1 | Differentiation Rules unit1::derivatives

1.1 | Review

- $\frac{d}{dx}cu = c\frac{d}{dx}u$
- $\frac{d}{dx}u + v = \frac{d}{dx}u + \frac{d}{dx}v$

1.2 | Product Rule

Differentiating a product of functions: rule is $(uv)' = u'v + uv'$

PROOF

$\Delta(uv) = u(x + \Delta x)v(x + \Delta x) - u(x)v(x)$

$= (u(x + \Delta x) - u(x))v(x + \Delta x) + u(x)v(x + \Delta x) - u(x)$

$= (\Delta u)(v(x + \Delta x) + u(x)\Delta v$

$\frac{\Delta(uv)}{\Delta x} = \frac{\Delta(u)}{\Delta x}v(x + \Delta x) + u\frac{\Delta v}{\Delta x}$

Take limit to get $\frac{d(uv)}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$

1.3 | Quotient Rule

Differentiating a quotient of functions: rule is $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$

PROOF

TIMESTAMP: ~15:00 Lecture 4 *Type me out later!*

Shows us that Power Rule works for negative powers!

1.4 | Chain Rule / Composition Rule

EXAMPLE $y = sin(x)^{10}$

Solution is to add intermediate variable names.

EXAMPLE $u = sin(x)$, $y = u^{10}$

PROOF

$\frac{\Delta y}{\Delta x} = \frac{\Delta y}{\Delta u}\frac{\Delta u}{\Delta x}$

The Δu cancels!

As Δx goes to 0:

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Differentiation of a composition is a product.

EXAMPLE $y = sin(x)^{10}$ Introduce intermediate variable: $u = sin(x)$, $y = u^{10}$ $\frac{dy}{du} = cos(x)$ $\frac{du}{dy} = 10x^9$ Multiply to get: $\frac{dy}{dx} = 10u^9 cos(x)$ Substitute u to end up with: $\frac{dy}{dx} = 10(sin(x))^9 cos(x)$

WARN: Variable names are confusing…

You can skip the intermediate calculations when trying to to calculate it quickly.

1.5 | Higher Derivatives

Rinse and repeat.

$u = u(x)$ u' is a new function which can be differentiated again to get u'' Trigonometric Derivativestrigderiv $u = \sin(x)$, $u' = \cos(x)$, $u'' = -\sin(x)$, $u''' = -\cos(x)$... Sometimes notation is $u(4) instead of u''''.

The other notation, specifically l$\frac{d}{dx}$, has an "operator" d which is applied to a function to get another function (that is the derivative). This can be just D instead of a fraction sometimes.

Lots of notation. $u'' = \frac{d}{dx}\frac{du}{dx} = \frac{d}{dx}\frac{d}{dx}u = \left(\frac{d}{dx}\right)^2 u = \frac{d^2}{(dx)^2}u = \frac{d^2u}{dx^2}$

2 | Links

More complex differentiation is covered in Implicit Differentiation. Further review can be found in MIT SVC Exam Review (Unit 1).