

## 1 | Solving Limits with Elimination

With solving limits via elimination, we are typically analyzing a rational function that needs factoring of a term out of the polynomials on the top and/or the bottom to get out of the indeterminate form  $(\frac{0}{0})$ .

- Try factoring both the top and bottom
  - $(x \pm 1)$
  - $(x \pm 2)$
- Rationalize all of the square roots

Tip for picking factors

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**Tip!** If you plug in a value to an expression, and out pops 0, that value is a **zero** of the expression. It is helpful like this

Factor:  $(x^6 - 1)$

As you could see, plugging  $x = 1$  yields 0, meaning that  $(x - 1)$  is a **zero** of  $(x^6 - 1)$ , and hence would be able to be factored out.

To factor it out, either do synthetic division or long division.

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Let's do a problem solve for  $\lim_{x \rightarrow 2} \frac{(x^2 - 4)}{(x - 2)}$

1. First, notice the fact this function will have a hole at  $x = 2$ . This is especially important because after we simplify we will lose this hole.
2. Ok, now let's simplify.  $\frac{(x^2 - 4)}{(x - 2)} = \frac{(x + 2)((x - 2))}{(x - 2)} = (x + 2)$
3. Great! So, we know that this function behaves linearly with simply a hole at 2.
4. Doing the double-sided limits...
  - Evaluating  $\lim_{x \rightarrow 2^+}$ , the value will be 4 because  $2 + 2 = 4$ .
  - Evaluating

Here's another one!  $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$

1. First, notice that if we are going to solve this problem, we have to divide the top thing  $(\sqrt{x+4} - 2)$  by  $x$ , somehow
2. The only thing we could do here is rationalize the top by multiplying the whole fraction by a fancy one  $\frac{\sqrt{x+4} + 2}{\sqrt{x+4} + 2}$ .
3. This results in  $\frac{x+4-4}{x \times (\sqrt{x+4} + 2)}$ , which simplifies to  $\frac{x}{x \times (\sqrt{x+4} + 2)}$
4. Plugging in  $x = 0$ , you get  $\frac{1}{4}$ .

**If there is no factors, you got yourself a vertical asymptote. Refer to #missing #disorganized for solution!**