1 | **Electric Change**

We are finally taking a surface integral! This is essentially multiplying the surface area of the shape of the function to the value of the function itself.

Firstly, taking the area dA by dV:

$$\begin{aligned}
dA &= \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} && (1)\\
&= \sqrt{1 + (3)^2 + (2)^2} && (2)\\
&= \sqrt{14} && (3)
\end{aligned}$$

Supplying the value into the function:

$$\begin{aligned}
&\int_0^7 \int_0^{11} (3x + 2y + 7)\sqrt{14}\ dy\ dx && (4)\\
\Rightarrow&\sqrt{14} \int_0^7 \int_0^{11} (3x + 2y + 7)\ dy\ dx && (5)\\
\Rightarrow&\sqrt{14} \int_0^7 \left(3xy + y^2 + 7y\right)\Big|_0^{11}\ dy\ dx && (6)\\
\Rightarrow&\sqrt{14} \left(\frac{33x^2}{2} + 198x\right)\Bigg|_0^7 && (7)\\
\Rightarrow&\frac{4389\sqrt{14}}{2} && (8)
\end{aligned}$$

The charge, therefore, is proportional to $\frac{4389\sqrt{14}}{2}\rho$.

2 | **Infinite wire**

Recall first that a semicircle with radius 7 can be defined as:

$$\begin{aligned}
y &= \sqrt{7^2 - x^2} && (9)\\
&= \sqrt{49 - x^2} && (10)
\end{aligned}$$

Let's first figure the value of this function dA:

$$\begin{aligned}
dA &= \sqrt{1 + \left(\frac{d}{dx}\sqrt{49 - x^2}\right)^2} && (11)\\
&= \sqrt{1 + \left(\frac{d}{dx}\sqrt{49 - x^2}\right)^2} && (12)\\
&= \sqrt{1 - \frac{x^2}{x^2 - 49}} && (13)
\end{aligned}$$

We will take the line integral of this function, and proceed to multiply by the value of xy at that point.

$$\int_0^7 \int_0^7 xy\sqrt{1-\frac{x^2}{x^2-49}}\,dx\,dy \tag{14}$$

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f(x,y) = x*y*sqrt(1-x^2/(x^2-49))
f.integrate(x, 0,7).integrate(y,0,7)
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Looks like the solution for the wire's weight is about $\frac{2401}{2}$ grams.

3 | More Difficult Polar Coordinates

Recall that, to figure the unit sphere volume, we can convert an $\mathbb{R}^2 \to \mathbb{R}^1$ result into circular coordinates. That, by pythagoras, $x^2+y^2=r^2$. Therefore, the expression of:

$$f(x,y) = \frac{1}{(x^2+y^2)^k} \Rightarrow f(r,\theta) = \frac{1}{r^{2k}} \tag{15}$$

We also note that, due to the correction factor, $dA = r\ dr\ d\theta$.

Taking the actual integral, therefore, will result in:

$$\int_0^{2\pi}\int_0^1 r^{-k}\ dr\ d\theta \tag{16}$$

$$\Rightarrow \int_0^{2\pi} \lim_{x\to 0}\left(\frac{1}{-k+1} - \frac{1}{x^{k-1}}\frac{1}{-k+1}\right)\ d\theta \tag{17}$$

Evidently, when $k \leq 1$, the second term would become infinity large.

Now, we essentially want to take this idea and expand it to n dimensions, to figure the correct spherical coordinates.

Turns out, the naïve version of the n sphere integral is the same correction factor multiplied by $\sin^{n-\{2\ldots(n-1)\}}$. Therefore, the same logic from above actually holds for n volcano as well: that, by very high dimension Pythagoras, ${x_1}^2+{x_2}^2+\ldots+{x_n}^2=r^2$.

Therefore:

$$\frac{1}{({x_1}^2+{x_2}^2+\cdots+{x_n}^2)^k} = \frac{1}{r^{2k}} \tag{18}$$

We will again note than the correction factor: $dA = r^{n-1}\ sin^{n-2}\ sin^{n-3}\ dr\ d\theta_1\ \ldots\ d\theta_n$.

Therefore:

$$\int\ldots\int_0^1 \frac{1}{r^{2k-n+1}}\ sin^{n-2}\ sin^{n-3}\ dr\ d\theta_1\ \ldots\ d\theta_n \tag{19}$$

$$\Rightarrow \int\ldots\int_0^1 r^{-2k+n-1}\ sin^{n-2}\ sin^{n-3}\ dr\ d\theta_1\ \ldots\ d\theta_n \tag{20}$$

$$\Rightarrow \int\ldots\int \left.\frac{r^{-2k+n}}{-2k+n}\right|_0^1\ sin^{n-2}\ sin^{n-3}\ dr\ d\theta_1\ \ldots\ d\theta_n \tag{21}$$

At this point, we can analyze the solution. The first term will always be 1 over a certain value. The second term, however, is more interesting.

Case 1: $-2k + n < 0$. If the value of $-2k + n < 0$, the r would have to be transported below the fraction. Therefore, taking $\lim_{r\to 0} \frac{1}{r^{2k-n}(-2k+n)}$ would $= +\infty$.

Case 2: $-2k + n >= 0$. This would render no problem: the second term would be simply 0.

Therefore, if the terms that make up the infinite volcano $-2k+n > 0$, the results would be infinite. Otherwise, the results are finite.