1 | Deriving Rotational KE and Inertia

Given m_i, mass, $\vec{r_i}'$, location of the center of mass, l_i, ω, the angular velocity, figure a $KE_{tot,rot}$.

Because of the fact that the value ω is in units $\frac{d\theta}{dt}$, the rate of radians change, and we know of a radius of the spin l_i, we could figure the velocity at which it is moving by simply scaling the change in radians up to a circle of radius l_i, that is:

$$V_i' = l_i\omega \tag{1}$$

(note that, to understand this, radians $\frac{arclength}{radius}$)

And so, substituting into the statement of $\sum_{i=1}^{N} \frac{1}{2}m_i\vec{v_i}'^2$

$$\begin{aligned} KE_{rot} &= \sum_{i=1}^{N} \frac{1}{2}m_i\vec{v_i}'^2 && (2)\\ &= \sum_{i=1}^{N} \frac{1}{2}m_i(l_i\omega)^2 && (3)\\ &= \sum_{i=1}^{N} \frac{1}{2}m_il_i^2\omega^2 && (4)\\ &= \frac{1}{2}\omega^2\sum_{i=1}^{N}(m_il_i^2) && (5) \end{aligned}$$

1.1 | Rotational Inertia

The right sum — the mass times the distance away from maxis of rotation ($\sum_{i=1}^{N}(m_il_i^2)$) — is defined as the rotational (moment) of inertia (spinny mass). That is,

$$I = \sum_{i=1}^{N}(m_il_i^2) \tag{6}$$

Replacing that value in the prior statement, the statement of KE_{rot} is defined as:

$$KE_{rot} = \frac{1}{2}\omega^2 I \tag{7}$$

1.2 | Rotational Inertia for a Ring

For a ring (that's perfectly circular) rotating on an axis perpendicular to the plane of the ring, the l_i — distance from axis of rotation — is the same value: namely, the radius R as the radius of a circle is the same for all positions. Meaning,

$$l_i = R \tag{8}$$

regardless of which value i.

Hence, the value of KE_{rot} would be evaluated as…

$$KE_{rot} = \sum_{i=1}^{N}(m_i l_i^2) \tag{9}$$
$$= \sum_{i=1}^{N}(m_i R^2) \tag{10}$$
$$= R^2 \sum_{i=1}^{N} m_i \tag{11}$$
$$\tag{12}$$

Substituting M as the sum of all masses in the ring ($M = \sum_{i=1}^{N} m_i$), the statement is therefore:

$$KE_{rot} = MR^2 \tag{13}$$