

#flo #ref

## 1 | Spans

concept introduced in KBxChapter2AReading  
notes, on as explained by professor dave.

title: review: subspace  
a vector space contained inside another vector space

eg.  $S$  is a subspace of  $V$   
that means every element in  $S$  is also in  $V$

which means, the only things we need to check that aren't inherited from the parent space are:

- if  $S$  is closed
  - $a$  in  $S$ , then  $ca$  is in  $S$  // closed under scalar multiplication
  - $a$  in  $S$ ,  $b$  in  $S$ , then  $a+b$  in  $S$  // closed under addition

1. checking a subspace eg. subspace:  $\mathbb{R}^3 S = [x, 0, -x]$  multiply by  $c$ :  $[cx, 0, -cx]$ , still in the same form.  
add another vector:  $[x, 0, -x] + [y, 0, -y] = [x+y, 0, -(x+y)]$  still in the same form so it's closed under addition and SCAMUL! therefore it's a subspace

### 1.0.1 | defining the span

$\vec{v}_1, \vec{v}_2, \dots, \vec{v}_N$  in  $V$

sum of these elements multiplied by some scalars:  $a_1 \vec{v}_1 + a_2 \vec{v}_2 + \dots a_n \vec{v}_N$

is called a **linear combination**

the set of all linear combinations is called the span

eg.

$$\vec{v}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$\text{span}(\vec{v}_1, \vec{v}_2, \vec{v}_3) = \begin{bmatrix} 2a \\ a \\ -a \end{bmatrix} + \begin{bmatrix} 0 \\ 2b \\ 2b \end{bmatrix} + \begin{bmatrix} -c \\ -c \\ -c \end{bmatrix} = \begin{bmatrix} 2a & +0 & -c \\ a & +2b & -c \\ -a & +2b & -c \end{bmatrix}$$

the span of any number of elements of vector space  $V$  is also a subspace of  $V$  actually, it is the *smallest subspace* of  $V$  that contains the set of elements that you ran the span on it is the intersection of all subspaces that contain them? **span: important for describing vector spaces**