

# 1 | Series Convergence

## 1.1 | Geometric Series

In  $\sum_{k=0}^{\infty} a(r^k)$ , where  $|r| < 1$ , the series converges to  $\sum_{k=0}^{\infty} a(r^k) = \frac{a}{1-r}$

In  $\sum_{k=0}^n a(r^k)$ ,  $\sum_{k=0}^n a(r^k) = \frac{a-ar^{n+1}}{1-r}$

## 1.2 | nth term divergence test

If  $\lim_{n \rightarrow \infty} a_n$  is not zero, the series **will** diverge. The inverse is not necessarily true; that is, if this fails, use another test to test convergence.

## 1.3 | Integral Test

If the integral to infinity is convergent, the sequence is convergent as long as the sequence is continuous, positive, and decreasing. The inverse applies, too.

## 1.4 | Power Series

$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$

If a p-series has a  $p > 1$ , the p-series will converge

If a p-series has a  $p \leq 1$ , the p-series will diverge

## 1.5 | Comparison Test

Both provided that  $a_n, b_n \geq 0$  &  $a_n \leq b_n$

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Also, if  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = C$  ( $0 < c < \infty$ ), the two series will either both converge or both diverge. So you only need to test one.

## 1.6 | Alternating Series Test

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## 1.7 | Ratio Test

In a geometric series, the common ratio is simply  $r = \frac{r^{n+1}}{r^n}$ .

If  $r$  is an real value,  $|r| < 1$ , then series converges. If  $|r| \geq 1$ , the series diverges.

As limit goes to infinity in the  $r$ , if the common ratio approaches  $<1$ , that means that the ratio will get smaller and smaller, just like if  $r$  were to be a real value and it was smaller than one. Meaning that the series **converges**.

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And so, formally.

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The inverse is true, too.

**However, if the ratio is equal to one, the test is inconclusive.**

Absolute Convergence => series who converge and whose absolute value converges

Conditional Convergence => series who converge and whose absolute value does not converge

## 1.8 | So what is the error of a Taylor series? (Lagrange Error)

The error at point  $x$  of a  $n$ th degree Taylor polynomial centered at  $a$  modeling a function with an absolute maximum value of  $M$  in its  $n + 1$ th derivative between a bound containing  $x$  and  $a$ :

$$|E(x)| \leq \frac{M(x-a)^{n+1}}{(n+1)!}$$

## 1.9 | Power Series

$$f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n = a_n(x-c)^0 + a_n(x-c)^1 \dots$$

For instance, a geometric series is a special power series...

$$g(x) = \sum_{n=0}^{\infty} ax^n$$

This geometric series converges if  $|x| < 1$ , and so it has an interval of convergence of  $-1 < x < 1$ . If this converges, this function will converge to  $\frac{a}{1-x}$

**Interval of Convergence:** at what values of  $x$  does the series converge?

**Radius of Convergence:** at what absolute distance from  $c$  (the "centering" of the series) will the series converge?

To figure the interval of convergence, simply use the ratio test and solve for  $x$  that makes the ratio  $< 1$ . Then, think about the inconclusive cases whereby ratio = 1 — then, use the comparison test, or integral test.

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\*Derivatives, integrals have the same radius of convergence as the parent function, but their interval may be different due to different behavior at endpoints\*