3 | $\int \ln x dx$

$$\begin{aligned}\int \ln x dx &= \int 1 \ln x dx \\ &= x \ln x - \int x \frac{1}{x} dx \\ &= x \ln x - \int 1 dx \\ &= \boxed{x \ln x - x}\end{aligned}$$

4 | $\int \tan^{-} x dx$

$$\begin{aligned}\int \tan^{-} x dx &= x \tan^{-} x - \int x \frac{1}{x^2+1} dx \\ &= x \tan^{-} x - \frac{1}{2} \int \frac{du}{u} \\ &= x \tan^{-} x - \frac{1}{2} \ln u + C \\ &= x \tan^{-} x - \frac{1}{2} \ln(x^2+1) + C\end{aligned}$$

5 | $\int x \sec^2 x dx$

$$\begin{aligned}\int x \sec^2 x dx &= x \tan x - \int \tan x dx \\ &= x \tan x + \ln|\cos x| + C\end{aligned}$$

6 | $\int x^2 e^{5x} dx$

$$\begin{aligned}\int x^2 e^{5x} dx &= x^2 \frac{1}{5} e^{5x} - \int 2x \frac{1}{5} e^{5x} dx \\ &= x^2 \frac{1}{5} e^{5x} - 2x \frac{1}{25} e^{5x} + \int 2 \frac{1}{25} e^{5x} dx \\ &= \frac{1}{5} e^{5x} (x^2 - \frac{2}{5} x + \frac{2}{25}) + C\end{aligned}$$

7 | $\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$

Find $f(x)$

$$f(x) = x^2 \sin x$$

8 | $\int x \cos x dx$

$$\begin{aligned}\int x \cos x dx &= x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C\end{aligned}$$

9 | $\int x^2 \sin x dx$

$$\begin{aligned}\int x^2 \sin x dx &= -x \cos x - \int - \cos x dx \\ &= -x \cos x - \sin x dx + C\end{aligned}$$

10 | $\int x^3 e^{x^2} dx$

Let $u = x^2$

$$\begin{aligned}\int x^3 e^{x^2} dx &= \int x^2 x e^{x^2} dx \\ &= \int u \frac{1}{2} du e^u \\ &= \frac{1}{2} \int u e^u du \\ &= \frac{1}{2} u e^u - \frac{1}{2} \int e^u du \\ &= \frac{1}{2} u e^u - \frac{1}{2} e^u + C \\ &= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2}\end{aligned}$$

11 | $\int x^2 \ln x dx$

$$\begin{aligned}\int x^2 \ln x dx &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \frac{1}{x} dx \\ &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx \\ &= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C\end{aligned}$$

12 | $\int \cos \ln x dx$

$$\begin{aligned}\int 1 \cos \ln x dx &= x \cos \ln x + \int \sin \ln x dx \\ &= x \cos \ln x + x \sin \ln x - \int \cos \ln x dx \\ 2\int \cos \ln x dx &= x \cos \ln x - x \sin \ln x \\ \int \cos \ln x dx &= \frac{1}{2}\left(x \cos \ln x - x \sin \ln x\right) + C\end{aligned}$$

Or you could use $u = \ln x$, apparently.

13 | **multiple parts**

13.1 | **e**

13.2 | $\int e^{2x} \cos 3x dx$

$$\begin{aligned}\int e^{2x} \cos 3x dx &= \cos 3x \frac{1}{2} e^{2x} + \int 3 \sin 3x \frac{1}{2} e^{2x} dx \\ &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \int 3 \cos 3x \frac{1}{4} e^{2x} dx \\ &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \frac{9}{4}\int e^{2x} \cos 3x dx \\ \frac{13}{4}\int e^{2x} \cos 3x dx &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} + C \\ \int e^{2x} \cos 3x dx &= \frac{4}{13}\left(\cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x}\right) + C \\ &= \frac{2}{13} e^{2x}\left(\cos 3x + \frac{3}{2} \sin 3x\right) + C\end{aligned}$$

13.3 | **evaluate previous from** $[0, \frac{pi}{6}]$

$$\frac{3}{13} e^{\frac{pi}{3}} - \frac{2}{13}$$

14 | $\int \sec^3 x dx$

$$\begin{aligned}
\int \sec^3 x dx &= \int \sec x \sec^2 x dx \\
&= \sec x \tan x - \int \sec x \tan^2 x dx \\
&= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx \\
&= \sec x \tan x - \int \sec^3 x dx - \int \sec x dx \\
2 \int \sec^3 x dx &= \sec x \tan x - \int \sec x dx \\
&= \sec x \tan x - \ln|\sec x + \tan x| + C \int \sec^3 x dx \quad = \frac{1}{2} (\sec x \tan x - \ln|\sec x + \tan x|) + C
\end{aligned}$$