## 1 | Export

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## 2 | Exercise 7

Suppose  $T \in \mathcal{L}(V)$  has a diagonal matrix A with respect to some basis of V and that  $\lambda \in \mathbb{F}$ . Prove that  $\lambda$  appears on the diagonal of A precisely  $E(\lambda, T)$  times.

## 3 | **Proof**

We will show that for each eigenvalue  $\lambda$ , there are at least  $E(\lambda,T)$  occurrences of that eigenvalue and at most  $E(\lambda,T)$  occurrences.

Suppose first that  $\dim E(\lambda,T)=m$  and  $v_1,\ldots,v_m$  is a basis of  $E(\lambda,T)$ . In the diagonal matrix, the column corresponding to each of the m eigenvectors is comprised of the coefficients of

$$Tv_i = \lambda v_i$$

The coefficients of an eigenvector linear combination are just the eigenvalue, so the associated eigenvalue  $(\lambda)$  appears once for each eigenvector. Thus,  $\lambda$  appears on the diagonal at least m times.

Suppose then that  $\lambda$  is on the diagonal m times. Each of those occurrences corresponds to where the diagonal matrix sends a (linearly independent) basis eigenvector, which implies that the basis of V has at least m eigenvectors corresponding to  $\lambda$ . These m eigenvectors can be extended to a basis of  $E(\lambda,T)$ , implying that  $\dim E(\lambda,T) \geq m$ .