

I chose to use Region B (bounded by $x=0$, $y=\sqrt{x}$, and $x=9$) and the semicircle as my cross section. The integral to calculate the volume is the integral of the area of each slice.

$$\begin{aligned} & \int_0^9 \pi r_x^2 dx \\ &= \int_0^9 \pi \left(\frac{\sqrt{x}}{2} \right)^2 dx \\ &= \frac{\pi}{4} \int_0^9 x dx \\ &= \frac{\pi}{8} 9^2 \\ &= \boxed{\frac{81\pi}{8}} \end{aligned}$$

This value is corroborated to four decimal points using the slice generator. The final model can be downloaded and viewed in OpenSCAD or seen here:



