We first set up the basic assumptions and variables.

```
GRAV <- 9.8 # gravity (m/s^2)

MASS <- 1 # mass (kg)

I_CM <- 1/12 # roational inertia at the centre of gravity (kg m^2)

L1 <- 0.5 # distance from rotation point to CoM (m)

L2 <- 1 # distance from rotation point to tension (m)

PHI <- pi/6 # angle of Ft relative to floor (parallel) (rad)

FT <- 12 # tension force (N)

OMEGA <- 0.1 # angle of line orthogonal to floor relative to gravity (rad) (because shifted axis)
```

Additionally, we set the time interval and seed values for time and theta (distance from flat):

```
dt <- 0.0001
t_max <- 5

vx <- 0
vy <- 0

x <- 0
y <- 0

theta <- 0
thetadot <- 0
time <- 0</pre>
```

Great. Let's start generating the table! We essentially write a for loop to appends to a few different vectors. Variables appended with c reflect the column vectors that we will put together.

```
cTime = NULL
cTheta = NULL
cDDTheta = NULL
cDTheta = NULL
cTorqueNet = NULL
cAccelX = NULL
cAccelY = NULL
cVelX = NULL
cVelY = NULL
cPosX = NULL
cPosY = NULL
cFFriction = NULL
cFNormal = NULL
# debugging values
cFNetY = NULL
cFTensionPhiComponent = NULL
cFGravityPhiComponent = NULL
cMuStatic = NULL
cKERot = NULL
cKETrans = NULL
```

Awesome. Let's now run a lovely little for loop to actually populate the values recursively.

```
for (i in 0:(t max/dt)) {
   # We first populate the time column with the time, theta column with theta
    cTime[i] = time
    cTheta[i] = theta
   I_ROT <- I_CM + MASS * (L1*cos(theta))^2 # we calculate I_ROT using
    # the Parallel axis theorem
   torque <- L2 * FT * cos(theta + PHI) - L1 * MASS * GRAV * cos(theta - OMEGA)
   # Given the theta value, we calculate the net torque and set that
    cTorqueNet[i] = torque
    # Now that we know the net torque, we could know how much the angular
   # acceleration is by just dividing out the rotational inertia
   thetadotdot <- torque/I_ROT
    cDDTheta[i] = thetadotdot
    # We could also multiply the theta acceleration by time to get the
   # velocity at that point
   thetadot <- dt*thetadotdot + thetadot
   cDTheta[i] = thetadot
    # We could therefore component-ize the acceleration in theta into
   # ax and ay
   ax <- -1 * L1 * sin(theta) * thetadotdot
    cAccelX[i] = ax
   ay <- L1 * cos(theta) * thetadotdot
   cAccelY[i] = ay # @mark isn't sin and cos backwards?
   # We also tally the components seperately for velocity
   vx \leftarrow ax*dt + vx
   vy <- ay*dt + vy
   # We finally tally the positions as well
   x \leftarrow vx*dt + x
   y \leftarrow vy*dt + y
   cPosX[i] = x
    cPosY[i] = v
   # Based on these accelerations, we therefore could calculate the relative
   # force of friction and normal force by subtracting the force in that direction
   # out of net
   ffriction <- FT*sin(PHI) + MASS*GRAV*sin(OMEGA)-MASS*ax
   fnormal <- MASS*ay-FT*cos(PHI)+MASS*GRAV*cos(OMEGA)</pre>
    cFNetY[i] = MASS*ay
    cFTensionPhiComponent[i] = FT*cos(PHI)
    cFGravityPhiComponent[i] = -MASS*GRAV*cos(OMEGA)
    cFFriction[i] = ffriction
    cFNormal[i] = fnormal
   # Then, we calculate the energies
    cKERot[i] = 0.5 * I_ROT * thetadot^2
    cKETrans[i] = 0.5 * MASS * (vx^2+vy^2)
    # Dividing the friction force by the normal force, of course, will result in
```

```
# the (min?) friction coeff
            cMuStatic[i] = ffriction/fnormal
            # We incriment the time and also increment theta by multiplying the velocity
            # by dt to get change in the next increment
            time <- dt + time
            theta <- dt*thetadot + theta
}
We now put all of this together in a dataframe.
rotating_link <- data.frame(cTime,</pre>
            cTheta,
            cDTheta,
            cDDTheta,
            cTorqueNet,
            cAccelX,
            cAccelY,
            cPosX,
            cPosY,
            cFFriction,
            cFNormal,
            cMuStatic,
            cKERot,
            cKETrans)
names(rotating_link) <- c("time",</pre>
     "theta",
      "d.theta",
      "dd.theta",
      "net.torque",
      "accel.x",
      "accel.y",
      "pos.x",
      "pos.y",
      "friction.force",
      "normal.force",
      "friction.coeff",
      "ke.rot",
      "ke.trans")
Let's import some visualization tools, etc.
library(tidyverse)
Let's first see the head of this table:
head(rotating_link)
1e-04 1.65503533066528e-07 0.00331007033913572 16.5503500847044 5.51678336156803 -1.36957070625324e-06
8.27517504235209 \ -1.36957070625324 \\ e-14 \ 2.48255283490049 \\ e-07 \ 6.97836885270962 \ 7.63391101666348 \\ e-14 \ 2.48255283490049 \\ e-15 \ 6.97836885270962 \\ e-15 \ 6.9783688527099 \\ e-15 \ 6.9783688527099 \\ e-15 \ 6.9783688527099 \\ e-15 \ 6.9783688527099 \\ e-15 \ 6.978368852709 \\ 
0.91412761263225 1.82609427500431e-06 1.36957070625325e-06
```

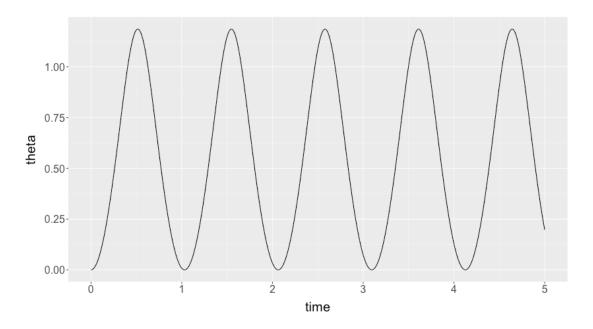
2e-04 4.965105669801e-07 0.00496510470321662 16.5503436408089 5.51678121360196 -4.1087102524066e-06 8.27517182040345 -6.84785166491308e-14 4.96510518650869e-07 6.97837159184917 7.63390779471484 0.914128357258976 4.10871078564987e-06 3.08153308923784e-06 3e-04 9.93021037301762e-07 0.00662013810071352 16.550333974969 5.51677799165225 -8.21741490575579e-06 8.27516698748041 -2.05435475293287e-13 8.27517423686492e-07 6.97837570055382 7.6339029617918 0.914129474199641 7.30437141208106e-06 5.47827855906406e-06 4e-04 1.65503484737311e-06 0.00827517020943236 16.5503210871885 5.51677369571816 -1.36956790672492e-05 8.2751605435829 -4.79349224609936e-13 1.24127593415794e-06 6.97838117881798 7.6338965178943 0.914130963454935 1.14130736658227e-05 8.55980524938151e-06 5e-04 2.48255186831635e-06 0.00993020070717963 16.5503049774726 5.51676832579872 -2.0543495271494e-05 8.27515248871082 -9.58697926641525e-13 1.73778596951651e-06 6.97838802663419 7.63388846302222 0.914132825025777 1.64348143474025e-05 1.23261107605995e-05 6e-04 3.47557193903431e-06 0.0115852292717624 16.550285645828 5.5167618818927 -2.87608541867632e-05 8.27514282286404 -1.72565517054075e-12 2.31704743310371e-06 6.9783962439931 7.63387879717543 0.91413505891332 2.23695895463475e-05 1.67771921598887e-05

Before we start graphing, let's set a common graph theme.

 $\label{lem:default.theme <- theme(text = element_text(size=20), axis.title.y = element_text(margin = margin(t = 0, axis.title.y = element_text(margin =$

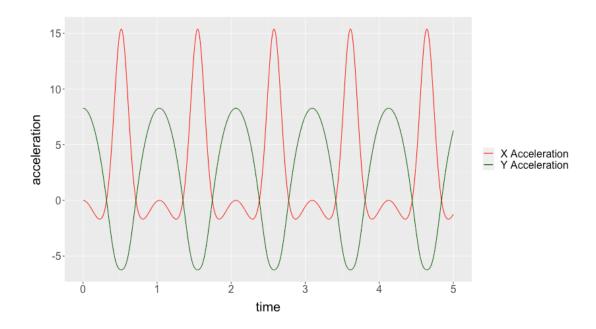
Cool! We could first graph a function for theta over time.

rotating_link %>% ggplot() + geom_line(aes(x=time, y=theta)) + default.theme



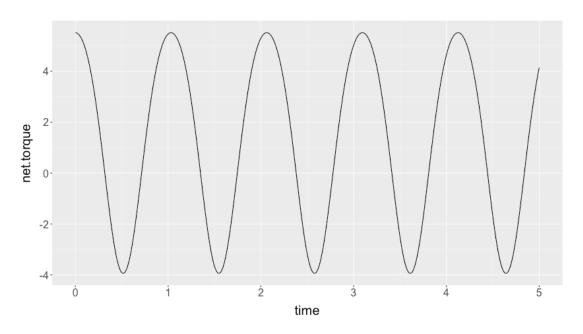
And, similarly, we will graph ax and ay on top of each other:

rotating_link %>% ggplot() + geom_line(aes(x=time, y=accel.x, colour="X Acceleration")) + geom_line(aes



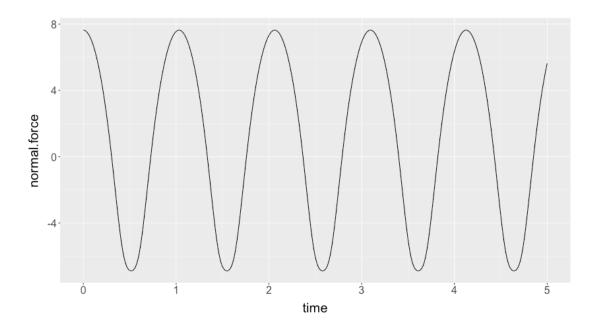
Let's also plot torque as well.

rotating_link %>% ggplot() + geom_line(aes(x=time, y=net.torque)) + default.theme



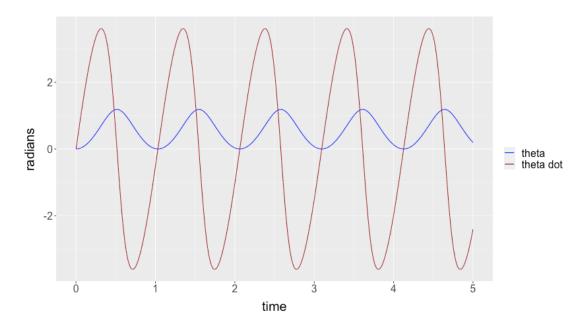
And. Most importantly! Let's plot the normal force.

 $\verb|rotating_link \%>\%| ggplot() + geom_line(aes(x=time, y=normal.force)) + default.theme|$



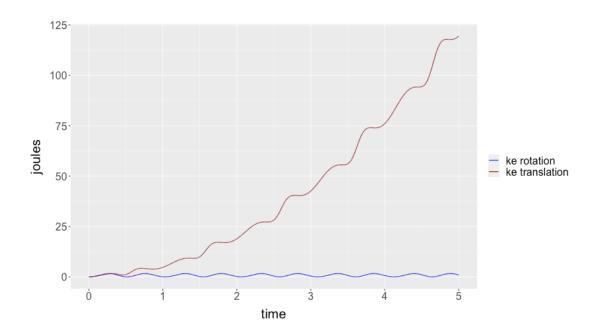
Obviously, after the normal force becomes negative, this graph stops being useful. Theta dot atop theta:

rotating_link %>% ggplot() + geom_line(aes(x=time, y=theta, colour="theta")) + geom_line(aes(x=time, y=theta, theta))) + geom_line(aes(x=time, y=theta, theta)) + geom_line(aes(x=time, y=theta))) + geom_line(aes(x=time, y=theta)) + geom_line(aes(x=time, y=theta))) + geom_



We finally, plot KE rotation and translation

rotating_link %>% ggplot() + geom_line(aes(x=time, y=ke.rot, colour="ke rotation")) + geom_line(aes(x=time, y=ke.rot, colour="ke rotation")) + geom_line(aes(x=time, y=ke.rot, colour="ke rotation")) + geom_line(aes(x=time, y=ke.rot, colour="ke rotation"))



rotating_link %>% ggplot() + geom_line(aes(x=time, y=pos.x, colour="x position")) + geom_line(aes(x=time))

