1 | **1)**

$\vec{L} = \vec{p} \times m\vec{v}$

The circle has a circumference of $2\pi R$, and it takes $\frac{2\pi}{\omega}$ seconds to travel that distance, so the tangential velocity must be $\vec{v} = 2\pi R \div \frac{2\pi}{\vec{\omega}} = R\vec{\omega}$ Therefore, $\vec{L} = \vec{R} \times mR\vec{\omega}$

The vector is pointing out of the page, as the object is rotating counterclockwise. $\begin{aligned}|\vec{L}| &= mR|\vec{R}||\vec{\omega}| \\ &= mR^2\omega\end{aligned}$

We know that $\sin\theta$ is 1 because the vectors are perpendicular.

2 | **2)**

We can think of $\vec{r}$ as the sum of some initial position vector and velocity times the time.

$\vec{r} = \vec{r}_0 + \vec{v}t$

Then, we can look at angular momentum:

$$\begin{aligned}\vec{L} &= \vec{r} \times m\vec{v} \\ &= (\vec{r}_0 + \vec{v}t) \times m\vec{v} \\ &= (\vec{r}_0 \times m\vec{v}) + (\vec{v}t \times m\vec{v}) \\ &= \vec{r}_0 \times m\vec{v}\end{aligned}$$

We see that angular momentum is not reliant on t. As such, it is conserved.

3 | **3)**

We are given:
$$\begin{aligned}\vec{L} &= \vec{p}\vec{v} \\ \frac{d\vec{L}}{dt} &= \frac{d\vec{p}}{dt} \times m\vec{v} + \frac{d\,m\vec{v}}{dt} \times \vec{p} \\ &= \vec{v} \times m\vec{v} + m\vec{a} \times \vec{p} \qquad = 0 + \vec{p} \times m\vec{a} \\ &= \vec{p} \times \vec{F} \\ &= \vec{\tau}\end{aligned}$$