1 | Problem

What happens if the Gram–Schmidt Procedure is applied to a list of vectors that is not linearly independent?

2 | Answer

Suppose the list $v_1, \ldots, v_n$ is linearly dependent. Then, there exists some v_j s.t. $v_1, \ldots, v_{j-1}$ is linearly independent while $v_1, \ldots, v_j$ is not. Then, $v_j \in$ span$(v_1, \ldots, v_{j-1})$

Because the Gram-Schmidt procedure preserves prefix spans,

$$v_j \in \text{span}(e_1, \ldots, e_{j-1})$$

Because of how a vector is written as a linear combination of an orthonormal basis, the denominator in the j-th step of the procedure is equivalent to

$$\|v - v\| = \|0\| = 0$$

and a division by zero occurs. Thus, the Gram-Schmidt procedure cannot be used on a linearly dependent list.