

1 | Tiling the Pringlehouse

As a review, our pringles shaped house has the following parametres:

$$\begin{cases} x(t) = 30\cos(t) \\ y(t) = 20\sin(t) \end{cases} \quad (1)$$

and the roof is defined by:

$$r(x, y) = \frac{1}{400} (\sqrt{3}x - y)^2 - \frac{1}{400} (\sqrt{3}y - x)^2 + 10 \quad (2)$$

We will first convert the above function into rectangular bounds to take the area of.

$$x = 30\cos(t) \quad (3)$$

$$\Rightarrow \frac{x}{30} = \cos(t) \quad (4)$$

$$\Rightarrow t = \arccos\left(\frac{x}{30}\right) \quad (5)$$

Supplying this back to the original expression for y:

$$y = 20\sin\left(\arccos\left(\frac{x}{30}\right)\right) \quad (6)$$

$$= 20\sqrt{1 - \left(\frac{x}{30}\right)^2} \quad (7)$$

Therefore, the actual integral:

$$\int_{-30}^{30} \int_{-20\sqrt{1 - \left(\frac{x}{30}\right)^2}}^{20\sqrt{1 - \left(\frac{x}{30}\right)^2}} 1 dy dx \quad (8)$$

We will endeavor now to use technology.

```
var("x y")
f(x,y) = 1
f.integrate(y, -20*sqrt(1-(x/30)^2), 20*sqrt(1-(x/30)^2)).integrate(x, -30,30)
```

It appears that the area of the floor is 600π .

We can do this something for the function of the roof. We will first figure correction factor dA , then take the integral as prescribed.

$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \quad (9)$$

At this point, we realize that the actual function will turn to be much too complicated to integrate by hand at this moment; therefore, we will create the expression digitally.

```

r(x,y) = (1/400)*(sqrt(3)*x-y)^2 - (1/400)*(sqrt(3)*y+x)^2 + 10
da = sqrt(1+r.diff(x)^2+r.diff(y)^2)
monte_carlo_integral(da.integrate(y, -20*sqrt(1-(x/30)^2), 20*sqrt(1-(x/30)^2)), [-30], [30], 10000000)

```

Looks like the result is converting to about 2002.2 for this shape.

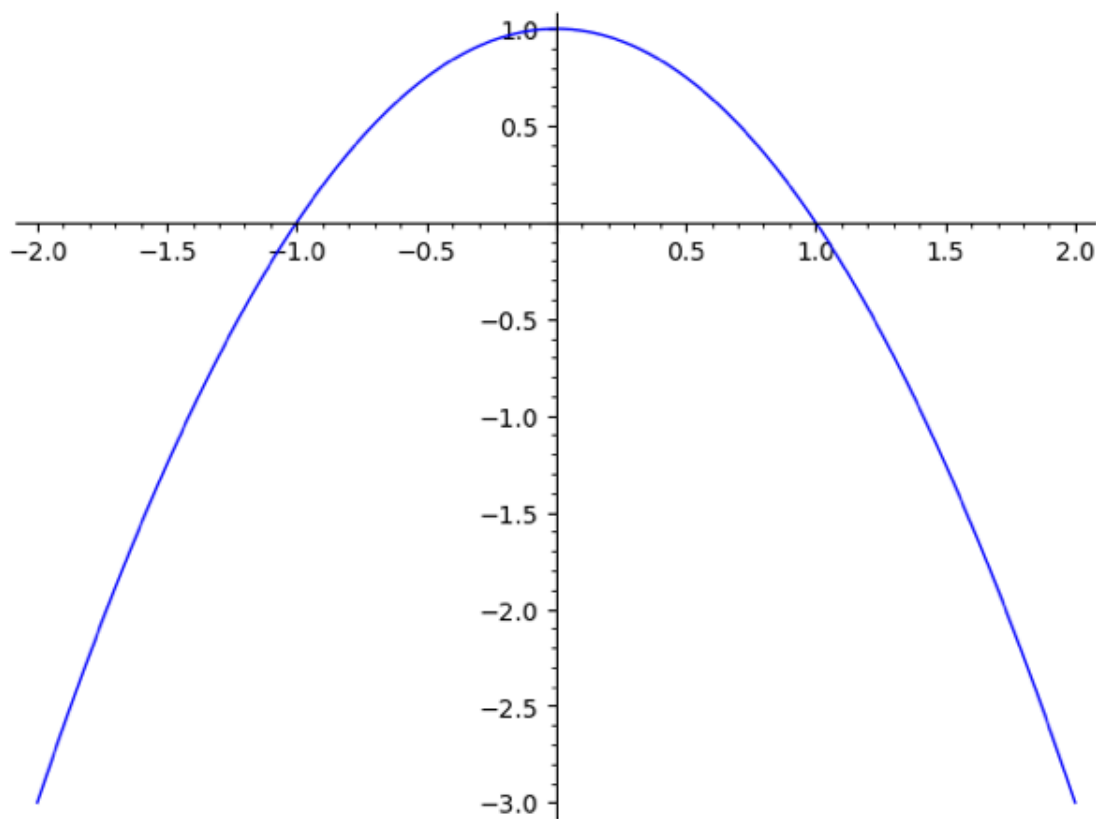
2 | Three Dimensional Region!

Slowly adding up the arguments to this figure reveals its general shape:

```

f(x) = 1-x^2
plot(f, -2, 2)

```

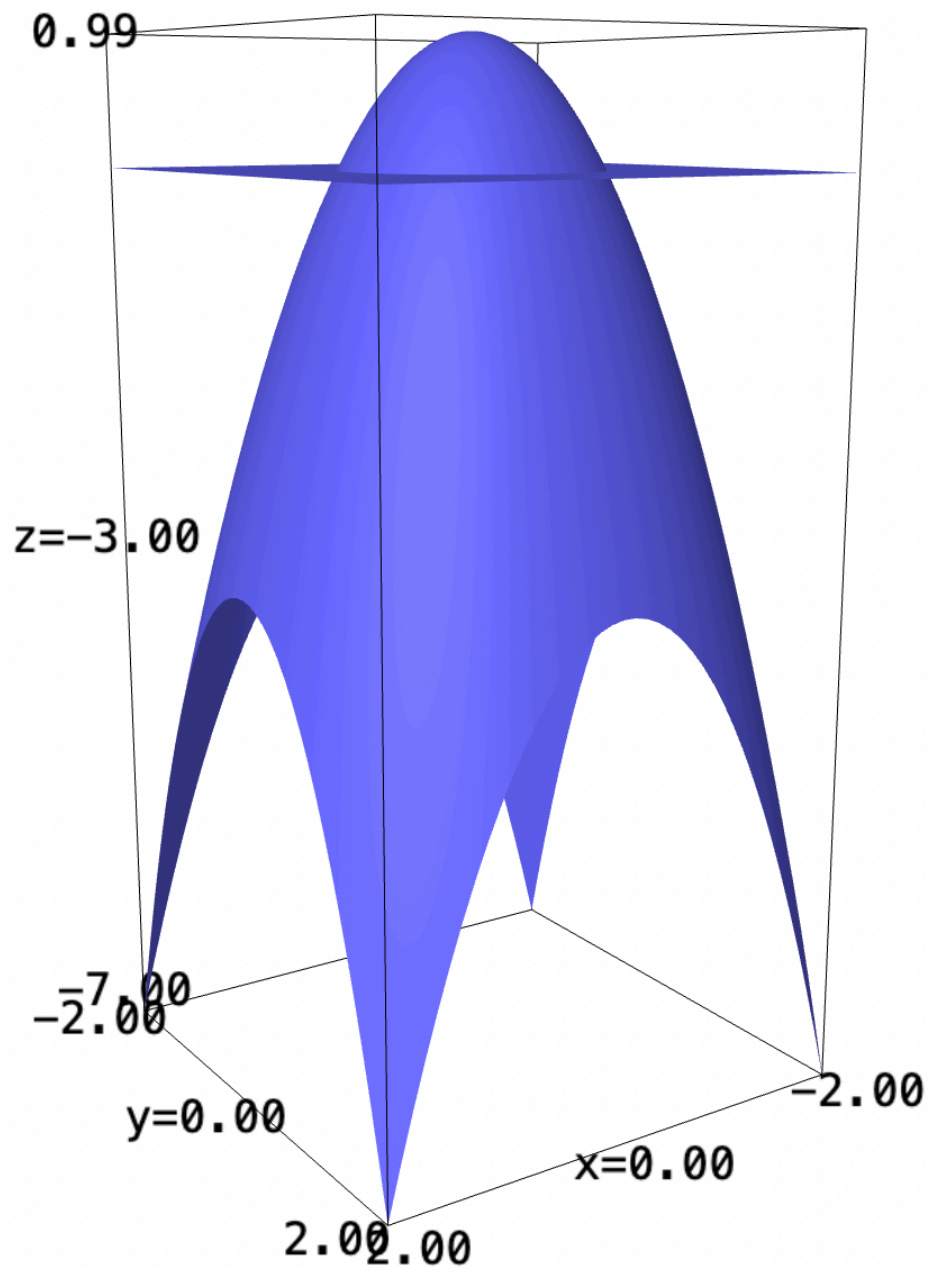


And, adding up the y component:

```

f(x,y) = 1-x^2-y^2
plot3d(f, (x,-2,2), (y,-2,2)) + plot3d(0, (x,-2,2), (y,-2,2))

```



We can see that the positive component exists at $x = [-1, 1]$, $y = [-1, 1]$. If the same pattern holds, then, the maximum volume would be over areas:

$$x = [-1, 1], y = [-1, 1], z = [-1, 1] \quad (10)$$

Taking the actual integral:

```
a(x,y,z) = 1-x^2-y^2-z^2
a.integrate(x,-1,1).integrate(y,-1,1).integrate(z,-1,1)
```

Shockingly, the resulting integral is 0. However, we can actually move the bounds to see that all other manifestations about this point would actually result in even more negative values.

```
a(x,y,z) = 1-x^2-y^2-z^2  
a.integrate(x,-1,1).integrate(y,-1,1).integrate(z,-1,1)
```

Taking the Jacobian matrix of this expression would reveal that:

$$\nabla a = \begin{bmatrix} -2x \\ -2y \\ -2z \end{bmatrix} \quad (11)$$

We see that there is no other point but $(0, 0, 0)$ is the maxima, and—by the pattern of the function—holding any two variables constant and squaring the remaining one will approach 0 (the smallest non-negative value) at bounds $[-1, 1]$. Therefore, it makes sense that the bounds prescribed would be the actual bounds desired.

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