$3 \mid \int \ln x dx$

$$\begin{split} \int \ln x dx &= \int 1 \ln x dx \\ &= x \ln x - \int x \frac{1}{x} dx \\ &= x \ln x - \int 1 dx \\ &= \boxed{x \ln x - x} \end{split}$$

4 | $\int \tan^- x dx$

$$\begin{split} \int \tan^- x dx &= x \tan^- x - \int x \frac{1}{x^2 + 1} dx \\ &= x \tan^- x - \frac{1}{2} \int \frac{du}{u} \\ &= x \tan^- x - \frac{1}{2} \ln u + C \\ &= x \tan^- x - \frac{1}{2} \ln(x^2 + 1) + C \end{split}$$

 $5 \mid \int x \sec^2 x dx$

$$\int x \sec^2 x dx = x \tan x - \int \tan x dx$$
$$= x \tan x + \ln|\cos x| + C$$

 $6 \mid \int x^2 e^{5x} dx$

$$\begin{split} \int x^2 e^{5x} dx &= x^2 \frac{1}{5} e^{5x} - \int 2x \frac{1}{5} e^{5x} dx \\ &= x^2 \frac{1}{5} e^{5x} - 2x \frac{1}{25} e^{5x} + \int 2 \frac{1}{25} e^{5x} dx \\ &= \frac{1}{5} e^{5x} (x^2 - \frac{2}{5}x + \frac{2}{25}) + C \end{split}$$

 $7 \mid \int x^2 \cos x dx = f(x) - \int 2x \sin x dx$

Find f(x)

$$f(x) = x^2 \sin x$$

 $8 \mid \int x \cos x dx$

$$\int x \cos x dx = x \sin x - \int \sin x dx$$
$$= x \sin x + \cos x + C$$

 $9 \mid \int x^2 \sin x dx$

$$\int x^2 \sin x dx = -x \cos x - \int -\cos x dx$$
$$= -x \cos x - \sin x dx + C$$

10 |
$$\int x^3 e^{x^2} dx$$

Let $u=x^2$

$$\int x^3 e^{x^2} dx = \int x^2 x e^{x^2} dx$$

$$= \int u \frac{1}{2} du e^u$$

$$= \frac{1}{2} \int u e^u du$$

$$= \frac{1}{2} u e^u - \frac{1}{2} \int e^u du$$

$$= \frac{1}{2} u e^u - \frac{1}{2} e^u + C$$

$$= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2}$$

11 | $\int x^2 \ln x dx$

$$\begin{split} \int x^2 \ln x dx &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \frac{1}{x} dx \\ &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx \\ &= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C \end{split}$$

12 | $\int \cos \ln x dx$

$$\begin{split} \int 1\cos\ln x dx &= x\cos\ln x + \int \sin\ln x dx \\ &= x\cos\ln x + x\sin\ln x - \int \cos\ln x dx \\ 2\int \cos\ln x dx &= x\cos\ln x - x\sin\ln x \\ \int \cos\ln x dx &= \frac{1}{2}\left(x\cos\ln x - x\sin\ln x\right) + C \end{split}$$

Or you could use $u = \ln x$, apparently.

13 | multiple parts

13.1 | **e**

13.2 | $\int e^{2x} \cos 3x dx$

$$\begin{split} \int e^{2x} \cos 3x dx &= \cos 3x \frac{1}{2} e^{2x} + \int 3 \sin 3x \frac{1}{2} e^{2x} dx \\ &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \int 3 \cos 3x \frac{1}{4} e^{2x} dx \\ &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \frac{9}{4} \int e^{2x} \cos 3x dx \\ \frac{13}{4} \int e^{2x} \cos 3x dx &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} + C \\ \int e^{2x} \cos 3x dx &= \frac{4}{13} \left(\cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} \right) + C \\ &= \frac{2}{13} e^{2x} \left(\cos 3x + \frac{3}{2} \sin 3x \right) + C \end{split}$$

13.3 | evaluate previous from $[0, \frac{pi}{6}]$

$$\frac{3}{13}e^{\frac{pi}{3}} - \frac{2}{13}$$

14 | $\int \sec^3 x dx$

$$\int \sec^3 x dx = \int \sec x \sec^2 x dx$$

$$= \sec x \tan x - \int \sec x \tan^2 x dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$$

$$= \sec x \tan x - \int \sec^3 x dx - \int \sec x dx$$

$$2 \int \sec^3 x dx = \sec x \tan x - \int \sec x dx$$

$$= \sec x \tan x - \ln|\sec x + \tan x| + C \int \sec^3 x dx = \frac{1}{2} (\sec x \tan x - \ln|\sec x + \tan x|) + C$$

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