1 | Basic Concepts

1.1 | Limit Definition

Limits have not been rigorously defined in this class so the explanation that $\lim_{x \to y}$ is the value of $f(x)$ when x "tends toward" or gets infinitely close to y.

1.2 | Two-sided Limits

Left and right-hand limits are denoted with a + or - superscript in the limit, like $\lim_{x \to y^+}$ and indicate that x gets infinitely close to y from a specific direction. This is useful for functions that are discontinous or have a removable discontinuity.

1.3 | Discontinuity

Several types of discontinuity exist:

- Jump discontinuity where lim from left and right exist but are unequal
- Removable discontinuity where lim from left and right are the same but the point itself is not defined.

1.4 | Squeeze Theorem

See Squeeze Theorem definition

2 | Calculating Limits

2.1 | Graphically

Look at the graph to tell where the function is going around the limit.

2.2 | Numerically

Evaluate the function at values very close to where the limit is tending to (like 0.001, -0.001, etc)

2.3 | Algebraically

Plug in. EX: $\lim_{x \to 0} \frac{x}{2} = \frac{0}{2}$

What if it isn't defined at that point? Begin trying to simplify until it is.

Usually means factorization until you find like terms to cancel out.

What if there's a radical at the top? Multiply by the conjugate - example: $\lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} = \lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = \lim_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = \lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

What if it's not straightforward to factor, like $x^3 - 1$? Use fancy algebraic identities like $a^3 - b^3 = (a^3 - b^3)(a^2 + ab + b^2)$

What if I'm dying and it just won't factor? The limit probably doesn't exist. example: $\lim_{x \to -5} \frac{x^2+25}{x+5}$

2.4 | Trigonometric Limits

Use trigonometric identities combined with the fact that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ to get answers. The mosmt useful trig identities in this context are the double-angle identities and the pythagorean identities. Look for applications of them!

Beware the evil $\lim_{x \to 0} \frac{\sin Ax}{\sin Bx}$ where A and B are constants > 2 (otherwise just use double angle). The trick here is to nicely use fractions to your advantage: $\lim_{x \to 0} \frac{\sin Ax}{\sin Bx} = \frac{\sin Ax}{1} \frac{1}{\sin Bx} = \frac{\sin Ax}{1} \frac{A}{A} \frac{1}{\sin Bx} \frac{B}{B} = \frac{\sin Ax}{A} \frac{1}{A} \frac{B}{\sin Bx} \frac{B}{1} = \frac{B}{A} \frac{\sin Ax}{A} \frac{B}{\sin Bx} = \frac{B}{A}$ as we know $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $1^{-1} = 1$

Remember that $\sin^2 2x = \sin 2x \cdot \sin 2x$ and don't make silly mistakes.

2.5 | Applying Squeeze Theorem

Look for obvious functions that have the same limit yet are always smaller/larger than f(x). Remember that this gets a whole lot easier when it tends towards zero. $\lim_{x \to 0} \frac{\sin x}{x} = 1$ is your friend. Limit laws may help as well.

3 | Things To Remember

3.1 | Vocab

Does not exist describes double-sided limits when the limits from both sides are different *Not defined* describes f(x) where x is not in the domain of f as well as points of the graph f(x) outside of its range. Holes are *not defined*. *Undefined* describes invalid operations like 27/0

3.2 | Standard stuff

Double triple quadruple check everything, it's a take-home test so you may as well ensure you got it correct. Write out all of your steps in an organized fashion so you don't get mixed up and teacher gets what you're doing. DO NOT RUSH THE AVG CHANGE QUESTION(S?). A BUG IN CODE OR A SIMPLE MISTAKE WILL GET THE WHOLE PROBLEM WRONG AND IT'S EASY SO YOU WILL HAVE NOBODY TO BLAME BUT YOURSELF.