1 | Rotation

Ask me for a picture of the whiteboard or something

2 | Temperature

The gradient of the function T is as follows:

$$\nabla T(x,y,z) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix}$$
$$= \begin{bmatrix} 12yz \\ 12xz \\ 12xy \end{bmatrix}$$
$$\nabla T(1,1,1) = \begin{bmatrix} 12 \\ 12 \\ 12 \end{bmatrix}$$

We know that the direction of steepest slope is in the direction of the gradient, or in the direction of $12\hat{i} + 12\hat{j} + 12\hat{k}$ in our case.

To get the rate of change in a particular direction, we first can take the dot product of the gradient and the position vector. Doing this, we get $12 \cdot 1 + 12 \cdot 1 + 12 \cdot 1 = 36$. Then, because we know that the vector $\hat{i} + \hat{j} + \hat{k}$ has a magnitude of $\sqrt{3}$, we can divide our value by that value to get $\frac{36}{\sqrt{3}} = 12\sqrt{3}$.

IDK how to explain this next part, just follow along

$$0 = 12a + 12b + 12c$$
$$1 = \sqrt{a^2 + b^2 + c^2}$$

We can conceptualize this as the intersection between a sphere and a plane. This, of course, is a circle. We now do algebra:

$$\begin{aligned} 0 &= a + b + c \\ a &= -b - c \\ 1 &= \sqrt{(-b-c)^2 + b^2 + c^2} \\ &= \sqrt{(b+c)^2 + b^2 + c^2} \\ &= \sqrt{b^2 + 2bc + c^2 + b^2 + c^2} \\ &= \sqrt{2b^2 + 2bc + 2c^2} \\ &= \sqrt{2} \cdot \sqrt{b^2 + bc + c^2} \\ \sqrt{b^2 + bc + c^2} &= \cos\frac{\pi}{2} \end{aligned}$$

All values of b, c are on this equation of an ellipse. In fact, all values of a, b and a, c will also satisfy this equation because of the property of the plane. Cool stuff.