

#flo

## 1 | Polynomials

- See KBrefPolynomial

### 1.1 | 0 polynomial

- Has degree  $-\infty$
- Degrees are usually positive, except for the 0 degree
- "that's too hard, and we're not going to do it here"

### 1.2 | Identically zero

- Like 0 or  $0x^0$
- Most polynomials are sometimes zero, but polynomials that are "identically zero" means that it's always zero (instead of just sometimes zero)

### 1.3 | $\mathcal{P}_m(F)$

- Polynomials with coefficients in  $F$  whose highest degree is  $m$
- It can't be "whose degree is exactly  $m$ " because otherwise you won't have the identity and it won't be closed under addition (in the case where coefficient sum  $a_m + b_m = 0$ )

#### 1.3.1 | It's a finite dimensional vector space

- $$a_0z^0 + \dots + a_mz^m + b_0z^0 + \dots + b_mz^m = (a_0 + b_0)z^0 + \dots + (a_m + b_m)z^m$$

### 1.4 | Proof of 2.16

- Structure: proof by contradiction

## 2 | Linear Independence

- "non-trivial" means "simplest possible", which has usually got the most zeros
- See KB20math530refLinearIndependence

## 2.1 | 2.21 Linear Dependence Lemma 2.21

#toexpand - it's saying that any linearly independent list has a vector inside that doesn't "contribute anything", and that if you remove it you'll have the same span. Implicitly, maybe through induction?) if you remove a dependent vector enough times then you get a linearly independent list. - The list  $(1, 1, 1), (2, 2, 2), (3, 3, 3)$  is really dependent, but  $(0), (0), (0)$  is the most dependent (you have to remove all to get independence).

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## 3 | Exercise 2.A.1

### 3.1 | Lemma

If vectors  $v_1, v_2, v_3, v_4$  span  $V$ , then the list

$$v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4$$

also spans  $V$ .

### 3.2 | Proof

We prove the lemma by showing that any vector  $v \in V$  can be written in the form  $a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4$  can also be written as a linear combination of the form

$$b_1(v_1 - v_2) + b_2(v_2 - v_3) + b_3(v_3 - v_4) + b_4v_4$$

If we set  $\begin{cases} b_1 = a_1 \\ b_2 = a_1 + a_2 \\ b_3 = a_1 + a_2 + a_3 \\ b_4 = a_1 + a_2 + a_3 + a_4 \end{cases}$  then the two combinations will be equivalent:

$$\begin{aligned} & a_1(v_1 - v_2) + (a_1 + a_2)(v_2 - v_3) + (a_1 + a_2 + a_3)(v_3 - v_4) + (a_1 + a_2 + a_3 + a_4)v_4 \\ &= a_1v_1 - a_1v_2 + a_1v_2 + a_2v_2 - (a_1 + a_2)v_3 + (a_1 + a_2)v_3 + a_3v_3 - (a_1 + a_2 + a_3)v_4 + (a_1 + a_2 + a_3 + a_4)v_4 \\ &= a_1v_1 + a_2v_2 + a_3v_3 + a_4v_4 \end{aligned}$$