1 | **Problem 1**

1.1 | $(1a)$

$PE = -W$

\[$W = \int_{R_e}^{\infty} F(r)\,dr$ \] We know that the force applied to a point mass m by the gravitational field of the earth (with mass M_e) with distance x is modeled by

$$F(r) = \frac{GmM_e}{r^2}$$

. Therefore, our work integral can be modified to be \[

$$\begin{aligned} W &= \int_{R_e}^{\infty} \frac{GmM_e}{r^2}\,dr \\ &= GmM_e \int_{R_e}^{\infty} \frac{1}{r^2}\,dr \\ &= GmM_e[-\frac{1}{r}]_{R_e}^{\infty} \\ &= -\frac{GmM_e}{R_e} \\ PE &= \frac{GmM_e}{R_e} \end{aligned}$$

\]

1.2 | $(1b)$

$KE = \frac{1}{2}mv^2$

$KE = PE$

\[$\frac{1}{2}mv^2 = \frac{GmM_e}{R_e}$ \]

$v = \sqrt{\frac{2GM_e}{R_e}}$

1.3 | $(1c)$

$v = \sqrt{\frac{2GM_e}{R_e}}$

\[$= \sqrt{\frac{2 \cdot 6.674 \cdot 10^{-11} \cdot 5.972\times10^{24}}{6.371 \cdot 10^{6}}}$ \]

$= 11185.7 m/s$

$= 25020.1 mph$

2 | **Problem 2**

\[
$$\begin{aligned}
\sum_{i=1}^{n} \vec{F}_{net,i} &= (\sum_{i=1}^{n} m_i)\ddot{\vec{r}}_{CM} \\
\sum_{i=1}^{n} m_i \ddot{\vec{r}}_i &= (\sum_{i=1}^{n} m_i)\ddot{\vec{r}}_{CM} \\
\int\int\sum_{i=1}^{n} m_i \ddot{\vec{r}}_i \, dt\, dt &= \int\int(\sum_{i=1}^{n} m_i)\ddot{\vec{r}}_{CM} \, dt\, dt \\
\int\sum_{i=1}^{n} m_i \dot{\vec{r}}_i \, dt + C_1 &= \int(\sum_{i=1}^{n} m_i)\dot{\vec{r}}_{CM} \, dt + C_1 \\
\sum_{i=1}^{n} m_i \vec{r}_i + C_1 t + C_2 &= (\sum_{i=1}^{n} m_i)\vec{r}_{CM} + C_1 t + C_2
\end{aligned}$$
\] Both constants are the same constant on both sides of the equation so they will cancel out. The sum of all mass is just M. \[$\vec{r}_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$ \]

3 | **Problem 3**

Any force within a system will have an opposite force applied as well (Newton's 3rd law). Therefore, forces within a system will cancel out and will have no effect on the center of mass.

4 | **Problem 4**

\[
$$\begin{aligned}
\vec{v} &= \frac{< 1, -4, 1 > +2 < -3, -2, 6 > +3 < 2, 5, -3 > +4 < -2, 4, 6 >}{1 + 2 + 3 + 4} \\
&= < -0.7, 2.3, 2.8 >
\end{aligned}$$
\]

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