1 | Precessional Velocity

Taking the setup, we can figure the sum of the angular momentums and leverage it to figure the spin angular momentum.

Let's first define a system: $\hat{i}$ is "right" on the figure, $\hat{j}$ "in" the page, $\hat{k}$ "up" the figure.

We note that the normal spin of the flywheel gives us:

$$\vec{L}_s = I\vec{\omega}_s\hat{i} \tag{1}$$

As the flywheel is rotating at a constant speed, we have actually no torque that this contributes to the net system — that is $\frac{d\vec{L}_s}{dt} = 0$.

Furthermore, we can figure torque—and subsequent angular momentum contribution—of gravity as follows:

$$\vec{\tau}_g = lmg\hat{j} \tag{2}$$

The total net torque on the system, then:

$$\vec{\tau}_{net} = \vec{\tau}_g + 0 \tag{3}$$
$$= \vec{\tau}_g \tag{4}$$

We also have that:

$$\vec{\tau}_{net} = \frac{\vec{L}_{net}}{dt} = \Delta\vec{L}_s = lmg \tag{5}$$

We see that, because of small-angle approximation, $\Delta\vec{L}_s = L_s\Omega$

Therefore, we can replace the values determined above and solve for Ω:

$$\Delta\vec{L}_s = L_s\Omega \tag{6}$$
$$\Rightarrow lmg = I\vec{\omega}_s\Omega \tag{7}$$
$$\Rightarrow \Omega = \frac{lmg}{I\vec{\omega}_s} \blacksquare \tag{8}$$

2 | Discussion Questions

2.1 | Gyro in the Opposite Direction

If ω_s was in the opposite direction, $\vec{L}_s = -\vec{L}_{sold}$ — by the right hand rule, it would be in the other direction.

The direction of procession would be in the same direction, "into" the page, by the $\hat{j}$ direction.

Therefore, the direction of $\vec{L}_s$ would be inching up and to the right—resulting in procession in the opposite ("clockwise") direction.

2.2 |