1 | **Thing**

First, we will define equations for the distance as a function of t. (Note that both h_0 and θ are parameters but aren't shown as arguments to the function.)

$$\begin{cases} x(t) &= v_0 \cos(\theta) t + h_0 \\ y(t) &= -\frac{1}{2} g t^2 + v_0 \sin(\theta) t + h_0 \end{cases}$$

Velocity is a function of h_0:

$$v_0 = \sqrt{2g(H - h_0)} \tag{1}$$

We can rewrite $x(t)$ and $y(t)$:

$$\begin{cases} x(t) &= \sqrt{2g(H-h_0)} \cos(\theta) t + h_0 \\ y(t) &= -\frac{1}{2} g t^2 + \sqrt{2g(H-h_0)} \sin \theta t + h_0 \end{cases}$$

We can set $y(t)$ equal to 0 and use the quadratic equation to solve for t:

$$0 = -\frac{1}{2} g t^2 + v_0 \sin(\theta) t + h_0$$

$$t = \frac{(v_0) \sin(\theta) \pm \sqrt{(v_0 \sin(\theta))^2 + 2gh_0}}{g}$$

In this case t is positive (because we are going forwards in time) so we will use the following equation for t:

$$t = \frac{(v_0) \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 2gh_0}}{g}$$

We can insert this into $x(t)$ to obtain $x_f = v_0 \cos(\theta) \frac{(v_0) \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 2gh_0}}{g} + h_0$

For Launch Condition 1, θ is equal to 0. We will use this for x_{f_1}:

$$x_{f_1} = v_0 \frac{\sqrt{2gh_0}}{g} + h_0$$

$$x_{f_1}^2 = \frac{(v_0 \sqrt{gh_0} + gh_0)^2}{g^2}$$

v_0, g, and h_0 are all positive:

$x_{f_1}^2 = (v_0 \sqrt{\frac{h_0}{g}} + h_0)^2$ We take the derivative:

$$\begin{aligned} \frac{d}{dx}[x_{f_1}^2] &= 2\left(\frac{v_0}{2g\sqrt{\frac{h_0}{g}}} + 1\right)\left(v_0 \sqrt{\frac{h_0}{g} + h_0}\right) \\ &= 2\left(\frac{\sqrt{2g(H-h_0)}}{2g\sqrt{\frac{h_0}{g}}} + 1\right)\sqrt{2g(H-h_0)}\sqrt{\frac{h_0}{g} + h_0} \end{aligned}$$