1 | A surface integral

We are defining a function:

$$f(x, y, z) = y^2 \tag{1}$$

and slicing out a vertical organ pipe shape with a sliced edge. That is:

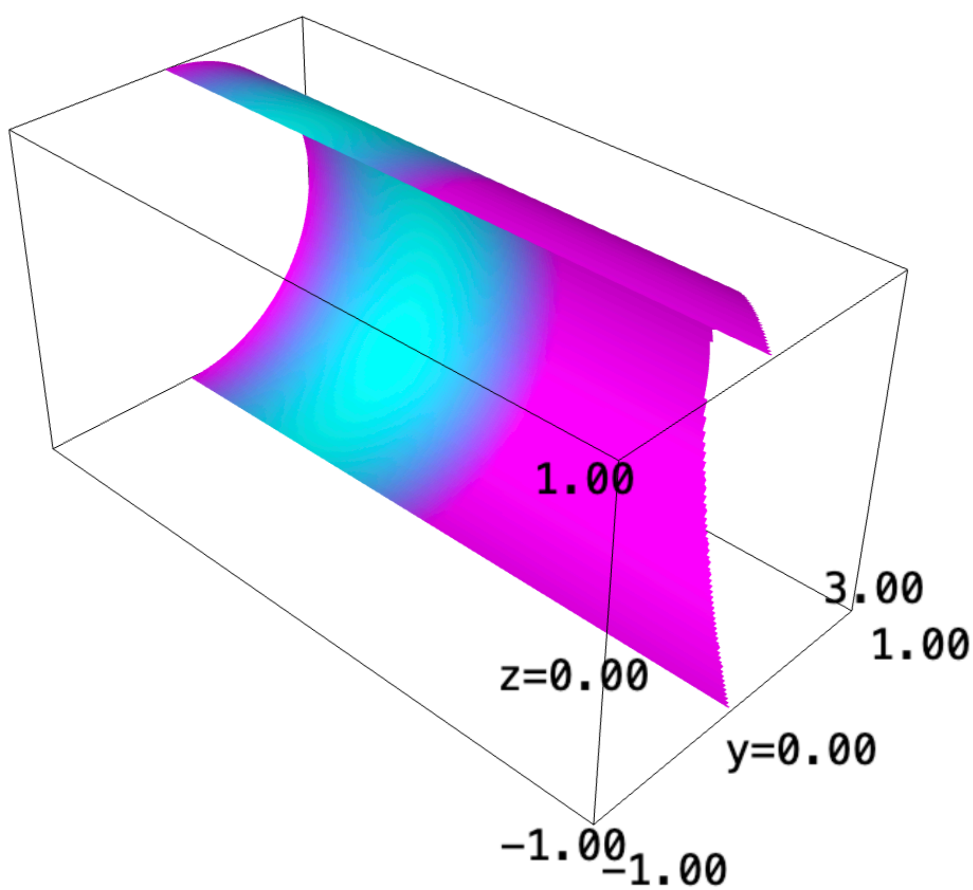
$$x^2 + z^2 = 1 \tag{2}$$

bounded by $y > 0$ and $y < 3 - x$.

Let's plot this:

```
var('x,y,z')
f = y^2

implicit_plot3d(x^2+z^2 == 1, (y,-1,3), (x,-1,1), (z, -1,1), region=(lambda x,y,z: y > 0 and y<3-x), co
```



that's honestly pretty cool!

Great, now let's take the actual surface integral. Note that, because the "pipe" does not have a defining ending point, we will set its end at the xy plane, that it ends at $x = 0$ at the other end in addition to $y = 3 - x$.

Looking at the actual function for which we are taking the integral, we have:

$$x^2 + z^2 = 1 \tag{3}$$

We will rearrange this expression in terms of z:

$$z = \sqrt{1 - x^2} \tag{4}$$

Fortunately, we see already that the function's derivative w.r.t. y is 0; indeed, it doesn't change along the y direction (the cylinder is centered around it after all.)

Taking the derivative in the x direction:

$$\begin{aligned} \frac{\partial z}{\partial x} &= \frac{\partial}{\partial x}\sqrt{1 - x^2} && (5) \\ &= \frac{-2x}{2\sqrt{-x^2+1}} && (6) \\ &= \frac{-x}{\sqrt{-x^2+1}} && (7) \end{aligned}$$

Squaring the expression below:

$$\frac{x^2}{-x^2+1} \tag{8}$$

And finally, we have the correction factor:

$$\begin{aligned} dA &= \sqrt{\frac{x^2}{-x^2+1} + 1}\, dV && (9) \\ &= \sqrt{\frac{1}{-x^2+1}}\, dV && (10) \end{aligned}$$

Lastly, we can multiply the actual value function to this to this expression to get the expression for the integral:

$$\iint_V y^2 \sqrt{\frac{1}{-x^2+1}}\, dx\ dy \tag{11}$$

Furthermore, our bounds are also a little complicated:

$$\int_0^3 \int_0^{3-x} y^2 \sqrt{\frac{1}{-x^2+1}}\, dx\ dy \tag{12}$$

Great, we will now ask Sage to take the actual integral for us:

TODO

2 | **Jacobian Matrix**

We will attempt to take the derivative matrix for this expression methodically.

Let's define the function as:

$$f(x, y, z) = (z^2 - \text{sin}(y))\hat{h} + (x + y + z)\hat{i} + (e^y + 7x)\hat{j} + (ln(x + y - 2z))\hat{k} \tag{13}$$

2.1 | f_x

Let's take the x dimension partials first.

- $f_x \cdot \hat{h} = 2$