1 | **boatman problem**

Target displacement: $\langle 3\text{km}, 2\text{km}\rangle$

We are working with the velocities of the boat and the river. The velocity of the river is defined as $r = \langle 0, -3.5\rangle$. We want to find vector $v = \langle v_x, v_y\rangle$ s.t.

$$\begin{aligned} |v| &= 13 \text{ km/h} \\ \lambda(v+r) &= \langle 3, 2\rangle \end{aligned}$$

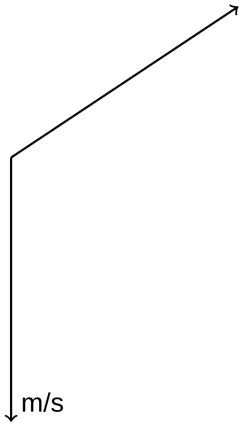
Where the trip will take λ hours

$$\begin{aligned} v_x^2 + v_y^2 &= 13^2 \\ \lambda(v_x + 0) &= 3 \\ \lambda(v_y + -3.5) &= 2 \end{aligned}$$

$$\begin{aligned} v_x &= \frac{3}{\lambda} \\ v_y &= \frac{2}{\lambda} + 3.5 \\ \frac{3^2}{\lambda^2} + \left(\frac{2}{\lambda} + 3.5\right)^2 &= 13^2 \\ \frac{3^2}{\lambda^2} + \frac{2^2}{\lambda^2} + 3.5^2 + \frac{4(3.5)}{\lambda} &= 13^2 \\ \frac{3^2+2^2}{\lambda^2} + \frac{4(3.5)}{\lambda} &= 13^2 - 3.5^2 \\ 3^2 + 2^2 + 4(3.5)\lambda &= \lambda^2\left(13^2 - 3.5^2\right) \\ 13 + 4(3.5)\lambda &= \lambda^2\,(156.75) \\ -156.75\lambda^2 + 14^2 + 13 &= 0 \\ \lambda &= \frac{-14 \pm \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} \\ \lambda &= \frac{-14 + \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} = -0.24676847741 \\ \lambda &= \frac{-14 - \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} = 0.336082671987 \end{aligned}$$

$$\boxed{\lambda \approx 0.33608 = 20\,\text{min}}$$

Maybe it's time to do it geometrically



Let θ be the angle difference that you paddle at, and ϕ be the angle that you are aiming for.

$$3.5^2\lambda^2 = 13 + 13\lambda - 2(13)13\lambda \cos\theta$$
$$\tan\phi = \frac{3}{2}$$
$$\sin(\theta + \phi) = \frac{3.5\lambda + 2}{13\lambda}$$

attempt 3: after getting help from leonard

$$\beta = \alpha + \frac{\pi}{2} = \tan^{-}\frac{2}{3} = 2.158$$
$$\frac{\sin\beta}{|v|} = \frac{\sin\gamma}{3.5}$$
$$\frac{\sin(2.158)}{13} = \frac{\sin\gamma}{3.5}$$
$$3.5\frac{\sin(2.158)}{13} = \sin\gamma$$
$$\gamma = 3.5\frac{\sin(2.158)}{13} = 0.2241$$
$$\alpha + \gamma = 0.588 + 0.2241 = \boxed{0.8121 \text{ radians}}$$

The speed

$$\frac{3}{13\cos 0.812} = 0.3353 \text{ hours}$$

dang it i was actually right the first time. apparently math isn't a democracy.

2 | circular motion

$$\theta'(t) = \omega_0$$
$$\theta(t) = \int \omega_0 dt \quad = \omega_0 t + \theta_0$$

2.1 | position

$$\begin{aligned}\vec{r}(t) &= (R\cos\theta(t), R\sin\theta(t)) \\ &= (R\cos(\omega_0 t+\theta_0), R\sin(\omega_0 t+\theta_0))\end{aligned}$$

2.2 | velocity

$$\vec{v}'(t) = (-\omega_0 R\sin(\omega_0 t+\theta_0), \omega_0 R\cos(\omega_0 t+\theta_0))$$

2.3 | acceleration

$$\begin{aligned}\vec{a}(t) &= \vec{v}'(t) \\ &= \frac{d}{dt}(-\omega_0 R\sin(\omega_0 t+\theta_0), \omega_0 R\cos(\omega_0 t+\theta_0)) \\ &= \left(-\omega_0^2 R\cos(\omega_0 t+\theta_0), -\omega_0^2 R\sin(\omega_0 t+\theta_0)\right)\end{aligned}$$

2.4 | perpendicular

$$\begin{aligned}\text{pos: } & \frac{\cancel{R}\sin(\omega_0 t+\theta_0)}{\cancel{R}\cos(\omega_0 t+\theta_0)} \\ \text{vel: } & \frac{\cancel{\omega_0 R}\cos(\omega_0 t+\theta_0)}{-\cancel{\omega_0 R}\sin(\omega_0 t+\theta_0)}\end{aligned}$$

See? They are negative reciprocals of eachother.

$$\begin{aligned}|\vec{v}| &= \sqrt{\omega_0^2 R^2\sin^2(\omega_0 t+\theta_0) + \omega_0^2 R^2\cos^2(\omega_0 t+\theta 0)} \\ &= \omega_0 R\sqrt{\sin^2(\omega_0 t+\theta_0)+\cos^2(\omega_0 t+\theta 0)} \\ &= \omega_0 R\sqrt{1} \\ &= \omega_0 R\end{aligned}$$

2.5 | acceleration

$$\begin{aligned}\text{accl } &= -\omega_0^2 R\left(\cos(\omega_0 t+\theta_0), \sin(\omega_0 t+\theta_0)\right) \\ \text{pos } &= R\left(\cos(\omega_0 t+\theta_0)+\sin(\omega_0 t+\theta_0)\right)\end{aligned}$$

The acceleration is a negative scalar multiple of the position, and thus points towards the center.

$$|\vec{a}| = -w_0^2 R\sqrt{1} = \frac{w_0^2 R^2}{R} = \frac{|\vec{v}|^2}{R}$$

2.6 | non uniform motion

$$\theta''(t) = a_0$$
$$\theta'(0) = 0$$
$$\theta(0) = 0$$

$$\theta'(t) = a_0 t + C_0 = a_0 t$$

2.7 | angular position

$$\theta(t) = \frac{1}{2}a_0 t^2 + C_1 = \frac{1}{2}a_0 t^2$$

2.8 | position

$$\begin{aligned}\vec{r}(t) &= (R\cos\theta(t), R\sin\theta(t)) \\ &= \left(R\cos\left(\frac{1}{2}a_0 t^2\right), R\sin\left(\frac{1}{2}a_0 t^2\right)\right)\end{aligned}$$

2.9 | velocity

$$\begin{aligned}\vec{v}(t) &= R\left(-\sin\left(\frac{1}{2}a_0 t^2\right)a_0 t, \cos\left(\frac{1}{2}a_0 t^2\right)a_0 t\right) \\ &= Ra_0 t\left(-\sin\left(\frac{1}{2}a_0 t^2\right), \cos\left(\frac{1}{2}a_0 t^2\right)\right)\end{aligned}$$

2.10 | acceleration

$$\begin{aligned}\vec{a}(t) = a_0 R\Bigg(&-\cos\left(\frac{1}{2}a_0 t^2\right)a_0 t^2 - \sin\left(\frac{1}{2}a_0 t^2\right), \\ &-\sin\left(\frac{1}{2}a_0 t^2\right)a_0 t^2 + \cos\left(\frac{1}{2}a_0 t^2\right)\Bigg)\end{aligned}$$

2.11 | velocity vector

Both magnitudes are non-zero, so we can use the dot product to show that the position and velocity are perpendicular.

$$\begin{aligned}\vec{r}(t)\cdot\vec{v}(t) &= R\left(-\cos\left(\frac{1}{2}a_0 t^2\right)\sin\left(\frac{1}{2}a_0 t^2\right)a_0 t + \sin\left(\frac{1}{2}a_0 t^2\right)\cos\left(\frac{1}{2}a_0 t^2\right)a_0 t\right) \\ &= R0 = 0\end{aligned}$$

Thus, the velocity is tangent to the circle.

Magnitude:

$$
\begin{aligned}
\vec{v}(t) &= Ra_0 t\sqrt{\sin^2\left(\frac{1}{2}a_0t^2\right) + \cos^2\left(\frac{1}{2}a_0t^2\right)} \\
&= Ra_0t\sqrt{1} = Ra_0t = R\theta'(t)
\end{aligned}
$$

2.12 | **acceleration**

We can split it up because we know the $|\vec{v}|^2$ term is the one with $a_0^2t^2$

$$
\begin{aligned}
\vec{a}(t) &= R\left(-a_0^2t^2\left(\cos\left(\frac{1}{2}a_0t^2\right), \sin\left(\frac{1}{2}a_0t^2\right)\right) + a_0\left(-\sin\left(\frac{1}{2}a_0t^2\right), \cos\left(\frac{1}{2}a_0t^2\right)\right)\right) \\
&= R\left(-\frac{|\vec{v}|^2}{R^{\cancel{2}}}\frac{\vec{r}}{R}\right) + Ra_0\frac{\vec{v}}{|\vec{v}|} \\
&= \left(-\frac{|\vec{v}|}{R}\frac{\vec{r}}{R}\right) + Ra_0\frac{\vec{v}}{|\vec{v}|}
\end{aligned}
$$

3 | **derivative distribution**

$$
\begin{aligned}
\frac{d\vec{A}}{dt} &= \lim_{h\to 0}\frac{\vec{A}(t+h) - \vec{A}(t)}{h} \\
&= \lim_{h\to 0}\frac{(A_x(t+h), A_y(t+h), A_z(t+h)) - (A_x(t), A_y(t), A_z(t))}{h} \\
&= \lim_{h\to 0}\frac{(A_x(t+h) - A_x(t), A_y(t+h) - A_y(t), A_z(t+h) - A_z(t))}{h} \\
&= \lim_{h\to 0}\left(\frac{A_x(t+h) - A_x(t)}{h}, \frac{A_y(t+h) - A_y(t)}{h}, \frac{A_z(t+h) - A_z(t)}{h}\right) \\
&= \left(\lim_{h\to 0}\frac{A_x(t+h) - A_x(t)}{h}, \lim_{h\to 0}\frac{A_y(t+h) - A_y(t)}{h}, \lim_{h\to 0}\frac{A_z(t+h) - A_z(t)}{h}\right) \\
&= \left(\frac{dA_x(t)}{dt}, \frac{dA_y(t)}{dt}, \frac{dA_z(t)}{dt}\right)
\end{aligned}
$$

4 | derivative linearity

$$
\begin{aligned}
\frac{d}{dt}\left(\alpha\vec{A}(t)+\beta\vec{B}(t)\right) &= \frac{d}{dt}\left(\alpha A_x(t), \alpha A_y(t)\right) + \left(\beta B_x(t), \beta B_y(t)\right) \\
&= \frac{d}{dt}\left(\alpha A_x(t)+\beta B_x(t), \alpha A_y(t)+\beta B_y(t)\right) \\
&= \left(\frac{d}{dt}\left(\alpha A_x(t)+\beta B_x(t)\right), \frac{d}{dt}\left(\alpha A_y(t)+\beta B_y(t)\right)\right) \\
&= \left(\left(\alpha\frac{d}{dt}A_x(t)+\beta\frac{d}{dt}B_x(t)\right), \left(\alpha\frac{d}{dt}A_y(t)+\beta\frac{d}{dt}B_y(t)\right)\right) \\
&= \left(\left(\alpha\frac{d}{dt}A_x(t), \alpha\frac{d}{dt}A_y(t)\right) + \left(\beta\frac{d}{dt}B_x(t), \beta\frac{d}{dt}B_y(t)\right)\right) \\
&= \alpha\frac{d}{dt}\left(A_x(t), A_y(t)\right) + \beta\frac{d}{dt}\left(B_x(t), B_y(t)\right) \\
&= \alpha\frac{d\vec{A}(t)}{dt} + \beta\frac{d\vec{B}(t)}{dt}
\end{aligned}
$$

5 | chain rule for derivatives

$$
\begin{aligned}
\frac{d}{dt}\vec{A}(u(t)) &= \left(\frac{d}{dt}A_x(u(t)), \frac{d}{dt}A_y(u(t))\right) \\
&= \left(\frac{dA_x(u)}{du}\frac{du}{dt}, \frac{dA_y(u)}{du}\frac{du}{dt}\right) \\
&= \left(\frac{dA_x(u)}{du}, \frac{dA_y(u)}{du}\right)\frac{du}{dt} \\
&= \frac{d\vec{A}(u)}{du}\frac{du}{dt}
\end{aligned}
$$