# 1 | diagonal matrix

def

A diagonal matrix is a square matrix that is zero everywhere except possibly along the diagonal.

## 1.1 | results

## 1.1.1 | every diagonal matrix is upper triangular

## 2 | diagonalizable

def

An operator  $T \in \mathcal{L}(V)$  is called *diagonalizable* if the operator has a diagonal matrix with respect to some basis of V.

## 2.1 | results

## 2.1.1 | Axler5.41 conditions equivalent to diagonalizability

Suppose V is finite-dimensional and  $T \in \mathcal{L}(V)$ . Let  $\lambda_1, \ldots, \lambda_m$  denote the distinct eigenvalues of T. Then the following are equivalent:

- 1. T is diagonalizable
- 2. V has a basis consisting of eigenvalues of T
- 3. there exist 1-dimensional subspaces  $U_1, \ldots, U_n$  of V, each invariant under T, s.t.

$$V = U_1 \oplus \cdots \oplus U_n$$

- 1.  $V = E(\lambda_1, T) \oplus \cdots \oplus E(\lambda_m, T)$  (V is the (direct) sum of eigenspaces)
- 2.  $V = E(\lambda_1, T) + \cdots + E(\lambda_m, T)$

### 2.1.2 | Axler5.44 Enough eigenvalues implies diagonalizability

If  $T \in \mathcal{L}(V)$  has V distinct eigenvalues, then T is diagonalizable.

1. intuition Because distinct eigenvalues correspond to linearly independent eigenvectors, so there will be enough linearly independent eigenvecs to form a basis and thus a diagonal matrix.

In fact, we just need the geometric multiplicities to add up (a result Axler promises in later chapters)

#### 2.1.3 | Relationship to non-diagonal matrix (in class 31 March 2021)

Suppose A is the original map (not diagonal), and that P is the matrix where each column is an eigenvector written in terms of the original basis (standard basis, usually). Then

$$AP = PD$$

where D is the diagonal matrix.

1. this (or a conjugation??) forms a similarity transform, which is a type of equivalence relation

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