

Here are four easy integrals.

1 | Single Value Function

$$f_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (1)$$

$$f_1(x, y) = 0 \quad (2)$$

What's the area of this function?

We can take the area of the shape, essentially by taking the volume by height 1: that is, for a rectangle of l, w, h , its top-area is simply $l \cdot w$, also known as $lw \cdot 1$. Therefore:

$$\int_0^7 \int_0^5 1 dx dy = 35 \quad (3)$$

The area of the shape is therefore 35.

2 | Area of the Plane

We want to first figure the correction per every given slice $dA = n dV$ to setup a surface integral. By pythagoras (i.e. projecting the changes to the parallelity of the surface), we have that:

$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dV \quad (4)$$

What's the area of the following function by (5, 7)?

$$f_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (5)$$

$$f_2(x, y) = 2x + 3y + 10 \quad (6)$$

$$dA = \sqrt{1 + 4 + 9} dV = \sqrt{14} dV \quad (7)$$

Therefore, taking the integral:

$$\int_0^5 \int_0^7 \sqrt{14}(2x + 3y + 10) dy dx \quad (8)$$

$$\Rightarrow \sqrt{14} \int_0^5 \int_0^7 (2x + 3y + 10) dy dx \quad (9)$$

At this point, we absolve the need to leverage a tonne of fractions by taking the inner integral digitally.

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f(x,y) = 2*x+3*y+10
f.integrate(y, 0, 7).integrate(x, 0,5)
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$$\sqrt{14} \int_0^5 \int_0^7 (2x + 3y + 10) dy dx \quad (10)$$

$$\Rightarrow \sqrt{14} \frac{1785}{2} \quad (11)$$

$$\Rightarrow 3339.43 \quad (12)$$

It appears that the surface area is about 3339.43 units.

3 | Area of a Parabola

What's the area of the following function by (5, 7)?

$$f_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (13)$$

$$f_3(x, y) = x^2 \quad (14)$$

We will again find the area correction factor:

$$dA = \sqrt{1 + 4x^2} dV \quad (15)$$

And therefore, taking the integral:

$$\int_0^5 \int_0^7 x^2 \sqrt{1 + 4x^2} dy dx \quad (16)$$

This problem is solvable by trig substitution followed by integration by parts. For now, however, we will leverage a calculator.

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f(x,y) = x^2*sqrt(1+4*x^2)
f.integrate(y, 0,7).integrate(x,0,5)
float(f.integrate(y, 0,7).integrate(x,0,5))
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Evidently, the surface area of the shape is about 2209 units.

4 | Another Surface Area

What's the area of the following function by (5, 7)?

$$f_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (17)$$

$$f_3(x, y) = x^2 - y^2 \quad (18)$$

We will find the correction factor, again:

$$dA = \sqrt{1 + 4x^2 + 4y^2} dV \quad (19)$$

We will again take this integral, digitally this time:

$$\int_0^5 \int_0^7 (x^2 - y^2) \sqrt{1 + 4x^2 + 4y^2} \, dy \, dx \quad (20)$$