1 | Taylor Series in e^x

Calculate, from the big scary formula, the Taylor series for e^x, centered around $x = 2$.

$$f(x) = e^x = e^2 + e^2(x-2) + \frac{e^2(x-2)^2}{2} + \frac{e^2(x-2)^3}{3} \cdots + \frac{e^2(x-2)^n}{n} \tag{1}$$

2 | Diff. in Higher Dimensions

2.1 | Derivative Matrix 14

Find the derivative matrix of

$$f : \mathbb{R}^4 \to \mathbb{R}^5; f(x_1, x_2, x_3, x_4) = \begin{bmatrix} x_1x_3 \\ \tan(x_4) \\ -\ln(x_2) \\ (3x_1-2)^4 \\ 1729 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} x_3 & 0 & x_1 & 0 \\ 0 & 0 & 0 & \sec^2(x_4) \\ 0 & \frac{-1}{x_2} & 0 & 0 \\ 12(3x_1-2)^3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

2.2 | Facing an Arbitrary Direction

Suppose you have a function $f(x, y); f : \mathbb{R}^2 \to \mathbb{R}^1$. Imagine you are standing at this function, at the point (x, y), facing θ. What is the slope? For what value is the slope greatest? Upwards? Downwards? Flat?

2.2.1 | Slope at point θ

The slope at point θ is as follows:

$$f_x(x, y)\cos(\theta) + f_y(x, y)\sin(\theta) \tag{4}$$

2.2.2 | Greatest slope Upwards

$$max. \frac{d}{d\theta}(f_x(x, y)\cos(\theta) + f_y(x, y)\sin(\theta)) \tag{5}$$

2.2.3 | **Greatest slope Downwards**

Given the max θ as derived above:

$$\pi - \theta \tag{6}$$

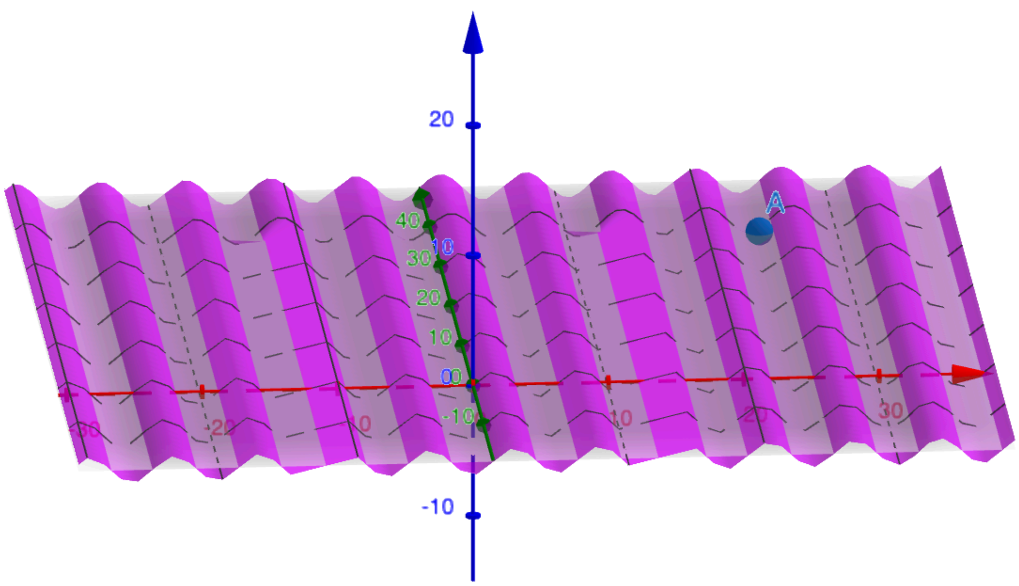
2.2.4 | **Angle of Flat Slope**

$$\theta = \text{arctan}(\frac{-f_x(x,y)}{f_y(x,y)}) \tag{7}$$

3 | **Sand Dunes**

You are trudging across a field of sand dunes, which the prevailing winds have blown into perfect, parallel, straight lines (or straight ridges, rather). You know that if you walk directly north-northeast, you'll make it to the oasis city of Iskendrebad. The landscape follows the function $f(x,y) = \sin(x)$; you're at the point with x coordinate $23\pi/3$ and y coordinate 37.

3.1 | **Make a Picture of the Situation**



3.2 | **What is your elevation**

At that point, you are at an elevation of $\sin(\frac{23\pi}{3}) = \frac{-\sqrt{3}}{2}$

3.3 | What does your hike look like?

"North-northeast" could translate an angle of roughly $68° \approx 0.0174533\ rad$. Slicing though the manifold with a line $y = 2.475x$, which represents the same angle...

We first parameterize the slice equation as follows:

$$\begin{aligned} y &= 2.475t \\ x &= t \end{aligned}$$

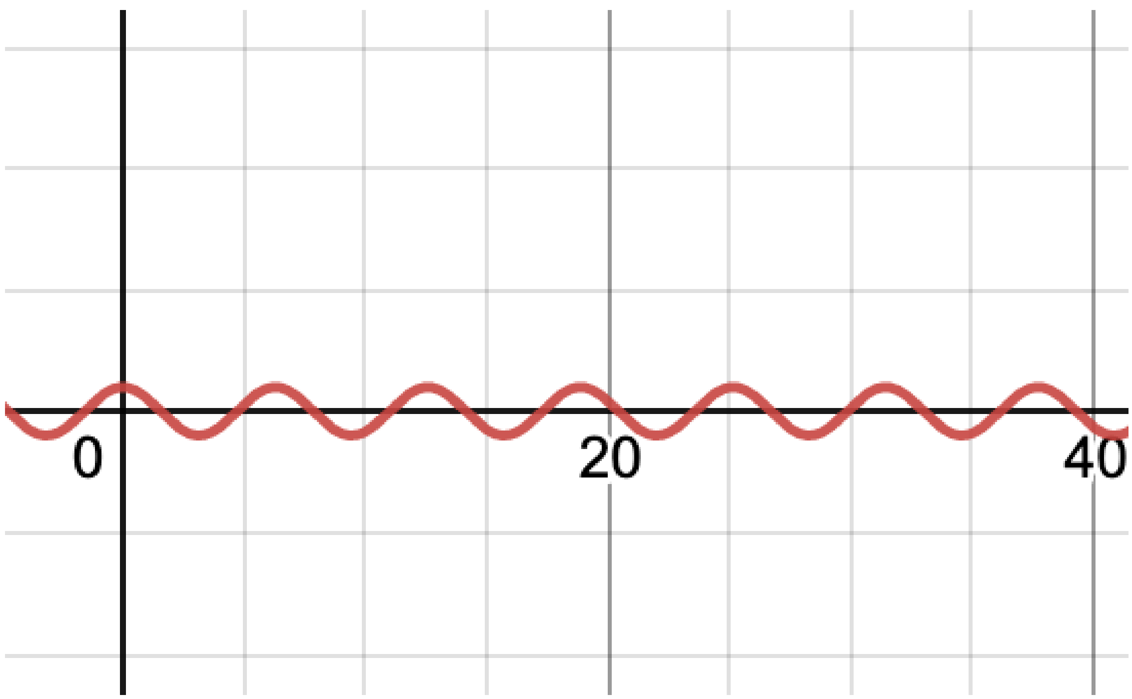
The function at $f(t, 2.475t)$, therefore, is:

$$f(t,t) = \sin(t) \tag{8}$$

Hence, the hike will also behave as $f(t) = \sin(t)$.

3.4 | What's the function for the slope along your hike?

$$f(t) = \cos(t) \tag{9}$$



3.5 | How steep is the sand dune at the point you're standing (in the direction you're hiking)?

As per above, the direction in which we are standing is at $68°$. This would represent a direction vector of:

$$\begin{bmatrix} 0.374606 \\ 0.927183 \end{bmatrix} \tag{10}$$

The gradient of the function at point at $(\frac{23\pi}{3}, 37)$:

$$\begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} \tag{11}$$

Therefore, the slope at that point is:

$$\begin{bmatrix} 0.374606 \\ 0.927183 \end{bmatrix} \cdot \begin{bmatrix} -\frac{1}{2} \\ 0 \end{bmatrix} = -0.1873032967 \tag{12}$$

This would amount to a slope of arctan$(-0.1873032967) \approx -10.609°$