1 | Review Sheet

1.1 | Problem 1

1.1.1 | (e)

\[
$$
\begin{aligned}
f(x) &= x(x^2+2) - \sin(x^4 - x^{90}) + e^{\sin(x)} + \ln\cos(x^2) \\
f'(x) &= 3x^2 + 2 - (4x^3 - 90x^{89})\cos(x^4 - x^{9}0) + \cos(x)e^{\sin(x)} + -\frac{2x\sin(x^2)}{\cos(x^2)}
\end{aligned}
$$
\]

1.1.2 | (f)

\[
$$
\begin{aligned}
y &= \frac{x^5 + x^{25}}{\sin(x)} + x^5\sin(x) + x^3\sin(x)e^{5x} \\
\frac{d}{dx}[y] &= \frac{\sin(x)(5x^4 + 25x^{24}) - \cos(x)(x^4 + x^{2}5)}{\sin^2(x)} + (5x^4\sin(x) + x^5\cos(x)) + ((3x^2\sin(x) + x^3\cos(x))e^{5x} + 5x^4\sin(x)e
\end{aligned}
$$
\]

1.2 | Problem 4

1.2.1 | (a)

\[
$$
\begin{aligned}
V &= 24.0Lmol^{-1} \\
V(t) &= 24t \\
R(t) &= \sqrt[3]{\frac{3}{4}V(t)} \\
&= \sqrt[3]{18t} \\
t &= 3 \\
V(3) &= 72L \\
R(3) &= 3\sqrt[3]{2} * 10cm & &= 30\sqrt[3]{2}cm \\
V'(t) &= 24 \\
R'(t) &= \frac{18}{\sqrt[3]{18t}^2} \\
V'(3) &= 24Ls^{-1} \\
R'(3) &= \frac{18}{\sqrt[3]{18(3)}^2} \\
&= \frac{18}{6\sqrt[3]{2}} * 10cms^{-1} & &= \frac{30}{\sqrt[3]{2}}cms^{-1}
\end{aligned}
$$
\]

1.2.2 | (b)

Assuming that the question is asking how much time would have passed when the radius is 3m: \[

$$
\begin{aligned}
3m &= 30 * 10cm \\
R(t) &= 30 \\
\sqrt[3]{18t} &= 30 \\
18t &= 30^3 \\
t &= \frac{30^3}{18} \\
&= 1500
\end{aligned}
$$

\]

1.3 | **Problem 5**

1.3.1 | (e)

\[

$$
\begin{aligned}
&\int (4y - y^2 + 4y^3 + 1)^{-2/3}(12y^2 - 2y + 4)\,dy \\
&= 3\int \frac{1}{3}(4y - y^2 + 4y^3 + 1)^{-2/3}(12y^2 - 2y + 4)\,dy \\
&= 3(4y - 4^2 + 4y^3 + 1)^{1/3} + C
\end{aligned}
$$

\]

1.3.2 | (f)

\[

$$
\begin{aligned}
\int 2x\cos(x)\,dx &= 2x\sin(x) - \int 2\sin(x)\,dx \\
&= 2x\sin(x) - 2\int \sin(x)\,dx \\
&= 2x\sin(x) + 2\cos(x)
\end{aligned}
$$

\]

2 | **Arc Length**

2.1 | **Problem 2**

$$\begin{aligned}
f(x) &= \frac{x^2}{8} - \ln x \\
f'(x) &= \frac{1}{4}x - \frac{1}{x} \\
L &= \int_1^2 \sqrt{1 + f'(x)^2}\, dx
\end{aligned}$$

\[$$= \int_1^2 \sqrt{1 + (\frac{1}{16}x^2 - \frac{1}{2} + \frac{1}{x^2})}\, dx$$ \]

$$\begin{aligned}
&= \int_1^2 \sqrt{\frac{1}{16}x^2 + \frac{1}{2} + \frac{1}{x^2}}\, dx \\
&= [\frac{\sqrt{\frac{(x^2+4)^2}{x^2}}(x^3 + 8x \log(x))}{8(x^2+4)}]_1^2 \\
&= \frac{3}{8} + \log 2
\end{aligned}$$

2.2 | **Problem 8**

$$\begin{aligned}
f(x) &= x^2 \\
f'(x) &= 2x
\end{aligned}$$

\[$f'(x) = 6$ \]

$$\begin{aligned}
2x &= 6 \\
x &= 3
\end{aligned}$$