1 | adjoint, T^* def

Suppose $T ∈ L(V, W). The *adjoint* of T is the function $T^* : W \to V$ s.t.

$$\langle Tv, w\rangle = \langle v, T^* w\rangle$$

Apparently there's another meaning for 'adjoint' in linear algebra too, but it's not covered here.

This definition makes sense because of the Riesz Representation Theorem… :question:

Adjoints are kind of like complex conjugates, as seen in Axler 7.10

2 | results

2.1 | Useful technique: 'flip T^* from one side of an inner product to become T on the other side'

You can always do this by definition of adjoint.

2.2 | Axler7.5 the adjoint is a linear map

If $T \in \mathcal{L}(V, W)$, then $T^* \in \mathcal{L}(W, V)$.

2.3 | Axler7.6 Properties of the adjoint

2.3.1 | $(S + T)^* = S^* + T^*$ for all $S, T \in \mathcal{L}(V, W)$

2.3.2 | $(\lambda T)^* = \overline{\lambda} T^*$ for all $\lambda \in \mathbb{F}$ and $T \in \mathcal{L}(V, W)$

2.3.3 | $(T^*)^* = T$ for all $T \in L(V, W)$

2.3.4 | $I^* = I$

2.3.5 | $(ST)^* = T * S*$ for all $T \in \mathcal{L}(V, W)$ and $S \in \mathcal{L}(W, U)$ where U is an inner product space over $\mathbb{F}$

2.4 | Axler7.7 null space and range of T^*

Suppose $T \in \mathcal{L}(V, W)$. Then,

2.4.1 | $T^* = (T)^\perp$

2.4.2 | $T^* = (T)^\perp$

2.4.3 | $T = (T^*)^\perp$

2.4.4 | $T = (T^*)^\perp$