We are given that the object m_1 collides with the rod with velocity v_0, and the rod is floating in free space. Given m_1, v_0, m_2, I_0, and r, we are to figure to the final velocity of m_1 after collision v_f, the velocity of m_2 after collision v_{CM}, and of course the rotation of the rod after collision ω.

We are assuming that this collision elastic.

We have, then, for conservation of linear momentum:

$$m_1 v_0 = m_1 v_f + m_2 v_{CM} \tag{1}$$

Furthermore, we understand that kinetic energy is also conserved here; therefore:

$$\frac{1}{2}m_1{v_0}^2 + \frac{1}{2}m_1{v_0}^2 = \left(\frac{1}{2}m_1{v_f}^2\right) + \left(\frac{1}{2}m_1{v_f}^2\right) + \left(\frac{1}{2}m_2{v_{CM}}^2\right) + \left(\frac{1}{2}I_0\omega^2\right) \tag{2}$$

$$\Rightarrow 2m_1{v_0}^2 = \left(2m_1{v_f}^2\right) + \left(m_2{v_{CM}}^2\right) + \left(I_0\omega^2\right) \tag{3}$$

as the point mass does not have any rotational inertia, and the rod is not rotating at the start.

Lastly, we understand that the angular momentum is conserved through a collision; letting the origin as the center of mass of the rod:

$$m_1 r^2 \left(\frac{v_0}{r}\right) = m_1 r^2 \left(\frac{v_f}{r}\right) + I_0\omega \tag{4}$$

$$\Rightarrow m_1 r v_0 = m_1 r v_f + I_0\omega \tag{5}$$

We now have a system of three equations that can be combined to solve for three unknowns v_f, v_{CM}, and ω.

Performing the actual solution digitally:

$$v_{cm} = \frac{4I_0 m_1 v_0}{m_1 m_2 r^2 + I_0 m_1 + 2I_0 m_2} \tag{6}$$

$$v_f = \frac{(m_1 m_2 r^2 + I_0 m_1 - 2I_0 m_2)v_0}{m_1 m_2 r^2 + I_0 m_1 + 2I_0 m_2} \tag{7}$$

and finally, we have

$$\omega = \frac{4m_1 m_2 r v_0}{m_1 m_2 r^2 + I_0 m_1 + 2I m_2} \tag{8}$$