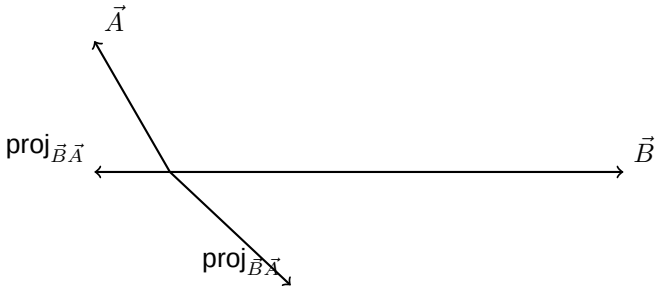
1 | vectors at an angle

1.1 | a sketch



scratch work: length: 6 cos 60 width: -6 cos 60 sin 60 height: 6 $\cos^2$ 60

1.2 | components

$$\text{comp}_{\vec{A}}\vec{B} = 6\cos 120$$
$$\text{comp}_{\vec{B}}\vec{A} = 2\cos 120$$

1.3 | dot product

$$\begin{aligned}\vec{A}\cdot\vec{B} &= |A||B|\cos\theta \\ &= 2\cdot 6\cdot\cos 120 = -3\end{aligned}$$

2 | proving expression for component

Lets redefine the coordinate axis so that $\vec{A}$ lies along the x-axis. Then,

$$\begin{aligned}\text{comp}_{\vec{A}}\vec{B} &= |\vec{B}|\cos\theta \\ &= \frac{|\vec{A}||\vec{B}|\cos\theta}{|\vec{A}|} \\ &= \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}\end{aligned}$$

3 | expression for projection

The projection is just a vector with length $\text{comp}_{\vec{A}}\vec{B}$ in the direction of $\vec{A}$.

$$\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|}\right)\frac{\vec{A}}{|\vec{A}|}$$

4 | expression for perpendicular

The part of $\vec{A}$ that is perpendicular to $\vec{B}$ is just the whole vector minus the part that is parallel:

$$\begin{aligned}\vec{A}_{\perp\vec{B}} &= \vec{A} - \text{proj}_{\vec{B}}\vec{A} \\ &= \vec{A} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)\vec{B}\end{aligned}$$

Checking using the dot product:

$$\begin{aligned}\left(\vec{A} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)\vec{B}\right)\cdot\vec{B} &= \vec{A}\cdot\vec{B} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)\vec{B}\cdot\vec{B} \\ &= \vec{A}\cdot\vec{B} - \left(\frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\right)|\vec{B}|^2 \\ &= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B} \\ &= 0\end{aligned}$$

5 | find angle using dot product

Well, the dot product already includes the angle, so let's just solve for that

$$\theta = \cos^{-}\left(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}\right)$$

The angle between:

$$\begin{aligned}\theta &= \cos^{-}\left(\frac{3+2-4}{\sqrt{1^2+2^2+2^2}\sqrt{3^2+1^2+2^2}}\right) \\ &= \cos^{-}\left((3+2-4)/(3*\sqrt{14})\right) = \cos^{-}(0.08908708) \quad \approx 84.8^\circ\end{aligned}$$

6 | dot product distribution over vector addition

$$\vec{A}\cdot\left(\vec{B}+\vec{C}\right) = |A|$$