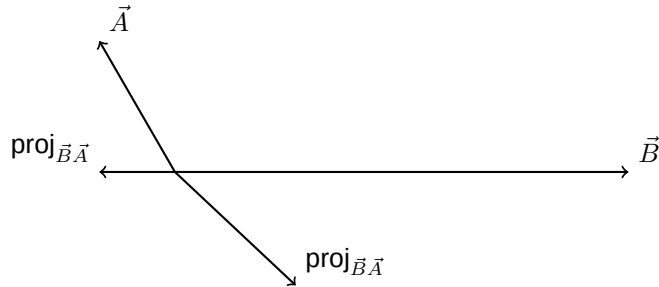


1 | vectors at an angle

1.1 | a sketch



scratch work: length: $6 \cos 60$ width: $-6 \cos 60 \sin 60$ height: $6 \cos^2 60$

1.2 | components

$$\text{comp}_{\vec{A}} \vec{B} = 6 \cos 120$$

$$\text{comp}_{\vec{B}} \vec{A} = 2 \cos 120$$

1.3 | dot product

$$\begin{aligned} \vec{A} \cdot \vec{B} &= |\vec{A}| |\vec{B}| \cos \theta \\ &= 2 \cdot 6 \cdot \cos 120 = -3 \end{aligned}$$

2 | proving expression for component

Lets redefine the coordinate axis so that \vec{A} lies along the x-axis. Then,

$$\begin{aligned} \text{comp}_{\vec{A}} \vec{B} &= |\vec{B}| \cos \theta \\ &= \frac{|\vec{A}| |\vec{B}| \cos \theta}{|\vec{A}|} \\ &= \frac{\vec{A} \cdot \vec{B}}{|\vec{A}|} \end{aligned}$$

3 | expression for projection

The projection is just a vector with length $\text{comp}_{\vec{A}} \vec{B}$ in the direction of \vec{A} .

$$\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}|} \right) \frac{\vec{A}}{|\vec{A}|}$$

4 | expression for perpendicular

The part of \vec{A} that is perpendicular to \vec{B} is just the whole vector minus the part that is parallel:

$$\begin{aligned}\vec{A}_{\perp \vec{B}} &= \vec{A} - \text{proj}_{\vec{B}} \vec{A} \\ &= \vec{A} - \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B}\end{aligned}$$

Checking using the dot product:

$$\begin{aligned}\left(\vec{A} - \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B} \right) \cdot \vec{B} &= \vec{A} \cdot \vec{B} - \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) \vec{B} \cdot \vec{B} \\ &= \vec{A} \cdot \vec{B} - \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{B}|^2} \right) |\vec{B}|^2 \\ &= \vec{A} \cdot \vec{B} - \vec{A} \cdot \vec{B} \\ &= 0\end{aligned}$$

5 | find angle using dot product

Well, the dot product already includes the angle, so let's just solve for that

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

The angle between:

$$\begin{aligned}\theta &= \cos^{-1} \left(\frac{3 + 2 - 4}{\sqrt{1^2 + 2^2 + 2^2} \sqrt{3^2 + 1^2 + 2^2}} \right) \\ &= \cos^{-1} \left((3 + 2 - 4) / (3 * \sqrt{14}) \right) = \cos^{-1}(0.08908708) \approx 84.8^\circ\end{aligned}$$

6 | dot product distribution over vector addition

$$\vec{A} \cdot (\vec{B} + \vec{C}) = |\vec{A}|$$