

1 | Electric Charge

We are finally taking a surface integral! This is essentially multiplying the surface area of the shape of the function to the value of the function itself.

Firstly, taking the area dA by dV :

$$dA = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \quad (1)$$

$$= \sqrt{1 + (3)^2 + (2)^2} \quad (2)$$

$$= \sqrt{14} \quad (3)$$

Supplying the value into the function:

$$\int_0^7 \int_0^{11} (3x + 2y + 7)\sqrt{14} \, dy \, dx \quad (4)$$

$$\Rightarrow \sqrt{14} \int_0^7 \int_0^{11} (3x + 2y + 7) \, dy \, dx \quad (5)$$

$$\Rightarrow \sqrt{14} \int_0^7 (3xy + y^2 + 7y) \Big|_0^{11} \, dy \, dx \quad (6)$$

$$\Rightarrow \sqrt{14} \left(\frac{33x^2}{2} + 198x \right) \Big|_0^7 \quad (7)$$

$$\Rightarrow \frac{4389\sqrt{14}}{2} \quad (8)$$

The charge, therefore, is proportional to $\frac{4389\sqrt{14}}{2}\rho$.

2 | Infinite wire

Recall first that a semicircle with radius 7 can be defined as:

$$y = \sqrt{7^2 - x^2} \quad (9)$$

$$= \sqrt{49 - x^2} \quad (10)$$

Let's first figure the value of this function dA :

$$dA = \sqrt{1 + \left(\frac{d}{dx} \sqrt{49 - x^2}\right)^2} \quad (11)$$

$$= \sqrt{1 + \left(\frac{d}{dx} \sqrt{49 - x^2}\right)^2} \quad (12)$$

$$= \sqrt{1 - \frac{x^2}{x^2 - 49}} \quad (13)$$

We will take the line integral of this function, and proceed to multiply by the value of xy at that point.

$$\int_0^7 \int_0^7 xy \sqrt{1 - \frac{x^2}{x^2 - 49}} dx dy \quad (14)$$

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f(x,y) = x*y*sqrt(1-x^2/(x^2-49))
f.integrate(x, 0,7).integrate(y,0,7)
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Looks like the solution for the wire's weight is about $\frac{2401}{2}$ grams.

3 | More Difficult Polar Coordinates

Recall that, to figure the unit sphere volume, we can convert an $\mathbb{R}^2 \rightarrow \mathbb{R}^1$ result into circular coordinates.

That, by pythagoras, $x^2 + y^2 = r^2$. Therefore, the expression of:

$$f(x, y) = \frac{1}{(x^2 + y^2)^k} \Rightarrow f(r, \theta) = \frac{1}{r^{2k}} \quad (15)$$

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