1 | Electric Change

We are finally taking a surface integral! This is essentially multiplying the surface area of the shape of the function to the value of the function itself.

Firstly, taking the area dA by dV:

$$
\begin{aligned}
dA &= \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} && (1)\\
&= \sqrt{1 + (3)^2 + (2)^2} && (2)\\
&= \sqrt{14} && (3)
\end{aligned}
$$

Supplying the value into the function:

$$
\begin{aligned}
&\int_0^7 \int_0^{11} (3x+2y+7)\sqrt{14}\; dy\; dx && (4)\\
\Rightarrow &\sqrt{14}\int_0^7 \int_0^{11} (3x+2y+7)\; dy\; dx && (5)\\
\Rightarrow &\sqrt{14}\int_0^7 \left(3xy+y^2+7y\right)\Big|_0^{11}\; dy\; dx && (6)\\
\Rightarrow &\sqrt{14}\left(\frac{33x^2}{2}+198x\right)\Bigg|_0^7 && (7)\\
\Rightarrow &\frac{4389\sqrt{14}}{2} && (8)
\end{aligned}
$$

The charge, therefore, is proportional to $\frac{4389\sqrt{14}}{2}\rho$.

2 | Infinite wire

Recall first that a semicircle with radius 7 can be defined as:

$$
\begin{aligned}
y &= \sqrt{7^2 - x^2} && (9)\\
&= \sqrt{49 - x^2} && (10)
\end{aligned}
$$

Let's first figure the value of this function dA:

$$
\begin{aligned}
dA &= \sqrt{1 + \left(\frac{d}{dx}\sqrt{49-x^2}\right)^2} && (11)\\
&= \sqrt{1 + \left(\frac{d}{dx}\sqrt{49-x^2}\right)^2} && (12)\\
&= \sqrt{1 - \frac{x^2}{x^2-49}} && (13)
\end{aligned}
$$

We will take the line integral of this function, and proceed to multiply by the value of xy at that point.

$$\int_0^7 \int_0^7 xy \sqrt{1 - \frac{x^2}{x^2 - 49}}\, dx\, dy \tag{14}$$

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f(x,y) = x*y*sqrt(1-x^2/(x^2-49))
f.integrate(x, 0,7).integrate(y,0,7)
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Looks like the solution for the wire's weight is about $\frac{2401}{2}$ grams.

3 | More Difficult Polar Coordinates

Recall that, to figure the unit sphere volume, we can convert an $\mathbb{R}^2 \to \mathbb{R}^1$ result into circular coordinates. That, by pythagoras, $x^2 + y^2 = r^2$. Therefore, the expression of:

$$f(x, y) = \frac{1}{(x^2 + y^2)^k} \Rightarrow f(r, \theta) = \frac{1}{r^{2k}} \tag{15}$$

We also note that, due to the correction factor, $dA = r\ dr\ d\theta$.

Taking the actual integral, therefore, will result in:

$$\int_0^{2\pi} \int_0^1 r^{-k}\, dr\, d\theta \tag{16}$$

$$\Rightarrow \int_0^{2\pi} \lim_{x \to 0} \left(\frac{1}{-k + 1} - \frac{1}{x^{k-1}} \frac{1}{-k + 1} \right) d\theta \tag{17}$$

Evidently, when $k \leq 1$, the second term would become infinity large.

Now, we essentially want to take this idea and expand it to n dimensions, to figure the correct spherical coordinates.

Turns out, the naïve version of the n sphere integral is the same correction factor multiplied by $\sin^{n-\{2\ldots(n-1)\}}$. Therefore, the same logic from above actually holds for n volcano as well: that, by very high dimension Pythagoras, ${x_1}^2 + {x_2}^2 + \ldots + {x_n}^2 = r^2$.