1 | diagonal matrix def

A *diagonal matrix* is a square matrix that is zero everywhere except possibly along the diagonal.

1.1 | results

1.1.1 | every diagonal matrix is upper triangular

2 | diagonalizable def

An operator $T \in \mathcal{L}(V)$ is called *diagonalizable* if the operator has a diagonal matrix with respect to some basis of V.

2.1 | results

2.1.1 | Axler5.41 conditions equivalent to diagonalizability

Suppose V is finite-dimensional and $T \in \mathcal{L}(V)$. Let $\lambda_1, \ldots, \lambda_m$ denote the distinct eigenvalues of T. Then the following are equivalent:

1. T is diagonalizable
2. V has a basis consisting of eigenvalues of T
3. there exist 1-dimensional subspaces $U_1, \ldots, U_n$ of V, each invariant under T, s.t.

$$V = U_1 \oplus \cdots \oplus U_n$$

1. $V = E(\lambda_1, T) \oplus \cdots \oplus E(\lambda_m, T)$ (V is the (direct) sum of eigenspaces)
2. $V = E(\lambda_1, T) + \cdots + E(\lambda_m, T)$

2.1.2 | Axler5.44 Enough eigenvalues implies diagonalizability

If $T \in \mathcal{L}(V)$ has V distinct eigenvalues, then T is diagonalizable.

1. intuition Because distinct eigenvalues correspond to linearly independent eigenvectors, so there will be enough linearly independent eigenvecs to form a basis and thus a diagonal matrix.

   In fact, we just need the geometric multiplicities to add up (a result Axler promises in later chapters)

2.1.3 | Relationship to non-diagonal matrix (in class 31 March 2021)

Suppose A is the original map (not diagonal), and that P is the matrix where each column is an eigenvector written in terms of the original basis (standard basis, usually). Then

$$AP = PD$$

where D is the diagonal matrix.

1. this (or a conjugation??) forms a similarity transform, which is a type of equivalence relation