1 | Escape Velocity

Earth's gravitational field extends infinitely and can be found with $\vec{F}_g = \frac{M_2 M_1 G}{r^2}\vec{r}$, where G is the universal gravitational constant. The vector $\vec{r}$ points from M_1 to M_2, $\vec{F}_g$ is force on M_2 , $\hat{r} = \frac{\vec{r}}{r}$, and finally r is the magnitude of $\vec{r}$.

This is equivalent to $\frac{-GM_1 M_2}{r^3}\vec{r}$.

1.1 | Derivation of GPE

$$\begin{gathered}\int_{r_e}^{\infty} \frac{GmM_e}{r^2} dr \\ GmM_e \int_{r_e}^{\infty} \frac{1}{r^2} dr \\ \frac{GmM_e}{r}\end{gathered}$$

1.2 | Derivation of Escape Velocity

$$\begin{aligned}W &= \Delta\text{KE} \\ \int_{r_e}^{\infty} \frac{-GmM_e}{r^2} dr &= -\frac{1}{2}mv_0^2 \\ -GmM_2 \int_{r_e}^{\infty} \frac{-1}{r^2} dr &= \frac{1}{2}mv_0^2 \\ 0 - \frac{-GmM_e}{r_e} &= \frac{1}{2}mv_0^2 \\ \frac{GM_e}{r_e} &= \frac{1}{2}v_0^2 \\ \frac{2GM_e}{r_e} &= v_0^2 \\ \sqrt{\frac{2GM_e}{r_e}} &= v_0 \\ \boxed{\sqrt{\frac{2GM_e}{r_e}} = v_0} \\ \boxed{v_0 \approx 11 \text{ km/s} \approx 24{,}000 \text{ mph}}\end{aligned}$$

2 | Potential Energy

PE can be defined to be 0 at a position of ∞ (similar to how we did it in Exploration of Fields/Voltage). PE is therefore less than 0 everywhere else. *Change* in potential energy (which is what we're usually concerned with, is either positive or negative depending on context.