

1 | cooling pizza

Compute

$$\int_0^5 -110e^{-0.4t} dt$$

to the nearest degree.

$$\int -110e^{-0.4t} dt = \frac{-110}{-0.4} e^{-0.4t} = 275e^{-0.4t}$$

Using the net change theorem,

$$\begin{aligned} \Delta \beta \int_0^5 -110e^{-0.4t} dt &= \int -110e^{-0.4(5)} dt - \int -110e^{-0.4(0)} dt \\ &= 275e^{-0.4(5)} - 275e^0 \\ &= 37.21720289 - 275 \\ &= 37.21720289 - 275 \\ &= 350 - 237.78279711 \approx \boxed{112F} \end{aligned}$$

2 | definite integral as area under a curve

The area in the triangle is 3 square units, so $5 + 3 = \boxed{8}$

3 | minimum value of $f(x) = \int_{-2}^{x^2-3x} e^{t^2} dt$

$$\begin{aligned} \frac{d}{dx} f(x) &= e^{(x^2-3x)^2} (2x-3) = 0 \\ \Rightarrow 2x-3 &= 0 \\ \Rightarrow 2x &= 3 \\ \Rightarrow x &= \frac{3}{2} \end{aligned}$$

4 | approximate area under the curve graphically

The function looks symmetric about $x = 12$, so I will focus on $[0, 12]$.

On the interval $[0, 6)$ a little under $6 \cdot 100$ barrels of oil flow through.

On the interval $[6, 12)$ a little over $6 \cdot 100 + \frac{1}{2}6 \cdot 100$ barrels flow through, for a total of

$$\approx 2(6 \cdot 100 + 6 \cdot 100 + \frac{1}{2}6 \cdot 100) = 3000$$

barrels of oil.

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5 | fundamental theorem of calculus but worded confusingly

$F(x)$ is the antiderivative of $f(x)$, so differences of its values are definite integrals. In this case,

$$F(3) - F(0) = \int_0^3 f(x) dx = \int_0^1 f(x) dx + \int_1^3 f(x) dx = 2 + 2.3 = \boxed{4.3}$$

6 | amusement park word problem

$$E(t) = \frac{15600}{t^2 - 24t + 160}$$

$$L(t) = \frac{9890}{t^2 - 38t + 370}$$

valid over the domain $[9, 23]$, and there are zero people in the park at $t = 9$.

6.1 | number of people who have entered the park by some time

$$\int_9^x E(t) dt = \int_9^x \frac{15600}{t^2 - 24t + 160} dt$$

$$= 15600 \ln(t^2 - 24t + 160)(2t - 24) \text{????}$$

I don't know how to integrate this symbolically, and WolframAlpha says it contains an inverse tangent. Thus, I will use a calculator:

$$\int_9^{17} \frac{15600}{t^2 - 24t + 160} dt \approx \boxed{6004}$$

6.2 | value of $H'(17)$

H represents the number of people in the amusement park. $H(17) \approx 3725$

$$\frac{d}{dx} \int_9^t (E(x) - L(x)) dx = E(x) - L(x)$$

$$E(17) - L(17) = \frac{15600}{t^2 - 24t + 160} - \frac{9890}{t^2 - 38t + 370} \approx -380$$

H' represents the change in the number of people in the park, per hour.