

## 1 | Algebraic and Geometric Multiplicities

I missed the last ten minutes of class and had to look up what the algebraic and geometric multiplicities are. I used this source.

Also it says something about

It is a fact that summing up the algebraic multiplicities of all the eigenvalues of an  $n \times n$  matrix  $A$  gives exactly  $n$ .

Which reminds me of the fundamental theorem of algebra...

$$1.1 \mid \begin{pmatrix} 4 & -12 \\ 2 & 0 \end{pmatrix}$$

### 1.1.1 | Geometric multiplicity

The null space is span  $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$  which is dimension 1.

### 1.1.2 | Algebraic multiplicity

The determinant of  $\begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix}$  is

$$-\lambda(4 - \lambda) - (-4) = \lambda^2 - 4\lambda + 4 = (\lambda - 2)^2$$

So the algebraic multiplicity is 2

$$1.2 \mid \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

### 1.2.1 | Geometric

Null space of 1  $((x, 0, 0))$  has dim 1. Null space of 3  $((x, \frac{-2x}{3}, \frac{4x}{3}))$  has dim 1 as well.

### 1.2.2 | Algebraic

The determinant simplifies to one factored term:

$$(1 - \lambda)^2(3 - \lambda)$$

So 1 has a multiplicity 2 and 3 has multiplicity 1?

$$1.3 \mid \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$$

### 1.3.1 | Geometric

For  $\lambda = 1$ , null space is  $(x, y, 0)$  so dim 2. For  $\lambda = 3$ , null space is  $(x, \frac{-x}{2}, x)$  so dim 1.

### 1.3.2 | Algebraic

The determinant is the same as the previous matrix, so once again, 1 has multiplicity 2 and 3 has multiplicity 1.