1 | **Problem 1**

1.1 | **a)**

Inorder for the boat to be going in the right direction we know that $\vec{C} + \vec{S} = \alpha \vec{D}$, where $\vec{C}$ is the current of the river, $\vec{S}$ is the speed of the boat, α is some scalar and $\vec{D}$ is the vector that goes from the boatman's starting point to their desired endpoint.

We can set the boatman's start point as $(0, 0)$, and thus $\vec{D} = \langle 3, 2 \rangle$. We also know that $\vec{C} = \langle 0, -3.5 \rangle$. Lastly, $\vec{S} = \langle 13 \sin(\theta), 13 \cos(\theta) \rangle$, where θ is the angle between the side of the river and $\vec{S}$.

We can then plug in these values into the equation written above:

$\vec{C} + \vec{S} = \alpha \vec{D}$
$\Rightarrow \langle 0, -3.5 \rangle + \langle 13 \sin(\theta), 13 \cos(\theta) \rangle = \alpha \langle 3, 2 \rangle$
$\Rightarrow \langle 13 \sin(\theta), -3.5 + 13 \cos(\theta) \rangle = \langle \alpha 3, \alpha 2 \rangle$
$\Rightarrow 13 \sin(\theta) = \alpha 3, -3.5 + 13 \cos(\theta) = \alpha 2$
$\Rightarrow 6\alpha = 26 \sin(\theta), 6\alpha = -10.5 + 39 \cos(\theta)$
$\Rightarrow 26 \sin(\theta) = -10.5 + 39 \cos(\theta)$

plug it into wolfram alpha:

$\theta \approx 0.75686$ radians or $\approx 43.36°$

1.2 | **b)**

The net velocity of the boat is $\vec{S} + \vec{C} = \langle 13 \sin(\theta), 13 \cos(\theta) - 3.5 \rangle$, where θ is the answer to part a. To get the speed of the boat we find the magnitude of this vector:

$|\vec{S} + \vec{C}| = \sqrt{(13 \sin(\theta))^2 + (13 \cos(\theta) - 3.5)} \approx 10.7282$ km/h

Now we need to find the distance traveled by the boat, which should be the magnitude of $\vec{D}$:

$|\vec{D}| = \sqrt{2^2 + 3^2} = \sqrt{4 + 9} = \sqrt{13} \approx 3.60555$ km

To get the time it took to take the trip we divide the distace by the speed:

$\frac{3.60555}{10.7282} = 0.336$ hours, which is 20.2 minutes

2 | **Problem 2**

2.1 | **a)**

$\vec{r}(t) = (R \cos(\omega_o t), R \sin(\omega_o t))$
This is because the x coordinate is defined as $r \cos(\theta)$ and the y coordinate is defined as $r \sin(\theta)$. In this case r or the radius is R and θ is $\omega_o t$, because $\omega_o = \frac{\theta}{t}$ (definition of angular velocity.

2.2 | **b)**

$\vec{v}(t) = \vec{r}'(t) = (\frac{d}{dt} R \cos(\omega_o t), \frac{d}{dt} R \sin(\omega_o t)) = (-R\omega_o \sin(\omega_o t), R\omega_o \cos(\omega_o t))$
Answer: $\vec{v}(t) = (-R\omega_o \sin(\omega_o t), R\omega_o \cos(\omega_o t))$

2.3 | c)

$\vec{a}(t) = \vec{r}''(t) = (\frac{d}{dt}(-R\omega_o\sin(\omega_o t)), \frac{d}{dt}R\omega_o\cos(\omega_o t)) = (-R\omega_o^2\cos(\omega_o t), -R\omega_o^2\sin(\omega_o t))$

Answer: $\vec{a}(t) = (-R\omega_o^2\cos(\omega_o t), -R\omega_o^2\sin(\omega_o t))$

2.4 | d)

The tangent:

Because $\vec{v}(t)$ is a vector it can be placed anywhere on the plane, so the only requierment for $\vec{v}(t)$ is that it has to be perpendicular to $\vec{r}(t)$, which is the radius of the circle. This means that the slope of $\vec{v}(t)$ has to be the opposite reciprocol of $\vec{r}(t)$:

Slope of $\vec{r}(t) = \frac{\Delta y}{\Delta x} = \frac{R\sin(\omega_o t)-0}{R\cos(\omega_o t)-0} = \frac{\sin(\omega_o t)}{\cos(\omega_o t)}$

Slope of $\vec{v}(t) = \frac{\Delta y}{\Delta x} = \frac{R\omega_o\cos(\omega_o t)-0}{-R\omega_o\sin(\omega_o t)-0} = -\frac{\cos(\omega_o t)}{\sin(\omega_o t)}$

The slopes are opposite reciprocols, thus $\vec{v}(t)$ is perpendicular to $\vec{r}(t)$, thus $\vec{v}(t)$ is tangent to the circle.

The magnitude:

$|\vec{v}(t)| = \sqrt{(-R\omega_o\cos(\omega_o t))^2 + (R\omega_o\sin(\omega_o t))^2} = \sqrt{R^2\omega_o^2\cos^2(\omega_o t + R^2\omega_o^2\sin^2(\omega_o t)} = \sqrt{R^2\omega_o^2(\cos^2(\omega_o t) + \sin^2(\omega_o t)} = \sqrt{R^2\omega_o^2(1)} = \sqrt{R^2\omega_o^2} = R\omega_o$

2.5 | e)

Point towards the center of the circle:

$\vec{a}(t)$ is a scalar multiple of $\vec{r}(t)$: $\vec{a}(t) = -\omega_o^2\cdot\vec{r}(t) = -\omega_o^2(R\cos(\omega_o t), R\sin(\omega_o t)) = (-R\omega^2\cos(\omega_o t), -R\omega_o^2\sin(\omega_o t)) = \vec{a}(t)$

because the scalar multiple is negative $\vec{v}(t)$ points in the opposite direction of $\vec{r}(t)$, which is towards the center of the circle because $\vec{r}(t)$ points from the center of the circle outwards.

The magnitude:

$|\vec{a}(t)| = \sqrt{(-R\omega^2\cos(\omega_o t))^2 + (-R\omega_o^2\sin(\omega_o t))^2} = \sqrt{R^2\omega_o^4\cos^2(\omega_o t) + R^2\omega_o^4\sin^2(\omega_o t)} = \sqrt{R^2\omega_o^4(\cos^2(\omega_o t) + \sin^2(\omega_o t))} = \sqrt{R^2\omega_o^4(1)} = \sqrt{R^2\omega_o^4} = R\omega_o^2$

$\frac{|\vec{v}(t)|^2}{R} = \frac{(R\omega_o)^2}{R} = \frac{R^2\omega_o^2}{R} = R\omega_o^2$

2.6 | f)

$\theta'(t) = \int \theta''(t)dt = \int \alpha_o dt = \alpha_o t + c$, where c is the constant of integration

Answer: $\alpha_o t + c$

2.7 | g)

$\theta(t) = \int \theta'(t)dt = \int (\alpha_o t + c)dt = \int \alpha_o t dt + \int c dt = \frac{\alpha_o t^2}{2} + ct + c'$ where c' is another constant of integration.

Answer: $\frac{\alpha_o t^2}{2} + ct + c'$

2.8 | h)

For simplicity's sake, I am going to assume that $c = 0$ and $c' = 0$:

$\vec{r}(t) = (R\cos(\frac{\alpha_o t^2}{2}), R\sin(\frac{\alpha_o t^2}{2}))$

2.9 | i)

$\vec{v}(t) = \frac{d}{dt}\vec{r}(t) = (\frac{d}{dt}R\cos(\frac{\alpha_o t^2}{2}), \frac{d}{dt}R\sin(\frac{\alpha_o t^2}{2})) = (-R\alpha_o t\sin(\frac{\alpha_o t^2}{2}), R\alpha_o t\cos(\frac{\alpha_o t^2}{2}))$

Answer: $\vec{v}(t) = (-R\alpha_o t\sin(\frac{\alpha_o t^2}{2}), R\alpha_o t\cos(\frac{\alpha_o t^2}{2}))$

2.10 | j)

$\vec{a}(t) = \vec{r}''(t) = (\frac{d}{dt} - R\alpha_o t\sin(\frac{\alpha_o t^2}{2}), \frac{d}{dt}R\alpha_o t\cos(\frac{\alpha_o t^2}{2}))$

It will be easier to visualize by doing the math in it's x and y component:

x component:

$\frac{d}{dt} - R\alpha_o t\sin(\frac{\alpha_o t^2}{2}) = -R\alpha_o(\sin(\frac{\alpha_o t^2}{2}) + \alpha_o t^2\cos(\frac{\alpha_o t^2}{2})) = -R\alpha_o\sin(\frac{\alpha_o t^2}{2}) - R\alpha_o^2 t^2\cos(\frac{\alpha_o t^2}{2})$

y component:

$\frac{d}{dt}R\alpha_o t\cos(\frac{\alpha_o t^2}{2}) = R\alpha_o(\cos(\frac{\alpha_o t^2}{2}) - \alpha_o t^2\sin(\frac{\alpha_o t^2}{2})) = R\alpha_o\cos(\frac{\alpha_o t^2}{2}) - R\alpha_o^2 t^2\sin(\frac{\alpha_o t^2}{2})$

Putting these two together we get:

$\vec{a}(t) = (-R\alpha_o\sin(\frac{\alpha_o t^2}{2}) - R\alpha_o^2 t^2\cos(\frac{\alpha_o t^2}{2}), R\alpha_o\cos(\frac{\alpha_o t^2}{2}) - R\alpha_o^2 t^2\sin(\frac{\alpha_o t^2}{2}))$

2.11 | k)