#flo #hw

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1 | **Inner product spaces!**

wait, we just got the definition of the dot product? in chapter 6??
we can generalize the dot product to get the **inner product**
the inner product is more fundemental than lenght, and can in fact lead to the concepts of len and angles
we denote this inner product with $\langle u, v \rangle$. now we get to the definition of the inner product:

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takes an inner product of two elements $\in V$ and goes to a number $\langle u,v \rangle \in F$
has the following properties:
**positivity**                      $\langle v, v \rangle \geq 0$ for all $v \in V$

**definiteness**                    $\langle v, v \rangle = 0$ iff $v = 0$

**additivity in first slot**        $\langle u+v, w \rangle = \langle u, w \rangle + \langle v, w \rangl

**homogeneity in the first slot**   $\langle \lambda v, u \rangle = \lambda \langle v, u \rangle$ for al

**conjugate symmetry**              $\langle u, v \rangle = \overline{\langle v, u \rangle}$ for all $u
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since all real numbers equal their complex conjugate, we can just say that in real vector spaces $\langle u, v \rangle = \langle v, u \rangle$
now with the inner product, we can define an **inner product space** which is just a

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vector space along with an inner product

V is an inner product space for the rest of the chapter
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the func that takes v to $\langle v, u \rangle$ is a linear map from V to F
each inner product also determines a norm, following the pattern $||v|| = \sqrt{\langle v, v \rangle}$
the norm is also also ~homogenous: $||\lambda v|| = |\lambda| \, ||v||$
we also get to define the concept of orthogonality:

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title: orthogonal
two vecs are othogonal if $\langle u, v \rangle = 0$
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iteresting, 0 is the only vec orthogonal to itself
with orthogonal decomposition, which i dont really get yet, we can prove the cauchy-schwarz inequality!

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title: cauchy-scharz inequality
$$|\langle u,v \rangle| \leq ||u|| \space ||v||$$
holds iff u or v is a scalar multiple of the other.
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and now we get to prove all kinds of geometric properties, like the triangle equality, the parelleogram equailty, ect

1.1 | 6.b! orthonormal bases

a list of vecs can be othernormal if they are all other and normal ie, every vector is orthogonal to all other vector in the list and they are of unit len (norm 1)

we also get orthonormal basis

and also, an orthonormal list of right len is an orthonormal basis

the thing with orthonormal basis is that writing out vectors as linear combinations of them is really easy

alright, how do we find them?

gram-schmidt procedure - takes an LID list and turns it into an orthonormal list with the same span - #review the actual procedure here..

we also know that every finite-dim inner product space has an orthonormal basis

and also that every orthonormal list can be extended to an orthonormal basis

schurs theorem states that for some finite-dim complex vec space V and some operator T, T has an upper triangular matrix w.r.t. some orthonormal basis of V

all linear functionals can be represented as inner products??

#review orthogonal projections

ooh, we can solve minimization problems with the orthogonal projection

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and we can aproximate functions wayyyy better this is sick! Screen Shot 2022-03-17 at 11.00.53 PM.png|300 Screen Shot 2022-03-17 at 11.00.20 PM.png|300 aproximating sin fun, first is the new way and second is taylor