

1 | Evaluating a Cylindrical Integral

Considering the function:

$$f(x, y, z) = \sqrt{x^2 + y^2} \quad (1)$$

To evaluate the integral, we will convert it to cylindrical coordinates. We note first that the integral is to be evaluated inside the cylinder of $x^2 + y^2 = 16$, which means that we wish to evaluate it in a circle with center at the origin with radius 4.

Furthermore, we understand that the bounds of the function are to be evaluated between $[-5, -4]$.

If we set up the integral, we will get:

$$\int_{-5}^{-4} \int_C \sqrt{x^2 + y^2} \, dx \, dy \, dz \quad (2)$$

This is convenient. We can evaluate the inner integral first like in $\mathbb{R}^2 \rightarrow \mathbb{R}^1$, then simply evaluate the other integral after.

Let's do so.

Note that the inner integral is a normal cylindrical coordinate setup. Therefore, we can take the following substitution:

$$\sqrt{x^2 + y^2} = r \quad (3)$$

Furthermore, that:

$$dx \, dy = dr \, d\theta \quad (4)$$

With the appropriate bounds, then:

$$\int_0^{2\pi} \int_0^4 r \, dr \, d\theta \quad (5)$$

$$\Rightarrow \int_0^{2\pi} \left. \frac{r^2}{2} \right|_0^4 d\theta \quad (6)$$

$$\Rightarrow \int_0^{2\pi} 8 \, d\theta \quad (7)$$

$$\Rightarrow 16\pi \quad (8)$$

Finally, we will take the integral of this value dz :

$$\int_{-5}^{-4} 16\pi \, dz = 16\pi \quad (9)$$

Therefore, the value of the integral is 16π .

2 | Uselessly Spherical Integral

We first recall that the differential volume can be written as:

$$dV = \rho^2 \sin \phi \, d\rho \, d\phi \, d\theta \quad (10)$$

To take this integral, then, we have to figure the distance ρ to a rectangle for every point (ϕ, θ) .

We will take this integral w.r.t. one quadrant of the four parts of the rectangle, then we will scale it up by 4.