0 | **meta**

This homework took ~3h whilst discussing with peers… I really need to practice this type of algebra.

1 | $\int \frac{\sqrt{x-1}}{x}dx$

Let $u = \sqrt{x-1}$, $du = \frac{1}{2\sqrt{x-1}}dx$

$$\int \frac{\sqrt{x-1}}{x}dx = \int \frac{u}{u^2+1}2u\,du$$
$$= 2\int \frac{(u^2+1)-1}{u^2+1}du$$
$$= 2\int \frac{u^2+1}{u^2+1} + \frac{-1}{u^2+1}du$$
$$= 2\int 1\,du - \frac{1}{u^2+1} + C$$
$$= 2\int 1\,du - \tan^- u + C$$
$$= 2u - \tan^- u + C$$
$$= \boxed{2\sqrt{x-1} - \tan^- \left(\sqrt{x-1}\right) + C}$$

Polynomial long division?
When you have a square root with a sum/difference inside, there's not much you can do. So, your best bet is to substitute either the stuff inside the root as u or the entire radical as u.

2 | $\int \frac{x^2}{x^2+1}dx$

Let $u = x^2 + 1$, $du = 2x\,dx$

$$\int \frac{x^3}{x^2+1}dx = \frac{1}{2}\int \frac{u-1}{u}du$$
$$= \frac{1}{2}\left(u - \int \frac{1}{u}du\right) + C$$
$$= \frac{1}{2}\left(u - \ln u\right) + C$$
$$= \boxed{\frac{1}{2}\left(x^2+1 - \ln(x^2+1)\right) + C}$$

JUST SPLIT THE FRACTION
AND LOOK FOR TANsr X

3 | $\int \frac{x-4}{x^2}dx$

$$\int \frac{x-4}{x^2}dx = \int \frac{x}{x^2}\frac{4}{x^2}dx$$
$$= \int \frac{1}{x}dx + 4\int \frac{1}{x^2}dx$$
$$= \boxed{\ln x - \frac{4}{x} + C}$$

4 | $\int (x+1)e^{x^2+2x}dx$

Let $u = x^2 + 2x$, $du = x + 1dx$

$$\int (x+1)e^{x^2+2x}dx = \frac{1}{2}\int e^u du$$
$$= \frac{1}{2}e^u$$
$$= \boxed{\frac{1}{2}e^{x^2+2x} + C}$$

5 | $\int \tan^2 x + 1dx$

$$\int \tan^2 x + 1dx = \int \sec^2 x - 1 + 1dx$$
$$= \int \sec^2 xdx$$

Let $u = x$, $du = dx$

$$= \int \sec^2 udu$$
$$= \tan u + C$$
$$= \boxed{\tan x + C}$$

6 | $\int \frac{6x^2-4}{x}dx$

$$\int \frac{6x^2-4}{x}dx = \int \frac{6x^2}{x}dx - 4\int \frac{1}{x}dx$$
$$= \int 3xdx - 4\ln|x| + C$$
$$= \boxed{3x^2 - 4\ln|x| + C}$$

7 | $\int \frac{e^x - 1}{e^x} dx$

$$\int \frac{e^x - 1}{e^x} dx = \int 1 - \frac{1}{e^x} dx$$
$$= \int 1 - e^{-x} dx$$
$$= x + e^{-x} + C$$
$$= \boxed{e^{-x} + x + C}$$

8 | $\int \frac{\sec^2 x}{\csc x} \sin x dx$

$$\int \frac{\sec^2 x}{\csc x} \sin x dx = \int \tan^2 x dx$$
$$= \int \sec^2 x - 1 dx$$
$$= \int \sec^2 x dx - \int 1 dx$$
$$= \boxed{\tan x - x + C}$$

9 | $\int \sin x \cos x dx$

Let $u = \sin x$, then $du = \cos x dx$

$$\int \sin x \cos x dx = \int u du$$
$$= \frac{1}{2} u^2$$
$$= \boxed{\frac{1}{2} \sin^2 x + C}$$

Product of $\sin $ and $\cos $, so we can use a double angle formula

$$\int \sin x \cos x dx = \int \frac{1}{2} \sin 2x dx$$
$$= \frac{1}{2} \int \sin 2x dx$$
$$= -\frac{1}{4} \cos 2x$$

10 | $\int \frac{e^{2\ln\sin x}+e^{2\ln\cos x}}{e^{2\ln\tan x}+e^{2\ln 1}}dx$

$$\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}}dx = \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1}dx$$

$$= \int \frac{1}{\tan^2 x + 1}dx$$

$$= \int \frac{1}{\sec^2 x}dx$$

$$= \int \cos^2 x\, dx$$

$$= \int \frac{1}{2}(\cos 2x + 1)dx$$

$$= \frac{1}{2}\int \cos 2x + 1\, dx$$

$$= \frac{1}{2}\left(\int \cos 2x\, dx + \int 1\, dx\right)$$

Let $u = 2x$, $du = 2dx$

$$\frac{1}{2}\int \cos 2x\, dx + \frac{x}{2} + C = \frac{1}{4}\int \cos u\, du + \frac{x}{2} + C$$

$$= \frac{1}{4}\sin u + \frac{x}{2} + C$$

$$= \boxed{\frac{1}{4}\sin 2x + \frac{x}{2} + C}$$

11 | $\int \frac{\sec x \tan x}{1+\sec^2 x}dx$

Let $u = \sec x$, $du = \sec x \tan x\, dx$

$$\int \frac{\sec x \tan x}{1 + \sec^2 x}dx = \int \frac{du}{1 + u^2}dx$$

$$= \int \frac{1}{1 + u^2}du$$

$$= \tan^- u + C$$

$$= \boxed{\tan^- \sec x + C}$$

12 | $\int x^2 e^{x^3} dx$

Let $u = x^3$, $du = 3x^2 dx$

$$\int x^2 e^{x^3} dx = \frac{1}{3} \int e^u \, du$$
$$= \frac{1}{3} e^u + C$$
$$= \boxed{\frac{1}{3} e^{x^3} + C}$$

13 | $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int e^u \, du$$
$$= 2e^u + C$$
$$= \boxed{2e^{\sqrt{x}} + C}$$