PS#28

Nueva Multivariable Calculus

1. Consider the function:

$$f : \mathbb{R}^3 \to \mathbb{R}^1$$
$$f(x, y, z) = \sqrt{x^2 + y^2}$$

And the region, E, in $\mathbb{R}^3$, given by:

$$E = \begin{array}{c}\text{the region}\\ \text{inside the cylinder } x^2 + y^2 = 16\\ \text{below } z = -4\\ \text{above } z = -5\end{array}$$

Evaluate (without a calculator!!!) the integral:

$$\iiint\limits_E f(x, y, z)\, dV$$

2. Find the volume of a rectangular box with side lengths a, b, and c.

Using an integral.

In spherical coordinates.

Without a computer.

...

Yes, I'm completely serious. We've done so many integrals that have involved forcing fundamentally-circular regions into rectangular coordinates, e.g.:

$$\int_{y=-5}^{y=+5} \int_{x=-\sqrt{5^2-y^2}}^{x=+\sqrt{5^2-y^2}} \text{blah blah}\, dx\, dy$$

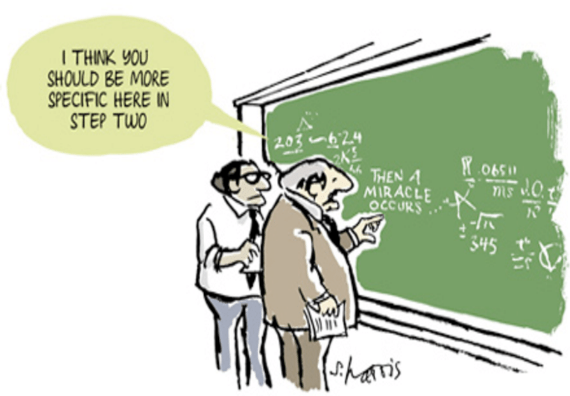
So gross. It's *time for the tables to be turned.* Let's take some fundamentally-rectangular region, and torture it by forcing it into a circular coordinate system.

To make this a bit easier, do this in two parts, starting with a simpler case:

(a) First, find the 2D area of a rectangle (a flat rectangle), using a polar double integral.

(b) And then, using a full triple integral in spherical, find the volume of a 3D box!

Note that, because we all know that the answer is $a \cdot b \cdot c$ (for a box), the whole point of this problem is *showing* how we get that answer, using this needlessly complex integral formulation. So don't just TEX up two lines of math and pat yourself on the back! We're trying to avoid this situation:



(a classic by the cartoonist Sidney Harris, printed in the *American Scientist*, Nov-Dec 1977.)