We are given that the object m_1 collides with the rod with velocity v_0, and the rod is floating in free space. Given m_1, v_0, m_2, I_0, and r, we are to figure to the final velocity of m_1 after collision v_f, the velocity of m_2 after collision v_{CM}, and of course the rotation of the rod after collision ω.

We are assuming that this collision elastic.

We have, then, for conservation of linear momentum:

$$m_1 v_0 = m_1 v_f + m_2 v_{CM} \tag{1}$$

Furthermore, we understand that angular momentum is also conserved here; therefore:

$$0 = \tag{2}$$

as the point mass does not have any rotational inertia, and the rod is not rotating at the start