1 | the equation

The goal is to add a bunch of slices, $d\theta$. For instance, you can approximate each slice as a sector of a circle:

$$dA = \underbrace{\left(\pi r^2\right)}_{\text{Area of the full circle}} \underbrace{\frac{d\theta}{2\pi}}_{\text{Fraction of the circle}}$$

$$dA = \left(\pi r^2\right) \frac{d\theta}{2\pi}$$

where The area of the full circle
The portion of the circle covered by$d\theta$

And so, the full equation would be

$$\boxed{A = \int_0^{2\pi} \frac{1}{2} r^2 d\theta}$$

Make sure you pick the bounds such that it only goes once around! If it's a cardiod with overlappy bits then you gotta make sure you know what area you're talking about.

1.1 | alternative formulation

You could also formulate the circle slice as an icosolese triangle, where the base is $rd\theta$ and the height is r. Then, the area of the triangle is

$$dA = \frac{1}{2}bh \quad = \frac{1}{2}r^2 d\theta$$

Which results in the same equation as above.