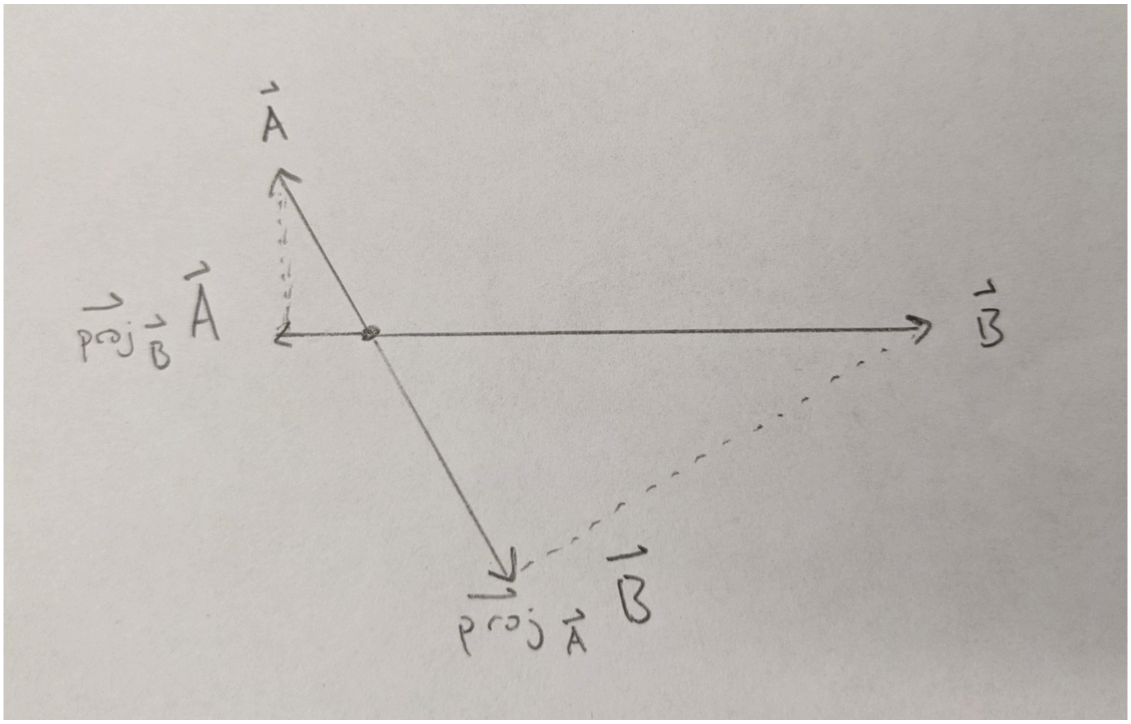
1 | **Problem 1:**

1.1 | **1.1)**



1.2 | **1.2)**

$comp_{\vec{A}}\vec{B} = |\vec{B}|\cos(\theta) = 6\cos(\frac{2\pi}{3}) = -3$ $comp_{\vec{B}}\vec{A} = |\vec{A}|\cos(\theta) = 2\cos(\frac{2\pi}{3}) = -1$

1.3 | **1.3)**

$\vec{A} \cdot \vec{B}$
$= |\vec{A}||\vec{B}|\cos(\theta) = 6 \cdot 2 \cdot (-0.5)$
$= -6$

2 | **Problem 2:**

$comp_{\vec{A}}\vec{B} = |\vec{B}|\cos(\theta)$
$= |\vec{B}|\cos(\theta) \times \frac{|\vec{A}|}{|\vec{A}|}$
$= \frac{|\vec{A}||\vec{B}|\cos(\theta)}{|\vec{A}|}$
$= \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|}$

3 | Problem 3:

The projection of $\vec{B}$ onto $\vec{A}$ would be the $\vec{A}$ component of $\vec{B}$ times the unit vector of $\vec{A}$ to give the component a direction and make it a vector: $\vec{proj}_{\vec{A}}\vec{B} = comp_{\vec{A}}\vec{B} \cdot \hat{A}$
$= |\vec{B}|\cos(\theta) \cdot \frac{\vec{A}}{|\vec{A}|}$
$= \frac{|\vec{B}|\cos(\theta)}{|\vec{A}|}\vec{A}$

4 | Problem 4:

Because of vector addition we know that:

$$\vec{A}_{\perp\vec{B}} = \vec{A} - \vec{A}_{\parallel\vec{B}}$$

and thus:
$= \vec{A} - |\vec{A}|\cos(\theta)\hat{B}$
$= \vec{A} - \frac{|\vec{A}|\cos(\theta)}{|\vec{B}|}\vec{B}$
$= \vec{A} - \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\vec{B}$

We can check this by taking the dot product of the vector above and $\vec{B}$. If the dot product equals zero then the two vectors apre perpendicular:

$$\vec{B} \cdot \vec{A}_{\perp\vec{B}} = \vec{B} \cdot (\vec{A} - \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|^2}\vec{B})$$

from problem 7 we know that the dot product is distributive, and thus:
$= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B}\frac{1}{|\vec{B}|^2} \cdot \vec{B}\cdot\vec{B}$
$= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B}\frac{1}{|\vec{B}|^2} \cdot |\vec{B}||\vec{B}|\cos(0)$
$= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B}\frac{1}{|\vec{B}|^2} \cdot |\vec{B}|^2$
$= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B}\frac{|\vec{B}|^2}{|\vec{B}|^2}$
$= \vec{A}\cdot\vec{B} - \vec{A}\cdot\vec{B}$
$= 0$

5 | Problem 5:

The dot product is defined as:

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos(\theta)$$

is this case we can solve for theta:
$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos(\theta)$
$\Rightarrow \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \cos(\theta)$
$\Rightarrow \theta = \cos^{-}(\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|})$

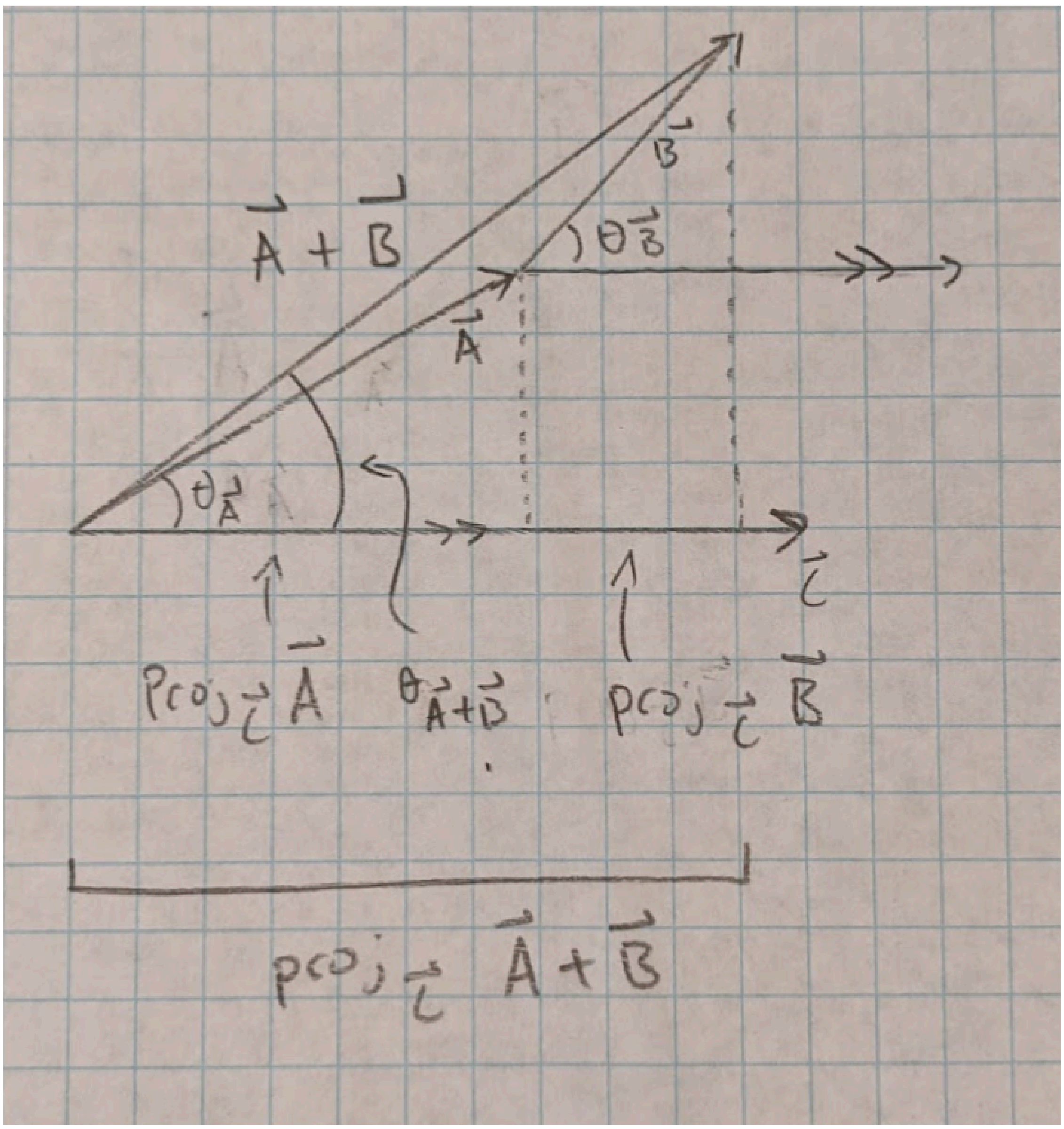
because we do not know theta we can use another definition of the dot product to get the numorator of the fraction:

$\Rightarrow \theta = \cos^{-}(\frac{A_xB_x + A_yB_y + A_zB_z}{|\vec{A}||\vec{B}|})$

Therefore the angle between the two vectors $(-1, 2, -2)$ and $(-3, 1, 2)$ is the following:

$\theta = \cos^{-}(\frac{3+2-4}{\sqrt{1+4+4}\cdot\sqrt{9+1+4}})$
$\Rightarrow \theta = \cos^{-}(\frac{1}{3\sqrt{14}})$
$\Rightarrow \theta \approx 1.48159$

6 | Problem 6:



Looking at the diagram above we see that:

$proj_{\vec{C}}\vec{A} + proj_{\vec{C}}\vec{B} = proj\vec{C}(\vec{A}+\vec{B})$
$\Rightarrow |\vec{C}|proj_{\vec{C}}\vec{A} + |\vec{C}|proj_{\vec{C}}\vec{B} = |\vec{C}|proj_{\vec{C}}(\vec{A}+\vec{B})$
$\Rightarrow |\vec{C}||\vec{A}|\cos(\theta_{\vec{A}}) + |\vec{C}||\vec{B}|\cos(\theta_{\vec{B}}) = |\vec{C}||\vec{A}+\vec{B}|\cos(\theta_{\vec{A}+\vec{B}})$
$\Rightarrow \vec{C}\cdot\vec{A} + \vec{C}\cdot\vec{B} = \vec{C}\cdot(\vec{A}+\vec{B})$

thus the dot product is distributive

this scales to the third dimention, because in a sense the diagram is the projection of 3D vectors onto a plane.

7 | Problem 7:

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$
$= A_x\hat{i}\cdot B_x\hat{i} + A_x\hat{i}\cdot B_y\hat{j} + A_x\hat{i}\cdot B_z\hat{k} + A_y\hat{j}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + A_y\hat{j}\cdot B_z\hat{k} + A_z\hat{k}\cdot B_x\hat{i} + A_z\hat{k}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k}$

the dot product between $A_x\hat{i}\cdot B_x\hat{i}$ would be: $|A_x||B_x|\cos(0)$, because the angle between $\hat{i}$ and $\hat{i}$ is 0 (they have the same direction), $\cos(0) = 1$, and thus the dot product equals A_xB_x. However, the dot product between and two unit vectors that are not the same would yield a theta of $\frac{\pi}{2}$, which means $\cos(\frac{\pi}{2}) = 0$ and thus that term would equal zero. This can be generalized as, if the two unit vectors are the same then it will yield a term equal to the product of their two coefficients, and if the two unit vectors are different, then the resulting term would be equal to zero. Therefore:

$= A_xB_x + 0 + 0 + A_yB_y + 0 + 0 + A_zB_z + 0 + 0$
$= A_xB_x + A_yB_y + A_zB_z$

8 | Problem 8:

$$\{\vec{r} = \vec{P}_o + t(\vec{v}_o), t \in \mathbb{R}\}$$

this works because the $\vec{P}_o$ term will position the vector so that the line passes through the point $\vec{P}_o$, and then the $t(\vec{v}_o)$ term will add and subtract and infinite amount of vectors in the direction and of the magnitude of $t(\vec{v}_o)$ from $\vec{P}_o$, thus creating a line that passes through $\vec{P}_o$

9 | Problem 9:

This is pretty similar to problem 8, the only differenc is that the $\vec{P}_o$ now equals one of the give vectors, and $\vec{v}_o$ is the vector between the two points:

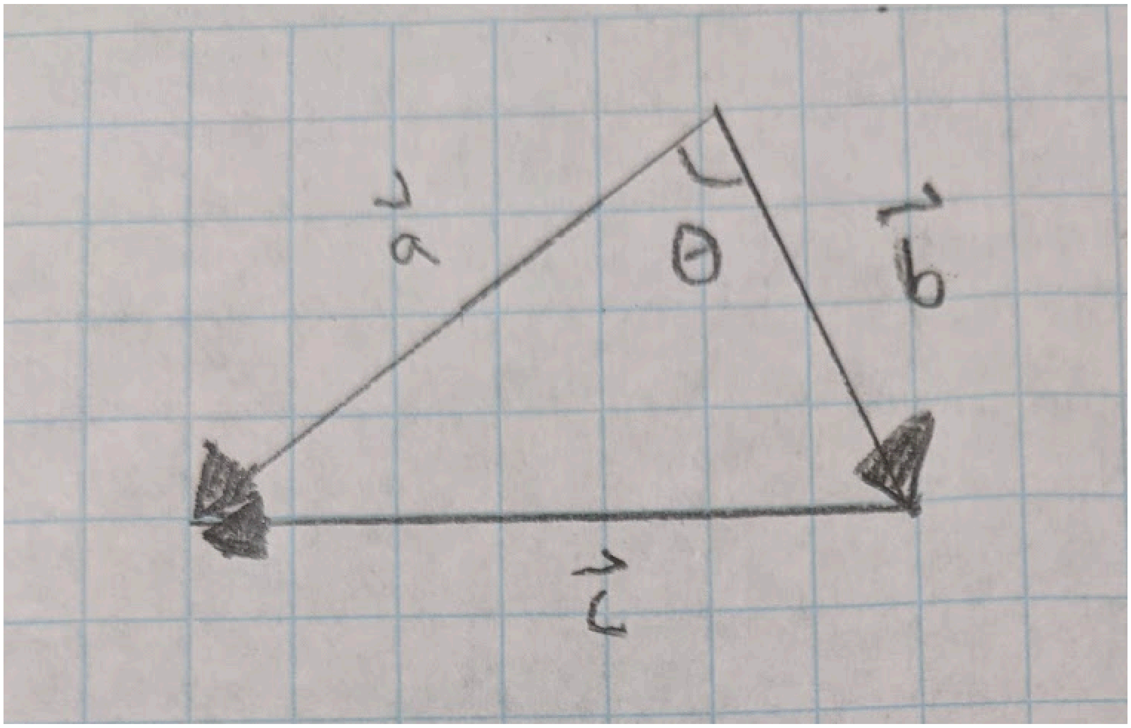
$\{\vec{r} = \langle 2,-5,-3\rangle + t\langle 1,-9,-4\rangle, t\in\mathbb{R}\} = \{\vec{r} = \langle 2,-5,-3\rangle + \langle t,-9t,-4t\rangle, t\in\mathbb{R}\}$
$= \{\vec{r} = \langle 2+t,-5-9t,-3-4t\rangle, t\in\mathbb{R}\}$

10 | Problem 10:

The law of cosines is as follows:

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$

If the vectors $\vec{c}, \vec{a}$ and $\vec{b}$ are the legs of the triangle then c, a and b are the magnitudes of the vectors respectively, and θ is the angle between vectors $\vec{a}$ and $\vec{b}$ tail to tail



looking at the image above we see that:
$\vec{c} = \vec{a} - \vec{b}$
$\Rightarrow \langle c_x, c_y, c_z \rangle = \langle a_x, a_y, a_z \rangle - \langle b_x, b_y, b_z \rangle$
$\Rightarrow c_x, c_y, c_z = \langle a_x - b_x, a_y - b_y, a_z - b_z \rangle$
We also know that by the definition of a vector:

$$\vec{a} = \langle a_x, a_y, a_z \rangle$$

$$\vec{b} = \langle b_x, b_y, b_z \rangle$$

We can use these numbers and plug them into the the law of cosines:
$c^2 = a^2 + b^2 - 2ab\cos(\theta)$
$\Rightarrow |\vec{c}| = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\theta)$
$\Rightarrow c_x^2 + c_y^2 + c_z^2 = a_x^2 + a_y^2 + a_z^2 + b_x^2 + b_y^2 + b_z^2 - 2|\vec{a}||\vec{b}|\cos(\theta)$
$\Rightarrow (a_x - b_x)^2 + (a_y - b_y)^2 + (a_z - b_z)^2 = a_x^2 + a_y^2 + a_z^2 + b_x^2 + b_y^2 + b_z^2 - 2|\vec{a}||\vec{b}|\cos(\theta)$
$\Rightarrow a_x^2 - 2a_xb_x + bx^2 + a_y^2 - 2a_yb_y + b_y^2 + a_z^2 + 2a_zb_z + b_z^2 = a_x^2 + a_y^2 + a_z^2 + b_x^2 + b_y^2 + b_z^2 - 2|\vec{a}||\vec{b}|\cos(\theta)$
$\Rightarrow -2a_xb_x - 2a_yb_y - 2a_zb_z = -2|\vec{a}||\vec{b}|\cos(\theta)$
$\Rightarrow -2(a_xb_x + a_yb_y + a_zb_z) = -2|\vec{a}||\vec{b}|\cos(\theta)$
$\Rightarrow a_xb_x + a_yb_y + a_zb_z = |\vec{a}||\vec{b}|\cos(\theta)$