1 | Jacobian Determinant for Polar

We are to determine (pun not intended) the polar correction factor for a double integral, $dA = r\ dr\ d\theta$.
To do this, we will have to first figure the change of bases expressions such that we can take:

$$f(x, y) = g(r, \theta) \tag{1}$$

Fortunately, this is already derived to use from before.

$$\begin{cases} x = r \cos\theta \\ y = r \sin\theta \end{cases} \tag{2}$$

Therefore, we have that:

$$f(x, y) = f(r \cos\theta, r \sin\theta) \tag{3}$$

And therefore, we can figure $J_{r,\theta}$:

$$J = \begin{bmatrix} cos\theta & -r \sin\theta \\ sin\theta & r \cos\theta \end{bmatrix} \tag{4}$$

Taking its determinant, then:

$$det(J) = r \cos^2\theta + r \sin^2\theta = r \tag{5}$$

And therefore, the change-of-basis result would be:

$$dx\ dy = r\ dr\ d\theta \tag{6}$$

2 | Jacobian Determinant for Spherical

We again need to figure a correction factor for $dx\ dy\ dz = \rho^2\ \sin\phi\ d\rho\ d\theta\ d\phi$.
We therefore have to figure a change of bases for the expression:

$$f(x, y, z) = g(\rho, \theta, \phi) \tag{7}$$

We can leverage the shape of the object to determine the parameterization:

$$\begin{cases} x = \rho \sin\phi \cos\theta \\ y = \rho \sin\phi \sin\theta \\ z = \rho \cos\phi \end{cases} \tag{8}$$

We will now figure the matrix for $J_{\rho,\theta,\phi}$:

$$J = \begin{bmatrix} sin\phi \cos\theta & -\rho\ sin\phi \sin\theta & \rho\ cos\phi \cos\theta \\ sin\phi \sin\theta & \rho\ sin\phi \cos\theta & \rho\ cos\phi \sin\theta \\ cos\phi & 0 & -\rho \sin\phi \end{bmatrix} \tag{9}$$

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var("rho phi theta")
M = matrix([[sin(phi)*cos(theta), -rho*sin(phi)*sin(theta), rho*cos(phi)*cos(theta)], [sin(phi)*sin(thet
M
M.det().full_simplify()
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Not quite sure why Sage didn't simply $(-\rho)^2$ into ρ^2, but, we can see that:

$$dx\ dy\ dz = \rho^2 \sin\phi\ d\rho\ d\theta\ d\phi \tag{10}$$

3 | Surface area in polar

Given the fact that:
$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$