

1 | Curve Sketching

unit2

Goal: Draw a graph of $f(x)$ using $f'(x)$ and $f''(x)$ (see Calculating Derivatives). *Warning!* Don't abandon precalculus knowledge and common sense! Lots of common sense is involved in this and the calculus mostly just fills in the gaps.

Two principles (really one) will aid this endeavour:

- If $f' > 0$, f is increasing (and vice versa).
- If $f'' > 0$, f' is increasing (and vice versa).

Ex 1. $f(x) = 3x - x^3$ $f'(x) = 3 - 3x^2 \rightarrow 3(1-x)(1+x)$ $f'(x) > 0$ in the range $-1 < x < 1$ and therefore f is increasing in that range. $f'(x) < 0$ in the range $1 < x$ and therefore f is decreasing in that range (similarly for the range $x < -1$).

General shape of function can be deduced from these facts.

DEFINITION If $f'(x_0) = 0$, x_0 is a *critical point*. At this point $f(x_0)$ is called the *critical value*.