1 | Probelem 1

1.1 | 1-1

Because all of the terms $\vec{A} \times \vec{B}$, $\vec{A} \times \vec{C}$, and $\vec{A} \times (\vec{B} + \vec{C})$ are all crossed with $\vec{A}$, all of the vectors are in the same plane which is perpendicular to $\vec{A}$. If we set $\vec{A} = (0, 0, A_z)$ to be on the z axis, then $\vec{A} \times \vec{B}$, $\vec{A} \times \vec{C}$, and $\vec{A} \times (\vec{B} + \vec{C})$ would all be in the xy plane.

1.2 | 1-2

Because $\vec{B}_{\perp \vec{A}}$, $\vec{C}_{\perp \vec{A}}$, $(\vec{B} + \vec{C})_{\perp \vec{A}}$ all are perpendicular to $\vec{A}$, they all must be coplaner, in the plane that is perpendicular to $\vec{A}$, which happens to be in the same plane as $\vec{A} \times \vec{B}$, $\vec{A} \times \vec{C}$, and $\vec{A} \times (\vec{B} + \vec{C})$.

1.3 | 1-3

Because of the definition of the cross product, we know that:

$$(\vec{A} \times \vec{B}) \perp \vec{B}$$

$$(\vec{A} \times \vec{C}) \perp \vec{C}$$

$$(\vec{A} \times (\vec{B} + \vec{C})) \perp (\vec{B} + \vec{C})$$

In tandem with the information form part 1-1 (all of the terms are perpendicular to $\vec{A}$), we know that:
$(\vec{A} \times \vec{B}) \perp \vec{A}\vec{B}$ plane
$(\vec{A} \times \vec{C}) \perp \vec{A}\vec{C}$ plane
$(\vec{A} \times (\vec{B} + \vec{C})) \perp \vec{A}(\vec{B} + \vec{C})$ plane
Thus, if we want to show that:

$$(\vec{A} \times \vec{B}) \perp (\vec{B}_{\perp \vec{A}})$$

$$(\vec{A} \times \vec{C}) \perp (\vec{C}_{\perp \vec{A}})$$

$$(\vec{A} \times (\vec{B} + \vec{C})) \perp (\vec{B} + \vec{C})_{\perp \vec{A}}$$

Then you need to show that:
$(\vec{B}_{\perp \vec{A}}) \in \vec{A}\vec{B}$ plane
$(\vec{C}_{\perp \vec{A}}) \in \vec{A}\vec{C}$ plane
$(\vec{B} + \vec{C})_{\perp \vec{A}} \in \vec{A}(\vec{B} + \vec{C})$ plane
To do this we can use the linear algebra definition of a plane. This definition states that a plane is defined as the locus of points that can be described by the linear combinatition of two vectors. In this case the two vectors are:

$\vec{A}$ and $\vec{B}$
$\vec{A}$ and $\vec{C}$
$\vec{A}$ and $(\vec{B}+\vec{C})$
We have defined:

$$\vec{A}=(0,0,A_z)$$

And can define:

$$\vec{B}=(B_x,B_y,B_z)$$

$$\vec{C}=(C_x,C_y,C_z)$$

Thus:

$$(\vec{B}+\vec{C})=(B_x+C_x,B_y+C_y,B_z+C_z)$$

Because $\vec{B}_{\perp\vec{A}}$, $\vec{C}_{\perp\vec{A}}$, and $(\vec{B}+\vec{C})_{\perp\vec{A}}$ are the projections of $\vec{B}$, $\vec{C}$, and $(\vec{B}+\vec{C})$ onto xy plane, respectively, they are defined as:

$$\vec{B}_{\perp\vec{A}}=(B_x,B_y,0)=(B_x,B_y,B_z)+n(0,0,A_z)=\vec{B}+n\vec{A}$$

$$\vec{C}_{\perp\vec{A}}=(C_x,C_y,0)=(C_x,C_y,C_z)+m(0,0,A_z)=\vec{C}+m\vec{A}$$

$(\vec{B}+\vec{C})_{\perp\vec{A}}=(B_x+C_x,B_y+C_y,0)$
$=(B_x+C_x,B_y+C_y,B_z+C_z)+p(0,0,A_z)=(\vec{B}+\vec{C})+p\vec{A}$
Thus, by the linear algebra definition of a plane:
$(\vec{B}_{\perp\vec{A}})\in\vec{A}\vec{B}$ plane
$(\vec{C}_{\perp\vec{A}})\in\vec{A}\vec{C}$ plane
$(\vec{B}+\vec{C})_{\perp\vec{A}}\in\vec{A}(\vec{B}+\vec{C})$ plane
Therefore:

$$(\vec{A}\times\vec{B})\perp(\vec{B}_{\perp\vec{A}})$$

$$(\vec{A}\times\vec{C})\perp(\vec{C}_{\perp\vec{A}})$$

$$(\vec{A}\times(\vec{B}+\vec{C}))\perp(\vec{B}+\vec{C})_{\perp\vec{A}}$$

1.4 | **1-4**

$(\vec{A}\times\vec{B})$ points in the direction of $\vec{R}_{90^\circ}(\vec{B}_{\perp\vec{A}})$
$(\vec{A}\times\vec{C})$ points in the direction of $\vec{R}_{90^\circ}(\vec{C}_{\perp\vec{A}})$
$\vec{A}\times(\vec{B}+\vec{C})$ points in the direction of $\vec{R}_{90^\circ}(\vec{B}+\vec{C})_{\perp\vec{A}}$

1.5 | **1-5**

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin(\theta) \cdot \frac{\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}})}{|\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}})|}$$

The last ”term” is there just to set the direction (from part 1-4).

The middle part, $|\vec{B}| \sin(\theta)$ is magnitude of component of of $\vec{B}$ that is perpendicular to $\vec{A}$.

We know that $\vec{B}_{\perp \vec{A}}$ is component of $\vec{B}$ that is perpendicular to $\vec{A}$ from the definition of projection and the facts that:

- $\vec{B}_{\perp \vec{A}}$ is in the xy plane
- $\vec{A}$ is perpendicular to the xy plane

Thus we know that:

$$|\vec{B}| \sin(\theta) = |\vec{B}_{\perp \vec{A}}|$$

Because $R_{90^\circ}()$ rotates the vector by 90 degrees:

$|\vec{B}_{\perp \vec{A}}| = |\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}})|$

Therefore:

$$\vec{A} \times \vec{B} = |\vec{A}||\vec{B}| \sin(\theta) \cdot \frac{\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}})}{|\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}})|} = |\vec{A}|\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}})$$

1.6 | **1-6**

As stated in problem 1-5:

$$\vec{B}_{\perp \vec{A}} = (B_x, B_y, 0)$$

$$\vec{C}_{\perp \vec{A}} = (C_x, C_y, 0)$$

$$(\vec{B} + \vec{C})_{\perp \vec{A}} = (B_x + C_x, B_y, C_y, 0)$$

This works by definition of projection, and the fact that the x values cannot influence the y or z values of a vector:

Therefore:

$$\vec{B}_{\perp \vec{A}} + \vec{C}_{\perp \vec{A}} = (\vec{B} + \vec{C})_{\perp \vec{A}}$$

1.7 | **1-7**

IMAGE

The rotation opperation is defined as:

$\vec{t} = (x, y, 0)$

$$\vec{R}_{90^\circ}(\vec{t}) = (-y, x, 0)$$

Where $\vec{t}$ is rotated 90 degrees counter clockwise in the xy plane.
Thus:

$$\vec{R}_{90^\circ}(\vec{B}_{\perp \vec{A}}) = (-B_y, B_x, 0)$$

$$\vec{R}_{90^\circ}(\vec{C}_{\perp \vec{A}}) = (-C_y, C_x, 0)$$