1 | Concept

For a function $f(x, y)$ and a vector in its input space $\vec{\mathbf{v}}$, the directional derivative of f along $\vec{\mathbf{v}}$ is the rate at which f changes as input moves along the vector.

While it is represented by a number of symbols, $\nabla_{\vec{\mathbf{v}}}$ will be used to denote a directional derivative along $\vec{\mathbf{v}}$ for these notes.

They can be thought of as generalized partial derivatives - as a partial derivative w/ respect to x tells us the amount a change in the input parallel to the x axis affects the output of the function, while this directional derivative describes how a change in any direction (as opposed to parallel to an axis) affects the change in the output in the function.

A partial derivative w/ respect to y can be thought of as a directional derivative along $\vec{\mathbf{v}} = \hat{\mathbf{j}}$ (so $\frac{\partial f}{\partial y} = \nabla_{\hat{\mathbf{j}}}$).

2 | Computation

Computing a directional derivative based on what we know so far is relatively simple.

Take the example vector $\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$.

$\nabla_{\vec{\mathbf{v}}} f = 2\frac{\partial f}{\partial x} + 3\frac{\partial f}{\partial y} + (-1)\frac{\partial f}{\partial z}$

This makes sense as $\frac{\partial f}{\partial x}$ is the amount a change in the output of the function with a small change in x, so a combination of each of the partial derivatives gives you change in the function for an arbitratry vector.

This also means that it can be computed via the gradient: $\nabla f \cdot \vec{\mathbf{v}}$ as ∇f is a vector of each of the partial derivatives and $\vec{\mathbf{v}}$ is a vector.

3 | Sources

This describes basic directional derivatives nicely.