

Give an example of an operator T on a complex vector space s.t. $T^9 = T^8$ but $T^2 \neq T$.

As opposed to a normal operator.

Let $T \in \mathcal{L}(\mathbb{C}^3)$ defined by

$$T(x, y, z) = (y, z, 0)$$

Then

$$T^2(x, y, z) = T(y, z, 0) = T(z, 0, 0) \neq (y, z, 0) = T$$

and yet

$$T^8(x, y, z) = (0, 0, 0) = T^9(x, y, z)$$