1 | **Precessional Velocity**

Taking the setup, we can figure the sum of the angular momentums and leverage it to figure the spin angular momentum.

Let's first define a system: $\hat{i}$ is "right" on the figure, $\hat{j}$ "in" the page, $\hat{k}$ "up" the figure.

We note that the normal spin of the flywheel gives us:

$$\vec{L}_s = I\vec{\omega}_s\hat{i} \tag{1}$$

As the flywheel is rotating at a constant speed, we have actually no torque that this contributes to the net system — that is $\frac{d\vec{L}_s}{dt} = 0$.

Furthermore, we can figure torque—and subsequent angular momentum contribution—of gravity as follows:

$$\vec{\tau}_g = lmg\hat{j} \tag{2}$$

The total net torque on the system, then:

$$\vec{\tau}_{net} = \vec{\tau}_g + 0 \tag{3}$$

$$= \vec{\tau}_g \tag{4}$$

We also have that:

$$\vec{\tau}_{net} = \frac{\vec{L}_{net}}{dt} = \Delta\vec{L}_s = lmg \tag{5}$$

We see that, because of small-angle approximation, $\Delta\vec{L}_s = L_s\Omega$

Therefore, we can replace the values determined above and solve for Ω:

$$\Delta\vec{L}_s = L_s\Omega \tag{6}$$

$$\Rightarrow lmg = I\vec{\omega}_s\Omega \tag{7}$$

$$\Rightarrow \Omega = \frac{lmg}{I\vec{\omega}_s} \blacksquare \tag{8}$$

2 | **Discussion Questions**

2.1 | **Gyro in the Opposite Direction**

If ω_s was in the opposite direction, $\vec{L}_s = -\vec{L}_{sold}$ — by the right hand rule, it would be in the other direction.

The direction of procession would be in the same direction, "into" the page, by the $\hat{j}$ direction.

Therefore, the direction of $\vec{L}_s$ would be inching up and to the right—resulting in procession in the opposite ("clockwise") direction.

2.2 | **Tilted Gyro**

In our expressions above, the thing that will change is the fact that the value of $r \times F$ for $\vec{\tau}_g$ would change to account for the angle ϕ: $\vec{\tau}_g = l \sin \tau mg$.

Therefore, the final expression Ω becomes:

$$\Omega = \frac{lmg \sin \tau}{I\vec{\omega}_s} \tag{9}$$

2.3 | **Biker**

If a bicyclist leans towards the right, they are adding a small "forward" change to the angular momentum originally pointing strictly towards the left. This would orient the axis of rotation a little bit more towards the forward direction from being completely towards the left, which points the axis a little more towards the front.

The bike then responds to this axis change by turning to an angle orthogonal (per the right hand rule) to the location a little "more forward", which is a little "more right."

2.4 | **Forces on the COM**

2.4.1 | **Normal Force and Torque**

When the gyro is precessing, it performs uniform circular motion by a radius l. We see that, as determined above, it is precessing at a constant speed of $\frac{lmg}{I\vec{\omega}_s}$.

Therefore:

$$\frac{v^2}{R} = a = \frac{lm^2g^2}{I^2\vec{\omega}_s^2} \tag{10}$$

The net horizontal force on the object can be modeled by:

$$\vec{F}_{net} = -T \tag{11}$$

Therefore:

$$ma = -T \tag{12}$$

$$\Rightarrow T = -\frac{lm^3g^2}{I^2\vec{\omega}_s^2} \tag{13}$$

Given, though tension, the gyro is exerting this amount of force upon the stand, the stand's normal force in the opposite direction (i.e. horizontal, in the "negative" direction) would be: $\frac{lm^3g^2}{I^2\vec{\omega}_s^2}$.

When the gyroscope is simply limp, it would hang with a weight of mg. Calculations with tension work in a similar manner, resulting in the fact that the normal force would be mg.

The gyroscope can't just fall, under this circumstance, when spinning as the