1 | projections or smt

1.1 | projections

[thick,->] (0,0) – (3,0) node[anchor=west] x; [thick,->] (0,0) – (0,3) node[above] y; [thick,->] (0, 0) – (1.5,2) node[above right] (3, 4); [thick,->] (0,0) – (3,0) node[anchor=west] y; [thick,->] (0,0) – (0,3) node[above] z; [thick,->] (0, 0) – (2,2.5) node[above right] (4, 5); [thick,->] (0,0) – (3,0) node[anchor=west] x; [thick,->] (0,0) – (0,3) node[above] z; [thick,->] (0, 0) – (1.5,2.5) node[above right] (3, 5);

1.2 | TODO language of projections?

2 | vectors problems

2.1 | adding two vectors $\langle a_1, a_2, a_3 \rangle$ and $\langle b_1, b_2, b_3 \rangle$

2.1.1 | the coordinates of the sum

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

2.1.2 | adding vectors

Geometrically, it is putting the vectors tip to tail. Follow one, then follow the other. Algebraically, it is adding each of the components. See the previous part

2.1.3 | subtracting vectors

We want to define $\vec{c} = \vec{a} - \vec{b}$ such that $\vec{b} + \vec{c} = \vec{a}$.

Geometrically, that means following $\vec{a}$, and then following $\vec{b}$ backwards (ie. we want to define a negative vector as the same vector backwards). Algebracially, we see that it inherits the properties from addition/subtraction.

2.2 | finding the vector between two points

Take the points as vectors, and subtract them.

$$\langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

2.3 | practice problems

2.3.1 | magnitude of a

$$|\langle 4, 0, 3 \rangle| = \sqrt{4^2 + 3^2} = 5$$

2.3.2 | magnitude of b

$$|\langle -2, 1, 5\rangle| = \sqrt{(-2)^2 + 1^2 + 5^2} = \sqrt{30} = 5.47722557505$$

2.3.3 | $\vec{a} + \vec{b}$

$$\langle 2, 1, 8\rangle$$

2.3.4 | $\vec{a} - \vec{b}$

$$\langle 6, -1, -2\rangle$$

2.3.5 | $3\vec{b}$

$$\langle -6, 3, 15\rangle$$

2.3.6 | $2\vec{a} + 5\vec{b}$

$$\langle -2, 5, 31\rangle$$

2.3.7 | $\hat{a}, \hat{b}$

$$\left\langle \frac{4}{5}, 0, \frac{3}{5} \right\rangle$$
$$\left\langle \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right\rangle$$

2.3.8 | $\theta_{\vec{a}x}$

Lets make a right triangle in the plane that contains the tip and tail of the vector and the x-axis.
The height will be from the x-axis to the tail, so we'll take the diagonal in the yz plane

$$h = \sqrt{a_y^2 + a_z^2}$$

The base of the triangle will be along the x-axis. So, the base is just the x component a_x.
And so, we can find theta using the tangent

$$\tan\theta = \frac{\sqrt{a_y^2 + a_z^2}}{a_x}$$

You could also do it with the cosine, as in dot product:

$$\cos\theta = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

2.4 | triangle proof

Lets let $\vec{a}, \vec{b}$ be the two sides and $\vec{c}$ be the middle side. This is the small triangle. Then, let's double each of the side lengths:

$$2\vec{a} + 2\vec{b} = 2(\vec{a} + \vec{b}) = 2\vec{c}$$

Thus, the middle line is half the magnitude of the longer third side.

3 | proving vector properties

You are really stretching my LaTeX abilities here

3.1 | $a + b = b + a$

[thick,->] (0,0) – (5.5,0) node[anchor=west] x; [thick,->] (0,0) – (0,4.5) node[above] y; [blue,thick,->] (0,0) – (1,3) node[midway,above left] a; [red,thick,->] (1,3) – (5,4) node[midway,above left] b; [red,thick,->] (0,0) – (4,1) node[midway,below right] b; [blue,thick,->] (4,1) – (5,4) node[midway, below right] a; [purple,thick,->] (0,0) – (5,4) node[midway, above left] a+b, b+a;

3.2 | $c(\mathbf{a} + \mathbf{b}) = c\mathbf{a} + c\mathbf{b}$

[thick,->] (0,0) – (7,0) node[anchor=west] x; [thick,->] (0,0) – (0,5) node[above] y;

[blue,very thick,->] (0,0) – (1,1.5) node[midway,right] a; [red,thick,->] (1,1.5) – (2,1) node[midway,above right] b; [blue,thick,->] (2,1) – (3,2.5) node[midway,above left] a; [red,thick,->] (3,2.5) – (4,2) node[midway,above right] b; [blue,thick,->] (4,2) – (5,3.5) node[midway,above left] a; [red,very thick,->] (5,3.5) – (6,3) node[midway,below left] b;

[blue,thick,->] (1,1.5) – (2,3) node[midway,above left] c

3.3 | $(cd)\mathbf{a} = c(d\mathbf{a})$

[thick,->] (0,0) – (6.5,0) node[anchor=west] x; [thick,->] (0,0) – (0,4) node[above] y;

[red, thick, ->] (0, 0.1) – (1, 0.6); [red, thick, ->] (1, 0.6) – (2, 1.1); [red, thick, ->] (2, 1.1) – (3, 1.6) node[above left] $(cd)\vec{\mathbf{a}}$; [red, thick, ->] (3, 1.6) – (4, 2.1); [red, thick, ->] (4, 2.1) – (5, 2.6); [red, thick, ->] (5, 2.6) – (6, 3.1); [purple, thick, ->] (0.1, 0) – (2.1,1); [purple, thick, ->] (2.1, 1) – (4.1,2) node[midway,below right] $c(d\vec{\mathbf{a}})$; [purple, thick, ->] (4.1, 2) – (6.1,3);