title: Premier Proof Presentation: Axler 1.C.12
course: 20math530
author: Exr0n
source: KBe2020math530refExr0nRetIndex

1 | Lemma

> Prove that the union of two subspaces of V is a subspace of V if and only if one of the subspaces is contained in the other.

#incomplete … this got deleted? I guess see KBe20math530PremierProofPresentation-export.pdf

2 | Working it out

xler

1.C ex 12 on 12 Sep 2020

A, B sub V Union of A, B is sub V

iff $A \cup B = A$ or $A \cup B = B$

↳ directly shows union is a subspace, because A and B are already subspaces

now, assume $A \cup B$ is a subspace:

thus, it must be closed under addition. if they were not contained one w/in other,

assume $a \in A, b \in B, a \notin B, b \notin A$

$\Rightarrow a + b \in A \cup B$ (subspace closed) $\therefore a + b \in A$ or $a + b \in B$

if $a + b \in A$ then $b = \cancel{A}\,(a+b) + (-a) \in A \therefore b \in A$ contradiction