1 | Electric Change

We are finally taking a surface integral! This is essentially multiplying the surface area of the shape of the function to the value of the function itself.

Firstly, taking the area dA by dV:

$$\begin{aligned} dA &= \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} && (1)\\ &= \sqrt{1 + (3)^2 + (2)^2} && (2)\\ &= \sqrt{14} && (3) \end{aligned}$$

Supplying the value into the function:

$$\begin{aligned} &\int_0^7 \int_0^{11} (3x + 2y + 7)\sqrt{14}\, dy\, dx && (4)\\ \Rightarrow &\sqrt{14} \int_0^7 \int_0^{11} (3x + 2y + 7)\, dy\, dx && (5)\\ \Rightarrow &\sqrt{14} \int_0^7 \left(3xy + y^2 + 7y\right)\Big|_0^{11}\, dy\, dx && (6)\\ \Rightarrow &\sqrt{14} \left(\frac{33x^2}{2} + 198x\right)\Bigg|_0^7 && (7)\\ \Rightarrow &\frac{4389\sqrt{14}}{2} && (8) \end{aligned}$$

2 | Infinite wire

Recall first that a semicircle with radius 7 can be defined as:

$$\begin{aligned} y &= \sqrt{7^2 - x^2} && (9)\\ &= \sqrt{49 - x^2} && (10) \end{aligned}$$

Let's first figure the

We will take the line integral of this function, and proceed to multiply by the value of xy at that point.

$$\int_0^7 \int_0^7 xy\, \sqrt{49 - x^2}\, dx\, dy \tag{11}$$