PS#13

Nueva Multivariable Calculus

FUN NEW CHALLENGE: Type this all up in LaTeX!!! This is how I want you to submit problem sets henceforth. Submit to Canvas both a PDF and a link to your Overleaf document (if you use Overleaf); sorry for requesting the redundancy. Be sure to include the original questions as well as your solutions; this will make it way easier for any reader (e.g., future you) to understand what's going on.

(Note that "challege" means "requirement.")

1. *(Given to me by Leonard.)* Consider the curve described by the following parametric equations:

$$\begin{aligned} x(t) &= t^2 \\ y(t) &= t^3 - ct \quad \text{(where } c \text{ is some real)} \end{aligned}$$

(a) Come up with a function—function*s*, rather—for this curve. In other words, convert it to $y =$ blah blah form ("rectangular").

(b) Try sketching it! (You can use a technological visualizer if/when you don't succeed.)

(c) Imagine you're a little particle on this curve, travelling from $t = 5$ to $t = 7$! What's your path (i.e., where do you start, where do you go, where do you end up)? And what's the total distance you travel?

2. You probably did something with "solids of revolution" in 1VC last year. Usually, finding the volumes of three-dimensional shapes is something we need higher-dimensional techniques for—but if we have a three-dimensional shape made by spinning it around an axis, you can use basic one-dimensional calculus techniques to find its area! (We'd say that such a shape has some sort of **rotational symmetry**, or maybe say that it's **radially symmetric**.)

So, consider the shape made by taking the function $f(x) = \frac{1}{x}$ from $x = 1$ out to ∞ and spinning it around the x-axis. Questions:

(a) What's the surface area of this shape?

(b) What's the volume of this shape? (You can go remind yourself how to do solids of revolution if you like/need.)

3. Read the (heavily redacted!) notes on "Differentiation in Higher Dimensions"!!! Then do #15 and #16, i.e., find all the first, second, and third partial derivatives (including the mixed ones) of:

- $f(x, y) = 7x + 2x^2y^3 + 10y^2$
- $f(x, y) = 3xy^3 + 8x^2y^4$