0 | meta

This homework took ~3h whilst discussing with peers... I really need to practice this type of algebra.

$$1 \mid \int \frac{\sqrt{x-1}}{x} dx$$

Let
$$u = \sqrt{x-1}$$
, $du = \frac{1}{2\sqrt{x-1}}dx$

$$\begin{split} \int \frac{\sqrt{x-1}}{x} dx &= \int \frac{u}{u^2+1} 2u du \\ &= 2 \int \frac{(u^2+1)-1}{u^2+1} du \\ &= 2 \int \frac{u^2+1}{u^2+1} + \frac{-1}{u^2+1} du \\ &= 2 \int 1 du - \frac{1}{u^2+1} + C \\ &= 2 \int 1 du - \tan^- u + C \\ &= 2u - \tan^- u + C \\ &= 2\sqrt{x-1} - \tan^- \left(\sqrt{x-1}\right) + C \end{split}$$

Polynomial long division?

When you have a square root with a sum/difference inside, there's not much you can do. So, your best bet is to substitute either the stuff inside the root as u or the entire radical as u.

$$2 \mid \int \frac{x^2}{x^2+1} dx$$

Let
$$u = x^2 + 1$$
. $du = 2xdx$

$$\int \frac{x^3}{x^2 + 1} dx = \frac{1}{2} \int \frac{u - 1}{u} du$$

$$= \frac{1}{2} \left(u - \int \frac{1}{u} du \right) + C$$

$$= \frac{1}{2} \left(u - \ln u \right) + C$$

$$= \left[\frac{1}{2} \left(x^2 + 1 - \ln(x^2 + 1) \right) + C \right]$$

JUST SPLIT THE FRACTION AND LOOK FOR TANST X

$$3 \mid \int \frac{x-4}{x^2} dx$$

$$\int \frac{x-4}{x^2} dx = \int \frac{x}{x^2} \frac{4}{x^2} dx$$
$$= \int \frac{1}{x} dx + 4 \int \frac{1}{x^2} dx$$
$$= \left[\ln x - \frac{4}{x} + C \right]$$

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$$\int (x+1)e^{x^2+2x}dx$$

Let $u = x^2 + 2x$, du = x + 1dx

$$\int (x+1)e^{x^2+2x}dx = \frac{1}{2} \int e^u du$$

$$= \frac{1}{2}e^u$$

$$= \left[\frac{1}{2}e^{x^2+2x} + C\right]$$

$$5 | \int \tan^2 x + 1 dx$$

$$\int \tan^2 x + 1 dx = \int \sec^2 x - 1 + 1 dx$$

$$= \int \sec^2 x dx$$
Let $u = x, du = dx$

$$= \int \sec^2 u du$$

$$= \tan u + C$$

$$= \boxed{\tan x + C}$$

$$6 \mid \int \frac{6x^2 - 4}{x} dx$$

$$\int \frac{6x^2 - 4}{x} dx = \int \frac{6x^2}{x} dx - 4 \int \frac{1}{x} dx$$
$$= \int 3x dx - 4 \ln|x| + C$$
$$= 3x^2 - 4 \ln|x| + C$$

$$7 \mid \int \frac{e^x - 1}{e^x} dx$$

$$\int \frac{e^x - 1}{e^x} dx = \int 1 - \frac{1}{e^x} dx$$
$$= \int 1 - e^{-x} dx$$
$$= x + e^{-x} + C$$
$$= \boxed{e^{-x} + x + C}$$

$$8 \mid \int \frac{\sec^2 x}{\csc x} \sin x dx$$

$$\int \frac{\sec^2 x}{\csc x} \sin x dx = \int \tan^2 x dx$$

$$= \int \sec^2 x - 1 dx$$

$$= \int \sec^2 x dx - \int 1 dx$$

$$= \boxed{\tan x - x + C}$$

$9 \mid \int \sin x \cos x dx$

Let $u = \sin x$, then $du = \cos x dx$

$$\begin{split} \int \sin x \cos x dx &= \int u du \\ &= \frac{1}{2} u^2 \\ &= \boxed{\frac{1}{2} \sin^2 x + C} \end{split}$$

Product of \$sin \$ and \$cos \$, so we can use a double angle formula

$$\int \sin x \cos x dx = \int \frac{1}{2} \sin 2x dx$$
$$= \frac{1}{2} \int \sin 2x dx$$
$$= -\frac{1}{4} \cos 2x$$

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$$\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}} dx$$

$$\int \frac{e^{2\ln\sin x} + e^{2\ln\cos x}}{e^{2\ln\tan x} + e^{2\ln 1}} dx = \int \frac{\sin^2 x + \cos^2 x}{\tan^2 x + 1} dx$$

$$= \int \frac{1}{\tan^2 x + 1} dx$$

$$= \int \frac{1}{\sec^2 x} dx$$

$$= \int \cos^2 x dx$$

$$= \int \frac{1}{2} (\cos 2x + 1) dx$$

$$= \frac{1}{2} \int \cos 2x + 1 dx$$

$$= \frac{1}{2} \left(\int \cos 2x dx + \int 1 dx \right)$$

Let u = 2x, du = 2dx

$$\begin{split} \frac{1}{2}\int\cos 2xdx + \frac{x}{2} + C &= \frac{1}{4}\int\cos udu + \frac{x}{2} + C \\ &= \frac{1}{4}\sin u + \frac{x}{2} + C \\ &= \boxed{\frac{1}{4}\sin 2x + \frac{x}{2} + C} \end{split}$$

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$$\int \frac{\sec x \tan x}{1+\sec^2 x} dx$$

Let $u = \sec x$, $du = \sec x \tan x dx$

$$\begin{split} \int \frac{\sec x \tan x}{1 + \sec^2 x} dx &= \int \frac{du}{1 + u^2} dx \\ &= \int \frac{1}{1 + u^2} du \\ &= \tan^- u + C \\ &= \boxed{\tan^- \sec x + C} \end{split}$$

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$$\int x^2 e^{x^3} dx$$

Let
$$u=x^3$$
, $du=3x^2dx$

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$$\int x^2 e^{x^3} dx = \frac{1}{3} \int e^u du$$
$$= \frac{1}{3} e^u + C$$
$$= \left[\frac{1}{3} e^{x^3} + C \right]$$

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$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$$

Let
$$u = \sqrt{x}$$
, $du = \frac{1}{2\sqrt{x}}dx$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = 2 \int e^{u} du$$
$$= 2e^{u} + C$$
$$= 2e^{\sqrt{x}} + C$$