

**Proof by induction\***

- Prove something is true for a smaller number and show that doing so  $\Rightarrow$  implies it is true for larger numbers.
- There are 5 steps.
  - Declare proof by induction
  - Declare inductive hypothesis
    - \* Inductive hypothesis is typically whatever you are trying to  $\Rightarrow$  prove.
  - Prove the base case
    - \* Like dominos where a proof of one number leads to the proof  $\Rightarrow$  of the next number
  - Show that  $P(n) \rightarrow P(n+1)$
  - Invoke induction.

**Proof:  $1+2+3+\dots+n = n(n+1)/2$** 

- Proof is by induction.

$P(n)$  is the hypothesis that  $1+2+3+\dots+n = n(n+1)/2$

$P(1)$ :

$$1 = 1(1+1)/2$$

$$1 = 2/2$$

$$1 = 1$$

$P(n+1)$ :

We need to show  $1+2+3+4+\dots+n+n+1 = ((n+1)(n+2))/2$

Assume that  $P(n)$  is true.

Left side simplifies to  $(n(n+1)/2) + (n+1)$

Algebraic manipulation leaves you with  $((n+2)(n+1))/2$

If  $P(n)$  is true then  $P(n+1)$  is true and  $P(1)$  is true therefore  $P(n)$  is true for all numbers.

**Proof by Strong Induction**

- Use  $P(1), P(2) \dots P(n)$  to prove  $P(n+1)$
- Same list but instead of proving  $P(n) \rightarrow P(n+1)$  it will be  $P(1), P(2) \dots P(n) \rightarrow P(n+1)$

Proof: Any group of students  $\geq 12$  can be divided into some combination of groups of 4 and groups of 5.

- This will be a proof by strong induction.

$$P(12) = 4+4+4$$

$$P(13) = 4+4+5$$

$$P(14) = 4+5+5$$

$$P(15) = 5+5+5$$

$$P(n) \rightarrow P(n+4)$$

If  $P(n-3)$  is true, then there exists a combination of groups of 4 and 5 to make up  $(n-3)$  students. Add a group of 4 and that gives us out combination for  $(n+1)$  students.

$$P(n-3) \rightarrow P(n+1)$$

By induction this is true for all  $n \geq 12$