1 | Problem 1

The circumference of the circle: $2\pi R$. To travel this circumference, it will mean traveling all of 2π under the speed of ω, which means it will take $\frac{2\pi}{\omega}$.

Finally, therefore, the tangential velocity will require traveling $2\pi R$ in $\frac{2\pi}{\omega}$:

$$V = 2\pi R \div \frac{2\pi}{\omega} = \frac{2\omega\pi R}{2\pi} = \omega R \tag{1}$$

And finally, in order to calculate angular momentum:

$$\vec{L}(t) = \vec{r} \times m\vec{V}_0 \tag{2}$$

We will find the magnitude of this expression first:

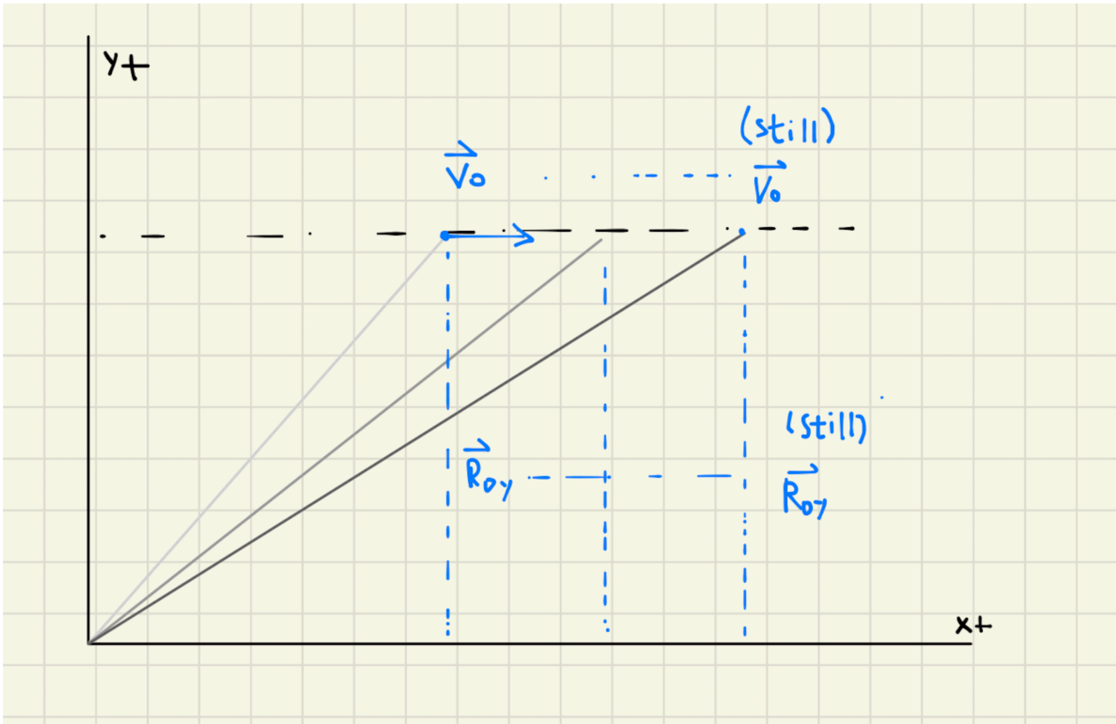
$$\begin{aligned} |\vec{L}| &= |\vec{r}||m\vec{v}_0| \sin\theta \qquad (3) \\ &= Rm\omega R \sin\left(\frac{\pi}{2}\right) \qquad (4) \\ &= m\omega R^2 \qquad (5) \end{aligned}$$

Furthermore, by the right hand rule, we can see that the direction of $\vec{L}$ is out of the page given the drawn figure.

2 | Problem 2

If a ball is traveling through space with no net force, given Newton's first law, it will travel linearly with velocity $\vec{v}_0$ at the direction thereof.

We can model this by the following figure:



We can see that if we define a coordinate system parallel to the direction of travel, we can have each $\sin(\theta)\vec{R_0}$ be equal to each other throughout the travel.

Therefore:

$$\vec{L}(t) = \vec{r}m\vec{V_0} \tag{6}$$
$$= |\vec{r}||m\vec{v_0}|\sin\theta \tag{7}$$
$$= |\vec{r}|\sin\theta|m\vec{v_0}| \tag{8}$$

We see that though $\sin\theta$ and $\vec{r}$ are the two non-constant terms in the expression, the projection of $\vec{r}\sin\theta$ would be equal if we are leveraging the equality as given above would stay constant as per the figure above because the y coordinate for every position is the same in our coordinate system (and hence the y projection would be the same).

Therefore, all elements of $\vec{L}(t)$ is constant—making the overall angular momentum constant.

3 | **Problem 3**

Proof:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \tag{9}$$

We are going to first take the expression for angular momentum:

$$\vec{L}(t) = \vec{r} \times m\vec{V} \tag{10}$$

And, as per prescribed to the right, take its derivative:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{V}) \tag{11}$$
$$= \frac{d}{dt}(\vec{r} \times m\vec{V}) \tag{12}$$
$$= \left(\frac{d\vec{r}}{dt} \times m\frac{d\vec{r}}{dt}\right) + \left(\vec{r} \times m\frac{d\vec{V}}{dt}\right) \tag{13}$$
$$= 0 + \left(\vec{r} \times m\frac{d\vec{V}}{dt}\right) \tag{14}$$
$$= \left(\vec{r} \times m\frac{d\vec{V}}{dt}\right) \tag{15}$$
$$= \vec{r} \times m\frac{d\vec{V}}{dt} \tag{16}$$

The only technique we took here is simply the cross-product product rule.

Lastly, therefore, we know that the first derivative of velocity is acceleration. Hence:

$$\frac{d\vec{L}}{dt} = \frac{d}{dt}(\vec{r} \times m\vec{V}) \tag{17}$$
$$= \vec{r} \times m\frac{d\vec{V}}{dt} \tag{18}$$
$$= \vec{r} \times m\vec{a} \tag{19}$$
$$= \vec{r} \times \vec{F} \tag{20}$$

Of course, $\vec{\tau}_{net} = \vec{r} \times \vec{F}_{net}$. Hence:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \blacksquare \tag{21}$$