

1 | shaded area problems

choices. Lets do sea shell:

$$\begin{aligned}
 r &= \sqrt{\theta} \\
 A &= \int_0^{2\pi} \frac{1}{2} \sqrt{\theta} d\theta \\
 &= \frac{1}{2} \int_0^{2\pi} \theta^{\frac{1}{2}} d\theta \\
 &= \frac{1}{2} \left[2\theta^{\frac{3}{2}} \right]_0^{2\pi} \\
 &= \boxed{\theta^{\frac{3}{2}}}
 \end{aligned}$$

Okay, now for the petal: OH WAIT I FORGOT TO DO THE one-half r-squared

$$\begin{aligned}
 r &= \sin 2\theta \\
 A &= \int_0^{\frac{\pi}{2}} \sin 2\theta d\theta \\
 A &= 2 \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta \\
 A &= 2 \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta \\
 A &= 2 \left(\sin^2 \theta - \int_0^{\frac{\pi}{2}} \cos \theta \sin \theta d\theta \right) \quad \text{Integration by parts} \\
 2 \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta &= 2 \left(\sin^2 \theta - \int_0^{\frac{\pi}{2}} \cos \theta \sin \theta d\theta \right) \\
 2 \int_0^{\frac{\pi}{2}} \sin \theta \cos \theta d\theta &= \sin^2 \theta
 \end{aligned}$$

???? i don't know how to integrate by parts

Anyway, Wolfram Alpha says that the area is $-\frac{1}{2} \cos 2\theta$

2 | intersection of two circles

$$\begin{aligned}
 r &= 1 \\
 r &= 2 \cos \theta
 \end{aligned}$$

The intersection

$$\begin{aligned}
 1 &= 2 \cos \theta \\
 \frac{1}{2} &= \cos \theta \\
 \theta &= \frac{\pi}{3}, -\frac{\pi}{3}
 \end{aligned}$$

So, lets take the integral between those, then subtract the isocelase triangle:

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} 1^2 d\theta = \frac{1}{2} \frac{\pi}{3} - -\frac{1}{2} \frac{\pi}{3} = \frac{\pi}{3}$$

Now, lets find the area of the triangle to subtract:

the base:

$$\begin{aligned} y_1 &= \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \\ y_2 &= -\frac{\sqrt{3}}{2} \\ b &= \sqrt{3} \end{aligned}$$

the height of the triangle:

$$h = \cos \frac{\pi}{3} = \frac{1}{2}$$

So the area of the triangle is $\frac{1}{2}bh = \frac{\sqrt{3}}{4}$
and thus,

$$A = 2 \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4} \right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

3 | overlap between $r = 1$ and $1 + 2 \cos 2\theta$

The plan: do the part outside the circle, then subtract it from the full big petal. Add the inner petal, and multiply by two.

3.1 | the bounds

Lets start with finding the points where things cross the origin

$$\begin{aligned} 1 + 2 \cos 2\theta &= 0 \\ 2 \cos 2\theta &= -1 \\ \cos 2\theta &= -\frac{1}{2} \\ 2\theta &= \frac{2\pi}{3}, -\frac{2\pi}{3} \\ \theta &= \frac{\pi}{3}, -\frac{\pi}{3} \end{aligned}$$

The bounds for the outer portion of the big petal is

$$\begin{aligned} 1 + 2 \cos 2\theta &= 1 \\ \cos 2\theta &= 0 \\ 2\theta &= \frac{\pi}{2}, -\frac{\pi}{2} \\ \theta &= \frac{\pi}{4}, -\frac{\pi}{4} \end{aligned}$$

3.2 | large petal area

$$A_p = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} (1 + 2 \cos 2\theta)^2 d\theta$$

3.3 | large petal outer area

$$A_o = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2} ((1 + 2 \cos 2\theta)^2 - 1) d\theta$$

3.4 | small petals

$$A_s = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{1}{2} (1 + 2 \cos 2\theta)^2 d\theta$$

3.5 | total area

$$A = 2(A_p - A_o + A_s)$$