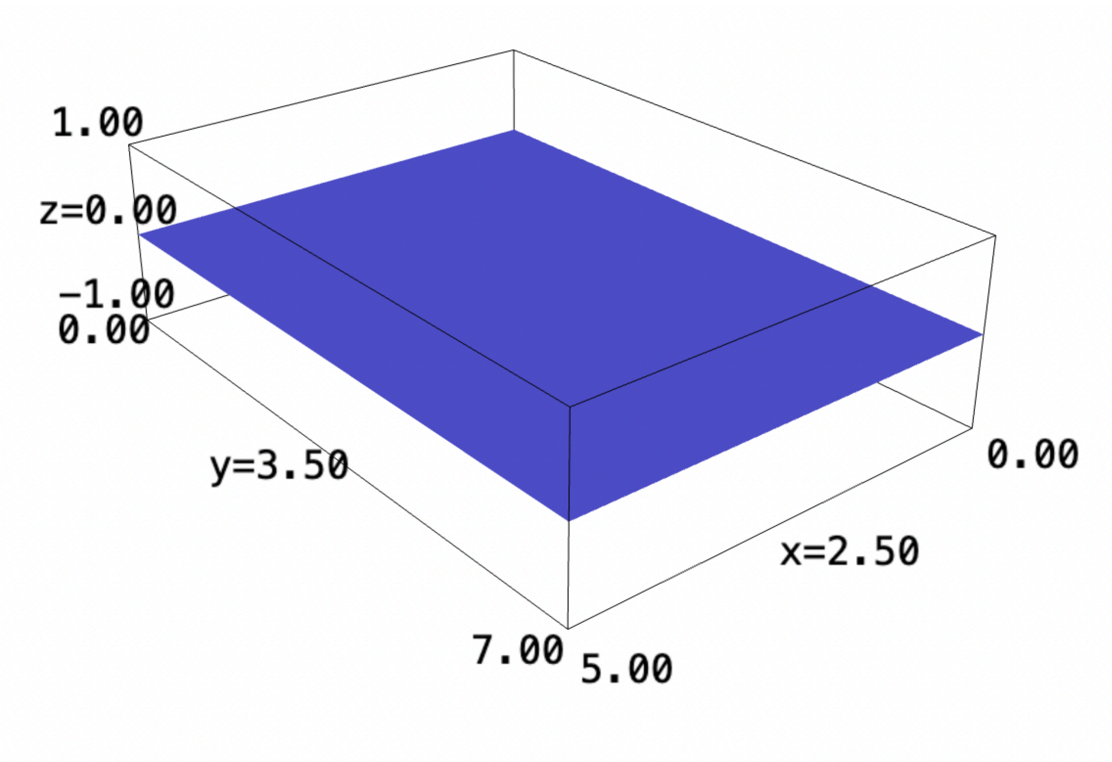
We

1 | Single Value Function

$$f_1 : \mathbb{R}^2 \to \mathbb{R}^1 \tag{1}$$
$$f_1(x, y) = 0 \tag{2}$$

What's the area of this function?

```
f(x,y) = 0
plot3d(f, (x,0,5), (y,0,7))
```



We can take the area of the shape, essentially by taking the volume by height 1: that is, for a rectangle of l, w, h, its top-area is simply $l \cdot w$, also known as $lw \cdot 1$. Therefore:

$$\int_0^7 \int_0^5 1 dx\, dy = 35 \tag{3}$$

The area of the shape is therefore 35.

2 | Area of the Plane

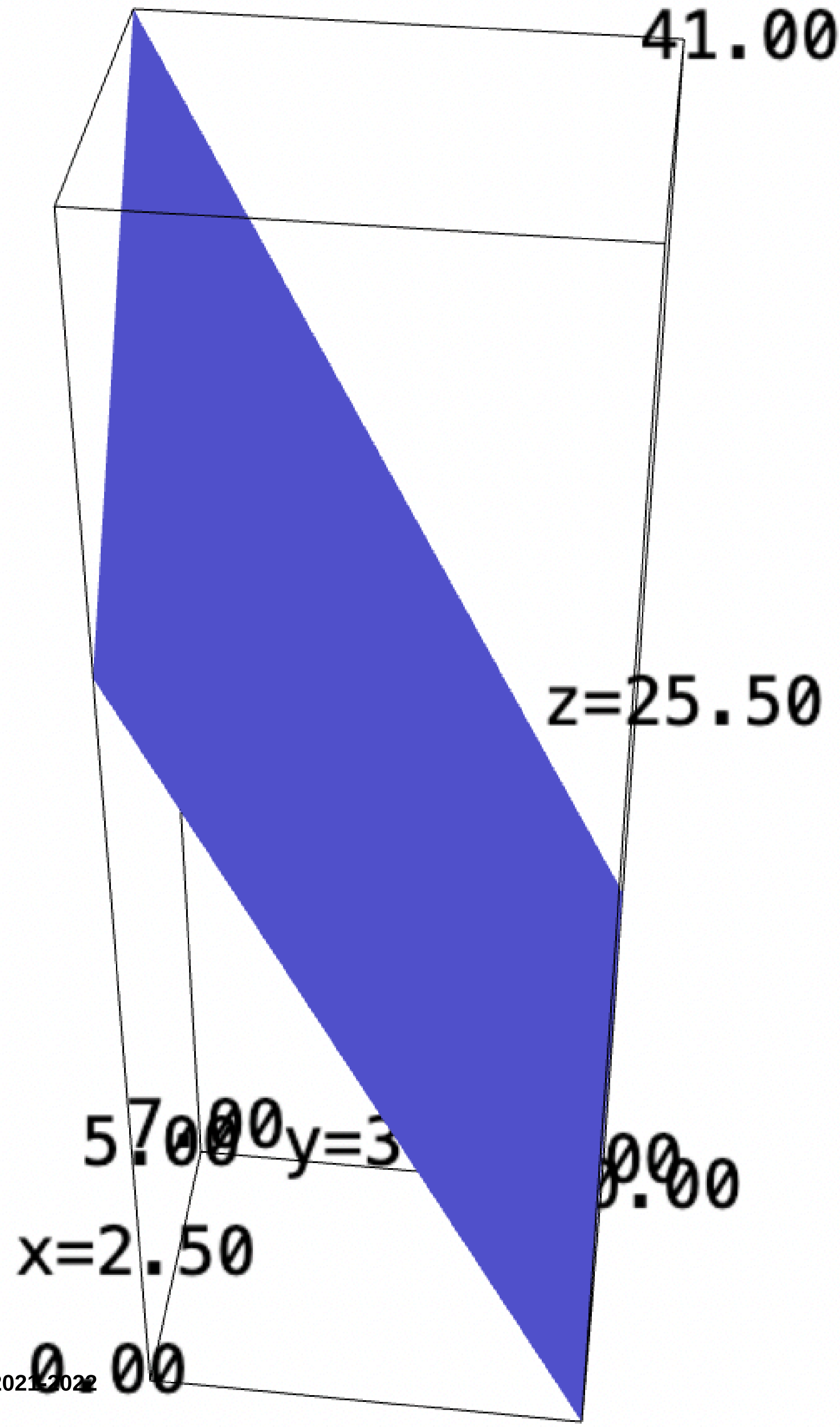
We want to first figure the correction per every given slice $dA = n\ dV$ to setup a surface integral. By pythagoras (i.e. projecting the changes to the parallelity of the surface), we have that:

$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dV \tag{4}$$

What's the area of the following function by $(5, 7)$?

$$f_2 : \mathbb{R}^2 \to \mathbb{R}^1 \tag{5}$$
$$f_2(x, y) = 2x + 3y + 10 \tag{6}$$

```
f(x,y) = 2*x+3*y+10
plot3d(f, (x,0,5), (y,0,7))
```


41.00
z=25.50
x=2.50
0.00

$$dA = \sqrt{1+4+9}dV = \sqrt{14}\, dV \tag{7}$$

Therefore, taking the integral:

$$\int_0^5 \int_0^7 \sqrt{14}\, dy\, dx \tag{8}$$
$$\Rightarrow 35\sqrt{14} \tag{9}$$

```
float(35*sqrt(14))
```

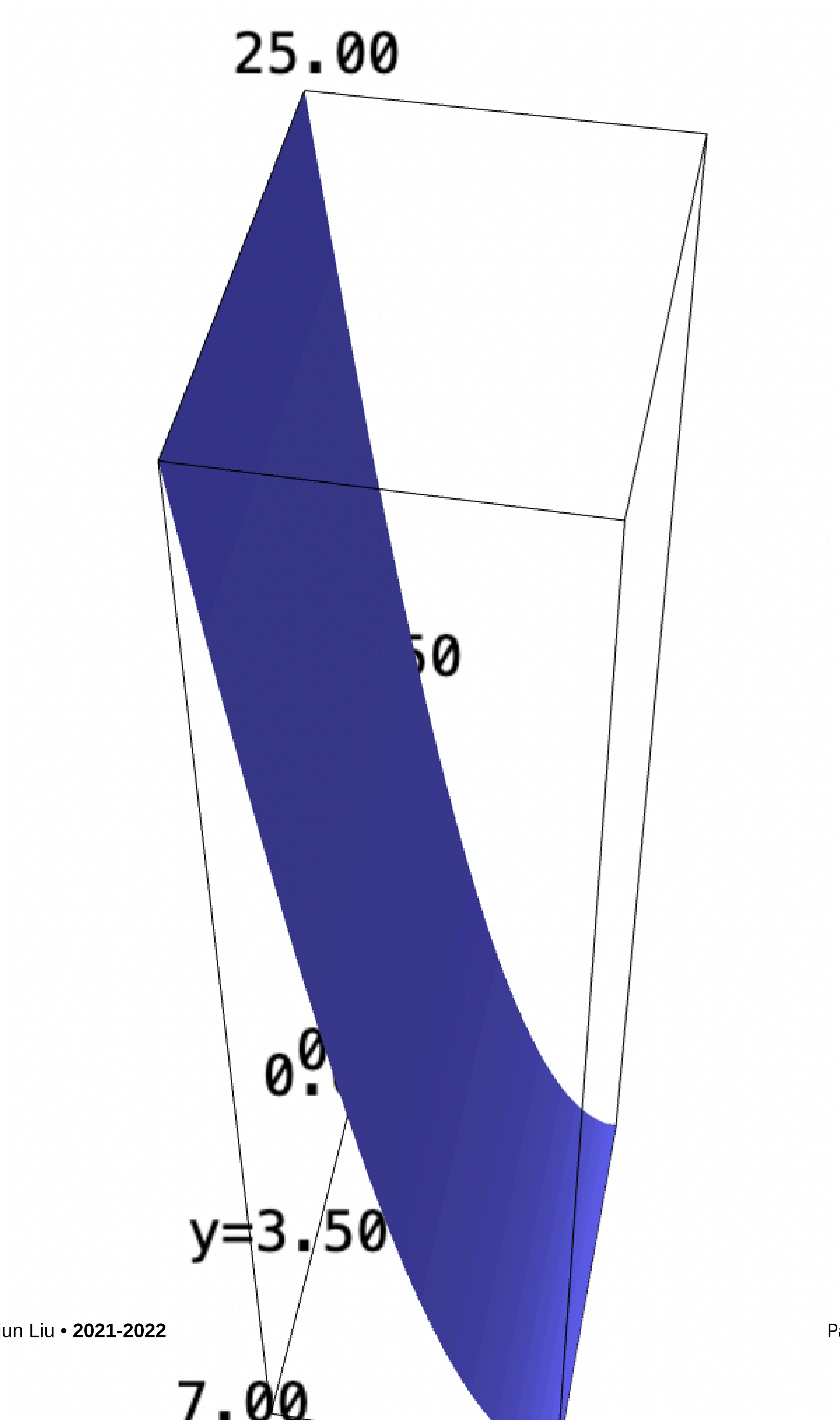
It appears that the surface area is about 130.958 units.

3 | Area of a Parabola

What's the area of the following function by $(5, 7)$?

$$f_3 : \mathbb{R}^2 \to \mathbb{R}^1 \tag{10}$$
$$f_3(x, y) = x^2 \tag{11}$$

```
f(x,y) = x^2
plot3d(f, (x,0,5), (y,0,7))
```


25.00
50
0.0
y=3.50
7.00

We will again find the area correction factor:

$$dA = \sqrt{1+4x^2}\, dV \tag{12}$$

And therefore, taking the integral:

$$\int_0^5 \int_0^7 \sqrt{1+4x^2}\, dy\, dx \tag{13}$$

This problem is solvable by trig substitution followed by integration by parts. For now, however, we will leverage a calculator.

```
f(x,y) = sqrt(1+4*x^2)
f.integrate(y, 0,7).integrate(x,0,5)
float(f.integrate(y, 0,7).integrate(x,0,5))
```

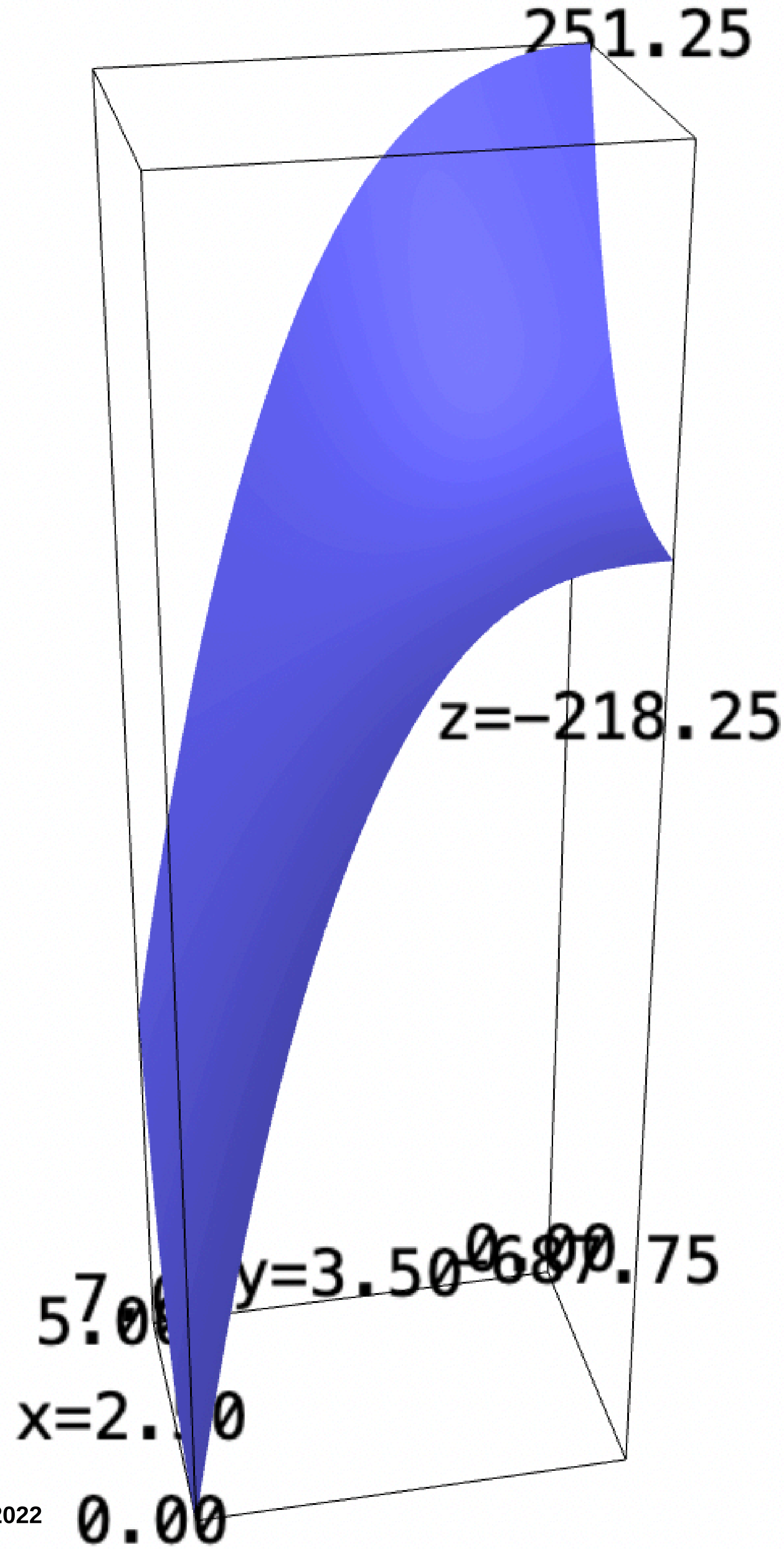
Evidently, the surface area of the shape is about 181.1197 units.

4 | Another Surface Area

What's the area of the following function by $(5, 7)$?

$$f_3 : \mathbb{R}^2 \to \mathbb{R}^1 \tag{14}$$
$$f_3(x, y) = x^2 - y^2 \tag{15}$$

```
f(x,y) = (x^2-y^2)*sqrt(1+4*x^2+4*y^2)
plot3d(f, (x,0,5), (y,0,7))
```


251.25
z=−218.25
0.00

Let's instead parameterize this function first to take its surface area. We will take the most basic parameterization.

$$\vec{v}(x,y) = x\hat{i} + y\hat{j} + (x^2 - y^2)\hat{k} \tag{16}$$

Taking, therefore, the partial derivatives:

$$\frac{\partial \vec{v}}{\partial x} = \hat{i} + 2x\hat{k} \tag{17}$$

$$\frac{\partial \vec{v}}{\partial y} = \hat{j} - 2y\hat{k} \tag{18}$$

Taking their cross product for the differential area, then:

$$(\hat{i} + 2x\hat{k}) \times (\hat{j} - 2y\hat{k}) \tag{19}$$

$$\Rightarrow 1 \tag{20}$$

We will find the correction factor, again:

$$dA = \sqrt{1 + 4x^2 + 4y^2}\ dV \tag{21}$$

We will again take this integral, digitally this time:

$$\int_0^5 \int_0^7 (x^2 - y^2)\sqrt{1 + 4x^2 + 4y^2}\ dy\ dx \tag{22}$$

```
f(x,y) = (x^2-y^2)*sqrt(1+4*x^2+4*y^2)
f.integrate(y, 0,7).integrate(x,0,5)
float(f.integrate(y, 0,7).integrate(x,0,5))
```

The shape is largely underneath the x-axis during the area on the rectangle. Therefore, we have an negative area! It is about 3719.47 units.