1 | loose definition

$$\int \frac{d}{dx} f(x) dx = f(x)$$

2 | formal definition

The theorem comes in two parts, apparently

2.1 | part 1

If f(x) is continuous over an interval [a, b], and the function F(x) is defined by

$$F(x) = \int_{a}^{x} f(t)dt$$

then F'(x) = f(x) over [a, b].

2.1.1 | intuition

Note that its $\int_a^x f(t)dt$ because x is an argument to the function and t is just the iteration variable.

Note that the integral can start anywhere to the left (arbitrary a) because that is removed as a constant when taking the derivative

Proof is by taking the limit form of a derivative of the integrals to x and x + h, and seeing that it collapses to the mean value. As the range of the mean value expression goes to zero, the value converges to itself.

2.1.2 | results

1. any integrable function and any continuous function has an anti-derivative

2.2 | part 2: the evaluation theorem

If f(x) is continuous over the interval [a,b] and F(x) is any anti-derivative of f(x), then

$$\int_{a}^{b} f(x)dx = F(b) - F(a)$$

2.2.1 | intuition

If you can find the anti-derivative, then the sum between the regions is just the difference in the anti-derivative, which makes sense. Basically contiguous areas add up.

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3 | an example

Imagine a function that has the bound of an integral as an argument:

$$g(x) = \int_0^x t \, dt = \frac{x^2}{2}$$
$$\frac{d}{dx}g(x) = \frac{d}{dx}\int_0^x t \, dt = \frac{d}{dx}\frac{x^2}{2} = x$$

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