## 1 | Concept

For a function f(x,y) and a vector in its input space  $\vec{\mathbf{v}}$ , the directional derivative of f along  $\vec{\mathbf{v}}$  is the rate at which f changes as input moves along the vector.

While it is represented by a number of symbols,  $\nabla_{\vec{\mathbf{v}}}$  will be used to denote a directional derivative along  $\vec{\mathbf{v}}$  for these notes.

They can be thought of as generalized partial derivatives - as a partial derivative w respect to x tells us the amount a change in the input parallel to the x axis affects the output of the function, while this directional derivative describes how a change in any direction (as opposed to parallel to an axis) affects the change in the output in the function.

A partial derivative w/ respect to y can be thought of as a directional derivative along  $\vec{\mathbf{v}} = \hat{\mathbf{j}}$  (so  $\frac{\partial f}{\partial y} = \nabla_{\hat{\mathbf{j}}}$ ).

## 2 | Computation

Computing a directional derivative based on what we know so far is relatively simple.

Take the example vector 
$$\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$
.

$$\nabla_{\vec{\mathbf{v}}} f = 2 \frac{\partial f}{\partial x} + 3 \frac{\partial f}{\partial y} + (-1) \frac{\partial f}{\partial z}$$

This makes sense as  $\frac{\partial f}{\partial x}$  is the amount a change in the output of the function with a small change in x, so a combination of each of the partial derivatives gives you change in the function for an arbitratry vector.

This also means that it can be computed via the gradient:  $\nabla f \cdot \vec{\mathbf{v}}$  as  $\nabla f$  is a vector of each of the partial derivatives and  $\vec{\mathbf{v}}$  is a vector.

## 3 | Sources

This describes basic directional derivatives nicely.

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