Given:

$$\begin{cases} f(x,y) : \mathbb{R}^2 \to \mathbb{R}^1 \\ f(x,y) = sin(x)cos(y) + 5 \end{cases} \tag{1}$$

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f(x,y) = sin(x)*cos(y) +5
plot3d(f, (x,-5,5), (y,-5,5))
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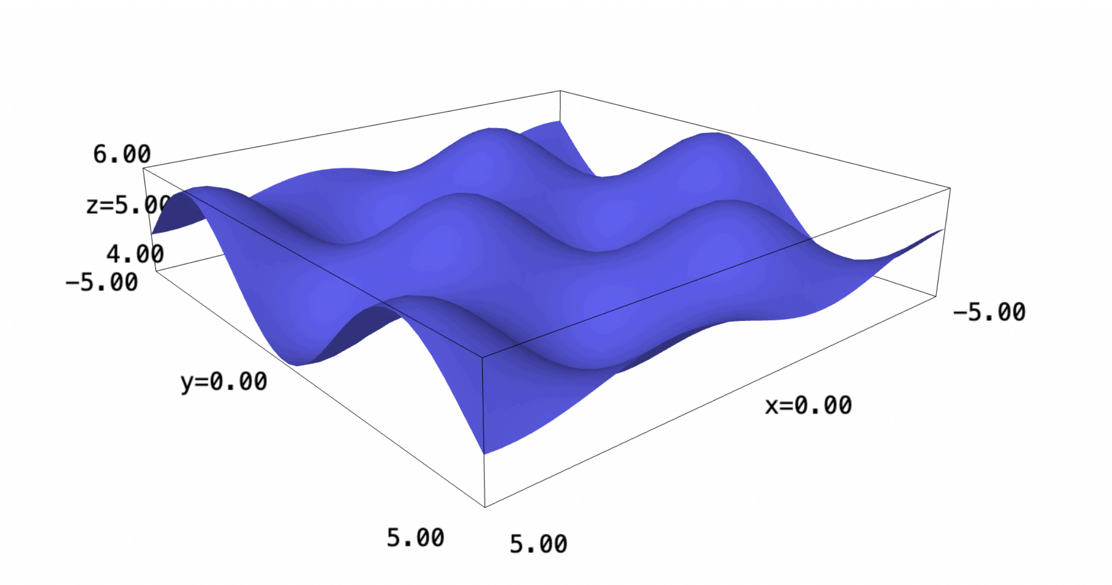

1 | Problem 1

What is the integral of the function along

$$y = 0, \{0 \leq x \leq 3\pi\} \tag{2}$$

We begin by the parameterization of the function, slicing along the bottom edge. We understand, therefore, that we are parameterize by:

$$\begin{cases} x = t \\ y = 0 \end{cases} \tag{3}$$

$t = 3\pi$ when $x = 3\pi$, meaning our bounds are $[0, 3\pi]$.

Performing the actual parameterization, then:

$$f(t,0) = sin(t) \cdot 1 + 5 \tag{4}$$
$$f(t) = sin(t) \cdot 1 + 5 \tag{5}$$

in units of t. We then figure the correction to $\frac{dy}{dx}$ for which it contributes. Every value of t, along the curve, contributes $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ of distance. We see from the above parameterization, that:

$$\begin{cases} \frac{dx}{dt} = 1 \\ \frac{dy}{dt} = 0 \end{cases} \tag{6}$$

Therefore:

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 1 \tag{7}$$

And finally, taking the integral:

$$\int_0^{3\pi} (sin(t) + 5) \cdot 1 \; dt = (-cos(t) + 5t)|_0^{3\pi} \tag{8}$$
$$= (1 + 15\pi) - (-1) \tag{9}$$
$$= 15\pi + 2 \tag{10}$$

2 | **Problem 2**

What is the integral of the function along

$$y = x, \{0 \leq x \leq 3\pi\}, \{0 \leq y \leq 3\pi\} \tag{11}$$

We begin by the parameterization of the function, slicing along the bottom edge. We understand, therefore, that we are parameterize by:

$$\begin{cases} x = t \\ y = t \end{cases} \tag{12}$$

$t = 3\pi$ when $y = x = 3\pi$, meaning our bounds are $[0, 3\pi]$.
Performing the actual parameterization, then:

$$f(t,t) = sin(t)cos(t) + 5 \tag{13}$$
$$f(t,t) = \frac{1}{2}(2sin(t)cos(t)) + 5 \tag{14}$$
$$f(t) = \frac{1}{2}sin(2t) + 5 \tag{15}$$

in units of t. We then figure the correction to $\frac{dy}{dx}$ for which it contributes. Every value of t, along the curve, contributes $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ of distance. We see from the above parameterization, that:

$$\begin{cases} \frac{dx}{dt} = 1 \\ \frac{dy}{dt} = 1 \end{cases} \tag{16}$$

Therefore:

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{2} \tag{17}$$

And finally, taking the integral:

$$\int_0^{3\pi} \left(\frac{1}{2}sin(2t) + 5\right) \cdot \sqrt{2}\, dt \tag{18}$$

$$= \int_0^{3\pi} \frac{1}{2}sin(2t)\sqrt{2}dt + \int_0^{3\pi} 5\sqrt{2}\, dt \tag{19}$$

$$= \left(\frac{-1}{4}cos(2t)\sqrt{2}\right)\Bigg|_0^{3\pi} + 5t\sqrt{2}|_0^{3\pi} \tag{20}$$

$$= 15\pi\sqrt{2} \tag{21}$$

3 | Problem 3

What is the integral of the function along

$$y = x^2, \{0 \leq x \leq 2\pi\}, \{0 \leq y \leq 4\pi^2\pi\} \tag{22}$$

We begin by the parameterization of the function, slicing along the bottom edge. We understand, therefore, that we are parameterize by:

$$\begin{cases} x = t \\ y = t^2 \end{cases} \tag{23}$$

$t = 2\pi$ when $y = 4\pi^2, x = 2\pi$, meaning our bounds are $[0, 2\pi]$.
Performing the actual parameterization, then:

$$f(t, t^2) = sin(t)cos(t^2) + 5 \tag{24}$$

$$f(t) = sin(t)cos(t^2) + 5 \tag{25}$$

in units of t. We then figure the correction to $\frac{dy}{dx}$ for which it contributes. Every value of t, along the curve, contributes $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ of distance. We see from the above parameterization, that:

$$\begin{cases} \frac{dx}{dt} = 1 \\ \frac{dy}{dt} = 2t \end{cases} \tag{26}$$

Therefore:

$$\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{1 + 4t^2} \tag{27}$$

And finally, taking the integral:

$$\int_0^{2\pi} \left(\frac{1}{2}sin(2t) + 5\right) \cdot \sqrt{1+4t^2}\ dt \tag{28}$$

We will now solve this integral analytically:

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t = var("t")
definite_integral((0.5*sin(2*t)+5)*sqrt(1+4*t^2), t, 0, 2*pi)
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It appears that the area under the parameterized curve is about 199 units.