1 | Problem

What happens if the Gram–Schmidt Procedure is applied to a list of vectors that is not linearly independent?

2 | Answer

Suppose the list v_1,\ldots,v_n is linearly dependent. Then, there exists some v_j s.t. v_1,\ldots,v_{j-1} is linearly independent while v_1,\ldots,v_j is not. Then, $v_j\in \text{span}(v_1,\ldots,v_{j-1})$

Because the Gram-Schmidt procedure preserves prefix spans,

$$v_j \in \mathsf{span}(e_1, \dots, e_{j-1})$$

Because of how a vector is written as a linear combination of an orthonormal basis, the denominator in the \$j\$-th step of the procedure is equivalent to

$$||v - v|| = ||0|| = 0$$

and a division by zero occurs. Thus, the Gram-Schmidt procedure cannot be used on a linearly dependent list.

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