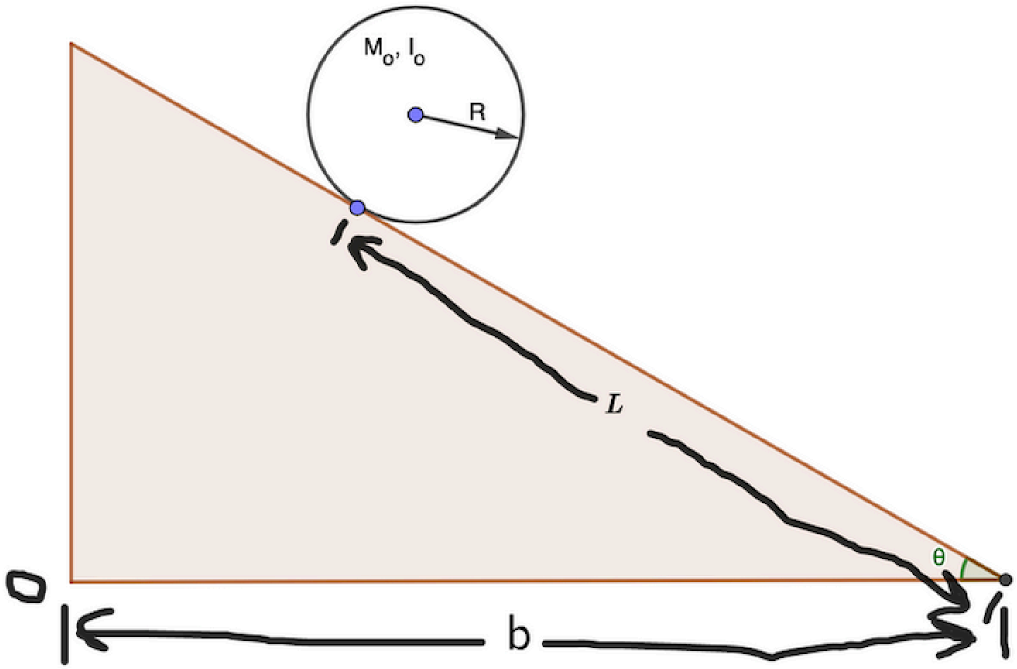

1 | Coefficient of Fiction

When given this scheme, we can understand that friction can be modeled by some:

$$F_f \leq \mu_s N \tag{1}$$

We will define a system via the picture exactly: up is $+y$, down is $-y$, right is $+x$, and left is $-x$. Let's first calculate the component of gravity perpendicular to the ramp—the only force causing the existence of a normal force.

We understand that force due to gravity on the object is:

$$F_g = -gM_0 \tag{2}$$

The component perpendicular to the ramp, then, would be:

$$N = -F_g \ cos\theta \tag{3}$$

Therefore, we can create an expression solving for the coefficient of friction:

$$F_f \leq \mu_s N \tag{4}$$
$$\Rightarrow F_f \leq -\mu_s F_g \ cos\theta \tag{5}$$
$$\Rightarrow F_f \leq \mu_s g M_0 \ cos\theta \tag{6}$$
$$\Rightarrow \mu_s \geq \frac{F_f}{gM_0 \ cos\theta} \tag{7}$$

Supplying the actual numerical values into the expressions, we get that:

$$\mu_s \geq \sim 0.236 \tag{8}$$

2 | Linear Acceleration

To figure the linear acceleration upon the system, we will need to create a summation of all forces. Fortunately, we realize that—due to the properties of the normal force—all components of the contact force perpendicular to the ramp cancel out.

Therefore, we simply have to figure the sum of all forces in the direction parallel to the ramp.

As the direction of motion is towards the $+x$ direction (rel. to ramp), we can see that the force of friction would therefore be applied towards the $-x$ direction.

Therefore:

$$F_{net_{rx}} = F_{gx} - F_f \tag{9}$$

The component of gravity parallel to the ramp would be:

$$F_{gx} = -F_g \sin\theta \tag{10}$$
$$= gM_0 \sin\theta \tag{11}$$

Taking these components together, then, we have that:

$$F_{net_{rx}} = gM_0 \sin\theta - F_f \tag{12}$$

Taking Newton's Second Law:

$$M_0 a_{rx} = gM_0 \sin\theta - F_f \tag{13}$$
$$\Rightarrow a_{rx} = \frac{gM_0 \sin\theta - F_f}{M_0} \tag{14}$$

Supplying the actual numerical values, we get that:

$$a_{rx} = 2.9\ \frac{m}{s^2} \tag{15}$$

The object, therefore, slides down the ramp at a linear acceleration of 2.9 metres a second squared.

3 | Angular Acceleration

To figure the angular acceleration we see that:

$$a = r\alpha \tag{16}$$

that is, given α is a value measured in radians per second squared, multiplying by r would be the acceleration over some distance traveled (just like how $r\theta$ is the circumference.)

Therefore:

$$a_{rx} = R\alpha \tag{17}$$
$$\Rightarrow \alpha = \frac{a_{rx}}{R} \tag{18}$$

Supplying the numerical values:

$$\alpha = 5.8\frac{rad}{s^2} \tag{19}$$

The object, therefore, has an angular acceleration of 5.8 radians per second squared.

4 | Coefficient of Friction

The coefficient of friction is static as the object, while its rolling down the ramp, never actually maintains content and slide the ramp. This results in no "sliding" motion of the object, and hence no kinetic friction.

Every time the object contacts the surface, it is "lifted" and replaced with the next segment of the circumference. This is also why the actual coefficient of friction is given as an inequality as static friction is derived as an inequality.

5 | Torque at Rest

We will define a system z such that "out of the page" is $+z$ and "into the page" is $-z$.

The torque on a rigid body exerted by gravity is equivalent to that obtained by exerting all of gravity force on the CoM. Therefore, all of $F_g = M_0 g$ is exerted at an angle θ tangent to the point of contact by the circle. The circle has additionally radius R; therefore:

$$\vec{\tau}_g = \vec{R} \times \vec{F_g} \tag{20}$$
$$= \vec{R} \times M_0 g(-\hat{y}) \tag{21}$$
$$= RM_0 g \sin(\theta)\hat{z} \tag{22}$$

Performing the same idea upon the force of friction, though it is exactly perpendicular to the radius:

$$\vec{\tau}_{Ff} = \vec{R} \times \vec{F_f} \tag{23}$$
$$= RF_f(-\hat{z}) \tag{24}$$

The net force on the center of mass, as per derived above, is:

$$F_{net_{rx}} = gM_0 \sin\theta - F_f \tag{25}$$

At the point indicated, we form another right triangle with hypotenuse b. The lever arm of the rotation of the center of cylinder is, therefore, the sum of R plus the side length of the triangle facing θ. Meaning: $R = b\sin\theta + R$.

The total $\vec{\tau}_{net}$, therefore, is simply the difference between these two torques:

$$\vec{\tau}_{net} = R(M_0 g \sin(\theta) - F_f) + (b\ sin\theta + R)(M_0 g \sin(\theta) - F_f) \quad (26)$$
$$= (2R + b\ sin\theta)(M_0 g \sin(\theta) - F_f) \quad (27)$$

6 | Derivative of Angular Momentum

We will work in the prime reference frame entirely first.

We understand that:

$$\vec{L}' = I_{CM}\vec{\omega}' \quad (28)$$

Furthermore, we see from previous derivations that:

$$\vec{L}_{sys} = \vec{R} \times M\vec{v}_{cm} + \vec{L}' \quad (29)$$
$$= \vec{R} \times M\vec{v}_{cm} + I_{CM}\vec{\omega}' \quad (30)$$

At the point indicated, we form another right triangle with hypotenuse b. The lever arm of the rotation of the center of cylinder is, therefore, the sum of R plus the side length of the triangle facing θ. Meaning: $R = b\sin\theta + R$.

The direction of the center of mass' velocity is down the ramp, forming exactly a $90°$ angle with the lever arm. Hence, this renders:

$$\vec{L}_{sys} = (b\ sin\theta + R)M_0 v_{cm} + I_{CM}\omega' \quad (31)$$

Therefore, taking the first derivative, we see that:

$$\frac{d\vec{L}'}{dt} = \vec{\tau}'_{net} = (b\ sin\theta + R)M_0 a_{CM} + I_{CM}\alpha' \quad (32)$$

7 | Torque and Rest is Equal to Derivative of Angular Momentum

To set up this equality, we will have to make a few substitutions.

First, we understand that:

$$\frac{d\vec{L}}{dt} = RM_0 a_{CM} + I_{CM}\alpha' \quad (33)$$

Recall that the angular inertia of an object about its center axis is:

$$I = MR^2 \quad (34)$$

Setting the variables for our situation, we have:

$$I_0 = M_0 R^2 \quad (35)$$

From previous derivation, we have that:

$$\alpha = \frac{a_{rx}}{R} \tag{36}$$

and that:

$$a_{rx} = \frac{gM_0 \sin\theta - F_f}{M_0} \tag{37}$$

Substituting in the previously-derived expression for a_{rx}, we have:

$$\alpha = \frac{gM_0 \sin\theta - F_f}{M_0 R} \tag{38}$$

Substituting the above-derived expressions here into the expression:

$$\begin{align}
\frac{d\vec{L}}{dt} &= I_{CM}\alpha' + (b\ sin\theta + R)M_0 a_{CM} \tag{39}\\
&= \left(M_0R^2\right)\left(\frac{gM_0 \sin\theta - F_f}{M_0R}\right) + \left((b\ sin\theta + R)M_0\frac{gM_0 \sin\theta - F_f}{M_0}\right) \tag{40}\\
&= \left(M_0R^2\right)\left(\frac{gM_0 \sin\theta - F_f}{M_0R}\right) + (b\ sin\theta + R)\left(gM_0 \sin\theta - F_f\right) \tag{41}\\
&= R\left(M_0 g \sin\theta - F_f\right) + (b\ sin\theta + R)\left(M_0 g \sin\theta - F_f\right) \tag{42}\\
&= (2R + b\ sin\theta)\left(M_0 g \sin\theta - F_f\right) \tag{43}
\end{align}$$

Finally, we found that:

$$\begin{align}
(2R + b\ sin\theta)\left(M_0 g \sin\theta - F_f\right) &= (2R + b\ sin\theta)\left(M_0 g \sin\theta - F_f\right) \tag{44}\\
\frac{d\vec{L}}{dt} &= \vec{\tau}_{net}\ \blacksquare \tag{45}
\end{align}$$