

## 1 | Axler 7.18 normal

def

Not to be confused with normal vectors, which have norm 1.

- An operator on an inner product space is called *normal* if it commutes with its adjoint.
- aka:  $T \in \mathcal{L}(V)$  is *normal* if

$$TT^* = T^*T$$

Every self adjoint operator is normal, because  $TT = TT$

## 2 | results

2.1 | **Axler 7.20**  $T$  is normal iff  $\|Tv\| = \|T^*v\|$  for all  $v$

This implies that  $T = T^*$  for all normal operators  $T$ .

2.2 | **Axler 7.21** For  $T$  normal,  $T$  and  $T^*$  have the same eigenvectors

And the corresponding eigenvalues are conjugates of one another.

2.3 | **Axler 7.22** Normal operators have orthogonal eigenvectors

Suppose  $T \in \mathcal{L}(V)$  is normal. Then eigenvectors of  $T$  corresponding to distinct eigenvalues are orthogonal.