1 | inner product **def**

An *inner product* on V is a function that takes each ordered pair (u, v) of elements of V to a number $\langle u, v\rangle \in$ and has the following properties

- **positivity** $\langle v, v\rangle \geq 0 \forall v \in V$
- **definiteness** $\langle v, v\rangle = 0 \iff v = 0$
- **additivity in first slot** $\langle u + v, w\rangle = \langle u, w\rangle + \langle v, w\rangle \forall u, v, w, \in V$
- **homogeneity in first slot** $\langle \lambda u, v\rangle = \lambda\langle u, v\rangle \forall \lambda \in, u, v \in V$
- **conjugate symmetry** $\langle u, v\rangle = \overline{\langle v, u\rangle} \forall u, v \in V$
  - Over the reals, this is equal to $\langle u, v\rangle = \langle v, u\rangle$

2 | motivation

2.1 | The norm of a complex number $\|z\|$ should be non-negative, so we can define it as

$$\|z\| = \sqrt{|z_1|^2 + \cdots + |z_n|^2}$$

Since the square of the absolute value is just a complex number times a conjugate, and because the norm squared should be the inner product of z with itself, maybe the inner product of $w, z \in^n$ should equal

$$w_1\overline{z_1} + \cdots + w_n\overline{z_n}$$

2.2 | positivity: we want inner product to be the size of the vector, so it should be a positive and real number

2.3 | notation

For a complex scalar $\lambda \in$, $\lambda \geq 0$ means λ is real and non-negative

$\langle u, v\rangle$ denotes an inner product.

3 | intuition

3.1 | additivity/homogeneity in the first slot also implies additivity in the second slot, and ’conjugate homogeneity in the second slot’

3.2 | we want the norm to be a real scalar, so we need to take the complex conjugate of the second one

3.2.1 | so, the Euclidean inner product is conjugate the second, then dot product

$$\langle u, v\rangle = u\overline{z}$$