PS#14: Smörgåsbord #2!

Nueva Multivariable Calculus

0. Read Andrew's solution notes to PS#13! Very good!
1. We've begun to have some fun with line integrals... but those are really just boring ol' 1D integrals. Who wants to be boring and use a 1D integral to find an area? Let's use a 2D integral to find a VOLUME!!! So, without looking anything up, try to calculate these two volumes:
   (a) the volume underneath the infinite egg carton, $f(x, y) = \sin(x)\cos(y)+5$, bounded by a rectangle with one corner at the origin and another corner at $(x = 3\pi, y = 5\pi)$
   (b) the volume underneath the infinite egg carton, $f(x, y) = \sin(x)\cos(y) + 5$, bounded by a 1/4 of a pizza with its tip at the origin and a radius of 5π (i.e., a 1/4th of a circle, centered at the origin, with a radius of 5π; the part of that circle in the upper-right quadrant)

   (Obviously make/draw some pictures, too.)
2. Suppose you have a rectangular box in the first **octant** (a really cool word! look it up if you don't know it!), such that it has a corner on the origin, sides lying along the positive x, y, and z axes (or parallel to them), and another corner, opposite the corner at the origin, lying on the plane:
$$2x + 4y + 5z = 20$$
   (Draw yourself a picture, or use technology, to graph this shape! Graph the plane, too.)

   What are the dimensions of the rectangle with these properties with the largest volume? (What is that volume?)

   (You might be able to do this problem with a straightforward substitution, but I want you to use a Lagrange multiplier here!)
3. Consider the 4D surface:
$$f : \mathbb{R}^3 \to \mathbb{R}^1$$
$$f(x, y, z) = -\sqrt{x^2 + y^2}$$
   and the path:
$$s : \mathbb{R}^1 \to \mathbb{R}^3$$
$$s(t) = (a\cos t)\hat{j} + (a\sin t)\hat{k}$$
$$\text{for } t \text{ between } 0 \text{ and } 2\pi$$
   What does this path look like? Draw a picture and describe it. Find the integral of $f(x, y, z)$, along this curve.