

1 | Problem 1

1.1 | KE

Given a distance from axis l_i , mass m_i , relative position \vec{r}'_i and angular velocity ω , you can get rotational KE via $\frac{1}{2} \sum m_i (l_i \omega)^2$. $l_i \theta$ would give arclength of the path of the mass, its so time derivative $l_i \omega$ would give its velocity. Additionally, this checks out in terms of dimensional analysis: radians are dimensionless so it's 1/s times meters to get m/s.

$$\begin{aligned}
 KE_r &= \frac{1}{2} \sum m_i (v'_i)^2 \\
 &= \frac{1}{2} \sum m_i (l_i \omega)^2 \\
 &= \frac{1}{2} \sum m_i l_i^2 \omega^2 \\
 &= \frac{1}{2} \omega^2 \underbrace{\sum m_i l_i^2}_I \\
 &= \frac{1}{2} I \omega^2
 \end{aligned}$$

1.2 | Rotational Inertia of Ring

$$\begin{aligned}
 I &= \sum m_i l_i^2 \\
 l_i &\text{ is constant on a ring and equal to } R. \\
 I &= \sum m_i R^2 \\
 I &= R^2 \sum m_i \\
 \sum m_i &\text{ is defined to be } M \text{ in previous problems.} \\
 I &= MR^2
 \end{aligned}$$

1.3 | Rotational Inertia of Sphere

I would expect it to be less than I_{disk} because more of the mass is spread out across the volume of the sphere, and more of it has a smaller l_i , meaning that $\sum m_i l_i^2$ will be smaller and therefore I_{sphere} is smaller.

2 | Problem 2

2.1 | $v(t)$ and $y(t)$

$$a(t) = a_0$$

$$\int a(t)dt = \int a_0$$

$$v(t) = a_0t + C$$

We know that $v(0) = v_0 = C$

$$\boxed{v(t) = a_0t + v_0}$$

$$v(t) = a_0t + v_0$$

$$\int v(t)dt = \int a_0t + v_0dt$$

$$y(t) = \frac{a_0}{2}t^2 + v_0t + C$$

We know that $y(0) = y_0 = C$

$$\boxed{y(t) = \frac{a_0}{2}t^2 + v_0t + y_0}$$

2.2 | Equation for $v^2(t)$

$$v(t) = a_0t + v_0$$

$$v^2(t) = (a_0t + v_0)^2$$

$$v^2(t) = a_0^2t^2 + 2a_0tv_0 + v_0^2$$

$$v^2(t) = v_0^2 + 2a_0\left(\frac{1}{2}a_0t^2 + v_0t\right)$$

$$v^2(t) = v_0^2 + 2a_0\left(\frac{1}{2}a_0t^2 + v_0t + y_0 - y_0\right)$$

We know from the previous problem that $\frac{1}{2}a_0t^2 + v_0t + y_0$ is equal to $y(t)$.

$$\boxed{v^2(t) = v_0^2 + 2a_0(y(t) - y_0)}$$

2.3 | Equation for Δy

$$\begin{aligned}
 v(t) &= a_0 t + v_0 \\
 \int_{t_1}^{t_2} v(t) &= \int_{t_1}^{t_2} a_0 t + v_0 \\
 y(t_2) - y(t_1) &= \left(\frac{a_0}{2} t_2^2 + v_0 t_2 \right) - \left(\frac{a_0}{2} t_1^2 + v_0 t_1 \right) \\
 \Delta y &= \frac{(a_0 t_2^2 + 2v_0 t_2) - (a_0 t_1^2 + 2v_0 t_1)}{2} \\
 \Delta y &= \frac{(a_0 t_2 + v_0) + (a_0 t_1 + v_0)}{2} (t_2 - t_1)
 \end{aligned}$$

We know from earlier that $v(t) = a_0 t + v_0$.

$$\begin{aligned}
 \Delta y &= \frac{v(t_2) - v(t_1)}{2} (t_2 - t_1) \\
 \boxed{\Delta y &= \frac{v(t_2) - v(t_1)}{2} \Delta t}
 \end{aligned}$$

3 | Problem 3

The constants that are flipped are a_0 , v_0 , and y_0 . The equations for $v(t)$ and $y(t)$ become flipped as a result:

$$\begin{aligned}
 a(t) &= -a_0 \\
 \int a(t) dt &= \int -a_0 \\
 v(t) &= -a_0 t + C \\
 \text{We know that } v(0) &= -v_0 = C \\
 \boxed{v(t) &= -a_0 t - v_0}
 \end{aligned}$$

$$\begin{aligned}
 v(t) &= -a_0 t - v_0 \\
 \int v(t) dt &= \int -a_0 t - v_0 dt \\
 y(t) &= -\frac{a_0}{2} t^2 - v_0 t + C \\
 \text{We know that } y(0) &= -y_0 = C \\
 \boxed{y(t) &= -\frac{a_0}{2} t^2 - v_0 t - y_0}
 \end{aligned}$$

The sign change initially doesn't matter during the $v^2(t)$ derivation due to the square, but the substitution

with $y(x)$ is flipped (see above) so part of the answer is flipped.

$$v(t) = -a_0 t - v_0$$

$$v^2(t) = (-a_0 t - v_0)^2$$

$$v^2(t) = a_0^2 t^2 + 2a_0 t v_0 + v_0^2$$

$$v^2(t) = v_0^2 + 2a_0 \left(\frac{1}{2} a_0 t^2 + v_0 t \right)$$

$$v^2(t) = v_0^2 + 2a_0 \left(\frac{1}{2} a_0 t^2 + v_0 t + y_0 - y_0 \right)$$

We know from the previous problem that $-\frac{1}{2}a_0 t^2 - v_0 t - y_0$ is equal to $y(t)$.

$$\boxed{v^2(t) = v_0^2 + 2a_0(-y(t) - y_0)}$$

Finally, the final equation in problem 2 would stay the same because it is for the *change* in position and addition is commutative.