1. Evaluate the following limit using Squeeze theorem (Think about the range of  $sin(\Box) \Box \Box cos(\Box)$ ) to find the enveloping functions

(a) 
$$\lim_{\theta \to \infty} -\frac{1}{\theta} \leq \lim_{\theta \to \infty} \frac{\sin \theta}{\theta} \leq \lim_{\theta \to \infty} \frac{1}{\theta}$$

$$0 \leq \lim_{\theta \to \infty} \frac{\sin \theta}{\theta} \leq 0$$

$$\lim_{\theta \to \infty} \frac{\sin \theta}{\theta} = 0 \text{ by the squeeze theorem}$$
(b) 
$$\lim_{\theta \to \infty} \frac{1 - \cos \theta}{\theta} = \lim_{\theta \to \infty} \frac{1}{\theta} - \lim_{\theta \to \infty} \frac{\cos \theta}{\theta}$$

$$= 0 - \lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = -\lim_{\theta \to \infty} \frac{\cos \theta}{\theta}$$

$$\lim_{\theta \to \infty} -\frac{1}{\theta} \leq -\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} \leq \lim_{\theta \to \infty} \frac{1}{\theta}$$

$$0 \leq -\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} \leq 0$$

$$-\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = 0 \text{ by squeeze theorem}$$

$$\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = 0$$
(c) 
$$\lim_{\theta \to \infty} \frac{\cos \theta}{\theta} = \infty$$

There are no functions that can serve as enveloping functions.

2. Prove that

$$\lim_{x\to 0}\frac{\sin\theta}{\theta}=1$$

using steps below and using the sketch of a unit circle where the angle  $\Box$  is in radians. K is a point on the unit circle.

(a) 
$$K = (\cos\theta, \sin\theta)$$
 b) 
$$\operatorname{Slope} \text{ of } OK = \frac{\sin\theta}{\cos\theta}$$
 c) 
$$OL: y - \sin\theta = \frac{\sin\theta}{\cos\theta}(x - \cos\theta)$$
 d) 
$$A = (1,0)$$
 e) 
$$L = (1, \frac{\sin\theta}{\cos\theta})$$
 f) 
$$\triangle OAK = \frac{\sin\theta}{2}$$

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g) 
$$\Box OAK = \frac{\theta}{2}$$

h) 
$$\triangle OAL = \frac{\sin\theta}{2\cos\theta}$$

i) 
$$\frac{\sin\theta}{2} \leq \frac{\theta}{2} \leq \frac{\sin\theta}{2\cos\theta}$$

$$\begin{split} &\lim_{\theta \to 0} 1 \leq \lim_{\theta \to 0} \frac{\theta}{\sin \theta} \leq \lim_{\theta \to 0} \frac{1}{\cos \theta} \\ &\lim_{\theta \to 0} \frac{1}{1} \leq \lim_{\theta \to 0} \frac{\sin \theta}{\theta} \leq \lim_{\theta \to 0} \cos \theta \\ &1 \leq \lim_{\theta \to 0} \frac{\sin \theta}{\theta} \leq 1 \end{split}$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$
 by the squeeze theorem

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