

1 | Tiling the Pringlehouse

As a review, our pringles shaped house has the following parametres:

$$\begin{cases} x(t) = 30\cos(t) \\ y(t) = 20\sin(t) \end{cases} \quad (1)$$

and the roof is defined by:

$$r(x, y) = \frac{1}{400} (\sqrt{3}x - y)^2 - \frac{1}{400} (\sqrt{3}y - x)^2 + 10 \quad (2)$$

We will first convert the above function into rectangular bounds to take the area of.

$$x = 30\cos(t) \quad (3)$$

$$\Rightarrow \frac{x}{30} = \cos(t) \quad (4)$$

$$\Rightarrow t = \arccos\left(\frac{x}{30}\right) \quad (5)$$

Supplying this back to the original expression for y:

$$y = 20\sin\left(\arccos\left(\frac{x}{30}\right)\right) \quad (6)$$

$$= 20\sqrt{1 - \left(\frac{x}{30}\right)^2} \quad (7)$$

Therefore, the actual integral:

$$\int_{-30}^{30} \int_{-20\sqrt{1 - \left(\frac{x}{30}\right)^2}}^{20\sqrt{1 - \left(\frac{x}{30}\right)^2}} 1 dy dx \quad (8)$$

We will endeavor now to use technology.

```
var("x y")
f(x,y) = 1
f.integrate(y, -20*sqrt(1-(x/30)^2), 20*sqrt(1-(x/30)^2)).integrate(x, -30,30)
```

It appears that the area of the floor is 600π .

We can do this something for the function of the roof. We will first figure correction factor dA , then take the integral as prescribed.

$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \quad (9)$$

At this point, we realize that the actual function will turn to be much too complicated to integrate by hand at this moment; therefore, we will create the expression digitally.

```
r(x,y) = (1/400)*(sqrt(3)*x-y)^2 - (1/400)*(sqrt(3)*y+x)^2 + 10
da = sqrt(1+r.diff(x)^2+r.diff(y)^2)
monte_carlo_integral(da.integrate(y, -20*sqrt(1-(x/30)^2), 20*sqrt(1-(x/30)^2)), [-30], [30], 10000000)
```

Looks like the result is converting to about 2002.2 for this shape.

2 | Three Dimensional Region!