1 | Problem

Suppose V is a real inner product space and $v_1, \ldots, v_m$ is a linearly independent list of vectors in V. Prove that there exist exactly 2^m orthonormal lists $e_1, \ldots, e_m$ of vectors in V that preserve the prefix spans.

2 | Proof Sketch

In general, during the Gram-Schmidt procedure, both e_j or $-e_j$ preserve orthonormality and prefix span equality. Thus, there are m independant binary choices and thus 2^m possibilities.

2.1 | But why does the vector space have to be real?

Because in the real numbers, there are only two scalars with magnitude 1 to choose from on each step. But in the complex numbers, the entire unit circle is fair game, so there are an infinite number of orthonormal bases.