1 | a plane through the origin

If we have a normal vector $\vec{n}$, then the plane is just all vectors that are orthogonal to that vector:

$$\vec{r} : \vec{r} \cdot \vec{n} = 0$$

In cartesian form:

$$xn_x + yn_y + zn_z = 0$$

2 | a plane through $\vec{p_0}$

We can just add $\vec{p}$ to everything:

$$\begin{aligned} \vec{p} + \vec{r} &: \vec{r} \cdot \vec{n} = 0 \\ \therefore \vec{r} &: (\vec{r} - \vec{p}) \cdot \vec{n} = 0 \end{aligned}$$

In cartesian form:

$$\begin{aligned} xn_x - p_x n_x + yn_y - p_y n_y + zn_z - p_z n_z = 0 \\ xn_x + yn_y + zn_z = p_x n_x + p_y n_y + p_z n_z \\ xn_x + yn_y + zn_z = \vec{p} \cdot \vec{n} \end{aligned}$$

3 | the distance

The distance vector is some multiple of the normal (because the distance is perpendicular to the plane) Thus, we just need to find the magnitude of some $\lambda\vec{n}$ such that $\lambda\vec{n}$ lies in the plane. In other words, find

$$\lambda\vec{n} : (\lambda\vec{n} - \vec{p_0}) \cdot \vec{n}$$

We can solve for λ like so:

$$\begin{aligned} \lambda\vec{n} \cdot \vec{n} - \vec{p_0} \cdot \vec{n} = 0 \\ \lambda\vec{n} \cdot \vec{n} = \vec{p_0} \cdot \vec{n} \\ \lambda = \frac{\vec{p_0} \cdot \vec{n}}{\vec{n} \cdot \vec{n}} \\ \lambda = \frac{\vec{p_0} \cdot \vec{n}}{|\vec{n}|^2} \end{aligned}$$

Finally, we need to multiply by the magnitude of $\vec{n}$:

$$\begin{aligned} d &= \lambda|\vec{n}| \\ &= \frac{\vec{p_0} \cdot \vec{n}}{|\vec{n}|} \end{aligned}$$