1 | Integration by Parts, Problem 1

Calculate the following anti-derivative

$$\int \ln(x)\, dx \tag{1}$$

We perform integration by parts on this expression:

$$\int ln(x)dx \tag{2}$$
$$\Rightarrow \int ln(x) \cdot 1dx \tag{3}$$
$$\Rightarrow xln(x) - \int \frac{1}{x} \cdot x\, dx \tag{4}$$
$$\Rightarrow xln(x) - \int 1dx \tag{5}$$
$$\Rightarrow xln(x) - x \tag{6}$$

2 | Integration by Parts, Problem 2

Calculate this anti-derivative, too, by hand

$$\int x^5 \sin(x)\, dx \tag{7}$$

We again perform integration by parts on this expression:

$$\int x^5 \sin(x)dx \tag{8}$$
$$\Rightarrow -x^5 \cos(x) + 5\int x^4 \cos(x)dx \tag{9}$$
$$\Rightarrow -x^5 \cos(x) + 5(x^4 \sin(x) - 4\int x^3 \sin(x))dx \tag{10}$$
$$\Rightarrow -x^5 \cos(x) + 5(x^4 \sin(x) - 4(-x^3 \cos(x) + 3\int x^2 \cos(x)))dx \tag{11}$$
$$\Rightarrow -x^5 \cos(x) + 5(x^4 \sin(x) - 4(-x^3 \cos(x) + 3(x^2 \sin(x) - 2(-x \cos(x) + \int \cos(x)))))dx \tag{12}$$
$$\Rightarrow -x^5 \cos(x) + 5(x^4 \sin(x) - 4(-x^3 \cos(x) + 3(x^2 \sin(x) - 2(-x \cos(x) + sin(x))))) + C \tag{13}$$
$$\Rightarrow -x^5 \cos(x) + 5x^4 \sin(x) + 20x^3 \cos(x) - 60x^2 \sin(x) - 120x \cos(x) + 120sin(x) + C \tag{14}$$

3 | Derivative Matrix Problems

Diff. Higher Dims, Number 9

$$f : \mathbb{R}^2 \to \mathbb{R}^1; f(x,y) = xtan(y) \tag{15}$$

$$\begin{bmatrix} \tan(y) \\ x\sec(2y) \end{bmatrix} \tag{16}$$

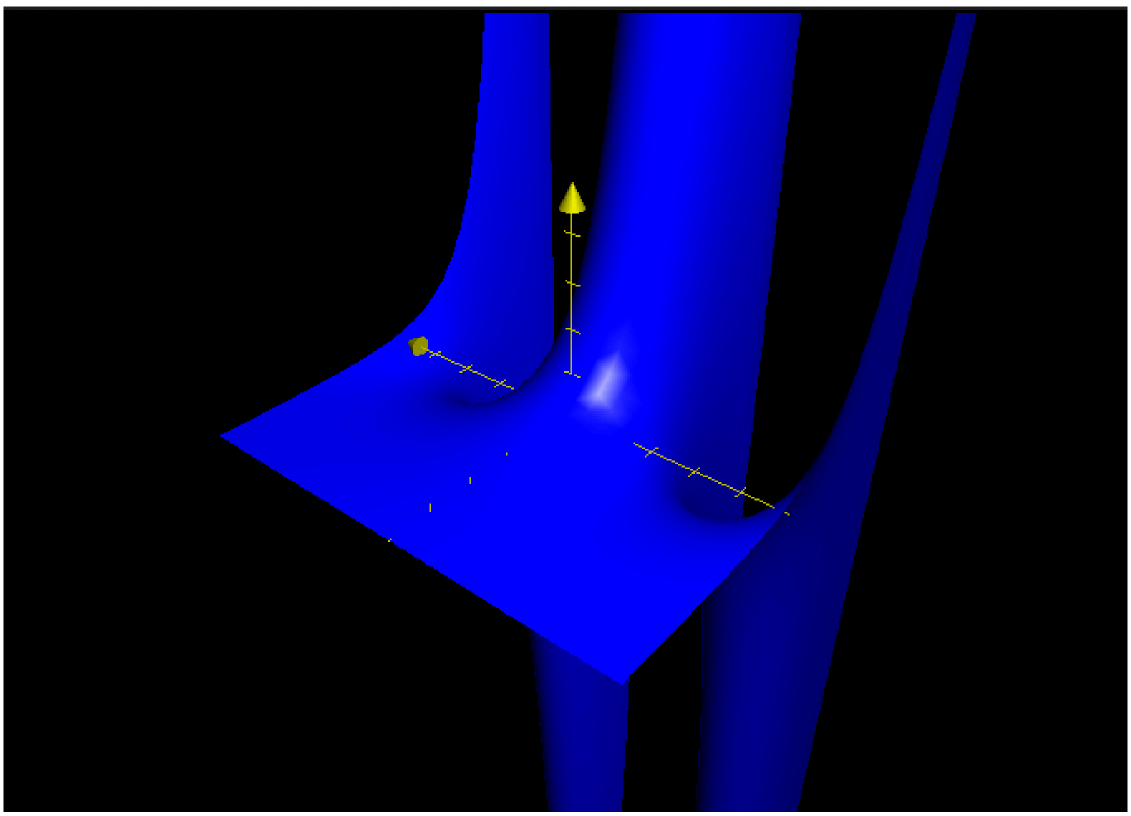
Diff. Higher Dims, Number 12

$$f : \mathbb{R}^3 \to \mathbb{R}^1; f(x, y, z) = x^2 + 7yz \tag{17}$$

$$\begin{bmatrix} 2x \\ 7z \\ 7y \end{bmatrix} \tag{18}$$

4 | **Slope at Point problem**

Consider the function $f(x, y) = e^x \cos(y)$. Graph it!



Suppose you are at the point $(1, \frac{\pi}{4})$ facing in the $\frac{\pi}{6}$ direction. How steep is the function? Give your answer both in normal slope units and as an angle.

Here's the gradient of the function:

$$\begin{bmatrix} e^x \cos(y) \\ -e^x \sin(y) \end{bmatrix} \tag{19}$$

At the point $(1, \frac{\pi}{4})$, therefore, the gradient is:

$$\begin{bmatrix} e\frac{\sqrt{2}}{2} \\ -e\frac{\sqrt{2}}{2} \end{bmatrix} \tag{20}$$

Projecting this vector upon $\frac{\pi}{6}$ direction, we arrive at the steepness of the function on that point:

$$\begin{bmatrix} e\frac{\sqrt{2}}{2} \\ -e\frac{\sqrt{2}}{2} \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{bmatrix} = \frac{e(\sqrt{6}-\sqrt{2})}{4} \tag{21}$$

This slope, in turn, represents an angle of roughly $35.128°$.