

1 | boatman problem

Target displacement: $\langle 3\text{km}, 2\text{km} \rangle$

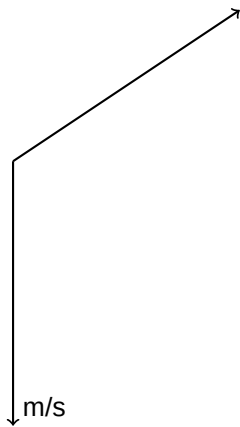
We are working with the velocities of the boat and the river. The velocity of the river is defined as $r = \langle 0, -3.5 \rangle$. We want to find vector $v = \langle v_x, v_y \rangle$ s.t.

$$\begin{aligned} |v| &= 13 \text{ km/h} \\ \lambda(v + r) &= \langle 3, 2 \rangle \end{aligned}$$

Where the trip will take λ hours

$$\begin{aligned} v_x^2 + v_y^2 &= 13^2 \\ \lambda(v_x + 0) &= 3 \\ \lambda(v_y + -3.5) &= 2 \\ v_x &= \frac{3}{\lambda} \\ v_y &= \frac{2}{\lambda} + 3.5 \\ \frac{3^2}{\lambda^2} + \left(\frac{2}{\lambda} + 3.5\right)^2 &= 13^2 \\ \frac{3^2}{\lambda^2} + \frac{2^2}{\lambda^2} + 3.5^2 + \frac{4(3.5)}{\lambda} &= 13^2 \\ \frac{3^2 + 2^2}{\lambda^2} + \frac{4(3.5)}{\lambda} &= 13^2 - 3.5^2 \\ 3^2 + 2^2 + 4(3.5)\lambda &= \lambda^2 (13^2 - 3.5^2) \\ 13 + 4(3.5)\lambda &= \lambda^2 (156.75) \\ -156.75\lambda^2 + 14^2 + 13 &= 0 \\ \frac{-14 \pm \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} & \\ \frac{-14 + \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} &= -0.24676847741 \\ \frac{-14 - \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} &= 0.336082671987 \end{aligned}$$

Maybe it's time to do it geometrically



Let θ be the angle difference that you paddle at, and ϕ be the angle that you are aiming for.

$$3.5^2 \lambda^2 = 13 + 13\lambda - 2(13)13\lambda \cos \theta$$

$$\tan \phi = \frac{3}{2}$$

$$\sin(\theta + \phi) = \frac{3.5\lambda + 2}{13\lambda}$$

attempt 3: after getting help from leonard

$$\beta = \alpha + \frac{\pi}{2} = \tan^{-1} \frac{2}{3} = 0.369$$

$$\frac{\sin \beta}{|v|} = \frac{\sin \gamma}{3.5}$$

$$\frac{\sin(0.369 + \frac{\pi}{2})}{13} = \frac{\sin \gamma}{3.5}$$

$$3.5 \frac{\sin(0.369 + \frac{\pi}{2})}{13} = \sin \gamma$$

$$\gamma = 3.5 \frac{\sin(0.369 + \frac{\pi}{2})}{13} = 0.2511$$