

# PS#21

## *Nueva Multivariable Calculus*

- Read this lengthy, detailed explanation of how and why the two-dimensional second derivative test works. It's long and detailed, so actually put some time and effort into reading it! I wouldn't ask you to read something that I hadn't written myself had I not read it and thought "Man, this is really good; I'm thrilled to read something so great, and also kinda annoyed that I didn't write it myself!"

Some of the broader context, note, which I hope I've mentioned in class, is that this ridiculously complicated two-dimensional second derivative test—or even doing 2D optimization by hand—isn't so much something we care about for its own sake. What we care about, in general, is optimization, most broadly. So we're doing most of this *as an example* of how ridiculously complicated optimization gets even in only just two dimensions.

- Read Andrew's notes on Physics in  $\mathbb{R}^3$  (linked on Canvas)!
- You are driving *very dangerously*. But that's OK, because you're in (on) the Bonneville Salt Flats in Utah, where there are no roads, and nothing to hit. Your position, as a function of time, at any time  $t$  is:

$$f : \mathbb{R}^1 \rightarrow \mathbb{R}^2$$
$$f(t) = \begin{bmatrix} t^2 - 9 \\ 12 \sin(t) + t \end{bmatrix}$$

for  $t$  between  $-2\pi$  and  $3\pi$

(Note that  $t$  is in minutes (around some arbitrary "zero" time);  $p(t)$ 's output is in miles north and east from some arbitrary origin point.)

Some questions:

1. Draw a map of your adventure. (You'll probably need to use a technology. For extra points, superimpose your graph on a satellite view of the salt flats.)
2. What's your velocity, as a function of time?
3. What's your acceleration, as a function of time?
4. When  $t = \pi$ , how fast are you going? (In other words, what's your speed, not in the  $x$ -direction or  $y$ -direction, but *in the direction you're going?*) (What are your  $x$  and  $y$ -coordinates, at that time? Make a new map, and label it.) Likewise, what's your acceleration in that direction at that moment? (Vocab question that I haven't figured out: velocity:speed::acceleration:???)
5. What's the fastest you ever go on this adventure?
6. What's the greatest your acceleration ever is? Is that survivable by a human?
7. How far do you actually travel over the course of this journey, from  $t = -2\pi$  to  $t = 3\pi$ ?)

On all these problems, show all your work and explain all your reasoning, thoroughly and completely! You probably want to include graphs of things, even when I haven't explicitly requested them.

- NO ONE HAS YET EVEN TRIED TO CALCULATE HOW MANY POSSIBLE SYLLABI THERE ARE!