1 | **Electric Change**

We are finally taking a surface integral! This is essentially multiplying the surface area of the shape of the function to the value of the function itself.

Firstly, taking the area dA by dV:

$$dA = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \tag{1}$$

$$= \sqrt{1 + (3)^2 + (2)^2} \tag{2}$$

$$= \sqrt{14} \tag{3}$$

Supplying the value into the function:

$$\int_0^7 \int_0^{11} (3x + 2y + 7)\sqrt{14} \, dy \, dx \tag{4}$$

$$\Rightarrow \sqrt{14} \int_0^7 \int_0^{11} (3x + 2y + 7) \, dy \, dx \tag{5}$$

$$\Rightarrow \sqrt{14} \int_0^7 \left(3xy + y^2 + 7y\right)\big|_0^{11} \, dy \, dx \tag{6}$$

$$\Rightarrow \sqrt{14} \left(\frac{33x^2}{2} + 198x\right)\bigg|_0^7 \tag{7}$$

$$\Rightarrow \frac{4389\sqrt{14}}{2} \tag{8}$$

The charge, therefore, is proportional to $\frac{4389\sqrt{14}}{2}\rho$.

2 | **Infinite wire**

Recall first that a semicircle with radius 7 can be defined as:

$$y = \sqrt{7^2 - x^2} \tag{9}$$

$$= \sqrt{49 - x^2} \tag{10}$$

Let's first figure the value of this function dA:

$$dA = \sqrt{1 + \left(\frac{d}{dx}\sqrt{49 - x^2}\right)^2} \tag{11}$$

$$= \sqrt{1 + \left(\frac{d}{dx}\sqrt{49 - x^2}\right)^2} \tag{12}$$

$$= \sqrt{1 - \frac{x^2}{x^2 - 49}} \tag{13}$$

We will take the line integral of this function, and proceed to multiply by the value of xy at that point.

$$\int_0^7 \int_0^7 xy \sqrt{1 - \frac{x^2}{x^2 - 49}} \; dx \; dy \tag{14}$$

```
f(x,y) = x*y*sqrt(1-x^2/(x^2-49))
f.integrate(x, 0,7).integrate(y,0,7)
```

Looks like the solution for the wire's weight is about $\frac{2401}{2}$ grams.

3 | **More Difficult Polar Coordinates**

Recall that, to figure the unit sphere volume, we can convert an $\mathbb{R}^2 \to \mathbb{R}^1$ result into circular coordinates. That, by pythagoras, $x^2 + y^2 = r^2$. Therefore, the expression of:

$$f(x, y) = \frac{1}{(x^2 + y^2)^k} \Rightarrow f(r, \theta) = \frac{1}{r^{2k}} \tag{15}$$

We also note that, due to the correction factor, $dA = r \; dr \; d\theta$.

Taking the actual integral, therefore, will result in:

$$\int_0^{2\pi} \int_0^1 r^{-k} \; dr \; d\theta \tag{16}$$

$$\Rightarrow \int_0^{2\pi} \lim_{x \to 0} \left(\frac{1}{-k+1} - \frac{1}{x^{k-1}} \frac{1}{-k+1} \right) \; d\theta \tag{17}$$

Evidently, when $k \leq 1$, the second term would become infinity large.

Now, we essentially want to take this idea and expand it to n dimensions, to figure the correct spherical coordinates.

Turns out, the naïve version of the n sphere integral is the same correction factor multiplied by $\sin^{n-\{2...(n-1)\}}$.