1 | Dot product:

- Name: dot product
- Result: Scalar
- Interpretation (what it measures): parallelity
  - the more parallel the larger the dot product
- Magnitude (with sign): $|\vec{a}||\vec{b}|cos(\theta)$
- Geometric magnitude: $|\vec{a}||\vec{b}_{||\vec{a}}|$
- Direction: no direction
- Algebraic form: $a_xb_x + a_yb_y + a_zb_z$
- Algebraic properties:
  - commutative
  - associative
  - distributive across addition

2 | Cross product:

- Name: Cross product
- Result: Vector
- Interpretation (what it measures): Orthgonality
  - the more orthogonal the longer the cross product
- Magnitude (with sign): $|\vec{a}||\vec{b}|\sin(\theta)$
- Geometric Magnitude: $|\vec{a}||\vec{b}_{\perp\vec{a}}|$
- Direction: perpendicular to the two vectors
  - by the right hand rule by rotating the first vector into the second vector
- Albraic form: $\langle a_yb_z - a_zb_y, a_xb_z - a_zb_x, a_xb_y - a_yb_x\rangle$
- Algebraic properties:
  - Anticommutative
  - $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$
  - $(\vec{A} \times \vec{B}) \perp \vec{A}$
  - $(\vec{A} \times \vec{B}) \perp \vec{B}$
  - Antiassociative

3 | Application of cross product:

- In physics there is something called torque, notated τ
  - Torqe is the net force of things that rotate, so:
    - $F_{net} = ma$
    - $\tau_{net} = I\omega$
- Somethings to note about τ:
  - It increases with a longer lever
  - It increases with a greater force
    - that is perpendicular to the lever
- Given these requirements we can make a formula:
  - $|\tau| = |\vec{r}||\vec{F}_{\perp\vec{r}}|$, where $\vec{F}$ is the force applied to the door, and $\vec{r}$ is the radius of the lever.
  - this, the right side of the equation, can be described using the dot product: $|\tau| = \vec{r} \times \vec{F}$

4 | Derivation of cross product algebraic form:

To start, we can define:

$$\vec{A} = (A_x, A_y, A_z) = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{B} = (B_x, B_y, B_z) = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Next we have to assume that the dot product is distributive across addition:

$\vec{A} \times \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_Z\hat{k}) \times (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$
$= A_xB_x\hat{i} \times \hat{i} + A_xB_y\hat{i} \times \hat{j} + A_xB_z\hat{i} \times \hat{k}$
$+ A_yB_x\hat{j} \times \hat{i} + A_yB_y\hat{j} \times \hat{j} + A_yB_z\hat{j} \times \hat{k}$
$+ A_zB_x\hat{k} \times \hat{i} + A_zB_y\hat{k} \times \hat{j} + A_zB_z\hat{k} \times \hat{k}$

From the definition of a cross product, we know that the cross product between any two vectors that are parallel is zero, thus:

$= A_xB_y\hat{i} \times \hat{j} + A_xB_z\hat{i} \times \hat{k}$
$+ A_yB_x\hat{j} \times \hat{i} + A_yB_z\hat{j} \times \hat{k}$
$+ A_zB_z\hat{k} \times \hat{i} + A_zB_z\hat{k} \times \hat{j}$