Desmos graphs

4 | witch of Maria Agnesi

Let B be the center of the orange circle with radius a, let D be the closest point to C on the x-axis, and let Q be the closest point to A on the y-axis.

4.1 | $x(t)$

$$\begin{aligned}
\tan\theta &= \frac{\overline{CD}}{\overline{OD}} \\
\cot\theta &= \frac{\overline{OD}}{\overline{CD}} \\
\overline{CD}\cot\theta &= \overline{OD} \\
2a\cot\theta &= x
\end{aligned}$$

4.2 | $y(t)$

First, note that the distances

$$\begin{aligned}
\overline{AB} &= \overline{BO} = a \\
\overline{PD} &= \overline{QO} = \overline{QB} + \overline{BO} = \overline{QB} + a = y
\end{aligned}$$

Using some geometry:

$$\begin{aligned}
\angle AOB &= 90 - \theta \\
\angle OAB &= 90 - \theta \qquad \text{(isocelese triangle)} \\
\angle ABO &= 2\theta
\end{aligned}$$

Which implies:

$$\begin{aligned}
\overline{QB} &= -a\cos(2\theta) \\
&= -a\left(1 - 2\sin^2\theta\right) \\
&= -a + 2a\sin^2\theta
\end{aligned}$$

By going back to the original distance relations, we have

$$\begin{aligned}
y &= \overline{QB} + a \\
&= a - a + 2a\sin^2\theta \quad = 2a\sin^2\theta
\end{aligned}$$

5 | parameterization of an elipse

`https://www.desmos.com/calculator/wcu1okhjyz`

$$\begin{aligned}
x(t) &= a\sqrt{c}\sin t \\
y(t) &= b\sqrt{c}\cos t
\end{aligned}$$

6 | mystery curve

it's just $(a\cos t, b\sin t)$ because of how the right triangle aligns with the axes.

8 | swallowtail catastrophe curves

Defined by

$$x = 2ct - 4t^3$$
$$y = -ct^2 + 3t^4$$

8.1 | features

8.1.1 | approaches a parabola-like shape above the y-axis

8.1.2 | approaches a parabola-like shape below the x-axis if $c > 0$

8.1.3 | has a cross-over in a triangle shape

1. gets bigger when c gets bigger

8.1.4 | it looks like a dorito that scales with the value of c

1. as c approaches zero from the positive direction, the swollowtail gets smaller

9 | Lissajous Figures

Defined by

$$x = a\sin(nt)$$
$$y = b\cos t$$

9.1 | features

9.1.1 | spring-like coil shape (almost like standing waves) with tighter "loops" at the ends

9.1.2 | a, b control the size of the coil (default $-1 \leq x, y \leq 1$ because of range of $\sin, \cos$

9.1.3 | number of y-intercepts is $n + 1$ except in the degenerate cases $n \leq 0$

11 | cycloid

Suppose instead that the circle slides along the surface and the point rotates at one radian per radian traveled. Let's start with the radian rotation…

$$\begin{aligned} x(t) &= \qquad r\sin t \\ y(t) &= r+ \quad r\cos t \end{aligned}$$

Then, we just have to move the origin as well:

$$\begin{aligned} x(t) &= t + r\sin t \\ y(t) &= r + r\cos t \end{aligned}$$

12 | first order derivative

I think I did not come to this conclusion on my own on 30 Aug. because I didn't realize we could assume we had $y(x)$.

$$\begin{aligned} y &= y(x(t)) \\ \frac{dy}{dt} &= y'(x(t))x'(t) = \frac{dy}{dx}\frac{dx}{dt} \qquad \text{(chain rule)} \\ \frac{dy}{dx} &= \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \end{aligned}$$

13 | second order derivative

$$\begin{aligned} x &= f(t) \\ y &= g(t) = g(f(t)) \end{aligned}$$

$$\begin{aligned} \frac{dy}{dt} &= \frac{dy}{dx}\frac{dx}{dt} \\ \frac{d^2y}{dt^2} &= \frac{dy}{dx}\frac{d}{dt}\frac{dx}{dt} + \frac{dx}{dt}\frac{d}{dt}\frac{dy}{dx} \\ &= \frac{dy}{dx}\frac{d^2x}{dt^2} + \frac{dx}{dt}\frac{d^2y}{dxdt(??)} \end{aligned}$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt}\frac{dx}{dt}$$

um… that seems like it didn't actually do anything. I'm kind of stuck… lets try working backwards:

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\dot{x}\ddot{y} - \dot{y}\ddot{x}}{(\dot{x})^3} \\ &= \dot{x}\frac{d}{dx}\left(\frac{\dot{y}}{\dot{x}}\right) \end{aligned}$$

why should the $\dot{x}$ in the bottom be cubed?

13.1 | **in class review**

$$
\begin{aligned}
\frac{d}{dx}\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right) = \frac{d}{dx}u = \frac{\frac{du}{dt}}{\frac{dx}{dt}} \\
= \frac{\frac{d}{dt}u}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\frac{\dot{y}}{\dot{x}}}{\dot{x}} \\
= \frac{\frac{\dot{x}\ddot{y}-\dot{y}\ddot{x}}{\dot{x}^2}}{\dot{x}}
\end{aligned}
$$