1 | **1)**

To finish the proof… Given two objects, A and B, with a force F between them, the torque on A and B is given by

$$\tau_A = \vec{r}_A \times \vec{F}_A$$
$$\tau_B = \vec{r}_B \times \vec{F}_B$$

where $\vec{F}_A$ is the foce applied by B on A, and vice versa. We know that because of N-3 $\vec{F}_A = -\vec{F}_B$. (We also know that the forces point towards each object.) Therefore,
$$\begin{aligned}\tau_{AB} &= \tau_A + \tau_B \\ &= \vec{r}_A \times \vec{F}_A + \vec{r}_B \times \vec{F}_B \\ &= \vec{r}_A \times \vec{F}_A + \vec{r}_B \times -\vec{F}_A\end{aligned}$$

We know that the direction of the two cross products are orthogonal to the plane that the two objects' position vectors and the origin of the system form.

$$\begin{aligned}\tau_{AB} &= \vec{r}_A \times \vec{F}_A + \vec{r}_B \times -\vec{F}_A \\ &= |\vec{r}_A||\vec{F}_A| \sin\theta_A - |\vec{r}_B||\vec{F}_A| \sin\theta_B \\ &= |\vec{r}_A| \sin\theta_A - |\vec{r}_B| \sin\theta_B\end{aligned}$$

The law of sines states that for a triangle $\triangle ABC$, $\frac{\overline{\text{BC}}}{\sin\theta_A} = \frac{\overline{\text{AC}}}{\sin\theta_B}$. We know that this applies in our particular proof because the objects A, B, and the origin form a triangle. As such,

$$\begin{aligned}|\vec{r}_A| \sin\theta_A &= |\vec{r}_B| \sin\theta B \\ \tau_{AB} &= 0\end{aligned}$$

The internal torque of any two objects of a system is zero, so the total internal torque must also be zero.

2 | **2)**

We know that for one of the $$s:
$$\begin{aligned}\vec{r} &= R\hat{i} + h\hat{k} \\ \vec{L}_1 &= \vec{r} \times m\vec{v} \\ \vec{v} &= R\omega\hat{j} \\ \vec{L}_1 &= (R\hat{i} + h\hat{k}) \times mR\omega\hat{j} \\ &= -hmR\omega\hat{i} + mR^2\omega\hat{k}\end{aligned}$$

We know that there are two masses, symmetric about the z-axis, so we know that the angular momentum of the other object can be derived just by multiplying the $\hat{i}$ and $\hat{j}$ terms by -1.

$$\begin{aligned}\vec{L}_2 &= \vec{L}_1 \cdot \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= hmR\omega\hat{i} + mR^2\omega\hat{k}\end{aligned}$$

We can add the two to get the aggregate angular momentum of the system:

$$
\begin{aligned}
\vec{L} &= \vec{L}_1 + \vec{L}_2 \\
&= (-hmR\omega\hat{\imath} + mR^2\omega\hat{k}) + (hmR\omega\hat{\imath} + mR^2\omega\hat{k}) \\
&= 2mR^2\omega\hat{k}
\end{aligned}
$$

3 | 3)

3.1 | a)