

## 1 | invertible, inverse

**def**

- A linear map  $T \in \mathcal{L}(V, W)$  is *invertible* if there exists a linear map  $S \in \mathcal{L}(W, V)$  such that  $ST$  equals the identity map on  $V$  and  $TS$  equals the identity map on  $W$ .
- A linear map  $S \in \mathcal{L}(W, V)$  satisfying  $ST = I$  and  $TS = I$  is called an *inverse* of  $T$
- If  $T$  is invertible,  $T^{-1}$  denotes the inverse of  $T$

### 1.1 | careful

#### 1.1.1 | the inverse of a map has to be commutative ( $TS = I$ and $ST = I$ )

#### 1.1.2 | the target identity is in one space on one side and in the other space on the other side

### 1.2 | results

#### 1.2.1 | unique

any invertible map has exactly one inverse

#### 1.2.2 | equivalent to injectivity and surjectivity (bijectivity)

See bijectivity. Iff a map is bijective, then it is invertible.

#### 1.2.3 | Equivalent Condition with eigenvalues

if a map has zero as an eigenvalue, then it is singular (5.A exercise 21)

#### 1.2.4 | non-singular matrices are invertible

#### 1.2.5 | operators that are injective or surjective are bijective

#### 1.2.6 | matrices with linearly independent columns and rows are bijective