1 | **Problem 1**

1.1 | $(1a)$

$$PE = -W$$

\[$W = \int_{R_e}^{\infty} F(r)\, dr$ \] We know that the force applied to a point mass m by the gravitational field of the earth (with mass M_e) with distance x is modeled by

$$F(r) = \frac{GmM_e}{r^2}$$

. Therefore, our work integral can be modified to be \[

$$\begin{aligned} W &= \int_{R_e}^{\infty} \frac{GmM_e}{r^2}\, dr \\ &= GmM_e \int_{R_e}^{\infty} \frac{1}{r^2}\, dr \\ &= GmM_e[-\frac{1}{r}]_{R_e}^{\infty} \\ &= -\frac{GmM_e}{R_e} \end{aligned}$$

\]

$$PE = \frac{GmM_e}{R_e}$$

1.2 | $(1b)$

$$KE = \frac{1}{2}mv^2$$

$$KE = PE$$

\[$\frac{1}{2}mv^2 = \frac{GmM_e}{R_e}$ \]

$$v = \sqrt{\frac{2GM_e}{R_e}}$$

1.3 | $(1c)$

$$v = \sqrt{\frac{2GM_e}{R_e}}$$

\[$= \sqrt{\frac{2 \cdot 6.674 \cdot 10^{-11} \cdot 5.972 \times 10^{24}}{6.371 \cdot 10^6}}$ \]

$$= 11185.7 m/s$$

$$= 25020.1 mph$$

2 | **Problem 2**

$$\sum_{i=1}^{n}\vec{F}_{net,i} = (\sum_{i=1}^{n} m_i)\ddot{\vec{r}}_{CM}$$

$$\sum_{i=1}^{n} m_i\ddot{\vec{r}}_i = (\sum_{i=1}^{n} m_i)\ddot{\vec{r}}_{CM}$$

\[$\displaystyle\int\int\sum_{i=1}^{n} m_i\ddot{\vec{r}}_i\,dt\,dt = \int\int(\sum_{i=1}^{n} m_i)\ddot{\vec{r}}_{CM}\,dt\,dt$ \] Both constants are the same constant on both sides

$$\int\sum_{i=1}^{n} m_i\dot{\vec{r}}_i\,dt + C_1 = \int(\sum_{i=1}^{n} m_i)\dot{\vec{r}}_{CM}\,dt + C_1$$

$$\sum_{i=1}^{n} m_i\vec{r}_i + C_1 t + C_2 = (\sum_{i=1}^{n} m_i)\vec{r}_{CM} + C_1 t + C_2$$

of the equation so they will cancel out. The sum of all mass is just M. \[$\vec{r}_{CM} = \dfrac{1}{M}\displaystyle\sum_{i=1}^{n} m_i\vec{r}_i$ \]

3 | **Problem 3**

Any force within a system will have an opposite force applied as well (Newton's 3rd law). Therefore, forces within a system will cancel out and will have no effect on the center of mass.

4 | **Problem 4**

\[$\displaystyle\vec{v} = \frac{< 1, -4, 1 > +2 < -3, -2, 6 > +3 < 2, 5, -3 > +4 < -2, 4, 6 >}{1+2+3+4}$ $= < -0.7, 2.3, 2.8 >$ \]

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