PS#26

Nueva Multivariable Calculus

(Read the solution set to the last pset, etc.!)

1. How much tile do you need to order to tile the floor of your Pringles-shaped house? How much paint do you need to paint the roof?

2. Suppose you have a (triple) integral over some three-dimensional region E such that the value of the entire resulting integral is maximal:

$$\iiint_E 1 - x^2 - y^2 - z^2 \, dV$$

   What is this three-dimensional region E?

3. On the last day before spring break, we saw that cool derivation for the 2D area beneath a Gaussian:

$$\text{area under a 1D bell curve} \quad = \quad \int_{-\infty}^{+\infty} e^{-x^2} \, dx \quad = \quad \sqrt{\pi}$$

   We figured that out in a weird/cool way: we took this 1D bell curve, spun it 360° around the origin to create a 2D surface, and found the volume of that surface. And then we used this cool algebraic relationship to go from that 3D volume to the 2D area we cared about:

$$\text{area under a 1D bell curve} = \sqrt{\text{volume under a 2D bell curve}}$$

   You probably don't remember the details; go back and re-read the solution set to PS#23 to refresh your memory.

   Anyway, we showed that this relationship holds for a bell curve/Gaussian—but how general is it? In other words, how many other functions does it apply to? If we take a 1D function, spin it 360° around the origin to create its radially-symmetric 2D version, is it always the case that:

$$\text{the area beneath a 1D function} = \sqrt{\begin{array}{c}\text{the volume beneath that function,}\\ \text{when spun around the origin}\\ \text{to create a radially-symmetric 2D version}\end{array}}$$

   This relationship holds for e^{-x^2}. Does it hold for all functions? Some functions? Which ones?