#flo #hw

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1 | Invertibility and Isomorphic Vector Spaces

1.0.1 | invertibility and relating sets

- what does invertible mean w.r.t. linear maps?

```
title: invertible, inverse
- A linear map $T \in L(V,W)$ is called *invertible* if there exists a linear map $S \in L(W,V)$ such th
- A linear map $S \in L(W,V)$ satisfying $ST = I$ and $TS=I$ is called an *inverse* of $T$ (note that th
```

relates to injectivity and surjectivity.. #extract

```
title: inverse is unique
An invertible linear map has a unique inverse
```

proof is just setting

$$S_1 = S_1 I = S_1(TS_2)... = S_2$$

long chain of equalitys based on the fact it acts as the inverse

```
title: $T^{-1}$
If $T$ is invertible, then its inverse is denoted by $T^{-1}$. In other words, if $T \in L(V,W)$ is inve
```

Just the inverse. oh, and here we get the thing i was wondering about earlier

```
title: invertibility is equivalent to injectivty and surjectivity
A linear map is invertivle iff it is injective and surjective
```

aka. bijective means invertible

pretty simple proof, just using the definitions very closely.

1.0.2 | maps that are not invertible

- multi by x^2 from P(R) to P(R) cus not surjective. #extract
  - this is because 1 is not in the range.
- backwards shift from f^{infin} to itself
  - not injective

1.0.3 | **isomorphic vec spaces**

'essentially the same,' bar the names of the elems

```
title: isomorphism, isomorphic
- An isomorphism is an invertible linear map
- two vector spaces are called isomorphic if there is an isomorphism from one vector space onto the oth
```

which means they are bijective.

essentially relabeling v as Tv.

```
isomorphic means equal shape in greek
isos -> equal, morph -> shape
```

but people use isomprhism when they want to emphazise how two spaces can be essentially the same

```
title: dimension shows whether vector spaces are isomorphic
Two finite-dim vec spaces over **F** are isomorphic iff they have the same dimension
```

makes sense.

uh,

```
title: L(V,W) and $F^(m,n)$ are isomorphic
Suppose $v_1, \dots, v_n$ is a basis of $V$ and $w_1, \dots, w_m$ is a basis of $W$. Then $M$ is an isom
```

amazing.

pluging in dimension results, we get

```
title: $\dim L(V,W)=(\dim V)(\dim W)$
Suppose $V$ and $W$ are finite-dim. Then $L(V,W)$ is finite-dim and
$$\dim L(V,W)=(\dim V)(\dim W)$$
```

Amazing proof.

1.0.4 | **linear maps though of as matrix multiplication**

not the matrix of a map like in KBxChapter3CReading, but the matrix of a vector.

```
title: matrix of a vector, M(v)
Suppose $v \in V$ and $v_1, \dots, v_n$ is a basis of $V$. The matrix of v w.r.t. this basis is the n-by
$$
M(v) = \begin{bmatrix}
c_1  \\
 \vdots \\
 c_n
 \end{bmatrix}.
$$

where $c_1, \dots, c_n$ are the scalars such that
$$ v = c_1 v_1 + \dots + c_n V_n. $$
```

again, about representing as a KBxLinearCombinations of the basis vecs.

```
title: $M(T)._{,k} = M(vk).$
Suppose $T \in L(V,W)$ and $v_1, \dots, v_n$ is a basis of $V$ and $w_1, \dots, w_m$ is a basis of $W$.
```

takin their word for this one.

now we fit together mat of lin map, mat of vec, and matmul.

```
title: linear maps act like matrix multiplication
Suppose $T \in L(V,W)$ and $v \in V$. Suppose $v_1, \dots, v_n$ is a basis of $V$ and $w_1, \dots, w_m$
$$M(Tv) = M(T)M(v).$$
```

every m-by-n mat induces another linear map from $F^{n,1}$ to $F^{1,m}$.

with isomorphisms, we can think of linear maps as multiplicaions on $F^{n,1}$ by some mat.

axler focuses on linear maps as matricie and vise versa

1.0.5 | operators

linear maps from a vector space to itself are *very* important

we give them their own notation,

```
title: operator, $L(V)$
- A linear map from a vector space to itself is called an *operator.*
- The notation $L(V)$ denotes the set of all operators on $V$. In other words, $L(V) = L(V,V).$
```

THIS IS IMPORTANT! says that the deepest and more important parts of linalg (and the rest of the book) deal with operators. #extract

- remember, neither injectivity nor surjectivity implies invertibility
- but it does for operators! normally, injectivity is easier so that is checked

```
title: injectivity is equivalent to surjectivity in finite dimensions
Suppose $V$ is finite-dim and $T \in L(V).$ Then the following are equivalent:

a. T is invertible
b. T is injective
c. T is surjective
```

proof via multiple uses of fundemental theorem of linear algebra

```
this seems like a very important chapter. need to do some extracting on this one.
big concepts were isomorphism and operators. extract later!
```