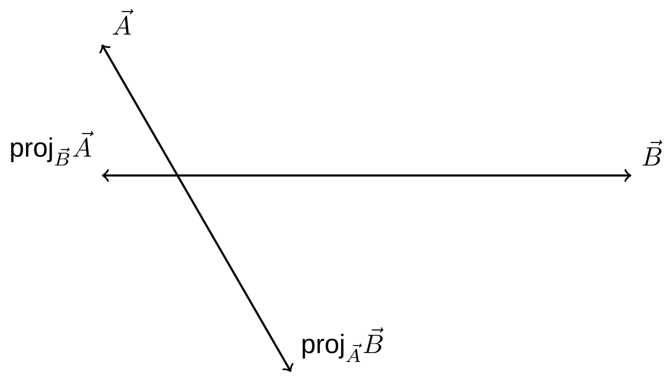
1 | vectors at an angle

1.1 | a sketch



1.2 | components

$$\text{comp}_{\vec{A}}\vec{B} = 6\cos 120$$
$$\text{comp}_{\vec{B}}\vec{A} = 2\cos 120$$

1.3 | dot product

$$\begin{aligned}\vec{A}\cdot\vec{B} &= |A||B|\cos\theta \\ &= 2\cdot 6\cdot\cos 120 = -6\end{aligned}$$

2 | proving expression for component

Lets redefine the coordinate axis so that $\vec{A}$ lies along the x-axis. Then,

$$\begin{aligned}\text{comp}_{\vec{A}}\vec{B} &= |\vec{B}|\cos\theta \\ &= \frac{|\vec{A}||\vec{B}|\cos\theta}{|\vec{A}|} \\ &= \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}\end{aligned}$$

3 | expression for projection

The projection is just a vector with length $\text{comp}_{\vec{A}}\vec{B}$ in the direction of $\vec{A}$.

$$\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}\right)\frac{\vec{A}}{|\vec{A}|}$$

4 | expression for perpendicular

The part of $\vec{A}$ that is perpendicular to $\vec{B}$ is just the whole vector minus the part that is parallel:

$$\begin{aligned}\vec{A}_{\perp\vec{B}} &= \vec{A} - \text{proj}_{\vec{B}}\vec{A} \\ &= \vec{A} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)\vec{B}\end{aligned}$$

Checking using the dot product:

$$\begin{aligned}\left(\vec{A} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)\vec{B}\right)\cdot\vec{B} &= \vec{A}\cdot\vec{B} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)\vec{B}\cdot\vec{B} \\ &= \vec{A}\cdot\vec{B} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)|\vec{B}|^2 \\ &= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B} \\ &= 0\end{aligned}$$

5 | find angle using dot product

Well, the dot product already includes the angle, so let's just solve for that

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

The angle between:

$$\begin{aligned}\theta &= \cos^{-}\left(\frac{3+2-4}{\sqrt{1^2+2^2+2^2}\sqrt{3^2+1^2+2^2}}\right) \\ &= \cos^{-}\left((3+2-4)/(3*\sqrt{14})\right) = \cos^{-}(0.08908708) \quad \approx 84.8^\circ\end{aligned}$$

6 | problems 6-8

See other files in the Canvas comments.

9 | vector equation that passes through the points

The vectors are

$$\begin{aligned}\vec{a} &= \langle -1, 4, 1\rangle \\ \vec{b} &= \langle 2, -5, -3\rangle\end{aligned}$$

Let's choose

$$\vec{r}(t) = \vec{p} + \vec{v}t$$

and make sure that $\vec{r}(0) = \vec{a}$, and $\vec{r}(1) = \vec{b}$. We can do this by setting

$$\vec{p} = \vec{a} = \langle -1, 4, 1 \rangle$$
$$\vec{v} = \vec{b} - \vec{a} = \langle 3, -9, -4 \rangle$$

Thus,

$$\boxed{\vec{r}(t) = \langle -1, 4, 1 \rangle + \langle 3, -9, -4 \rangle t}$$

This way,

$$\vec{r}(0) = \vec{p} = \vec{a}$$
$$\vec{r}(1) = \vec{p} + \vec{v} = \vec{a} + (\vec{b} - \vec{a}) = \vec{b}$$