1 | salt flats

1.1 | chart the adventure

Our adventure is of the form $f(t) = \begin{bmatrix} t^2 - 9 \\ 12\sin t - t \end{bmatrix}$. Here's a graph, to scale: `https://www.desmos.com/calculator/qchdfts4in`.

1.2 | velocity as a function of time

$$\frac{d}{dt} f(t) = \begin{bmatrix} 2t \\ 12\cos t - 1 \end{bmatrix}$$

1.3 | acceleration as a function of time

$$\frac{d^2}{dt^2} f(t) = \begin{bmatrix} 2 \\ -12\sin t \end{bmatrix}$$

1.4 | the moment $t = \pi$

$$\left| \frac{d}{dt} f(t) \Big|_\pi \right| = \sqrt{4\pi^2 + 144\cos^2 \pi + 1 - 24\cos t} = \sqrt{4\pi^2 + 144 + 1 + 24} \approx 208$$
$$\left| \frac{d^2}{dt^2} f(t) \Big|_\pi \right| = \sqrt{4 + 12^2 \sin^2 t} = \sqrt{4 + 144} \approx 12$$

1.5 | maximum speed

The acceleration is zero at the maximum speed (but also at the minimum speed).

$$\begin{aligned} \sqrt{4 + 12^2 \sin^2 t} &= 0 \\ 12^2 \sin^2 t &= -4 \\ \sin^2 t &= -\frac{4}{12^2} \\ \sin t &= -\frac{1}{6} \end{aligned}$$

It's hard to find exactly which value of t both lies in the bounds and corresponds to the maximal speed, though.

1.6 | maximum acceleration

$$\left| \frac{d^3}{dt^3} f(t) \right| = -12\cos t = 0$$

The maximum acceleration occurs when $t = (2n+1)\pi + \frac{\pi}{2}, n \in \mathbb{Z}$.

The acceleration at these maxima is 12 miles / hour squared, which is 0.0001 gs. This may not be the most joyous joyride.

1.7 | distance traveled by odometer

$$\int_{-2\pi}^{3\pi} \sqrt{4t^2 + 12^2 \cos^2 t - 24 \cos t + 1} dt \approx 189.5 \text{mi}$$

2 | sandy times

2.1 | a chart

2.2 | elevation

$$\sin \frac{23\pi}{3} = \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

2.3 | hike as a path

$$f(t) = \left\langle \frac{23\pi}{3} + t \cos \frac{3\pi}{8}, 37 + t \sin \frac{3\pi}{8}, \sin \left(\frac{23\pi}{3} + t \cos \frac{3\pi}{8} \right) \right\rangle$$

a chart!

2.4 | slope along the path

$$\begin{aligned} \frac{d}{dt} f_z(t) &= \frac{d}{dt} \sin \left(\frac{23\pi}{3} + t \cos \frac{3\pi}{8} \right) \\ &= \cos \left(\frac{23\pi}{3} + t \cos \frac{3\pi}{8} \right) \cos \frac{3\pi}{8} \end{aligned}$$

2.5 | slope when you start

$$\frac{d}{dt} f_z(t) \Big|_0 = \cos \frac{23\pi}{3} \cos \frac{3\pi}{8}$$

But you want an angle, so we have to take the arctan.

$$\tan^- \left(\cos \frac{23\pi}{3} \cos \frac{3\pi}{8} \right) \approx 0.189 \text{ rad}$$