

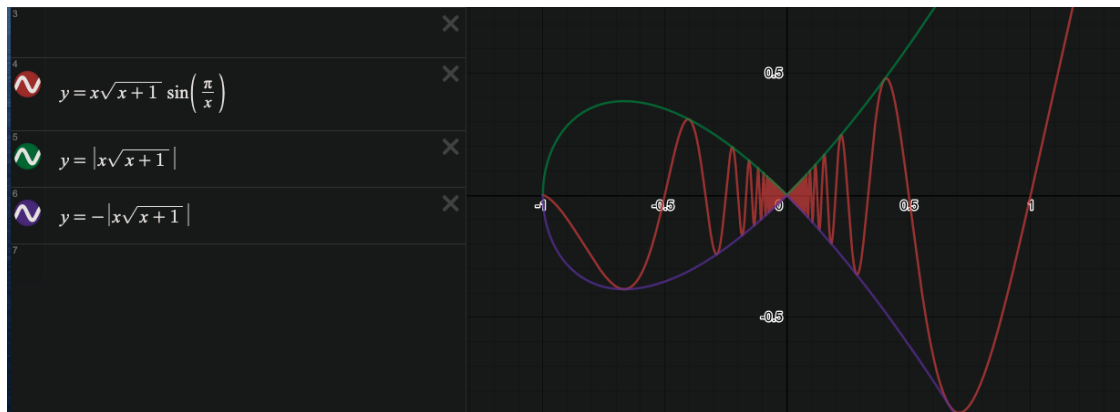
1 | Problems on slide 38

1.1 | Problem 36

$$\lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0$$

$$\backslash [\quad -1 \leq \sin \frac{\pi}{x} \leq 1 \quad \backslash]$$

$$\therefore -|x\sqrt{x+1}| \leq x\sqrt{x+1} \sin \frac{\pi}{x} \leq |x\sqrt{x+1}|$$



$$\lim_{x \rightarrow 0} -|x\sqrt{x+1}| = -|0\sqrt{1}| = 0$$

$$\backslash [\quad \lim_{x \rightarrow 0} |x\sqrt{x+1}| = |0\sqrt{1}| = 0 \quad \backslash]$$

$$\therefore \lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = \boxed{0}$$

1.2 | Problem 37

$$\lim_{x \rightarrow 4} 4x - 9 = 4(4) - 9 = 16 - 9 = 7$$

$$\lim_{x \rightarrow 4} x^2 - 4x + 7 = 4^2 - 4(4) + 7 = 7$$

$$\backslash [\quad 4x - 9 \leq f(x) \leq x^2 - 4x + 7 \quad \backslash]$$

$$\therefore \lim_{x \rightarrow 4} f(x) = \boxed{7}$$

1.3 | Problem 38

$$\lim_{x \rightarrow 1} 2x = 2(1) = 2$$

$$\backslash [\quad \lim_{x \rightarrow 1} x^4 - x^2 + 2 = 1 - 1 + 2 = 2 \quad \backslash]$$

$$2x \leq g(x) \leq x^4 - x^2 + 2$$

$$\therefore \lim_{x \rightarrow 1} g(x) = \boxed{2}$$

1.4 | Problem 39

This one is doable by just saying that $\lim_{x \rightarrow 0} x^4 \cos \frac{2}{x} = \lim_{x \rightarrow 0} x^4 \lim_{x \rightarrow 0} \cos \frac{2}{x} = 0 \left(\lim_{x \rightarrow 0} \cos \frac{2}{x} \right) = 0$.

$$-1 \leq \cos \frac{2}{x} \leq 1$$

But we can also do it properly: $\left[\begin{array}{l} \therefore -x^4 \leq x^4 \cos \frac{2}{x} \leq x^4 \\ \lim_{x \rightarrow 0} -x^4 = 0 = \lim_{x \rightarrow 0} x^4 \end{array} \right]$

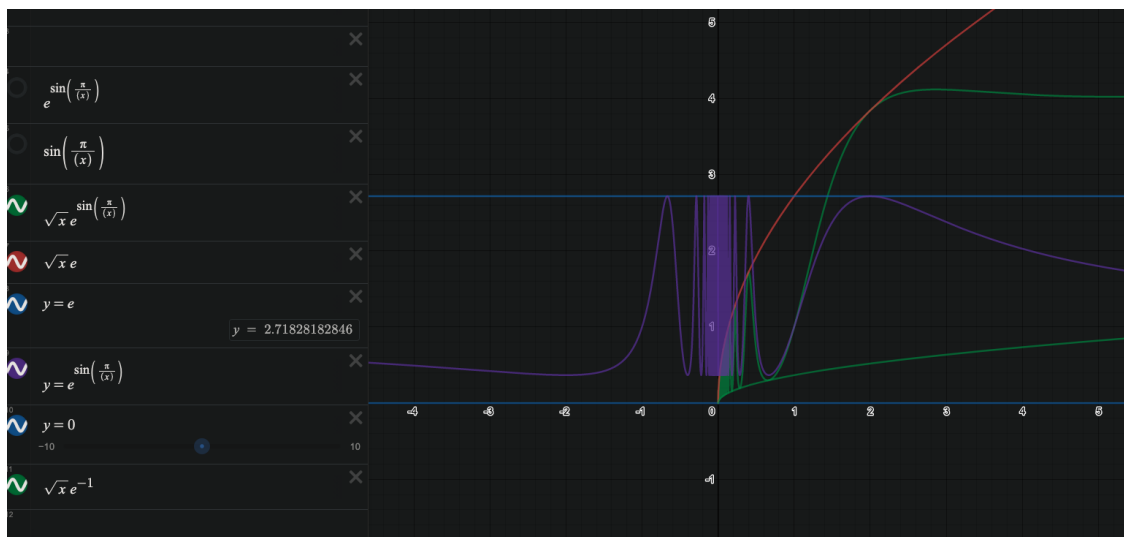
$$\therefore \lim_{x \rightarrow 0} x^4 \cos \frac{2}{x} = \boxed{0}$$

1.5 | Problem 40

- inspired by GUESS GOD

So originally you notice that $\sqrt{0}$ is just 0 so the thing is going to be zero in the end either way

But we can guess god our way to the nice functions using this graph



So we know from earlier that $-1 \leq \sin \frac{\pi}{x} \leq 1$ and like taking a positive number to a power is not gonna make it negative so like $e^{\sin \frac{\pi}{x}}$ is gonna be more than 0

oh and also because the sin power thing just makes it fluctuate - we can proly ignore that entire term and just try \sqrt{x}

except like it's too low it needs to be bigger

- maybe just multiply by e like $\sqrt{x}e$

great so now we have an upper bound and the lower bound is zero so that works except maybe we can make a sunglasses:er one? - guess god strats maybe it's the lower bound of the $\sin \frac{\pi}{x}$ exponent so like $\sqrt{x}e^{-1}$ and yup that's work guess god strat always wins

$$-1 \leq \sin \frac{\pi}{x} \leq 1$$

$$\therefore e^{-1} \leq e^{\sin \frac{\pi}{x}} \leq e^1$$

$$\therefore \sqrt{x}e^{-1} \leq \sqrt{x}e^{\sin \frac{\pi}{x}} \leq \sqrt{x}e$$

Here's the actual "writeup" \[$\lim_{x \rightarrow 0^+} \sqrt{x}e^{-1} = \sqrt{0}e^{-1} = 0$ \] thanks for coming

$$\lim_{x \rightarrow 0^+} \sqrt{x}e = \sqrt{0}e = 0$$

$$\therefore \lim_{x \rightarrow 0^+} \sqrt{x}e^{\sin \frac{\pi}{x}} = \boxed{0}$$

to my ted talk
