1 | Row Reduced Echelon Form

Null space is the same (because algebra). Then turn it into a system of equations and use those equations to find the null space.

2 | Factoring a vector

Say we have $\begin{pmatrix} -2x_3 - 4x_4 \\ -4x_3 - 7x_4 \\ x_3 \\ x_4 \end{pmatrix}$. Then you can write it as the linear combination

$$\begin{pmatrix} -2x_3 \\ -4x_3 \\ x_3 \\ 0 \end{pmatrix} + \begin{pmatrix} -4x_4 \\ -7x_4 \\ 0 \\ x_4 \end{pmatrix} = x_3 \begin{pmatrix} -2 \\ -4 \\ 1 \\ 0 \end{pmatrix} + x_4 \begin{pmatrix} -4 \\ -7 \\ 0 \\ 1 \end{pmatrix}$$

3 | #icr 3.C icr

3.1 | Matrix Definition

Old news (but lots of subscripts)

3.2 | Making a matrix from a map

Based on maps being uniquely determined

3.3 | Matrix addition and scalar multiplication

Not news

3.4 | The matrix for the derivative map (finite)

$$T \in \mathcal{L}\left(\mathcal{P}_5\left(\mathbb{R}\right), \mathcal{P}_4\left(\mathbb{R}\right)\right)$$

Start with standard bases: $\mathcal{P}_5 \to 1, x, x^2, x^3, x^4, x^5$, $\mathcal{P}_4 \to 1, x, x^2, x^3, x^4$ Now lets define the map:

$$\begin{aligned} T1 &= 0 \\ Tx &= 1 \\ Tx^2 &= 2x \\ Tx^3 &= 3x^2 \\ Tx^4 &= 4x^3 \\ Tx^5 &= 5x^4 \end{aligned}$$

And then we write each output as a linear combo of the basis of $\mathcal{P}_4$ then we can define the matrix:

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix}$$

Note that the matrix is 5×6 because we are going from dimension $6 \to 5$ (and the second dimension gets "consumed" in the multiplication)

3.5 | **Axler3.40 dimension of the matrix vector space** icr

Put a one in every location which forms a basis.

3.6 | **Axler3.49 column of matrix product equals matrix times column** icr

Makes sense if you draw it out.. basically a column in the product AC will have used all of A but only the one column in C.

$$(AC)_{\cdot,k} = A(C_{\cdot,k})$$

and

$$(AC)_{j,\cdot} = (A_{j,\cdot})C$$