## 1 | 1)

$$\vec{L} = \vec{p} \times m \vec{v}$$

The circle has a circumference of  $2\pi R$ , and it takes  $\frac{2\pi}{\vec{\omega}}$  seconds to travel that distance, so the tangential velocity must be  $\vec{v}=2\pi R\div\frac{2\pi}{\vec{\omega}}=R\vec{\omega}$  Therefore,  $\vec{L}=\vec{R}\times mR\vec{\omega}$ 

The vector is pointing out of the page, as the object is rotating counterclockwise.  $|\vec{L}| = mR|\vec{R}||\vec{\omega}| \\ = mR^2\omega$ 

We know that  $\sin \theta$  is 1 because the vectors are perpendicular.

## 2 | 2)

We can think of  $\vec{r}$  as the sum of some initial position vector and velocity times the time.

$$\vec{r} = \vec{r}_0 + \vec{v}t$$

Then, we can look at angular momentum:

$$\begin{split} \vec{L} &= \vec{r} \times m\vec{v} \\ &= (\vec{r}_0 + \vec{v}t) \times m\vec{v} \\ &= (\vec{r}_0 \times m\vec{v}) + (\vec{v}t \times m\vec{v}) \\ &= \vec{r}_0 \times m\vec{v} \end{split}$$

We see that angular momentum is not reliant on t. As such, it is conserved.

## 3 | 3)

$$\vec{L} = \vec{p}\vec{v}$$
 We are given: 
$$\frac{d\vec{L}}{dt} = \frac{d\vec{p}}{dt} \times m\vec{v} + \frac{d\,m\vec{v}}{dt} \times \vec{p}$$
 
$$= \vec{v} \times m\vec{v} + m\vec{a} \times \vec{p} \qquad = 0 + \vec{p} \times m\vec{a}$$
 
$$= \vec{p} \times \vec{F}$$
 
$$= \vec{\tau}$$