

#source Axler "Linear Algebra Done Right" chapter 2.B

#flo #ref #disorganized #incomplete

## 1 | Bases

### 1.1 | Summary

If it spans, and it's linearly independent, it's a basis!

### 1.2 | Axler2.27 #definition basis

A *basis* of  $V$  is a list of vectors in  $V$  that is linearly independent and spans  $V$ . - Basically a linearly independent spanning list, or the "minimum" amount of information contained in a vector space

#### 1.2.1 | Other Results

- Axler2.29 "criterion for a basis"
  - A list is a basis if and only if each vector in  $V$  can be written as exactly one linear combination of the list
- Axler2.31 all spanning lists contain a basis
  - Intuitive. A spanning list might not be linearly independent, but some subset of it must be.
- Axler2.32 Any finite dimensional vector space has a basis
  - Intuitive. It has a spanning list
  - Also, no infinite dimensional vector space has a basis, by definition
- Axler2.33 Linearly independent lists can be extended to a basis
  - Intuitive. Do this by adding in vectors that "bring new information"
- Axler2.34 Every subspace of  $V$  is part of a direct sum of  $V$ 
  - Intuitive. Kind of like saying there's an additive complement to every subspace of  $V$
  - Any vector space can be thought of the span of it's basis. Because  $V$  has a basis, and one of  $U$ 's bases can be written as a subsequence of  $V$ 's basis, that basis can be expanded and the expanded elements spanned to form the complement vecspace.