PS#17: More fun!!!

Nueva Multivariable Calculus S2022

(remember, folks, that "showing your work" means, inter alia*, drawing lots of pictures!!!!)*

0. Read Andrew's solution notes to PS#16! (SO MANY GREAT PICTURES!!!)
1. Consider the same sawed-off cylinder/organ pipe the side areas of which we found last time, above a circle of radius five, with a top described by the function $f(x, y) = 7 + x + y$. Find the *volume* of this shape! (Yes, the volume this time.) Don't look up how to do it—just use your existing knowledge!

   Try doing it with slices parallel to the axes, and also with *radial* slices. Do you get the same answer?
2. Consider the double integral, given to me by Veena:

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{\left(x^4\right)}\, dx\, dy$$

   What's the shape being described by this double integral? (Give pictures, a description, etc.) What's its volume? (I.e., actually calculate the integral. Don't use a technology!)
3. What's the average value of the function $f(x, y) = e^y \sqrt{x + e^y}$ on the rectangle with vertices at $(0, 0)$, $(4, 0)$, $(4, 1)$ and $(0, 1)$?
4. Mark Hurwitz now has a magic box, too! It's not filled with magical energy, but instead, it's filled with a strange material, such that when Mark places it so that it has one corner at the origin and the opposite corner at $(3, 3, 4)$, its density is given by the function:

$$d : \mathbb{R}^3 \to \mathbb{R}^1$$

$$d(x, y, z) = \frac{1}{z + 1}$$

   What's the total volume of Mark's magic box? What's its total mass? What's its average density?