1 | shaded area problems

choices. Lets do sea shell:

$$
\begin{aligned}
r &= \sqrt{\theta} \\
A &= \int_0^{2\pi} \frac{1}{2}\sqrt{\theta} d\theta \\
&= \frac{1}{2}\int_0^{2\pi} \theta^{\frac{1}{2}} d\theta \\
&= \frac{1}{\cancel{2}} \cancel{2}\theta^{\frac{3}{2}} \\
&= \theta^{\frac{3}{2}} \\
&= (2\pi)^{\frac{3}{2}}
\end{aligned}
$$

Okay, now for the petal:

$$
\begin{aligned}
r &= \sin 2\theta \\
A &= 4\int_0^{\frac{\pi}{2}} \frac{1}{2}\sin^2 2\theta d\theta \\
&= 2\int_0^{\frac{\pi}{2}} \sin^2 2\theta d\theta \\
&= 2\frac{1}{8}(4\theta - \sin 4\theta)\Big|_{\frac{\pi}{2}} \\
&= \frac{1}{4}\left(4\theta - \sin 4\theta\right)\Big|_{\frac{\pi}{2}} \\
&= \frac{1}{4}\left(2\pi - 0\right) \\
&= \frac{\pi}{2}
\end{aligned}
$$

2 | intersection of two circles

$$
\begin{aligned}
r &= 1 \\
r &= 2\cos\theta
\end{aligned}
$$

The intersection

$$
\begin{aligned}
1 &= 2\cos\theta \\
\frac{1}{2} &= \cos\theta \\
\theta &= \frac{\pi}{3}, -\frac{\pi}{3}
\end{aligned}
$$

So, lets take the integral between those, then subtract the isocelese triangle:

$$
\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}1^2 d\theta = \frac{1}{2}\frac{\pi}{3} - -\frac{1}{2}\frac{\pi}{3} \quad = \frac{\pi}{3}
$$

Now, lets find the area of the triangle to subtract:
the base:

$$\begin{aligned} y_1 &= \sin\frac{\pi}{3} \quad = \frac{\sqrt{3}}{2} \\ y_2 &= -\frac{\sqrt{3}}{2} \\ b &= \sqrt{3} \end{aligned}$$

the height of the triangle:

$$h = \cos\frac{\pi}{3} \quad = \frac{1}{2}$$

So the area of the triangle is $\frac{1}{2}bh = \frac{\sqrt{3}}{4}$
and thus,

$$A_{\text{intersection}} = 2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) \quad = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

We can subtrat the intersection from 2π to find the area of one circle excluding the area of the other:

$$A = \pi - \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) = \boxed{\frac{\pi}{3} + \frac{\sqrt{3}}{2}}$$

3 | overlap between $r = 1$ and $1 + 2\cos 2\theta$

The plan: do the part outside the circle, then subtract it from the full big petal. Add the inner petal, and multiply by two.

3.1 | the bounds

Lets start with finding the points where things cross the origin

$$\begin{aligned} 1 + 2\cos 2\theta &= 0 \\ 2\cos 2\theta &= -1 \\ \cos 2\theta &= -\frac{1}{2} \\ 2\theta &= \frac{2\pi}{3}, -\frac{2\pi}{3} \\ \theta &= \frac{\pi}{3}, -\frac{\pi}{3} \end{aligned}$$

The bounds for the outer portion of the big petal is

$$\begin{aligned} 1 + 2\cos 2\theta &= 1 \\ \cos 2\theta &= 0 \\ 2\theta &= \frac{\pi}{2}, -\frac{\pi}{2} \\ \theta &= \frac{\pi}{4}, -\frac{\pi}{4} \end{aligned}$$

3.2 | **large petal area**

$$A_p = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}(1 + 2\cos 2\theta)^2 d\theta$$

3.3 | **large petal outer area**

$$A_o = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}\left((1 + 2\cos 2\theta)^2 - 1\right) d\theta$$

3.4 | **small petals**

$$A_s = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{1}{2}(1 + 2\cos 2\theta)^2 d\theta$$

3.5 | **total area**

$$A = 2(A_p - A_o + A_s) = 2\pi - 4$$