1. Evaluate the following limit using Squeeze theorem (Think about the range of sin(□) □□ cos(□)) to find the enveloping functions

    (a)
    $$\lim_{\theta \to \infty} -\frac{1}{\theta} \le \lim_{\theta \to \infty} \frac{\sin \theta}{\theta} \le \lim_{\theta \to \infty} \frac{1}{\theta}$$
    $$0 \le \lim_{\theta \to \infty} \frac{\sin \theta}{\theta} \le 0$$
    $\lim_{\theta \to \infty} \frac{\sin \theta}{\theta} = 0$ by the squeeze theorem

    (b)
    $$\lim_{\theta \to \infty} \frac{1 - cos\theta}{\theta} = \lim_{\theta \to \infty} \frac{1}{\theta} - \lim_{\theta \to \infty} \frac{\cos \theta}{\theta}$$
    $$= 0 - \lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = -\lim_{\theta \to \infty} \frac{\cos \theta}{\theta}$$
    $$\lim_{\theta \to \infty} -\frac{1}{\theta} \le -\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} \le \lim_{\theta \to \infty} \frac{1}{\theta}$$
    $$0 \le -\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} \le 0$$
    $-\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = 0$ by squeeze theorem
    $$\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = 0$$

    (c)
    $$\lim_{\theta \to \infty} \theta^2 \cos \frac{1}{\theta^2} = \infty$$
    There are no functions that can serve as enveloping functions.

2. Prove that
    $$\lim_{x \to 0} \frac{\sin \theta}{\theta} = 1$$
    using steps below and using the sketch of a unit circle where the angle □ is in radians. K is a point on the unit circle.

    (a)
    $$K = (\cos \theta, \sin \theta)$$

    b)
    $$\text{Slope of } OK = \frac{\sin \theta}{\cos \theta}$$

    c)
    $$OL : y - \sin \theta = \frac{\sin \theta}{\cos \theta}(x - \cos \theta)$$

    d)
    $$A = (1, 0)$$

    e)
    $$L = (1, \frac{\sin \theta}{\cos \theta})$$

    f)
    $$\triangle OAK = \frac{\sin \theta}{2}$$

g)

$$\square OAK = \frac{\theta}{2}$$

h)

$$\triangle OAL = \frac{\sin\theta}{2\cos\theta}$$

i)

$$\frac{\sin\theta}{2} \le \frac{\theta}{2} \le \frac{\sin\theta}{2\cos\theta}$$

j)

$$\lim_{\theta\to 0} 1 \le \lim_{\theta\to 0} \frac{\theta}{\sin\theta} \le \lim_{\theta\to 0} \frac{1}{\cos\theta}$$

$$\lim_{\theta\to 0} \frac{1}{1} \le \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \le \lim_{\theta\to 0} \cos\theta$$

$$1 \le \lim_{\theta\to 0} \frac{\sin\theta}{\theta} \le 1$$

$\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$ by the squeeze theorem