Momentum Flow

January 26, 2022

1 Jumping Armadillos

Note: question (g) is an optional proof. Please complete the problems after it though, using the result.

N armadillos, each with mass m, stand on a railway flatcar of mass M. They jump off one end of the flatcar with velocity $\vec{u}$ *relative to the car's velocity* ***after*** *they jump.* The car rolls in the opposite direction without friction. This means that the velocity of the armadillo after it jumps is $(v_{car} - u)$ and the velocity of the car after the armadillo jumps is v_{car}. (We'll talk about this in class: why this description makes sense for "throwing" something out of vehicle).

(a) Draw a picture to describe this situation.

(b) What is the final velocity of the flatcar if all the armadillos jump off at the same time?

(c) What is the final velocity of the flatcar if they jump off one at a time? If you want a hint, see below. (The answer can be left in the form of a sum of terms.)

**Hint: Let v_j be the velocity of the car after the j^{th} armadillo has leapt from it.

**Hint: Conserve momentum after the first armadillo jumps, then after the 2nd jumps, or j=2. See if you can then generalize for the j^{th} armadillo.

(d) What happens as the mass of the flatcar approaches zero?

(e) Does jumping off simultaneously or one-by-one result in a higher velocity for the car? The same? Can you give a physical explanation for your answer?

(f) Now, consider the case where the armadillos undergo mitosis, doubling in number, and halving in volume, such that the total mass of the armadillos remain the same. Now they jump off the car one at a time. Is the final velocity of the car the same as before they underwent mitosis? Faster? Slower?

Note: question (g) is an optional proof. Required: Please complete the problems after part g), using the result from g).

(g) BONUS QUESTION: Let the mass of each armadillo decrease to some infinitesimal Δm. Additionally, let the time in between armadillos leaping Δt go to zero. Prove that your equation becomes

$$M\frac{dv}{dt} = u\frac{dm}{dt}$$

where M is the total mass of the rocket and remaining fuel, v(t) is the velocity of the rocket (with the fuel on the rocket) as a function of time, and dm is the differential mass that is ejected from the rocket (as a function of time).

REQUIRED QUESTIONS:

(h) Interpret the equation in (g). Which physical situations does this describe? Solve the differential equation to find an equation for the velocity of the car as a function of the mass ejected.

**Hint: Let $\frac{dv}{dt} = \frac{dv}{dm}\frac{dm}{dt}$

(i) Graph your equation. What happens to the car's final velocity as it expels all of the mass. What if the car's mass without the fuel approaches zero?

Does this result sync with what you discover in part d?