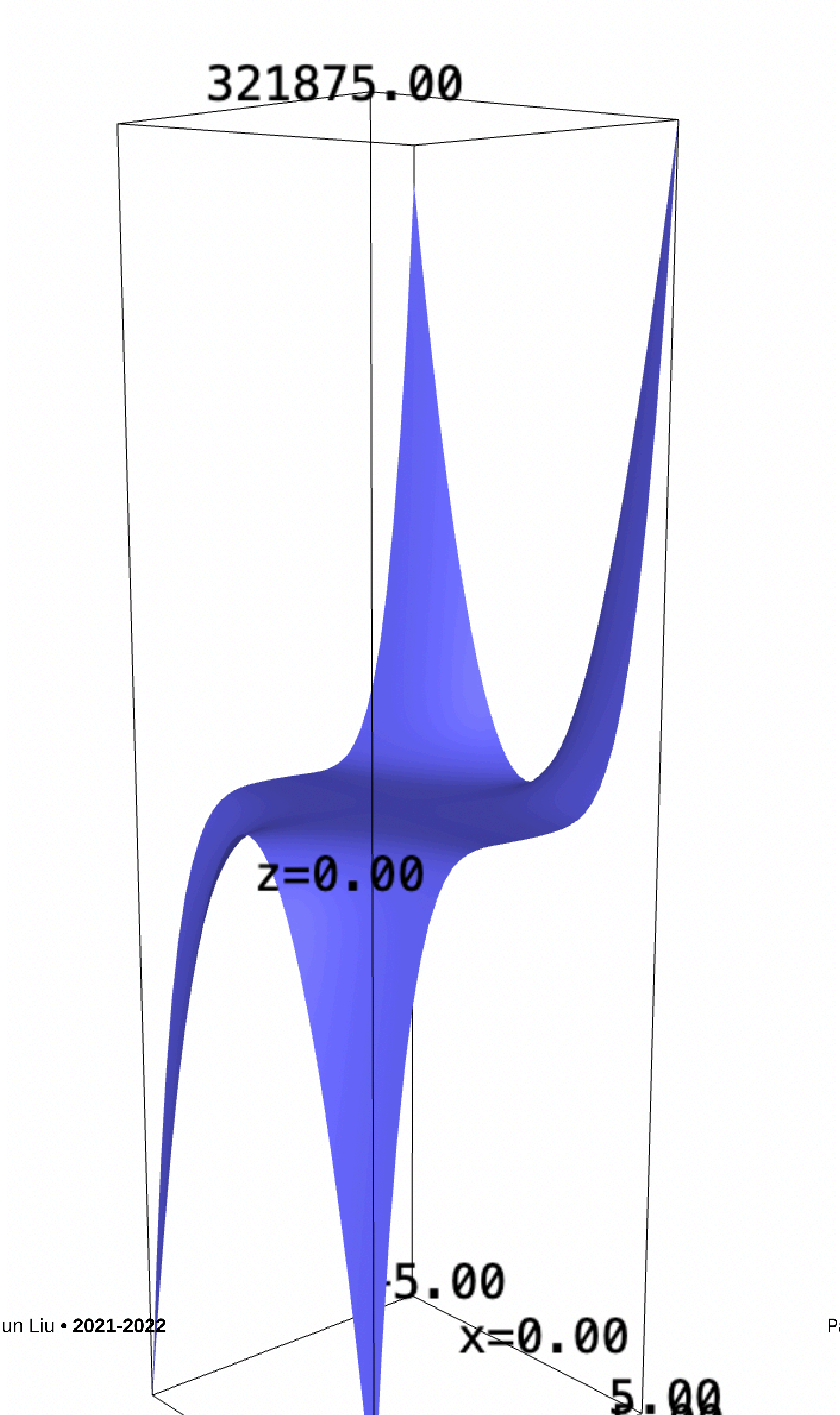
1 | Diff. in Higher Dimensions

1.1 | Partial Derivatives

Find all the first, second, and third partial derivatives. Also, draw a picture of them!

$$f(x, y) = 4x^2y^5 + 3x^3y^2 \tag{1}$$

```
f(x,y) = 4*x^2*y^5 + 3*x^3*y^2
plot3d(f, (x,-5,5), (y,-5,5))
```


321875.00
z=0.00
-5.00
x=0.00
5.00

- $f_x = 8xy^5 + 9x^2y^2$
- $f_y = 20x^2y^4 + 6x^3y$
- $f_{xx} = 8y^5 + 18xy^2$
- $f_{yy} = 80x^2y^3 + 6x^3$
- $f_{xy} = 40xy^4 + 18x^2y$
- $f_{xxx} = 18y^2$
- $f_{yyy} = 240x^2y^2$
- $f_{xxy} = 40y^4 + 36xy$
- $f_{yyx} = 160xy^3 + 18x^2$

1.2 | Number of Partial Derivatives

Suppose you have a function $\mathbb{R}^2 \to \mathbb{R}^1$. How many total first partial derivatives does it have? What about second partial derivatives? What about third partial derivatives? What about k'th partial derivatives?

A function from $\mathbb{R}^2 \to \mathbb{R}^1$ has…

- 2 first partial derivatives
- 3 second partial derivatives
- 4 third partial derivatives
- $k + 1$ kth partial derivatives

2 | Considering the Parameterization

Consider the parameterization of a straight line given by:

$$\begin{cases} x(t) = 3t + 4 \\ y(t) = 5t - 7 \\ for\ -\infty < t < +\infty \end{cases} \tag{2}$$

What is the equation of the line, in normal $y = mx + b$ form?

$$x = 3t + 4 \tag{3}$$
$$\Rightarrow x - 4 = 3t \tag{4}$$
$$\Rightarrow t = \frac{x - 4}{3} \tag{5}$$

$$
\begin{aligned}
&y = 5t - 7 && (6)\\
\Rightarrow &y = 5(\frac{x-4}{3}) - 7 && (7)\\
\Rightarrow &y = \frac{5x-20}{3} - 7 && (8)\\
\Rightarrow &y = \frac{5x-20-21}{3} && (9)\\
\Rightarrow &y = \frac{5x-41}{3} && (10)\\
\Rightarrow &y = \frac{5}{3}x - \frac{41}{3} && (11)
\end{aligned}
$$

Where does the parameterization start?

The parameterization starts at: $(x(0), y(0)) = (4, -7)$.

How fast is the parameterization moving, in the x direction?

The parameterization is moving at 3 units per unit t in the x direction.

How fast is the parameterization moving, in the y direction?

The parameterization is moving at 5 units per unit t in the y direction.

How fast is the parameterization moving, in the direction it's actually traveling.

The parameterization is moving at $\sqrt{5^2+3^2} = \sqrt{34}$ units on the line.