1 | boatman problem

Target displacement: $\langle 3\text{km}, 2\text{km}\rangle$

We are working with the velocities of the boat and the river. The velocity of the river is defined as $r = \langle 0, -3.5\rangle$. We want to find vector $v = \langle v_x, v_y\rangle$ s.t.

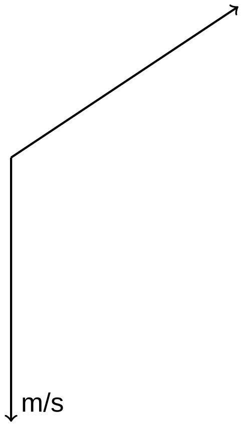
$$
\begin{aligned}
|v| &= 13 \text{ km/h} \\
\lambda(v+r) &= \langle 3, 2\rangle
\end{aligned}
$$

Where the trip will take λ hours

$$
\begin{aligned}
v_x^2 + v_y^2 &= 13^2 \\
\lambda(v_x + 0) &= 3 \\
\lambda(v_y + -3.5) &= 2
\end{aligned}
$$

$$
\begin{aligned}
v_x &= \frac{3}{\lambda} \\
v_y &= \frac{2}{\lambda} + 3.5 \\
\frac{3^2}{\lambda^2} + \left(\frac{2}{\lambda} + 3.5\right)^2 &= 13^2 \\
\frac{3^2}{\lambda^2} + \frac{2^2}{\lambda^2} + 3.5^2 + \frac{4(3.5)}{\lambda} &= 13^2 \\
\frac{3^2+2^2}{\lambda^2} + \frac{4(3.5)}{\lambda} &= 13^2 - 3.5^2 \\
3^2 + 2^2 + 4(3.5)\lambda &= \lambda^2\left(13^2 - 3.5^2\right) \\
13 + 4(3.5)\lambda &= \lambda^2\,(156.75) \\
-156.75\lambda^2 + 14^2 + 13 &= 0 \\
\frac{-14 \pm \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} \\
\frac{-14 + \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} &= -0.24676847741 \\
\frac{-14 - \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} &= 0.336082671987
\end{aligned}
$$

Maybe it's time to do it geometrically



Let θ be the angle difference that you paddle at, and ϕ be the angle that you are aiming for.

$$3.5^2\lambda^2 = 13 + 13\lambda - 2(13)13\lambda\cos\theta$$
$$\tan\phi = \frac{3}{2}$$
$$\sin(\theta+\phi) = \frac{3.5\lambda + 2}{13\lambda}$$

attempt 3: after getting help from leonard

$$\beta = \alpha + \frac{\pi}{2} = \tan^{-}\frac{2}{3} = 2.158$$
$$\frac{\sin\beta}{|v|} = \frac{\sin\gamma}{3.5}$$
$$\frac{\sin(2.158)}{13} = \frac{\sin\gamma}{3.5}$$
$$3.5\frac{\sin(2.158)}{13} = \sin\gamma$$
$$\gamma = 3.5\frac{\sin(2.158)}{13} = 0.2241$$
$$\alpha + \gamma = 0.588 + 0.2241 = 0.8121 \text{ radians}$$

The speed

$$\frac{3}{13\cos 0.812} = 0.3353 \text{ hours}$$

dang it i was actually right the first time. apparently math isn't a democracy.

2 | circular motion

$$\theta'(t) = \omega_0$$
$$\theta(t) = \int \omega_0 dt \quad = \omega_0 t + \theta_0$$

2.1 | position

$$\begin{aligned}\vec{r}(t) &= (R\cos\theta(t), R\sin\theta(t)) \\ &= (R\cos(\omega_0 t+\theta_0), R\sin(\omega_0 t+\theta_0))\end{aligned}$$

2.2 | velocity

$$\vec{v}'(t) = (-\omega_0 R\sin(\omega_0 t+\theta_0), \omega_0 R\cos(\omega_0 t+\theta_0))$$

2.3 | acceleration

$$\begin{aligned}\vec{a}(t) &= \vec{v}'(t) \\ &= \frac{d}{dt}(-\omega_0 R\sin(\omega_0 t+\theta_0), \omega_0 R\cos(\omega_0 t+\theta_0)) \\ &= \left(-\omega_0^2 R\cos(\omega_0 t+\theta_0), -\omega_0^2 R\sin(\omega_0 t+\theta_0)\right)\end{aligned}$$

2.4 | perpendicular

$$\begin{aligned}\text{pos: } & \frac{\cancel{R}\sin(\omega_0 t+\theta_0)}{\cancel{R}\cos(\omega_0 t+\theta_0)} \\ \text{vel: } & \frac{\cancel{\omega_0 R}\cos(\omega_0 t+\theta_0)}{-\cancel{\omega_0 R}\sin(\omega_0 t+\theta_0)}\end{aligned}$$

See? They are negative reciprocals of eachother.

$$\begin{aligned}|\vec{v}| &= \sqrt{\omega_0^2 R^2\sin^2(\omega_0 t+\theta_0)+\omega_0^2 R^2\cos^2(\omega_0 t+\theta 0)} \\ &= \omega_0 R\sqrt{\sin^2(\omega_0 t+\theta_0)+\cos^2(\omega_0 t+\theta 0)} \\ &= \omega_0 R\sqrt{1} \\ &= \omega_0 R\end{aligned}$$

2.5 | acceleration

$$\begin{aligned}\text{accl } &= -\omega_0^2 R(\cos(\omega_0 t+\theta_0), \sin(\omega_0 t+\theta_0)) \\ \text{pos } &= R(\cos(\omega_0 t+\theta_0)+\sin(\omega_0 t+\theta_0))\end{aligned}$$

The acceleration is a negative scalar multiple of the position, and thus points towards the center.