# 1 | linear approximations

## 1.1 | cube root

### 1.1.1 | approximation

$$(1+x)^{\frac{1}{3}} \to \frac{1}{3}(1+x)^{\frac{-2}{3}}$$

at $x=0$ is

$$\frac{1}{3}(1+0)^{\cdots} = \frac{1}{3}$$

so the linear approximation is

$$y \approx m(x-0) + f(0) = \frac{1}{3}x + 1$$

### 1.1.2 | estimations

| value | estimate |
|---|---|
| 0.05 | 1.016666 |
| -0.25 | 0.916666 |

These will be overestimates because the graph is concave down in this reigon.

## 1.2 | sin(x)

### 1.2.1 | approximation

$$y \approx \frac{d}{dx}\sin x\Big|_0 (x-0) + \sin 0 = x$$

### 1.2.2 | estimates

| value | estimate |
|---|---|
| -0.1 | -0.1 |
| 0.1 | 0.1 |

The first estimate will be an underestimate because $\sin x$ is concave up in that reigon. The opposite is true for the second estimate.

## 1.3 | unknown function (only some points known

### 1.3.1 | approximation

$$y \approx \frac{d}{dx}f(x)\Big|_c (x-c) + f(c)$$

plugging in $c=1$,

$$y \approx 5(x-1) - 4$$

### 1.3.2 | estimations

| value | estimate |
|---:|---:|
| 1.2 | -3 |

This will be an underestimate because the second derivative is positive and the graph is thus concave up.

# 2 | differentials

For a function $y = f(x)$, $dy$ and $dx$ are diferentials and the relationship is $dy = f'(x)dx = \frac{L(a+\Delta a)-L(a)}{dx}dx$.

For a function written $f(x) =$ (something), the differentials are $df$ and $dx$ and the relationship is the same: $df = f'(x)dx$.

## 2.1 | cube error

### 2.1.1 | differential

$$\begin{aligned} df &= f'(x)dx \\ &= 3x^2dx \end{aligned}$$

### 2.1.2 | volume error

If I understand the use of differentials corretly, then $x$ is the measured value $(2)$ and $dx$ is the uncertainty (delta x), or $0.2$ft. Then, the change in the volume (change in fuction or $df$) would be $3(2)^2(0.2) = 2.4$

### 2.1.3 | max error for some $\epsilon$

\[
$$\begin{aligned} df &\approx 3x^2dx \\ dx &\approx \frac{df}{3x^2} \\ &\approx \frac{1}{3(2)^2} \\ &\approx \frac{1}{12}\text{ ft} = 1\text{in} \end{aligned}$$
\]

## 2.2 | sphere measuring

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$$\begin{aligned} f(r) &= \frac{4}{3}\pi r^3 \\ \frac{df}{dr} &= 4\pi r^2 \\ df &= 4\pi r^2(dr) \\ &= 4\pi 21^2(0.05) = \pm 88.4\pi\text{ cm}^3 \end{aligned}$$
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