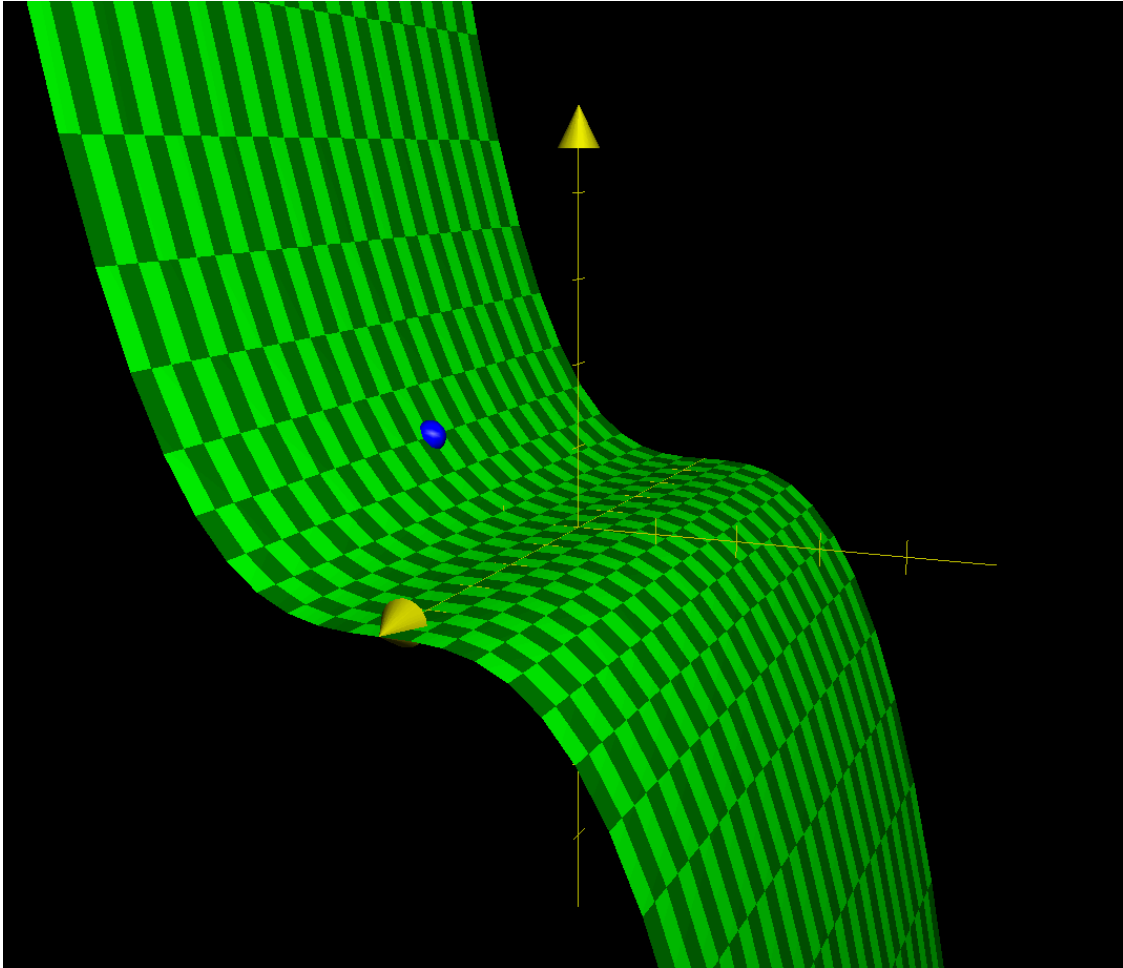


One possible design for one side of the roof would be a simple cubic function; that:

$$f(x, y) = \frac{1}{8}x^3 \{0 \leq x \leq 4, -5 \leq y \leq 5\} \quad (1)$$



## 1 | Slope in Middle

The "middle" of the roof, therefore, is the location  $(2, 0)$ , as indicated by the blue dot above.

Standing in the middle, and facing the "ridge"  $(+x)$  direction, we could calculate the slope of the roof.

The vector facing the ridge of the roof, to the positive  $x$  direction, is represented by

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (2)$$

the gradient of this function is represented by:

$$\begin{bmatrix} \frac{3}{8}x^2 \\ 0 \end{bmatrix} \quad (3)$$

Therefore, at the center point as indicated, the gradient is:

$$\begin{bmatrix} 6 \\ 0 \end{bmatrix} \quad (4)$$

Computing the dot product of the direction and the gradient as found, we arrive that — at the center — the slope is:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 0 \end{bmatrix} = 6 \quad (5)$$

Therefore, the slope as indicated is  $6 \approx 80.5^\circ$ .

## 2 | Facing the Peak

We first determine a vector that originates from the center of the roof, and facing towards one of the ridges; that:

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix} - \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \quad (6)$$

Normalizing this vector, we arrive at:

$$\begin{bmatrix} \frac{2}{\sqrt{29}} \\ \frac{5}{\sqrt{29}} \end{bmatrix} \quad (7)$$

We will then project the gradient at the center point atop this vector:

$$\begin{bmatrix} \frac{2}{\sqrt{29}} \\ \frac{5}{\sqrt{29}} \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 0 \end{bmatrix} = \frac{12}{\sqrt{29}} \quad (8)$$

Therefore, the slope as indicated is  $\frac{12}{\sqrt{29}} \approx 65.83^\circ$ .

## 3 | Maximizing the Angle

To face in the steepest direction, we will need to face the direction of the gradient. As the gradient is

$$\begin{bmatrix} 6 \\ 0 \end{bmatrix} \quad (9)$$

as derived above, the direction of the gradient is therefore:

$$\begin{bmatrix} \frac{6}{6} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad (10)$$