PS#11: Lagrange multipliers!!!

Nueva Multivariable Calculus

USE MATTHEW GILL'S BEAUTIFUL NUEVA LaTeX TEMPLATE AND MAKE IT LOOK REALLY, REALLY PRETTY. Write up all of your problems clearly and thoroughly, explaining all of your thought process!

1. I didn't write a solution set to Roofs 4, but read my solution set to the second optimization problem, and my brief notes on Lagrange multipliers!
2. Consider the classic 1D calc problem:

   > What's the rectangle with largest area that has a perimeter of 100 units? Give the dimensions and the area. You can do this using just the basic tricks of single-variable calculus—but do it using the method of Lagrange multipliers.

   (a) Without doing any math, and just trusting your gut, what's the answer?
   (b) Solve this using 1D calculus!
   (c) Unnecessary but fun fanciness: solve this using a LAGRANGE MULTIPLIER!!!!
3. The material for the bottom of an (open-topped) aquarium costs half as much as the high strength glass for the four sides. Find the shape of the cheapest aquarium that holds a given volume V.
4. Suppose that the temperature at any point in 3D-space is given by the function:

$$f : \mathbb{R}^3 \to \mathbb{R}^1$$

$$f(x, y, z) = x^2 + y^5 - 2z \quad \text{kelvin}$$

   (This is physically ridiculous, because then the temperature at $(-1, -1, -1)$ would be $(-1)^2 + (-1)^5 + 2 \cdot (-1) = -2$, which is of course impossible, but oh well.) Say you're at the point $(3, 2, 7)$. Some questions:

   (a) What's the temperature at that point?
   (b) Is it way too hot, or way too cold?
   (c) In what direction do you want to aim the thrusters on your jetpack in order to get more comfortable (either hotter or colder, depending) as quickly as possible?
   (d) OH NO!!!! It turns out the thruster control software on your jetpack is broken, and you can only move in one direction!!! If you can only move in one of the x, y, or z direction, starting from $(3, 2, 7)$, in which should you move in order to warm up the fastest? How fast does your temperature change?
   (e) Wait, now it's fixed. You can move in all three directions again. So, when you move in the direction you found in (c), how fast will the temperature change?