1 | covariance matrices

1.1 | expected value properties

$$E[X+Y] = E[X] + E[Y] E[\lambda X] = \lambda E[X]$$

1.2 | variance

$$\begin{aligned}
&E[(X-E[X])^2] && \text{mean squared error}\\
E[(X-E[X])^2] &= E[X^2 + E[X]^2 - 2XE[X]] && \text{expand}\\
&= E[X^2] + E[E[X]^2] - E[2XE[X]] && \text{additivity}\\
&= E[X^2] + E[X]^2 - E[2XE[X]] && \text{expected value of a scalar}\\
&= E[X^2] + E[X]^2 - 2E[X]E[X] && \text{homogeneity}\\
&= E[X^2] + \cancel{E[X]^2} - \cancel{2}E[X]^2 && \text{expected value of a scalar}\\
&= E[X^2] - E[X]^2 && \text{combine like terms}
\end{aligned}$$

1.3 | covarience

The same thing, just with X, Y instead of X, X.

$$\begin{aligned}
&E[(X-E[X])(Y-E[Y])] && \text{covarience}\\
E[(X-E[X])(Y-E[Y])] &= E[XY - XE[Y] - YE[X] + E[X]E[Y]] && \text{expand}\\
&= E[XY] - E[XE[Y]] - E[YE[X]] + E[E[X]E[Y]] && \text{additivity}\\
&= E[XY] - E[Y]E[X] - E[X]E[Y] + E[E[X]E[Y]] && \text{homogeneity}\\
&= E[XY] - E[Y]E[X] \cancel{- E[X]E[Y] + E[X]E[Y]} && \text{expected value of a scalar}\\
&= E[XY] - E[X]E[Y] && \text{combine like terms}
\end{aligned}$$