

#flo #hw

1 | Linear Maps

no one gets excited about vector spaces -axler

the interesting part: linear maps!

```
title: learning objectives
- fundamentals theorem of linear maps
- matrix of linear map w.r.t. given bases
- isomorphic vec spaces
- product spaces
- quotient spaces
- duals spaces
  - vector space
  - linear map
```

2 | The vector space of linear maps

key definition!

```
title: linear map
aka *linear transformation.*
```

a *linear map* from V to W is a function $T: V \rightarrow W$ with the following properties:

****additivity****

$T(u+v) = Tu + Tv$ for all $u, v \in V$;

****homogeneity****

$T(\lambda v) = \lambda(Tv)$ for all $\lambda \in F$ and $v \in V$.

the functional notation $T(V)$ is the same as the notation Tv when talking about linear maps.

```
title: notation --  $L(V, W)$ 
```

the set of all linear maps from V to W .

2.0.1 | examples of linear maps

- 0?
 - 0 is the func that takes each ele from some vec space to the additive iden of another vec space.
 - * $0v = 0$
 - * left: func from V to W , right: additive iden in W
 - * #question what does it mean for it to be a function from V to W ?
- identity, denoted I

- $Iv = v$
- maps each element to itself linear transformation like a `.map`?
- differentiation and integration!
- multiplication by x^2 (on polynomials)
- shifts! defined as, $T(x_1, x_2, x_3, \dots) = (x_2, x_3, \dots)$
 - #question this is an example, but how do we define it as a transformation? or is giving an example in the general case the same thing as defining a transformation?
- from $R^3 \rightarrow R^2$? #question what? how does this work?
- #review how this dimension shift works..

title: linear maps and basis of domain

Suppose v_1, \dots, v_n is a basis of V and $w_1, \dots, w_n \in W$. Then there exists a unique linear map T such that

$$Tv_j = w_j$$

for each $j=1, \dots, n$.

we can uniquely map between the basis of a subspace and a list of equal len in a diff subspace?