

Desmos graphs

4 | witch of Maria Agnesi

Let B be the center of the orange circle with radius a , let D be the closest point to C on the x-axis, and let Q be the closest point to A on the y-axis.

4.1 | $x(t)$

$$\begin{aligned}\tan \theta &= \frac{\overline{CD}}{\overline{OD}} \\ \cot \theta &= \frac{\overline{OD}}{\overline{CD}} \\ \overline{CD} \cot \theta &= \overline{OD} \\ 2a \cot \theta &= x\end{aligned}$$

4.2 | $y(t)$

First, note that the distances

$$\begin{aligned}\overline{AB} &= \overline{BO} = a \\ \overline{PD} &= \overline{QO} = \overline{QB} + \overline{BO} = \overline{QB} + a = y\end{aligned}$$

Using some geometry:

$$\begin{aligned}\angle AOB &= 90 - \theta \\ \angle OAB &= 90 - \theta \quad (\text{isocelase triangle}) \\ \angle ABO &= 2\theta\end{aligned}$$

Which implies:

$$\begin{aligned}\overline{QB} &= -a \cos(2\theta) \\ &= -a (1 - 2 \sin^2 \theta) \\ &= -a + 2a \sin^2 \theta\end{aligned}$$

By going back to the original distance relations, we have

$$\begin{aligned}y &= \overline{QB} + a \\ &= \cancel{a} - \cancel{a} + 2a \sin^2 \theta = 2a \sin^2 \theta\end{aligned}$$

5 | parameterization of an ellipse

<https://www.desmos.com/calculator/wcu1okhjyz>

$$\begin{aligned}x(t) &= a\sqrt{c} \sin t \\ y(t) &= b\sqrt{c} \cos t\end{aligned}$$

6 | mystery curve

it's just $(a \cos t, b \sin t)$ because of how the right triangle aligns with the axes.

8 | swallowtail catastrophe curves

Defined by

$$\begin{aligned}x &= 2ct - 4t^3 \\ y &= -ct^2 + 3t^4\end{aligned}$$

8.1 | features

8.1.1 | approaches a parabola-like shape above the y-axis

8.1.2 | approaches a parabola-like shape below the x-axis if $c > 0$

8.1.3 | has a cross-over in a triangle shape

1. gets bigger when c gets bigger

8.1.4 | it looks like a dorito that scales with the value of c

1. as c approaches zero from the positive direction, the swallowtail gets smaller

9 | Lissajous Figures

Defined by

$$\begin{aligned}x &= a \sin(nt) \\ y &= b \cos t\end{aligned}$$

9.1 | features

9.1.1 | spring-like coil shape (almost like standing waves) with tighter "loops" at the ends

9.1.2 | a, b control the size of the coil (default $-1 \leq x, y \leq 1$ because of range of \sin, \cos)

9.1.3 | number of y-intercepts is $n + 1$ except in the degenerate cases $n \leq 0$

11 | cycloid

Suppose instead that the circle slides along the surface and the point rotates at one radian per radian traveled. Let's start with the radian rotation...

$$\begin{aligned}x(t) &= r \sin t \\ y(t) &= r + r \cos t\end{aligned}$$

Then, we just have to move the origin as well:

$$\begin{aligned}x(t) &= t + r \sin t \\ y(t) &= r + r \cos t\end{aligned}$$

12 | first order derivative

I think I did not come to this conclusion on my own on 30 Aug. because I didn't realize we could assume we had $y(x)$.

$$\begin{aligned}y &= y(x(t)) \\ \frac{dy}{dt} &= y'(x(t))x'(t) = \frac{dy}{dx} \frac{dx}{dt} \quad (\text{chain rule}) \\ \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}}\end{aligned}$$

13 | second order derivative

$$\begin{aligned}x &= f(t) \\ y &= g(t) = g(f(t)) \\ \frac{dy}{dt} &= \frac{dy}{dx} \frac{dx}{dt} \\ \frac{d^2y}{dt^2} &= \frac{dy}{dx} \frac{d}{dt} \frac{dx}{dt} + \frac{dx}{dt} \frac{d}{dt} \frac{dy}{dx} \\ &= \frac{dy}{dx} \frac{d^2x}{dt^2} + \frac{dx}{dt} \frac{d^2y}{dx dt} \\ \frac{d^2x}{dt^2} &= \frac{d}{dt} \frac{dx}{dt}\end{aligned}$$

um... that seems like it didn't actually do anything. I'm kind of stuck... lets try working backwards:

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x})^3} \\ &= \dot{x} \frac{d}{dx} \left(\frac{\dot{y}}{\dot{x}} \right)\end{aligned}$$

why should the \dot{x} in the bottom be cubed?

13.1 | in class review

$$\begin{aligned}
 \frac{d}{dx} \frac{dy}{dx} &= \frac{d}{dx} \left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \right) = \frac{d}{dx} u = \frac{\frac{du}{dt}}{\frac{dx}{dt}} \\
 &= \frac{\frac{d}{dt} u}{\frac{dx}{dt}} = \frac{\frac{d}{dt} \dot{y}}{\dot{x}} \\
 &= \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{\dot{x}^2}
 \end{aligned}$$