1 | **Export**

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2 | **Exercise 7**

> Suppose $T \in \mathcal{L}(V)$ has a diagonal matrix A with respect to some basis of V and that $\lambda \in \mathbb{F}$. Prove that λ appears on the diagonal of A precisely $E(\lambda, T)$ times.

3 | **Proof**

We will show that for each eigenvalue λ, there are at least $E(\lambda, T)$ occurrences of that eigenvalue and at most $E(\lambda, T)$ occurrences.

Suppose first that dim $E(\lambda, T) = m$ and $v_1, \dots, v_m$ is a basis of $E(\lambda, T)$. In the diagonal matrix, the column corresponding to each of the m eigenvectors is comprised of the coefficients of

$$Tv_j = \lambda v_j$$

The coefficients of an eigenvector linear combination are just the eigenvalue, so the associated eigenvalue (λ) appears once for each eigenvector. Thus, λ appears on the diagonal at least m times.

Suppose then that λ is on the diagonal m times. Each of those occurrences corresponds to where the diagonal matrix sends a (linearly independent) basis eigenvector, which implies that the basis of V has at least m eigenvectors corresponding to λ. These m eigenvectors can be extended to a basis of $E(\lambda, T)$, implying that dim $E(\lambda, T) \geq m$.