1 | Types of Proofs

- Proof by Example/Counterexample
- Proof by Cases
- Proof by Contradiction
- Proof by Induction
- Proof by Strong Induction

Proof: The number of primes is infinite.

- This will be an example of proof by contradiction.

Assume for the purposes of contradiction that the number of primes is finite.
Then we can list them:

- {P~1~, P~2~, …}

Consider the number S = P~1~ x P~2~ x P~3~ x … + 1
Case 1: S is prime

- Contradiction as S is not in the set.

Case 2: S is not prime

- If S is not prime it must be divisible by at least two prime > numbers.
- However, all primes are in that list, so if S is divided by any > number in the list the remainder will be one.
- Contradiction!

All cases have contradictions so our assumptions must be false.
Q.E.D.
Proof by induction

- Prove something is true for a smaller number and show that doing so > implies it is true for larger numbers.
- There are 5 steps.
  - Declare proof by induction
  - Declare inductive hypothesis
    * Inductive hypothesis is typically whatever you are trying to > prove.
  - Prove the base case
    * Like dominos where a proof of one number leads to the proof > of the next number
  - Show that P(n) $\rightarrow$ P(n+1)
  - Invoke induction.

Proof: 1+2+3…+n = n(n+1)/2

- Proof is by induction.

P(n) is the hypothesis that 1+2+3…+n = n(n+1)/2

P(1):

1 = 1(1+1)/2

1 = 2/2

1 = 1

P(n+1):

We need to show 1+2+3+4…+n+n+1 = ((n+1)(n+2))/2

Assume that P(n) is true.

Left side simplifies to (n(n+1)/2) + (n+1)

Algebraic manipulation leaves you with ((n+2)(n+1))/2

If P(n) is true then P(n+1) is true and P(1) is true therefore P(n) is true for all numbers.

Proof by Strong Induction

- Use P(1), P(2) …. P(n) to prove P(n+1)
- Same list but instead of proving P(n) → P(n+1) it will be P(1), > P(2)\… P(n) → P(n+1)

Proof: Any group of students ≥ 12 can be divided into some combination of groups of 4 and groups of 5.

- This will be a proof by strong induction.

P(12) = 4+4+4

P(13) = 4+4+5

P(14) = 4+5+5

P(15) = 5+5+5

P(n) → P(n+4)

If P(n-3) is true, then there exists a combination of groups of 4 and 5 to make up (n-3) students. Add a group of 4 and that gives us out combination for (n+1) students.

P(n-3) → P(n+1)

By induction this is true for all n ≥ 12

2 | **Links**

See Induction for more examples.