

1 | Jacobian Determinant for Polar

We are to determine (pun not intended) the polar correction factor for a double integral, $dA = r dr d\theta$.

To do this, we will have to first figure the change of bases expressions such that we can take:

$$f(x, y) = g(r, \theta) \quad (1)$$

Fortunately, this is already derived to use from before.

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \end{cases} \quad (2)$$

Therefore, we have that:

$$f(x, y) = f(r \cos \theta, r \sin \theta) \quad (3)$$

And therefore, we can figure $J_{r,\theta}$:

$$J = \begin{bmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{bmatrix} \quad (4)$$

Taking its determinant, then:

$$\det(J) = r \cos^2 \theta + r \sin^2 \theta = r \quad (5)$$

And therefore, the change-of-basis result would be:

$$dx dy = r dr d\theta \quad (6)$$

2 | Jacobian Determinant for Spherical

We again need to figure a correction factor for $dx dy dz = \rho^2 \sin \phi d\rho d\theta d\phi$.

We therefore have to figure a change of bases for the expression:

$$f(x, y, z) = g(\rho, \theta, \phi) \quad (7)$$

We can leverage the shape of the object to determine the parameterization:

$$\begin{cases} x = \rho \sin \phi \cos \theta \\ y = \rho \sin \phi \sin \theta \\ z = \rho \cos \phi \end{cases} \quad (8)$$

We will now figure the matrix for $J_{\rho,\theta,\phi}$:

$$J = \begin{bmatrix} \sin \phi \cos \theta & -\rho \sin \phi \sin \theta & \rho \cos \phi \cos \theta \\ \sin \phi \sin \theta & \rho \sin \phi \cos \theta & \rho \cos \phi \sin \theta \\ \cos \phi & 0 & -\rho \sin \phi \end{bmatrix} \quad (9)$$