Hello, fellow person that comes across this. I have had one brief exposure with Linear Algebra following MATH 21-1 at UCSC. However, Axler is just so cool, so I am trying to learn a bit of linalg on the side to supplement my much more traditional linalg experience at the UC.

A few things of note. This whole thing is very ”partial”: in the sense that its contents contain many a parts of things omitted which I feel like I have a good grasp on from 21-1 such that I don't need to be reminded again; I only include things that maybe useful to me later either b/c I don't know it or I want to be reminded of it. As such, I don't think this will be helpful for most people.

1 | 1.A

1.1 | Things of Note

- $\lambda \in \mathbb{F}$ is called a ”scalar”. I mean duh but still.

1.1.1 | Defining a list

A list of length n is a collection of n elements (any mathematical object?) separated by commas.

”Identical” lists are established when lists have:

- the same length
- same elements
- in the same order.

Its also called a n-tuple.

n must be a finite non-negative value. Therefore, an ”infinitely long list” is not a list.

1.1.2 | Sets vs Lists

Lists have order and repetition. In sets, order and repetitions don't matter.

1.1.3 | $\mathbb{F}$

- A set
- Containing 2 elements $0, 1$
- Operators of ”addition” and ”multiplication” that satisfy the following properties

1. Properties of $\mathbb{F}$ That, with $\alpha, \beta, \lambda \in \mathbb{F}$:
   - **Commutativity** $\alpha + \beta = \beta + \alpha$ and $\alpha\beta = \beta\alpha$
   - **Associativity** $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda)$ and $(\alpha\beta)\lambda = \alpha(\beta\lambda)$
   - **Existence of Identities** $\lambda + 0 = \lambda$ and $\lambda 1 = \lambda$
   - **Additive Inverse** for every α, $\exists \beta$ s.t. $\alpha + \beta = 0$
   - **Multiplicative Inverse** for every $\alpha \neq 0$, $\exists \beta$ s.t. $\alpha\beta = 1$
   - **Distribution** $\lambda(\alpha + \beta) = \lambda\alpha + \lambda\beta$

1.1.4 | $\mathbb{F}^n$

$$\mathbb{F}^n = \{(x_1, \ldots, x_n) : x_j \in \mathbb{F}\ for\ j = 1, \ldots, n\} \tag{1}$$

We say x_j is the j^{th} coordinate of $(x_1, \ldots, x_n)$.
In $\mathbb{F}^n$...

1. Addition
$$(x_1, \ldots, x_n) + (y_1, \ldots, y_n) = (x_1 + y_1, \ldots, x_n + y_n) \tag{2}$$
2. Scalar Multiplication
$$\lambda(x_1, \ldots, x_n) = (\lambda x_1, \ldots, \lambda x_n) \tag{3}$$
3. Zero
$$0 = (0, \ldots, 0) \tag{4}$$
4. Additive Inverse ...of $x \in \mathbb{F}^n$:
$$x + (-x) = 0 \tag{5}$$
That:
$$x = (x_1, \ldots, x_n), -x = (-x_1, \ldots, -x_n) \tag{6}$$

1.2 | **In-Text Exercises**

1.2.1 | **Verify that** $i^2 = -1$

$(0 + 1i)(0 + 1i) = (0 + 0 + 0 + ii) = -1$

1.2.2 | **Defining subtraction and division**

$\alpha, \beta \in \mathbb{C}$
Subtraction could be defined in that:

- Let $-\alpha$ be defined as the additive inverse of α
- Subtraction, therefore, is defined $\beta - \alpha = \beta + (-\alpha)$

Division could be defined in that:

- Let $1/\alpha$ be defined as the multiplicative inverse of α
- Subtraction, therefore, is defined $\beta/\alpha = \beta(1/\alpha)$

1.3 | **Actual Exercises**

1: Suppose $a, b \in \mathbb{R}$, $a, b \neq 0$, find $c, d \in \mathbb{R}$ s.t. $\frac{1}{(a+bi)} = c + di$

$$\frac{1}{(a+bi)} = \frac{(a-bi)}{(a+bi)(a-bi)} = \tag{7}$$

$$\Rightarrow \frac{a-bi}{a^2-(bi)^2} = c + di \tag{8}$$

$$\Rightarrow \frac{a-bi}{a^2+b^2} = c + di \tag{9}$$

$$\Rightarrow \frac{a}{a^2+b^2} - \frac{bi}{a^2+b^2} = c + di \tag{10}$$

Therefore:

$$c = \frac{a}{a^2+b^2} \tag{11}$$

$$d = \frac{-b}{a^2+b^2} \tag{12}$$

2: Show that $\frac{-1+\sqrt{3}i}{2}$ is the cube root of 1.

$$(\frac{-1+\sqrt{3}i}{2})^3 \tag{13}$$

$$\Rightarrow (\frac{-1+\sqrt{3}i}{2})(\frac{-1+\sqrt{3}i}{2})(\frac{-1+\sqrt{3}i}{2}) \tag{14}$$

$$\Rightarrow \frac{(-1+\sqrt{3}i)(-1+\sqrt{3}i)(-1+\sqrt{3}i)}{8} \tag{15}$$

$$\Rightarrow \frac{(1-2\sqrt{3}i-3)(-1+\sqrt{3}i)}{8} \tag{16}$$

$$\Rightarrow \frac{(1-2\sqrt{3}i-3)(-1+\sqrt{3}i)}{8} \tag{17}$$

$$\Rightarrow \frac{8}{8} = 1 \tag{18}$$

3: Find two distinct square roots of i

?

4: Show that $\alpha + \beta = \beta + \alpha, \forall \alpha, \beta \in C$

Let:

$\forall a, b, c, d \in \mathbb{R}$

- $\alpha = (a + bi)$
- $\beta = (c + di)$

$$\alpha + \beta = (a + bi) + (c + di) \tag{19}$$
$$= (a + c) + (b + d)i \tag{20}$$
$$= (c + a) + (d + b)i \tag{21}$$
$$= (c + di) + (a + bi) \tag{22}$$
$$= \beta + \alpha \blacksquare \tag{23}$$

5: Show that $(\alpha + \beta) + \lambda = \alpha + (\beta + \lambda), \forall \alpha, \beta, \lambda \in \mathbb{C}$

Let:

$\forall a, b, c, d, e, f \in \mathbb{R}$

- $\alpha = (a + bi)$
- $\beta = (c + di)$
- $\lambda = (e + fi)$

$$(\alpha + \beta) + \lambda = ((a + bi) + (c + di)) + (e + fi) \tag{24}$$
$$= ((a + c) + (b + d)i) + (e + fi) \tag{25}$$
$$= (a + c + e) + (b + d + f)i \tag{26}$$
$$= (a + (c + e)) + (b + (d + f))i \tag{27}$$
$$= (a + bi) + (c + e) + (d + f)i \tag{28}$$
$$= (a + bi) + ((c + di) + (e + fi)) \tag{29}$$
$$= \alpha + (\beta + \lambda) \blacksquare \tag{30}$$

2 | **1.B**

2.1 | **Things of Note**

2.1.1 | **Vector Spaces** V

A ”vector”/”point” is a member of a vector space. The exact nature of scalar multiplication depends on which $\mathbb{F}$ we are working it; hence, when being precise, we say that V is a vector space ”over $\mathbb{F}$”.

1. Motivation
    - Addition is commutative, associative, and has identity
    - Every element has additive inverse
    - Scalar multiplication is associative
    - Addition and scalar multiplication is connected by distribution
2. Basic Operators
    - **Addition** on set V is a function that assigns $u + v \in V$ to each pair $u, v \in V$

- **Scalar Multiplication** on set V is a function that assigns $\lambda v \in V$ to each $\lambda \in \mathbb{F}$ and $v \in V$. Note that this is different than a field, because if you can't multiply two different things in and out of the field and expect it to remain. But you could multiply an element in field $\mathbb{F}$ and a vector in vector space V and expect it to stay in V.

Note also "Multiplication" is not defined as 1) there are two and 2) they behave very differently.

3. Properties For $u, v, w \in V$ and $a, b \in \mathbb{F}$.
   - **Commutativity** $u + v = v + u$
   - **Associativity** $(u + v) + w = u + (v + w)$ and $(ab)v = a(bv)$.
   - **Additive Identity** $\exists 0 \in V \ s.t. \ v + 0 = v, \forall v \in V$
   - **Additive Inverse** $\forall v \in V, \exists w \in V \ s.t. \ v + w = 0$
   - **Multiplicative Identity** $1v = v$
   - **Distribution** $a(u + v) = au + av$ and $(a + b)v = av + bv$
4. Unique Additive Identity The additive identity ("zero") in a vector space must be unique. (i.e. there cannot be two distinct zeros 0 and $0'$ which both are $\in V$). This is because:

$$0 = 0 + 0' = 0' + 0 = 0' \quad (31)$$

   That — if both 0 and $0'$ are additive identities, $0 = 0'$.
5. Unique Additive Inverse Every element in a vector space has an unique additive inverse (i.e. there cannot be two distinct additive inverses of $v \in V$ w and w' which both are $\in V$).

   Suppose w and w' are both additive inverses of v, then it holds that:

$$w = w + 0 = w + (v + w') = (w + v) + w' = 0 + w' = w' \quad (32)$$

   That — if both w and w' exists in V, $w = w'$.
6. Zero and Vectors $0v = 0$ for $v \in V$. $a\vec{0} = \vec{0}$ for $a \in \mathbb{F}$.

2.1.2 | $\mathbb{F}^\infty$

Wait but aren't $\mathbb{F}^n$ supposed to be made of lists, which has finite length?

I guess its just sequences of all of everything in F.

$$\mathbb{F}^\infty = \{(x_1, x_2, \ldots) : x_j \in \mathbb{F} \ for \ j = 1, 2, \ldots\} \quad (33)$$

2.1.3 | $\mathbb{F}^S$

$\mathbb{F}^S$ is defined as the set of functions that maps elements in set S to $\mathbb{F}$. It is a vector space.

1. Addition Addition between $f, g \in \mathbb{F}^S$ is defined by:

$$(f + g)(x) = f(x) + g(x), \ \forall x \in S \quad (34)$$

2. Scalar Multiplication Multiplication between $\lambda \in \mathbb{F}$ and $f \in \mathbb{F}^s$, $\lambda f \in \mathbb{F}^s$ is defined as:

$$(\lambda f)(x) = \lambda f(x), \ \forall x \in S \quad (35)$$

3. $\mathbb{F}^n$ and $\mathbb{F}^\infty$ are special cases of $\mathbb{F}^s$ …this is because a list $\{x_1, x_2, x_3, \ldots, x_n\}$ is actually a bijective mapping between $\{1, 2, 3, \ldots, n\}$ (the indexes) and the values of the list, which are all $\in \mathbb{F}$. so :tada:!

2.2 | **In-Text Exercises**

2.2.1 | **Verify that $\mathbb{F}^n$ is a vector space over $\mathbb{F}$**

Not going to write this one out, but:

- Commutativity: via rules addition, commutation (in $\mathbb{F}$), then undoing addition
- Associativity: addition, communication, then undoing addition
- Additive Identity: addition + definition of "zero" in $\mathbb{F}^n$
- Additive Inverse: addition + additive inverse (in $\mathbb{F}$)
- Multiplicative Identity: scalar multiplication (by 1) and then identity (in $\mathbb{F}$)
- Distribution: definition of addition in $\mathbb{F}^n$, scalar multiplication, undoing definition of addition again

2.3 | **Actual Exercises**

1: Proof that $-(-v) = v,\ \forall v \in V$

Step	Explanation
$v = v + 0$	Additive identity
$v = v + (-v + -(-v))$	Additive inverse
$v = (v + -v) + -(-v)$	Associative property
$v = 0 + -(-v)$	Additive inverse
$v = -(-v)$ ■	Additive Identity

2: Suppose $a \in \mathbb{F}, v \in V$, and $av = 0$. Proof $a = 0$ or $v = 0$.

Let $a \neq 0$. We define the multiplicative inverse of a as a^{-1}.

Step	Explanation
$v = 1v$	Multiplicative identity
$v = aa^{-1}v$	Multiplicative inverse
$v = ava^{-1}$	Commutativity
$v = 0a^{-1}$	Given
$v = 0$ ■	Number times 0

If $a = 0$, ■.

3: Suppose $v, w \in V$, explain why $\exists$ unique $x \in V$ s.t. $v + 3x = w$

Let $x = \frac{1}{3}(w - v)$; by addition and scalar multiplication, $\exists x \in V$.

Step	Explanation
$v + 3x = w$	Given
$v + 3(\frac{1}{3}(w - v)) = w$	Defined
$v + (w - v) = w$	Multiplication in $\mathbb{F}$
$v - v + w = w$	Commutativity
$w = w$ ■	Additive Inverse

Therefore, $\exists x \in V$ that satisfies the needed property.

Suppose there exists more than 1 x which satisfies this property. We call them x and x'. This would tell us the following equalities:

$v + 3x = w$, $v + 3x' = w$.

It follows from the equalities that:

$3x = w - v$, $3x' = w - v$

Then, it follows that

$3x = 3x'$

Therefore:

$$x = x' \tag{36}$$

Hence, if given that there exists $v + 3x = w$, $v + 3x' = w$, $x = x'$. Hence, there is only one unique x such that $v + 3x = w$.

3 | 1.C

Axler, in his infinite wisdom, has crammed everything that's interesting to note in Chapter 1.c.

3.1 | Things of Note

3.1.1 | Subspaces

(Woo hoo!)

A subset $U \subset V$ is called a "subspace of V" if U is also a vector space using the same addition and scalar multiplication operators.

1. Checking for Subspaces Check for three conditions:

   For $U \subset V$

   - **Additive Identity** $0 \in U$. (also could be defined as "set is nonempty", b/c if nonempty, and its closed under scalar multiplication, multiplying any element by 0 will do the trick. But often showing 0 is in it is actually simpler.)
   - **Closed Under Addition** $u, w \in U$ implies $u + w \in U$
   - **Closed Under Scalar Multiplication** $a \in \mathbb{F}$ and $u \in U$ implies $au \in U$

3.1.2 | Summing Subsets

Suppose $U_1, \dots, U_m$ are subsets of V. The "sum" of the subsets ($U_1 + \dots + U_m$) is the set of all possible sums of elements in $U_1, , U_m$. That is:

$$U_1 + \dots + U_m = \{u_1 + \dots + u_m : u_1 \in U_1, \dots, u_m \in U_m\} \tag{37}$$

1. Properties of the Sums of Subspaces Suppose $U_1, \dots, U_m$ are subspaces of V. $U_1 + \dots + U_m$ is the smallest subspace of V containing all of $U_1, \dots, U_m$.

3.1.3 | Direct Sum

Suppose $U_1, \dots, U_m$ are subspaces of V

- Sum $U_1+\dots+U_m$ is a direct sum if $U_1+\dots+U_m$ can be only written only one way as a sum $u_1+\dots+u_m$
- Direct sum is noted as $U_1 \oplus \dots \oplus U_m$

"Sums is the union, direct sums is the disjoint union".

1. Checking for Direct Sums Suppose $U_1, \dots, U_m$ are subspaces of V. $U_1 + \dots + U_m$ is a direct sum iff the only way to write 0 as a sum $u_1 + \dots + u_m$ is by taking each u_j equaling to 0. This could be implied from the definition of a direct sum: that there is only one way to write a 0 as $u_1 + \dots + u_m$, and being closed scalar multiplication means that you could multiply 0 to each subspace individually and they still have to add up to 0.
2. Direct Sum of Two Subspaces Suppose U, W are subspaces of V.

   $U + W$ is a direct sum iff $U \cap W = \{0\}$.

3.2 | In-Text Excercises

3.2.1 | Summing Subspaces, an Example

Suppose $U = \{(x, x, y, y) \in \mathbb{F}^4 : x, y \in \mathbb{F}\}$ and $W = \{(x, x, x, y) \in \mathbb{F}^4 : x, y \in \mathbb{F}\}$. Then:

$$U + W = \{(x, x, y, z) \in \mathbb{F}^4 : x, y, z \in \mathbb{F}\} \tag{38}$$

We verify this by writing out the sum.

$$\begin{aligned} U + W &= \{(x_u, x_u, y_u, y_u) + (x_w, x_w, x_w, y_w) : x_u, y_u \in U, x_w, y_w \in W\} && (39) \\ &= \{(x_u + x_w, x_u + x_w, y_u + x_w, y_u + y_w) : x_u, y_u \in U, x_w, y_w \in W\} && (40) \\ &= \{\left(\underbrace{x}_{x_u+x_w}, x, \underbrace{y}_{y_u+x_w}, \underbrace{z}_{y_u+y_w}\right) : x, y, z \in \mathbb{F}\} && (41) \end{aligned}$$

3.2.2 | Verify sums equal to spaces

Suppose U, W are subspaces of $\mathbb{F}^3$.

$$\begin{aligned} U &= \{(x, y, 0) \in \mathbb{F}^3 : x, y \in \mathbb{F}\} && (42) \\ W &= \{(0, 0, z) \in \mathbb{F}^3 : z \in \mathbb{F}\} && (43) \end{aligned}$$

Verify $\mathbb{F}^3 = U \oplus W$

$$U + W = \{(x, y, z) \in F^3 : x, y, z \in F\} = \mathbb{F}^3 \tag{44}$$

:tada:?

3.3 | **Actual Excercises**

1: For each of the following subsets of $\mathbb{F}^3$, determine if it is a subspace of $\mathbb{F}^3$.

$$U = \{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 + 2x_2 + 3x_3 = 0\} \tag{45}$$

We could see that $(0, 0, 0) \in U$.

Let $u_1 = (x_1, x_2, \frac{-x_1-2x_2}{3}), u_2 = (x_3, x_4, \frac{-x_3-2x_4}{3}).u_1, u_2 \in U$.

$u_1 + u_2 = (x_1 + x_3, x_2 + x_4, \frac{-(x_1+x_3)-2(x_2+x_4)}{3})$. Define $x_1 + x_3 = x$, $x_2 + x_4 = y$. Therefore: $u_1 + u_2 = (x, y, \frac{-x-2y}{3})$.

$x + 2y - x - 2y = 0$.

Therefore, U is closed under addition.

Let $u_1 = (x_1, x_2, \frac{-x_1-2x_2}{3})$. We scale each element by scalar factor λ.

$\lambda u_1 = (\lambda x_1, \lambda x_2, \lambda\frac{-x_1-2x_2}{3})$.

$\lambda x_1 + 2\lambda x_2 - \lambda x_1 - 2\lambda x_2 = 0$. Therefore, U is closed under scalar multiplication.

Therefore, it is a subspace of $\mathbb{F}^3$.

10: Suppose that U_1 and U_2 are subspaces of V. Prove that the intersection $U_1 \cap U_2$ is a subspace of V.

Given U_1 and U_2 are both subspaces, 0 must be in both subsets hence it must be in their intersection.

Let $u_1 = (x_1, y_1, z_1)$. Let $u_2 = (x_2, y_2, z_2)$. Finally, let $u_1, u_2 \in U_1 \cap U_2$. By this last fact, we could derive the fact that $u1, u2 \in U_1, U_2$.

As U_1 is a subspace, U_1 closed under addition. Since $u_1, u_2 \in U_1$, $u_1 + u_2 \in U_1$. As U_2 is a subspace, U_2 closed under addition. Since $u_1, u_2 \in U_2$, $u_1 + u_2 \in U_2$.

As $u_1 + u_2 \in U_1, U_2$, it is additionally $u_1 + u_2 \in U_1 \cap U_2$. $U_1 \cap U_2$ is therefore closed under addition.

By the same token….

As U_1 is a subspace, U_1 is closed under scalar multiplication. Since $u_1 \in U_1$, $\lambda u_1 \in U_1$. As U_2 is a subspace, U_2 is closed under scalar multiplication. Since $u_1 \in U_2$, $\lambda u_1 \in U_2$.

Therefore, as $\lambda u_1 \in U_1, U_2$, it is additionally true that $u_1 \in U_1 \cap U_2$, it is therefore closed under multiplication.

4 | **2.A**

What's with the balancing of these chapters? Like Span and Linear Independence is squished in one, but then Bases gets a whole chapter and so does dimension.

4.1 | **Things of Note**

- Lists of vectors are usually denoted as a list without parentheses

4.1.1 | **Linear Combination**

A linear combination of a list is a sum of vectors in the form:

$$a_1v_1 + \cdots + a_mv_m \tag{46}$$

where, $a_1, \ldots, a_m \in \mathbb{F}$.

1. Verifying Linear Combinations You could note that a linear combination is actually just a linear system of equations. That:

   To check (x, y, z) is a linear combination of (v_1, v_2, v_3), (w_1, w_2, w_3), figure if there exists a pair (a_1, a_2) such that…

   $$\begin{cases} x = a_1v_1 + a_2w_1 \\ y = a_1v_2 + a_2w_2 \\ z = a_1v_3 + a_2w_3 \end{cases} \tag{47}$$

   If so, there exists the requisite scalars such that the linear combination is possible.

4.1.2 | **Linear Span**

The span is the set of all linear combinations of a list of vector. That:

$$span(v_1, \ldots, v_m) = \{a_1v_1 + \cdots + a_mv_m : a_1, \ldots, a_m \in \mathbb{F}\} \tag{48}$$

The span of an empty list of vectors $()$ is defined to be $\{0\}$. Due to the fact that any subspace must be closed under scalar multiplication + addition, the span of a list of vectors in V is the smallest subspace of V containing all the vectors in that list.

1. Spanning List If $span(v_1, \ldots, v_m) = V$, we say that $v_1, \ldots, v_m$ spans V.
2. Finite-Dimensional Vector Space A vector space is ”finite-dimensional” if some list of vector in it could span the space. i.e. there exists a list of vectors that span the space. Otherwise, it is called ”infinite dimensional”.

   Every subspace of a finite dimentional vector space is finite dimentional.

4.1.3 | **Polynomials**

We quickly recap the definition of a polynomial. A function $p : \mathbb{F} \to \mathbb{F}$ is called a polynomial if $\exists a_0, \ldots, a_m \in \mathbb{F}$ s.t.

$$p(z) = a_0 + a_1z + a_2z^2 + \cdots + a_mz^m, \forall z \in \mathbb{F} \tag{49}$$

By the same token, $\mathcal{P}($

1. Degree of a Polynomial A polynomial $p \in \mathcal{P}(\mathbb{F})$ has degree m if $\exists a_0, a_1, \ldots, a_m \in \mathbb{F}$ with $a_m \neq 0$ such that…

   $$p(z) = a_0 + a_1z + \cdots + a_mz^m, \forall z \in \mathbb{F} \tag{50}$$

   If p has degree m, we write $deg\ p = m$. A zero-polynomial is said to have degree $-\infty$.

2. $\mathcal{P}_m(\mathbb{F})$ For a non-negative integer m, $\mathcal{P}_m(\mathbb{F})$ is defined as the set of all polynomials with coefficients in $\mathbb{F}$ and degree at most m.

   You will therefore notice, then, that:

$$\mathcal{P}_m(\mathbb{F}) = span(1, z, \dots, z^m) \tag{51}$$

4.1.4 | Linear Independence

Linear independence exists by a only *unique* choice of scalars $a_1, \dots, a_m$ to form any given $v \in span(v_1, \dots, v_m)$. This could also be stated as that:

that the only choice $a_1, \dots, a_m \in \mathbb{F}$ that makes $a_1v_1 + \dots + a_mv_m = 0$ is the "trivial" case whereby $a_1 = \dots = a_m = 0$.

The empty set is also defined as linearly independent.

4.1.5 | Linear Dependence

A list of vectors in V is linearly dependent if its not linearly independent.

A list $v_1, \dots, v_m$ of vectors is linearly dependent if there exist $a_1, \dots, a_m \in \mathbb{F}$ that's not all 0, such that $a_1v_1 + \dots + a_mv_m = 0$.

1. Linear Dependence Lemma Suppose $v_1, \dots, v_m$ is an linearly dependent list in V. Then, exists a $j \in \{1, 2, \dots, m\}$ such that:

   (a) $v_j \in span(v_1, \dots, v_{j-1})$

   (b) If the j^{th} term is removed, the span remains the same

2. Lengths of Lin. Indp List In a finite-dimentional vector space, the length of every linearly independent list is less than or equal to the length of every spanning list.

4.2 | In-Text Excercises

4.2.1 | $\mathcal{P}(\mathbb{F})$ is a subspace of $\mathbb{F}^{\mathbb{F}}$

Zero exists in the set as, for a polynomial, $(a_0, \dots, a_m) = (0, \dots, 0)$ would create a function $f : \mathbb{F} \to 0$.

Due to commutativity, we could group and factor-out input-variable z such that the sum of two polynomials become $(a_{0a} + a_{0b}) + (a_{1a} + a_{1b})z + \dots + (a_{ma} + a_{mb})z^m$, which would be another polynomial. This would be closed under addition.

Due to distribution, a scalar λ multiplied to a polynomial would just scale every value by λ resulting in $\lambda a_0 + \lambda a_1 z + \dots + \lambda a_m z^m$, which would be another polynomial. This would be closed under scalar multiplication.

Therefore, the set of polynomials in $\mathbb{F}$ is as subspace.

4.2.2 | Show that $\mathcal{P}(\mathbb{F})$ in infinite-dimensional

Any list $U \subset \mathcal{P}(\mathbb{F})$ would contain the highest-degree polynomial with degree m. Therefore, the element $z^{m+1} \in \mathcal{P}(\mathbb{F})$ would not be in the span of the list: making the list not span the entire space. Therefore, there could not be a list that spans $\mathcal{P}(\mathbb{F})$, making it infinite-dimentional.