

#ret #hw

## 1 | 2.A Exercises

Please reconsider the questions from Friday now that we have discussed them and part of Chapter 2.A. Do

Be sure to try a few problems, so you have some ideas to share with your classmates on Thursday! Ideally,

And if you haven't brought in your old quizzes, please be sure to do so!

### 1.0.1 | Linear Dependence Lemma

- Why do we care that  $j$  is the largest element? #question
  - So we can add up everything before it? Just arbitrary?
- How does 2.22 work? #question
  - To get to 2.22, subtract everything but  $a_j v_j$  from both sides of  $a_1 v_1 + \dots + a_m v_m = 0$
  - Everything past  $v_j$  has to equal 0.
  - So we get  $a_j v_j = -a_1 v_1 - \dots - a_{j-1} v_{j-1}$
  - Divide by  $a_j$  and we get 2.22
  - Thus,  $v_j$  is a linear combination of the other vectors
  - And in the  $\text{span}(v_1, \dots, v_{j-1})$
- What  $v_j$  is it replacing? #question
  - It's replacing what's in the "...", which is unclear.. is  $v_j$  actually in the equation then? Or just in the value? #question
  - Now, we can remove the  $j^{\text{th}}$  finally, and represent it as the linear combination of the previous elements
  - $\therefore$  any element of the span can be represented without  $v_j$

### 1.0.2 | A few problems

~Fibonacci!

1. ex. 3 Find a number  $t$  such that  $(3, 1, 4), (2, -3, 5), (5, 9, t)$  is not linearly independent in  $\mathbb{R}^3$  \* Set up system of equations,  $3a + 2b = 5$   $a - 3b = 9$   $4a + 5b = t$   
 solve, get  $b = -2$  and  $a = 3$  plug it back in,  $4(3) + 5(-2) = 2 = t$   
**answer: 2**  
*ah, 2.2 != 2.20 - nice.*
2. ex. 5
  - (a) show that if we think of  $\mathbb{C}$  as a vector space over  $\mathbb{R}$ , then the list  $(1+i, 1-i)$  is linearly **independent**.
  - (b) show that if we think of  $\mathbb{C}$  as a vector space over  $\mathbb{C}$ , then the list  $(1+i, 1-i)$  is linearly **dependent**.

Means: use scalars from  $\mathbb{R}$  in the vector space  $\mathbb{C}$ ? \*

$$(a) \quad a(1+i) + b(1-i) = 0$$

prove that the only values of  $a$  and  $b$  are 0, thus satisfying the linear independence definition.

move  $i$  to only one side,  $a + b = i(b - a)$  since  $a + b$  comes from  $\mathbb{R}$ , and  $\mathbb{R}$  is closed under addition,  $a + b$  cannot have a complex component.  $\therefore a$  and  $b$  must  $= 0$

$$(a) \quad a(1+i) = b(1-i)$$

let  $b = i$  let  $a = 1$

$i(1-i) = i - i^2 = 1 + i \therefore$  we can represent  $(1-i)$  in terms of  $(i+1)$  with scalars from  $\mathbb{C}$ , and thus, it is linearly dependent.

3. ex. 8 prove or give a counterexample: If  $v_1, v_2, \dots, v_m$  is a linearly independent list of vectors in  $V$  and  $\lambda \in F$  with  $\lambda \neq 0$ , then  $\lambda v_1, \lambda v_2, \dots, \lambda v_m$  is linearly independent. \*

$a_1 v_1 + a_2 v_2 + \dots + a_m v_m = 0$  only if all scalars are equal to 0, as given in the definition

$\lambda(a_1 v_1 + a_2 v_2 + \dots + a_m v_m) = 0 \quad \lambda \cdot 0 = 0 \quad \lambda a_1 v_1 + \lambda a_2 v_2 + \dots + \lambda a_m v_m = 0$  only if all scalars are equal to 0  $\therefore \lambda v_1, \lambda v_2, \dots, \lambda v_m$  is linearly independent.

Draws from: KBxLinearIndependence KBxSpansLinAlg