Desmos graphs

4 | witch of Maria Agnesi

Let B be the center of the orange circle with radius a, let D be the closest point to C on the x-axis, and let Q be the closest point to A on the y-axis.

4.1 | $x(t)$

$$\begin{aligned}
\tan\theta &= \frac{\overline{CD}}{\overline{OD}} \\
\cot\theta &= \frac{\overline{OD}}{\overline{CD}} \\
\overline{CD}\cot\theta &= \overline{OD} \\
2a\cot\theta &= x
\end{aligned}$$

4.2 | $y(t)$

First, note that the distances

$$\begin{aligned}
\overline{AB} &= \overline{BO} = a \\
\overline{PD} &= \overline{QO} = \overline{QB} + \overline{BO} = \overline{QB} + a = y
\end{aligned}$$

Using some geometry:

$$\begin{aligned}
\angle AOB &= 90 - \theta \\
\angle OAB &= 90 - \theta \qquad \text{(isocelese triangle)} \\
\angle ABO &= 2\theta
\end{aligned}$$

Which implies:

$$\begin{aligned}
\overline{QB} &= -a\cos(2\theta) \\
&= -a\left(1 - 2\sin^2\theta\right) \\
&= -a + 2a\sin^2\theta
\end{aligned}$$

(we need to take the negative cosine to get a positive length, because the angle is in the second quadrant and thus the cosine will be negative unless we flip it.)

By going back to the original distance relations, we have

$$\begin{aligned}
y &= \overline{QB} + a \\
&= \cancel{a - a} + 2a\sin^2\theta \quad = 2a\sin^2\theta
\end{aligned}$$

Click me: witch of of maria agnesi sketch

5 | parameterization of an elipse

https://www.desmos.com/calculator/wcu1okhjyz

$$x(t) = a\sqrt{c}\sin t$$
$$y(t) = b\sqrt{c}\cos t$$

6 | mystery curve

it's just $(a\cos t, b\sin t)$ because of how the right triangle aligns with the axes.

8 | swallowtail catastrophe curves

Defined by

$$x = 2ct - 4t^3$$
$$y = -ct^2 + 3t^4$$

8.1 | features

8.1.1 | approaches a parabola-like shape above the y-axis

8.1.2 | approaches a parabola-like shape below the x-axis if $c > 0$

8.1.3 | has a cross-over in a triangle shape

1. gets bigger when c gets bigger

8.1.4 | it looks like a dorito that scales with the value of c

1. as c approaches zero from the positive direction, the swollowtail gets smaller

9 | Lissajous Figures

Defined by

$$x = a\sin(nt)$$
$$y = b\cos t$$

9.1 | features

9.1.1 | spring-like coil shape (almost like standing waves) with tighter "loops" at the ends

9.1.2 | a, b **control the size of the coil (default** $-1 \leq x, y \leq 1$ **because of range of** sin, cos

9.1.3 | **number of y-intercepts is** $n+1$ **except in the degenerate cases** $n \leq 0$

11 | cycloid

Suppose instead that the circle slides along the surface and the point rotates at one radian per radian traveled. Let's start with the radian rotation…

$$\begin{aligned} x(t) &= \quad r \sin(t-\pi) \\ y(t) &= r + \quad r \cos(t-\pi) \end{aligned}$$

Then, we just have to move the origin as well:

$$\begin{aligned} x(t) &= rt + r \sin(t-\pi) \\ y(t) &= r + r \cos(t-\pi) \end{aligned}$$

12 | first order derivative

I think I did not come to this conclusion on my own on 30 Aug. because I didn't realize we could assume we had $y(x)$.

$$\begin{aligned} y &= y(x(t)) \\ \frac{dy}{dt} &= y'(x(t))x'(t) = \frac{dy}{dx}\frac{dx}{dt} \qquad \text{(chain rule)} \\ \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \end{aligned}$$

13 | second order derivative

$$\begin{aligned} x &= f(t) \\ y &= g(t) = g(f(t)) \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{dy}{dx}\frac{dx}{dt} \\ \frac{d^2y}{dt^2} &= \frac{dy}{dx}\frac{d}{dt}\frac{dx}{dt} + \frac{dx}{dt}\frac{d}{dt}\frac{dy}{dx} \\ &= \frac{dy}{dx}\frac{d^2x}{dt^2} + \frac{dx}{dt}\frac{d^2y}{dxdt(??)} \end{aligned}$$

13.1 | **in class review**

$$\begin{aligned}
\frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right) = \frac{d}{dx}u = \frac{\frac{du}{dt}}{\frac{dx}{dt}} \\
= \frac{\frac{d}{dt}u}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\frac{\dot{y}}{\dot{x}}}{\dot{x}} \\
= \frac{\frac{\dot{x}\ddot{y}-\dot{y}\ddot{x}}{\dot{x}^2}}{\dot{x}}
\end{aligned}$$