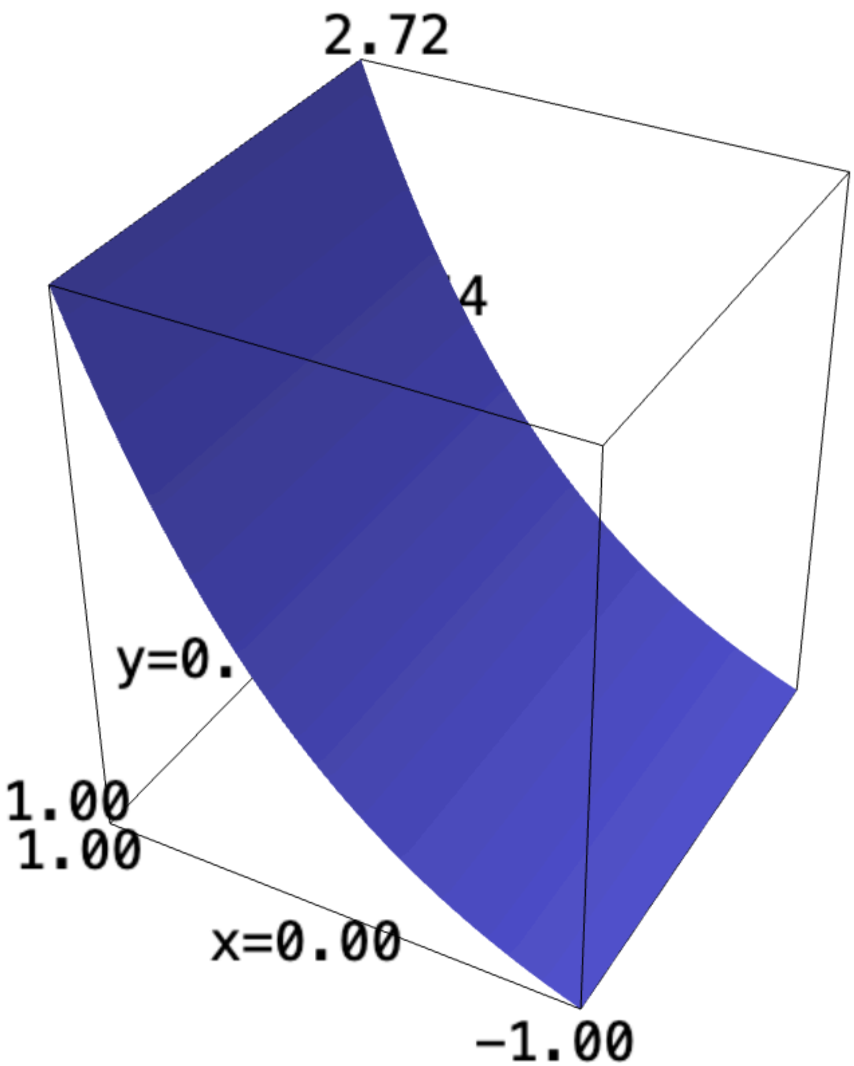
1 | Triangle Bottom

Let's plot the region first:

```
f(x,y) = e^x
plot3d(f, (x, -2, 2), (y, -2,2))
```



We can see that the shape is symmetric along the y axis. Hence, a triangle along it would be symmetric and divide an area under exactly by half. Therefore, we can simply take the integral $x, y \in [0, 1]$, and divide the result by half.

$$\int_0^1 \int_0^1 e^x \, dx \, dy \tag{1}$$
$$\Rightarrow \int_0^1 e - 1 \, dy \tag{2}$$
$$\Rightarrow ey - y \,|_0^1 \tag{3}$$
$$\Rightarrow e - 1 \tag{4}$$

The area under the prescribed triangle, then, would be:

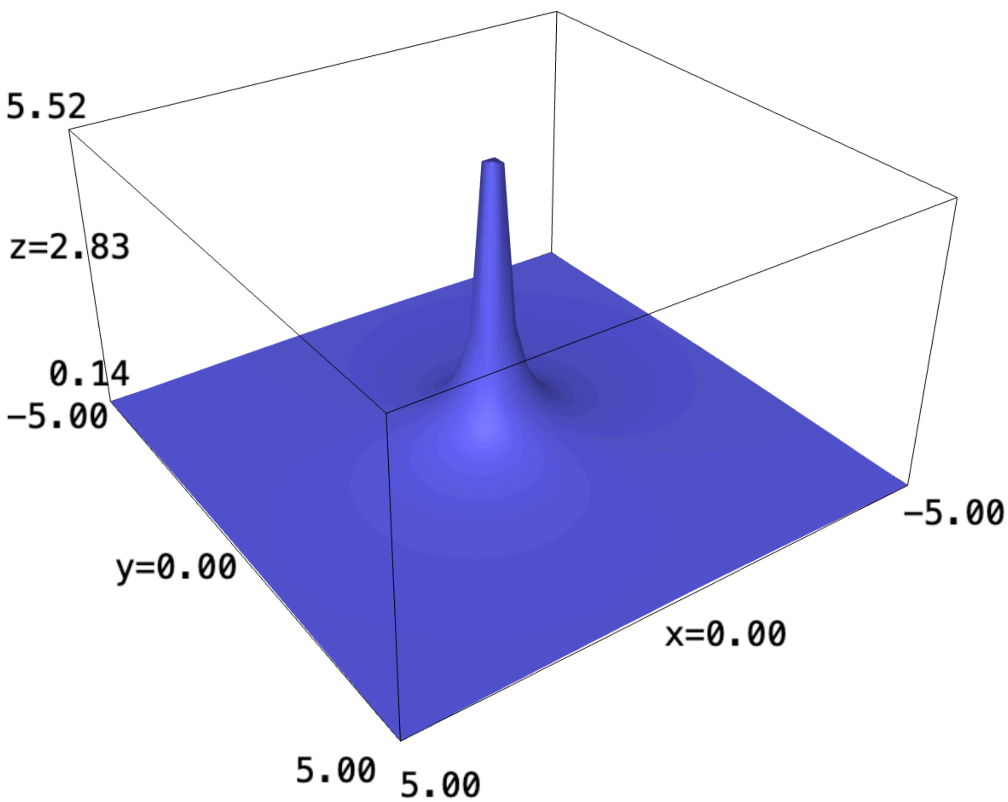
$$\frac{e-1}{2} \tag{5}$$

2 | **Polar Function**

Take the function:

$$f(x,y) = \frac{1}{\sqrt{x^2+y^2}} \tag{6}$$

```
f(x,y) = 1/sqrt(x^2+y^2)
plot3d(f, (x,-5,5), (y,-5,5))
```



Evidently, it is actually much easier to manipulate this shape in polar form (note the bottom squaring-and-add). As a reminder, the parameterization into polar is as follows:

$$\begin{cases} y = r\ sin(\theta) \\ x = r\ cos(\theta) \end{cases} \tag{7}$$

Supplying these parameterizations:

$$f(x, y) = \frac{1}{\sqrt{x^2 + y^2}} \tag{8}$$

$$\Rightarrow f(r, \theta) = \frac{1}{\sqrt{(r\ cos(\theta))^2 + (r\ sin(\theta))^2}} \tag{9}$$

$$\Rightarrow f(r, \theta) = \frac{1}{\sqrt{r^2(cos^2(\theta) + sin^2(\theta))}} \tag{10}$$

$$\Rightarrow f(r, \theta) = \frac{1}{\sqrt{r^2}} \tag{11}$$

$$\Rightarrow f(r, \theta) = \frac{1}{|r|} \tag{12}$$

We are going to be integrating over $[0, 2\pi]$, a circle, of radius 1 $[0, 1]$. We will note that these values never approaches being negative, making the absolute value have no utility.

When integrating over a radius, we also note that change in theta $d\theta$ must be multiplied by r to get the circumference on each ring. $\frac{r}{|r|}$, therefore—given that the values never approach negative—we have $\frac{r}{|r|} = 1$.

Therefore, taking the actual integral:

$$\int_0^1 \int_0^{2\pi} 1\ d\theta\ dr \tag{13}$$

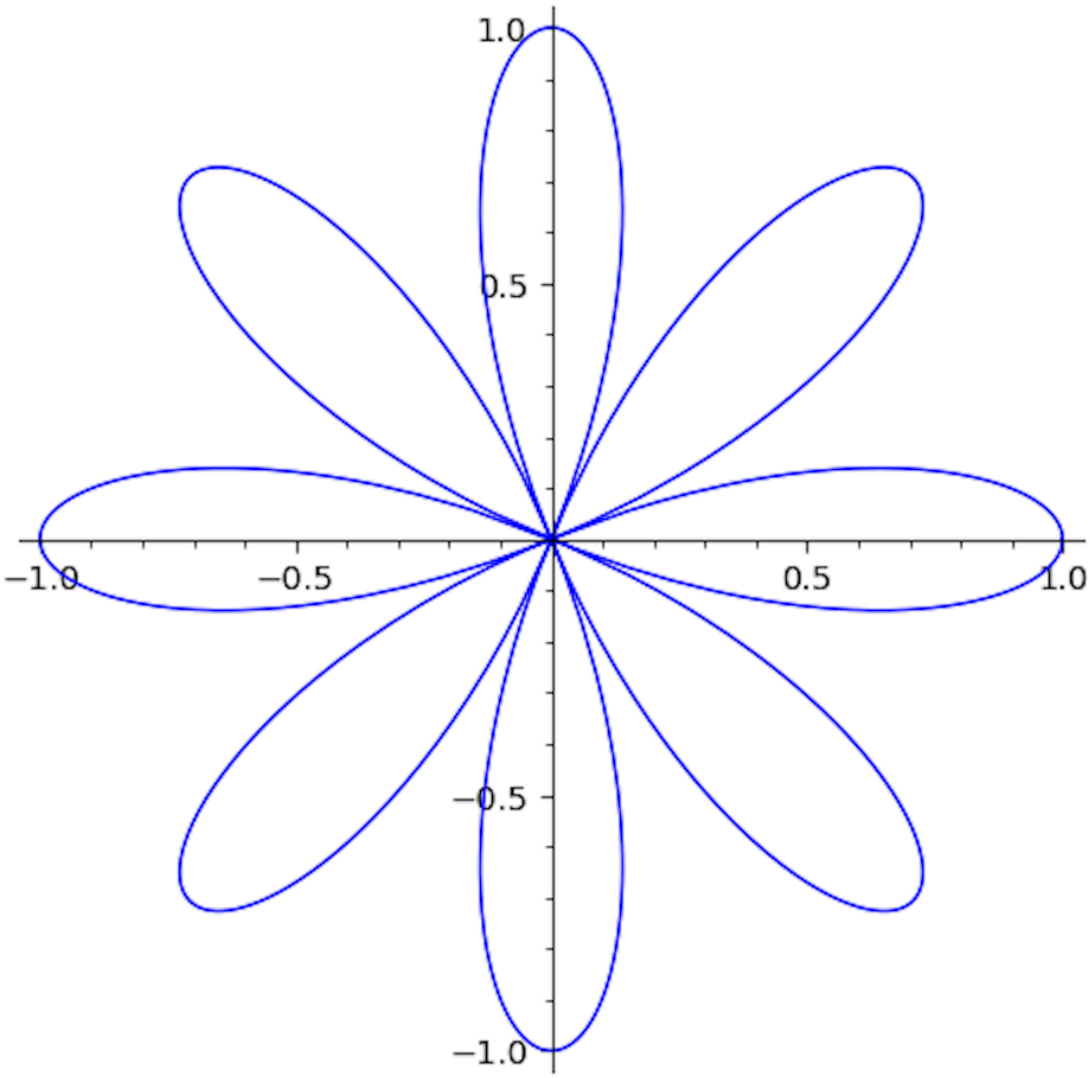
$$\Rightarrow \int_0^1 2\pi\ dr \tag{14}$$

$$\Rightarrow 2\pi \tag{15}$$

3 | Rose

Taking the actual rose, and plotting it first:

```
r(theta) = cos(4*theta)
polar_plot(r, (theta, 0, 2*pi))
```



Observing the shape, and the domain of cos being $[-1, 1]$, we can see that the diametre of the flower is 2 inches.

I am not quite sure what the surface area of the expression in $\mathbb{R}^1 \to \mathbb{R}^1$ means, but one possible solution would be the total area of the pedals, times two—resulting in both sides.

We know that, for instance, $cos(\theta) = 0$ when $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$. Therefore, $cos(4\theta)$ would be 0 when $\theta = \frac{\pi}{8}, \frac{3\pi}{8}$. This lines up with the lower-right pedal.

Let's take the integral, then:

$$\int_{\frac{\pi}{8}}^{\frac{3\pi}{8}} cos(4\theta)\ d\theta \tag{16}$$

$$\Rightarrow \frac{1}{2} sin(4\theta) \Big|_{\frac{\pi}{8}}^{\frac{3\pi}{8}} \tag{17}$$

$$\Rightarrow -0.5 - 0.5 \tag{18}$$

$$\Rightarrow -1 \tag{19}$$

The negative value simply indicates its below the x axis, but each pedal has an area of 1. Multiplying this value by the number of pedals 8, we have a total area of 8 on one side.

Multiplying this again by 2 to account for both sides, we have 16 square inches as the surface area of the entire flower.