

1 | inner product space

def

An *inner product space* is a vector space V along with an inner product on V .

When $V = \mathbb{R}^n$, assume the inner product is the Euclidean inner product

$$\langle (w_1, \dots, w_n), (z_1, \dots, z_n) \rangle = w_1 \overline{z_1} + \dots + w_n \overline{z_n}$$

2 | results

2.1 | Axler 6.7 properties

2.1.1 | **For each fixed $u \in V$, the function that takes v to $\langle v, u \rangle$ is a linear map from V to \mathbb{C}**

2.1.2 | $\langle 0, u \rangle = 0 = \langle u, 0 \rangle \forall u \in V$

2.1.3 | $\langle u, v + w \rangle = \langle u, v \rangle + \langle u, w \rangle$ **for all** $u, v, w \in V$

2.1.4 | $\langle u, \lambda v \rangle = \overline{\lambda} \langle u, v \rangle$ **for all** $\lambda \in \mathbb{C}$ **and** $u, v \in V$