

1 | Deriving Rotational KE and Inertia

Given m_i , mass, \vec{r}_i' , location of the center of mass, l_i , ω , the angular velocity, figure a $KE_{tot,rot}$.

Because of the fact that the value ω is in units $\frac{d\theta}{dt}$, the rate of radians change, and we know of a radius of the spin l_i , we could figure the velocity at which it is moving by simply scaling the change in radians up to a circle of radius l_i , that is:

$$V_i' = l_i \omega \quad (1)$$

(note that, to understand this, radians $\frac{arclength}{radius}$)

And so, substituting into the statement of $\sum_{i=1}^N \frac{1}{2} m_i \vec{v}_i'^2$

$$KE_{rot} = \sum_{i=1}^N \frac{1}{2} m_i \vec{v}_i'^2 \quad (2)$$

$$= \sum_{i=1}^N \frac{1}{2} m_i (l_i \omega)^2 \quad (3)$$

$$= \sum_{i=1}^N \frac{1}{2} m_i l_i^2 \omega^2 \quad (4)$$

$$= \frac{1}{2} \omega^2 \sum_{i=1}^N (m_i l_i^2) \quad (5)$$

1.1 | Rotational Inertia

The right sum — the mass times the distance away from maxis of rotation ($\sum_{i=1}^N (m_i l_i^2)$) — is defined as the rotational (moment) of inertia (spiny mass). That is,

$$I = \sum_{i=1}^N (m_i l_i^2) \quad (6)$$

Replacing that value in the prior statement, the statement of KE_{rot} is defined as:

$$KE_{rot} = \frac{1}{2} \omega^2 I \quad (7)$$

1.2 | Rotational Inertia for a Ring

For a ring (that's perfectly circular) rotating on an axis perpendicular to the plane of the ring, the l_i — distance from axis of rotation — is the same value: namely, the radius R as the radius of a circle is the same for all positions. Meaning,

$$l_i = R \quad (8)$$

regardless of which value i .

Hence, the value of KE_{rot} would be evaluated as...

$$KE_{rot} = \sum_{i=1}^N (m_i l_i^2) \quad (9)$$

$$= \sum_{i=1}^N (m_i R^2) \quad (10)$$

$$= R^2 \sum_{i=1}^N m_i \quad (11)$$

$$(12)$$

Substituting M as the sum of all masses in the ring ($M = \sum_{i=1}^N m_i$), the statement is therefore:

$$KE_{rot} = MR^2 \quad (13)$$

1.3 | Rotational Inertia of a Solid Sphere

I believe that the rotational inertia of I_{sphere} to be less than I_{disk} . This is because, as the dimension of the object increases, it would be easier to change its velocity (a disk is easier to spin than a ring, etc.). Hence, my intuition states that I_{sphere} would be lower than I_{disk} .

Mathematically, as M is staying at the same value, in the disk case has more mass closer to the axis of rotation — meaning that the $m_i R^2$ term would be smaller in more of the point masses than that of an object at a lower dimension. Hence, the sphere would have more points with lower $m_i R^2$ terms than that of disk; hence, I_{sphere} would be less than I_{disk} .

2 | Kinematics Equations

Given $a = a_0$, we derive the kinematics equations.