1 | Notes from Within the Lecture

$$CM = \frac{\sum m_i \vec{r_i}}{\sum m_i} = \frac{1}{M} \sum m_i \vec{r_i} \tag{1}$$

i.e.: the centre of mass is the weighted average of the centers. "First moment."

$$I = M \frac{\sum m_i {r_i}^2}{M} = \sum m_i {r_i}^2 \tag{2}$$

i.e.: the rotational inertia is M times the weighted average, squared. "Second moment."

- 1st moment: Mean
- 2nd moment: Standard Deviation
- 3rd moment: Skew
- 4th moment: Kurtosis

If I take an axis, and shift the direction of the axis, we can compute the rotational inertia about the center of mass and then move it

2 | Proving the Parallel Axis Theorem

We will begin with the definition of rotational inertia about an origin:

$$I = \sum_i m_i {l_i}^2 \tag{3}$$

As per defined by the problem $l_i' = x_i'\hat{i} + y_i'\hat{j}$, the displacement vector from l_i to the CM.
We also understand that $\vec{R}_{CM} = X_{CM}\hat{i} + Y_{CM}\hat{j}$, the components to the location of the center of mass.
Therefore, the actual position $\vec{l_i}$ of the axis of rotation can be expressed as:

$$\begin{aligned} l_i &= \vec{R}_{CM} + \vec{l_i'} \qquad (4) \\ &= (X_{CM}\hat{i} + Y_{CM}\hat{j}) + (x_i'\hat{i} + y_i'\hat{j}) \qquad (5) \end{aligned}$$