a sketch



### 1.2 | components

$$\text{comp}_{\vec{A}} \vec{B} = 6 \cos 120$$

$$\text{comp}_{\vec{B}} \vec{A} = 2 \cos 120$$

### 1.3 | dot product

$$\begin{aligned} \vec{A} \cdot \vec{B} &= |\vec{A}| |\vec{B}| \cos \theta \\ &= 2 \cdot 6 \cdot \cos 120 = -3 \end{aligned}$$

## 2 | proving expression for component

Lets redefine the coordinate axis so that  $\vec{A}$  lies along the x-axis. Then,

$$\begin{aligned} \text{comp}_{\vec{A}} \vec{B} &= |\vec{B}| \cos \theta \\ &= \frac{|\vec{A}| |\vec{B}| \cos \theta}{|\vec{A}|} \\ &= \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|} \end{aligned}$$

## 3 | expression for projection

The projection is just a vector with length  $\text{comp}_{\vec{A}} \vec{B}$  in the direction of  $\vec{A}$ .

$$\left( \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|} \right) \frac{\vec{A}}{|\vec{A}|}$$

## 4 | expression for perpendicular

The part of  $\vec{A}$  that is perpendicular to  $\vec{B}$  is just the whole vector minus the part that is parallel:

$$\begin{aligned}\vec{A}_{\perp \vec{B}} &= \vec{A} - \text{proj}_{\vec{B}} \vec{A} \\ &= \vec{A} - \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B}\end{aligned}$$

Checking using the dot product:

$$\begin{aligned}\left( \vec{A} - \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B} \right) \cdot \vec{B} &= \vec{A} \cdot \vec{B} - \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B} \cdot \vec{B} \\ &= \vec{A} \cdot \vec{B} - \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) |\vec{B}|^2 \\ &= \vec{A} \cdot \vec{B} - \vec{A} \cdot \vec{B} \\ &= 0\end{aligned}$$

## 5 | find angle using dot product

Well, the dot product already includes the angle, so let's just solve for that

$$\theta = \cos^{-1} \left( \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

The angle between:

$$\begin{aligned}\theta &= \cos^{-1} \left( \frac{3 + 2 - 4}{\sqrt{1^2 + 2^2 + 2^2} \sqrt{3^2 + 1^2 + 2^2}} \right) \\ &= \cos^{-1} \left( (3 + 2 - 4) / (3 * \sqrt{14}) \right) = \cos^{-1}(0.08908708) \approx 84.8^\circ\end{aligned}$$

## 6 | problems 6-8

See other files in the Canvas comments.

## 7 | vector equation that passes through the points

The vectors are

$$\begin{aligned}\vec{a} &= \langle -1, 4, 1 \rangle \\ \vec{b} &= \langle 2, -5, -3 \rangle\end{aligned}$$

Let's choose

$$\vec{r}(t) = \vec{p} + \vec{v}t$$

and make sure that  $\vec{r}(0) = \vec{a}$ , and  $\vec{r}(1) = \vec{b}$ . We can do this by setting

$$\begin{aligned}\vec{p} &= \vec{a} = \langle -1, 4, 1 \rangle \\ \vec{v} &= \vec{b} - \vec{a} = \langle 3, 9, 4 \rangle\end{aligned}$$

This way,

$$\begin{aligned}\vec{r}(0) &= \vec{p} = \vec{a} \\ \vec{r}(1) &= \vec{p} + \vec{v} = \vec{a} + (\vec{b} - \vec{a}) = \vec{b}\end{aligned}$$