PS#13: Smörgåsbord of fun!

Nueva Multivariable Calculus

(This might be a little longer than recent problem sets, so take that into account as you plan your schedule! Lots of great problems, though.)

1. Read Andrew's solution notes to those three intro line integrals problem!!!! (This is a very good, long and detailed, solution set, so do read it carefully! It's on Canvas.)

2. Consider the function:
$$f(x,y) = ye^x$$
Find the area under this function above the straight line from $(1,2)$ to $(4,7)$. (Include (a) picture(s) of some sort—as you should always do, without me having to tell you to do so!)

3. Consider the paraboloid:
$$f(x,y) = x^2 + y^2$$
Find the area under this function, along a circle of radius 1 centered at the origin. (Picture??)

   (This doesn't really *require* multivariable calculus to figure out—you can probably figure it out with just basic geometry! But I do want you to do a line integral here, just to see how it works. The trick is coming up with a good parameterization of the circle—I guarantee that with the right parameterization, you'll end up with an integral that's quite easy to work out.)

4. *(Same problem some of you did the last day before break!)* You are running a hat factory. Your cost is primarily in seal furs, which cost \$170, and in wages for the little raccoons that sew the furs into hats (raccoons have tiny opposable thumbs, so they're very dexterous; plus, they're nocturnal, so you can run them on the graveyard shift without paying them more). Suppose your revenue R is loosely modelled by the equation:
$$R(h,s) = 200h^{2/3}s^{1/3}$$
where h is the total raccoon-hours of labor and s is the tons of seal furs you use. If you have \$20,000 to spend running this factory, what's your maximum possible revenue? How many tons of seal furs do you use, and how much do you pay the raccoons?

5. *(A 1D Taylor series problem!)* **Percentages are nonlinear**. Suppose it takes you six hours to drive from SF to LA going 60mph. If you increase your speed by 100%—i.e., double it—then the time it takes to get to LA doesn't go down by 100%. You don't get to LA in zero hours by driving 120mph. (Among other things, this would violate conservation of energy.) Even if you increase your speed by only 50%, you don't decrease the amount of time it takes to get to LA by 50%.

   (a) Why not? How long would it take? Figure it out.

   On the other hand, if you're dealing only with *small* differences in speed, increasing your speed by x% *will* decrease your travel time by x% (roughly, approximately).

   (c) Verify that this is true in this specific example. For instance, if you increase your speed by 2%, how long will it take to get to LA? How does that compare to 98% of six hours? (You can use a small percentage other than 2% if you like!)

   (d) Most importantly, *why* is this true? Make an argument for why, as long as x is reasonably small, if you increase your speed by x%, your travel time will decrease by about x%.

   (*Inspired by a conversation on this topic, as reported to me, between Scott Kovach and Aamnah Khalid while driving from SF to Death Valley; the conversation apparently involved them trying to compute a particular Taylor series out loud, without pen or paper, while driving... which is something of a hint.*)