1. Consider a group of 17 students. Every pair of students may be classified as one of three things: friends, acquaintances, or strangers. Prove that there must be three students such that all three students are either friends, acquaintances, or strangers with each other.

2. Call a number *sensible* if it can be expressed as the square root of a rational number; i.e., a sensible number S can be expressed as the square root of a/b, or

$$S = \sqrt{\frac{a}{b}}$$

where a and b are nonzero integers. Prove that $\sqrt[3]{2}$ is not sensible.

3. a, b, and n are nonnegative real numbers where a*b = n. Prove that either a or b must be less than or equal to $\sqrt{n}$.

4. Define the Fibonachichi sequence, F(n), as follows:

- for n <= 4, F(n) = 1
- for n > 4, F(n) = F(n-1) + F(n-2) + F(n-3) + F(n-4)

So, this sequence looks like: 1, 1, 1, 1, 4, 7, 13, 25, ...

Are any Fibonachichi numbers divisible by 3? Prove your answer.

5. Define an *n*-team tournament as an event where each team plays each other team exactly once, and every game ends in a win or loss. Prove by induction that if no team wins all of its games, that there exists a cycle of teams T_1, T_2, …, T_k such that, for 1 <= i < k, T_i beats T_{i+1} and T_k beats T_1, for some k <= n.

For example, if the Giants beat the Dodgers, who beat the Yankees, who beat the Mets, who beat the Giants, then Giants > Dodgers > Yankees > Mets > Giants is a 4-team cycle. You are trying to prove that a cycle of some size <= n must exist.

Note: it's a bit cumbersome to say "tournament where no team wins all of its games", so you may refer to this as a "valid tournament".

(There is also a proof that does not use induction; that proof is optional.)