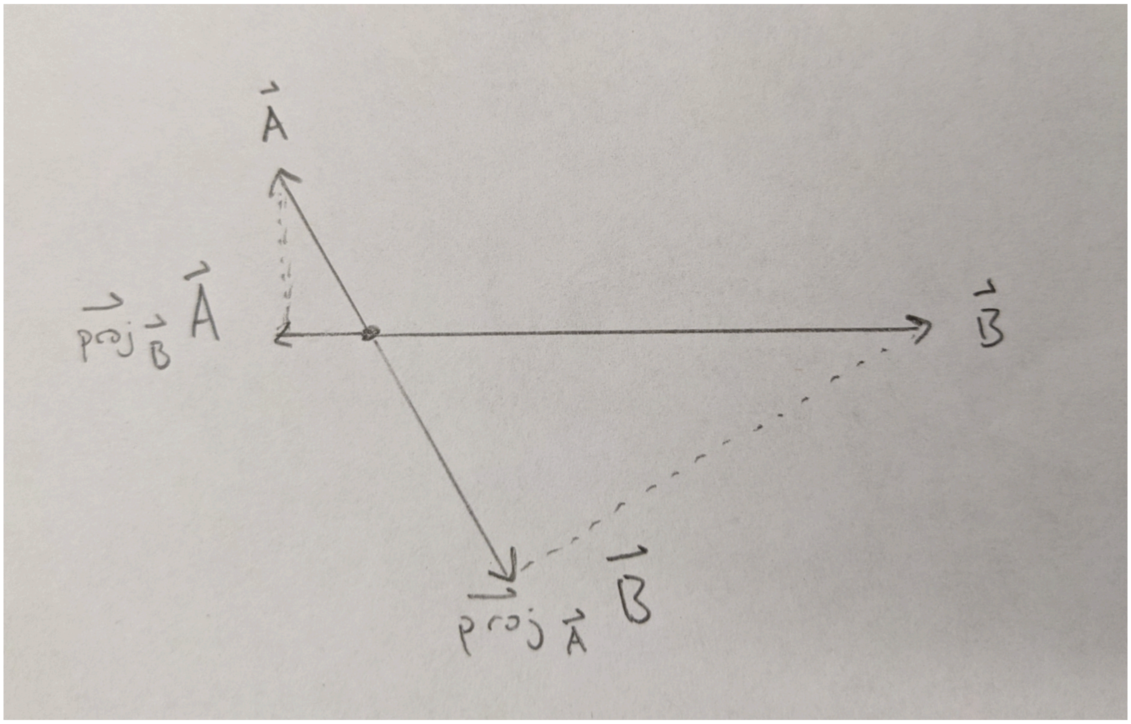
1 | Problem 1:

1.1 | 1.1)



1.2 | 1.2)

$comp_{\vec{A}}\vec{B} = |\vec{B}|\cos(\theta) = 6\cos(\frac{2\pi}{3}) = -3$ $comp_{\vec{B}}\vec{A} = |\vec{A}|\cos(\theta) = 2\cos(\frac{2\pi}{3}) = -1$

1.3 | 1.3)

$\vec{A}\cdot\vec{B}$
$= |\vec{A}||\vec{B}|\cos(\theta) = 6\cdot 2\cdot(-0.5)$
$= -6$

2 | Problem 2:

$comp_{\vec{A}}\vec{B} = |\vec{B}|\cos(\theta)$
$= |\vec{B}|\cos(\theta) \times \frac{|\vec{A}|}{|\vec{A}|}$
$= \frac{|\vec{A}||\vec{B}|\cos(\theta)}{|\vec{A}|}$
$= \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}$

3 | Problem 3:

The projection of $\vec{B}$ onto $\vec{A}$ would be the $\vec{A}$ component of $\vec{B}$ times the unit vector of $\vec{A}$ to give the component a direction and make it a vector: $\vec{proj}_{\vec{A}}\vec{B} = comp_{\vec{A}}\vec{B} \cdot \hat{A}$
$= |\vec{B}|\cos(\theta) \cdot \frac{\vec{A}}{|\vec{A}|}$
$= \frac{|\vec{B}|\cos(\theta)}{|\vec{A}|}\vec{A}$

4 | Problem 4:

The vector component of $\tilde{A}$ onto the vector perpendicular to $\tilde{B}$ is the $\vec{proj}_{\perp\tilde{B}}\tilde{A}$, where $\perp\ \tilde{B}$ is a vector perpendicular to $\tilde{B}$. If we set $\tilde{B}$ as the x axis, then the y axis would be $\perp\ \tilde{B}$ and the "y component of A" would be $\vec{proj}_{\perp\tilde{B}}\tilde{A}$. Thus:

$\tilde{A}_{\perp\tilde{B}} = \vec{proj}_{\perp\tilde{B}}\tilde{A} = \tilde{A}\sin(\theta)$ where θ is the angle between $\tilde{A}$ and $\tilde{B}$.

To prove that this is perpendicular we can take the dot product of $\tilde{A}_{\perp\tilde{B}}$ and $\tilde{B}$:

$|\tilde{A}_{\perp\tilde{B}}||\tilde{B}|\cos(\theta_1) = |\tilde{A}_{\perp\tilde{B}}||\tilde{B}|\cos(\frac{\pi}{2}) = |\tilde{A}_{\perp\tilde{B}}||\tilde{B}| \cdot 0 = 0$

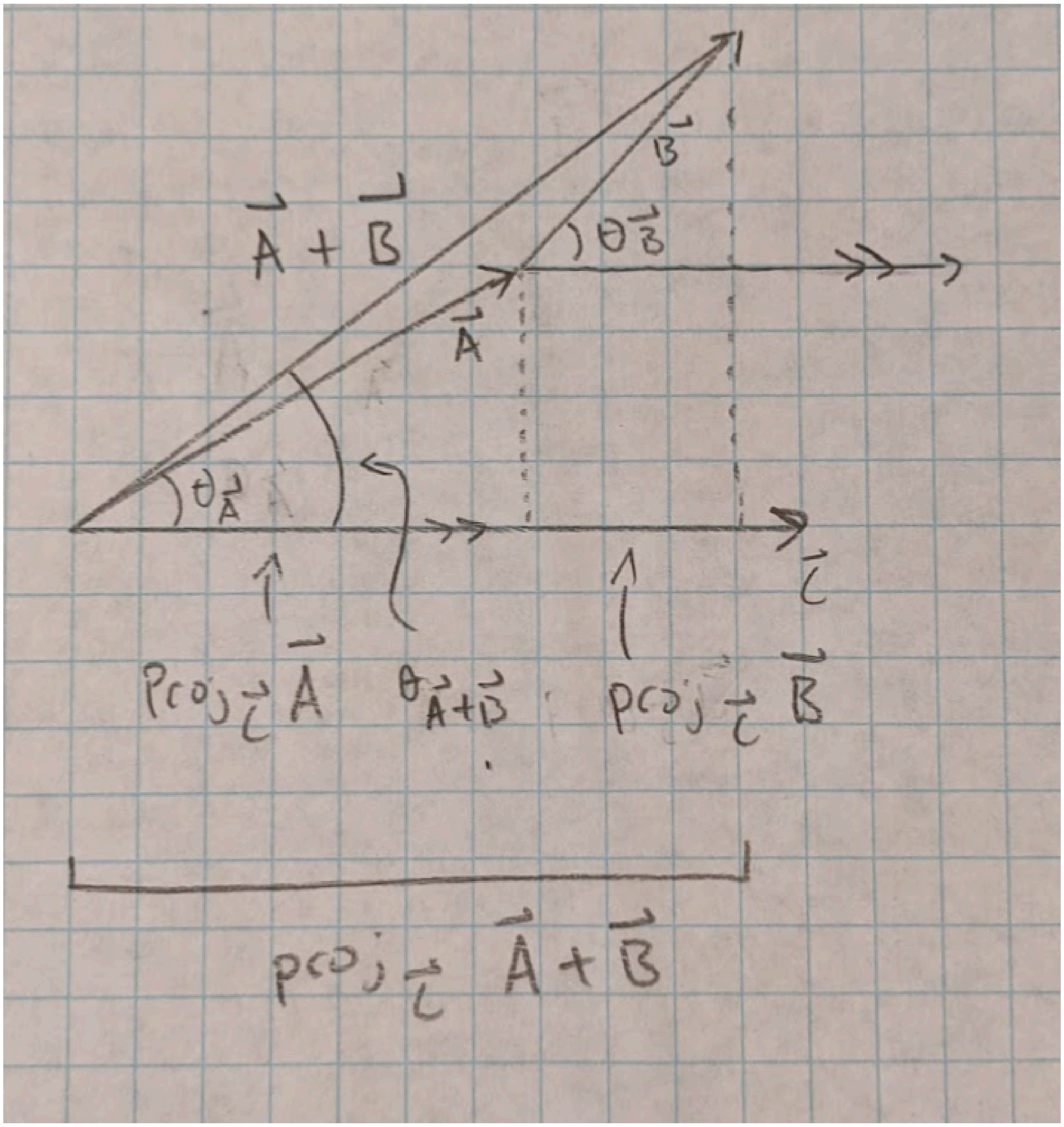
5 | Problem 5:

The dot product is defined as:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos(\theta)$$

is this case we can solve for theta:
$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos(\theta)$
$\Rightarrow \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \cos(\theta)$
$\Rightarrow \theta = \cos^{-}(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|})$
because we do not know theta we can use another definition of the dot product to get the numorator of the fraction:
$\Rightarrow \theta = \cos^{-}(\frac{A_xB_x + A_yB_y + A_zB_z}{|\vec{A}||\vec{B}|})$
Therefore the angle between the two vectors $(-1, 2, -2)$ and $(-3, 1, 2)$ is the following:
$\theta = \cos^{-}(\frac{3+2-4}{\sqrt{1+4+4} \cdot \sqrt{9+1+4}})$
$\Rightarrow \theta = \cos^{-}(\frac{1}{3\sqrt{14}})$
$\Rightarrow \theta \approx 1.48159$

6 | Problem 6:



Looking at the diagram above we see that:

$proj_{\vec{C}}\vec{A} + proj_{\vec{C}}\vec{B} = proj\vec{C}(\vec{A}+\vec{B})$
$\Rightarrow |\vec{C}|proj_{\vec{C}}\vec{A} + |\vec{C}|proj_{\vec{C}}\vec{B} = |\vec{C}|proj_{\vec{C}}(\vec{A}+\vec{B})$
$\Rightarrow |\vec{C}||\vec{A}|\cos(\theta_{\vec{A}}) + |\vec{C}||\vec{B}|\cos(\theta_{\vec{B}}) = |\vec{C}||\vec{A}+\vec{B}|\cos(\theta_{\vec{A}+\vec{B}})$
$\Rightarrow \vec{C}\cdot\vec{A} + \vec{C}\cdot\vec{B} = \vec{C}\cdot(\vec{A}+\vec{B})$

thus the dot product is distributive

this scales to the third dimention, because in a sense the diagram is the projection of 3D vectors onto a plane.

7 | **Problem 7:**

$\vec{A} \cdot \vec{B} = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \cdot (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$
$= A_x\hat{\imath} \cdot B_x\hat{\imath} + A_x\hat{\imath} \cdot B_y\hat{\jmath} + A_x\hat{\imath} \cdot B_z\hat{k} + A_y\hat{\jmath} \cdot B_x\hat{\imath} + A_y\hat{\jmath} \cdot B_y\hat{\jmath} + A_y\hat{\jmath} \cdot B_z\hat{k} + A_z\hat{k} \cdot B_x\hat{\imath} + A_z\hat{k} \cdot B_y\hat{\jmath} + A_z\hat{k} \cdot B_z\hat{k}$

the dot product between $A_x\hat{\imath} \cdot B_x\hat{\imath}$ would be: $|A_x||B_x|$ cos(0), because the angle between $\hat{\imath}$ and $\hat{\imath}$ is 0 (they have the same direction), cos$(0) = 1$, and thus the dot product equals A_xB_x. However, the dot product between and two unit vectors that are not the same would yield a theta of $\frac{\pi}{2}$, which means cos$(\frac{\pi}{2}) = 0$ and thus that term would equal zero. This can be generalized as, if the two unit vectors are the same then it will yield a term equal to the product of their two coefficients, and if the two unit vectors are different, then the resulting term would be equal to zero. Therefore:

$= A_xB_x + 0 + 0 + A_yB_y + 0 + 0 + A_zB_z + 0 + 0$
$= A_xB_x + A_yB_y + A_zB_z$