

1 | Escape Velocity

Earth's gravitational field extends infinitely and can be found with $\vec{F}_g = \frac{M_2 M_1 G}{r^2} \vec{r}$, where G is the universal gravitational constant. The vector \vec{r} points from M_1 to M_2 , \vec{F}_g is force on M_2 , $\hat{r} = \frac{\vec{r}}{r}$, and finally r is the magnitude of \vec{r} .

This is equivalent to $\frac{-GM_1 M_2}{r^3} \vec{r}$.

1.1 | Derivation of GPE

$$\int_{r_e}^{\infty} \frac{GmM_e}{r^2} dr$$

$$GmM_e \int_{r_e}^{\infty} \frac{1}{r^2} dr$$

$$\frac{GmM_e}{r}$$

1.2 | Derivation of Escape Velocity

$$W = \Delta KE$$

$$\int_{r_e}^{\infty} \frac{-GmM_e}{r^2} dr = -\frac{1}{2}mv_0^2$$

$$-GmM_2 \int_{r_e}^{\infty} \frac{-1}{r^2} dr = \frac{1}{2}mv_0^2$$

$$0 - \frac{-GmM_e}{r_e} = \frac{1}{2}mv_0^2$$

$$\frac{GM_e}{r_e} = \frac{1}{2}v_0^2$$

$$\frac{2GM_e}{r_e} = v_0^2$$

$$\sqrt{\frac{2GM_e}{r_e}} = v_0$$

$$\boxed{\sqrt{\frac{2GM_e}{r_e}} = v_0}$$

$$\boxed{v_0 \approx 11 \text{ km/s} \approx 24,000 \text{ mph}}$$

2 | Potential Energy

PE can be defined to be 0 at a position of ∞ (similar to how we did it in Exploration of Fields/Voltage). PE is therefore less than 0 everywhere else. *Change* in potential energy (which is what we're usually concerned with, is either positive or negative depending on context).