1 | **Complete the Representation**

Function	First four terms	Generalized
$\frac{1}{1-2x}$	$1 + 2x + 4x^2 + 8x^3 + \cdots$	$\sum_{k=0} 2^k x^k$
$\cos(3x)$	$1 - \frac{9x^2}{2!} + \frac{81x^4}{4!} - \frac{729x^6}{6!} + \cdots$	$\sum_{k=0} \frac{(-1)^k (3x)^{2k}}{2k!}$
$\frac{e^x}{e^2}$	$\frac{1}{e^2} + \frac{x}{e^2} + \frac{x^2}{e^2 2!} + \cdots$	$\sum_{k=0} \frac{x^k}{e^2 k!}$
$\sin(x^2)$	$x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} + \frac{x^{14}}{7!} + \cdots$	$\sum_{k=0} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$
$\frac{1}{1+x^4}$	$1 - x^4 + x^8 - x^{16} + \cdots$	$\sum_{k=0} (-x^4)^k$
$e^{((x-1)^2)}$	$1 + (x-1)^2 + \frac{(x-1)^4}{2!} + \frac{(x-1)^6}{3!} + \cdots$	$\sum_{k=0} \frac{(x-1)^{2k}}{k!}$
$\frac{\cos(x)-1}{x^2}$	$-\frac{1}{2!} + \frac{x^2}{4!} - \frac{x^4}{6!} + \cdots$	$\sum_{k=1} \frac{(-1)^k x^{2(k-1)}}{(2k)!}$
$2x \ln(1+2x)$	$(2x)(2x) - \frac{(2x)(2x)^2}{2} + \frac{(2x)(2x)^3}{3} - \frac{(2x)(2x)^4}{4} + \cdots$	$\sum_{k=1} \frac{2x(-1)^{k-1}(2x)^k}{k}$
$\frac{2x}{1+x^2}$	$2x - 2x^3 + 2x^5 - 2x^7 + \cdots$	$\sum_{k=0} 2x(-1)^k x^{2k}$

2 | **page 3**

2.1 | **a: skipped**

2.2 | **find maclaurin series for** $f'(x)$ **where** $f(x) = \sum_{k=0} \frac{(2x)^{k+1}}{k+1}$

$$\cancel{\frac{d}{dx}\frac{(2x)^{n+1}}{n+1} = \frac{(n+1)^2 (2x)^n (2)}{(n+1)^2} = 2(2x)^n}$$

Instead of using the quotient rule, $\frac{1}{k+1}$ is a constant for each term so we can just use the chain and power rules:

$$\frac{d}{dx}\frac{(2x)^{k+1}}{k+1} = \frac{1}{k+1}\frac{d}{dx}(2x)^{k+1} = \frac{1}{\cancel{k+1}}\cancel{(k+1)}(2x)^k (2) = 2(2x)^k$$

So, our series is just

$$\sum_{k=0} 2(2x)^k = 2 + 4x + 8x^2 + 16x^3 + \cdots$$

2.3 | **estimate** $f'\left(-\frac{1}{3}\right)$

When only using the first 4 terms:

$$2 + 4\frac{-1}{3} + 8\left(\frac{-1}{3}\right)^2 + 16\left(\frac{-1}{3}\right)^2 = \frac{10}{3}$$

For the entire sequence:

$$\sum_{k=0} 2\left(\frac{-2}{3}\right)^k = 2\sum_{k=0}\left(-\frac{2}{3}\right)^k = \frac{2}{1 - -\frac{2}{3}} = \frac{2}{\frac{5}{3}} = \frac{6}{5}$$

because the series is geometric.