

## 1 | orthogonal

**def**

Two vectors  $u, v \in V$  are called *orthogonal* if  $\langle u, v \rangle = 0$

## 2 | results

### 2.1 | orthogonal $\sim$ perpendicular

### 2.2 | Axler 6.12 orthogonality and zero

#### 2.2.1 | $0$ is orthogonal to every vector in $V$

#### 2.2.2 | $0$ is the only vector in $V$ that is orthogonal to itself

### 2.3 | Axler 6.13 Pythagorean Theorem

Suppose  $u$  and  $v$  are orthogonal vectors in  $V$ . Then

$$\|u + v\|^2 = \|u\|^2 + \|v\|^2$$

#### 2.3.1 | proof with more algebra written out

$$\begin{aligned}\|u + v\|^2 &= \langle u + v, u + v \rangle \\ &= \langle u, u + v \rangle + \langle v, u + v \rangle \\ &= \langle u, u \rangle + 0\langle u, v \rangle + 0\langle v, u \rangle + \langle v, v \rangle \\ &= \|u\|^2 + \|v\|^2\end{aligned}$$