1 | A Problem

Normal or "easy" limits are rather simple, as examples like $\lim_{x\to 4} \frac{x+3}{x^2+1}$ just need some plugging in. Derivatives are usually harder as $\lim_{x\to 0} \frac{f(x_0+\Delta x)-f(x_0)}{x-x_0}$ always evaluates to $\frac{0}{0}$, and needs some cancellation.

2 | Some Notation

DEFINITION Right hand limit or $\lim_{x\to x_0^+} f(x)$ indicates that x is greater than x_0 (or that x begins on the right side of the number line).

DEFINITION Left hand limit or $\lim_{x\to x_0^-} f(x)$ indicates that x is less than x_0 (or that x begins on the left side of the number line). These notations will make dealing with limits of these functions more convienient.

EXAMPLE Take the following example of a conditional function:

$$\begin{split} &\text{if } x>0,\, f(x)=x+1\\ &\text{if } x<0,\, f(x)=-x+2\\ &lim_{x\to x_0^+}f(x)=lim_{x\to x_0}x+1=1\\ &lim_{x\to x_0^-}f(x)=lim_{x\to x_0}-x+2=2 \end{split}$$

We did not need a x = 0 value to compute these limits!

3 | Nested Limits

A checklist for what to do before dealing with nested limits.

EXAMPLE: $\sin \sqrt{x}$

- \square Check domain + range of inner function (in this case $[0,\infty)$, $[0,\infty)$).
- \Box Check domain + range of outer function as well as what it takes in. (takes in $[0,\infty)$, range is [-1,1])
- ☐ Restrict domain based on requirements of inner + outer functions

EXAMPLE: $\ln \sin x$

- \boxtimes Domain of $\sin x$ is \$(-\infty, \infty), range is [-1, 1].
- \boxtimes Domain of $\ln x$ is $[0,\infty)$, range is $(-\infty,\infty)$.
- \boxtimes As $\ln x$ takes only positive values, the restricted domain for the composite function is $[0,\pi]$, $[2\pi,3\pi]$, etc. The range of the composite function would be $(-\infty,0]$.

4 | Links

Adjacent to this: Continuity

Building upon this: Calculating Derivatives

Further reference can be found at Limits and Continuity Practice.

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