

#source Axler2.C

#ref #disorganized #incomplete

## 1 | #definition dimension

The dimension of  $V$  (denoted  $\dim V$ ) is the length of a basis in  $V$  - This relies on Axler2.35: Basis length does not depend on the basis Any two bases of a finite-dimensional vector space have the same length

### 1.1 | Results

#### 1.2 | Axler2.38 Dimension of a subspace

If  $V$  is finite-dimensional and  $U$  is a subspace of  $V$ , then  $\dim U \leq \dim V$  - Intuitive. The basis of a subspace is shorter than the basis of the original vecspace, because otherwise it's span would be larger than the original vecspace (because bases are linearly independent + len lin-indep  $\leq$  len span).

#### 1.3 | Axler2.39 Linearly independent list of the right length is a basis

All linearly independent lists of the length  $\dim V$  are bases. - Intuitive. If it's already linearly independent meaning each element brings "new information", then if there's that many elements then there should be that much information.

#### 1.4 | Axler2.43 Dimension of a sum

If  $U_1$  and  $U_2$  are subspaces of a finite dimensional vecspace, then

$$\dim(U_1 + U_2) = \dim U_1 + \dim U_2 - \dim(U_1 \cap U_2)$$

- This inducts into something analogous to PIE! KBrefPrincipleInclusionExclusion