1 | find the third order partials of $f(x,y) = 4x^2y^5 + 3x^3y^2$

$$\begin{aligned}
\frac{\partial}{\partial x} &= 8xy^5 + 9x^2y^2 \\
\frac{\partial}{\partial y} &= 20x^2y^4 + 6x^3y \\
\frac{\partial^2}{\partial x^2} &= 8y^5 + 18xy^2 \\
\frac{\partial^2}{\partial x \partial y} &= 40xy^4 + 18x^2y \\
\frac{\partial^2}{\partial y^2} &= 80x^2y^3 + 6x^3 \\
\frac{\partial^3}{\partial x^3} &= 18y^2 \\
\frac{\partial^3}{\partial^2 x \partial y} &= 40y^4 + 36xy \\
\frac{\partial^3}{\partial x \partial^2 y} &= 160xy^3 + 18x^2 \\
\frac{\partial^3}{\partial y^3} &= 240x^2y^2
\end{aligned}$$

2 | for some function of $\mathbb{R}^n \to \mathbb{R}^m$, the k-th order partial

can have up to $\frac{\binom{kn}{k}}{k!}$ possible partials???? I was going to tackle this but I realized that I have significant amounts of one-night homework. Will return.

Well, for $n = 2$, there can be up to $k + 1$ k-th order partials.

3 | the parameterization $x(t) = 3t + 4, y(t) = 5t - 7, -\infty < t < \infty$

3.1 | equation of the line

I feel like there must be a better way of doing this. However, I shall have to find the slope and y-intercept manually.

y-intercept:

$$\begin{aligned}
x(t) &= 3t + 4 = 0 \\
3t &= -4 \\
t &= -\frac{4}{3} \\
y\left(-\frac{4}{3}\right) &= -\frac{20}{3} - 7 = -\frac{41}{3}
\end{aligned}$$

Slope:

$$\frac{d}{dt}y(t) = 5$$
$$\frac{d}{dt}x(t) = 3\frac{\frac{dy}{dt}}{\frac{dx}{dt}} \quad = \frac{dy}{dx} = \frac{5}{3}$$

Thus, the equation for the line is

$$\boxed{y = \frac{5}{3}x - \frac{41}{3}}$$

3.2 | parameterization start

$$(x(0), y(0)) = (4, -7)$$

3.3 | speed in x direction

$$\frac{d}{dt}x(t) = \boxed{3}$$

3.4 | speed in y direction

$$\frac{d}{dt}y(t) = \boxed{5}$$

3.5 | actual speed

Lets find the velocity vector first

$$\vec{v} = \frac{d}{dt}(x(t), y(t)) = (3, 5)$$

The magnitude of that velocity vector is

$$|\vec{v}| = \sqrt{3^2 + 5^2} = \boxed{5.83}$$