1 | Tiling the Pringlehouse

As a review, our pringles shaped house has the following parametres:

$$\begin{cases} x(t) = 30cos(t) \\ y(t) = 20sin(t) \end{cases} \tag{1}$$

and the roof is defined by:

$$r(x,y) = \frac{1}{400}\left(\sqrt{3}x - y\right)^2 - \frac{1}{400}\left(\sqrt{3}y - x\right)^2 + 10 \tag{2}$$

We will first convert the above function into rectangular bounds to take the area of.

$$x = 30cos(t) \tag{3}$$

$$\Rightarrow \frac{x}{30} = cos(t) \tag{4}$$

$$\Rightarrow t = arccos\left(\frac{x}{30}\right) \tag{5}$$

Supplying this back to the original expression for y:

$$y = 20sin\left(arccos\left(\frac{x}{30}\right)\right) \tag{6}$$

$$= 20\sqrt{1 - \left(\frac{x}{30}\right)^2} \tag{7}$$

Therefore, the actual integral:

$$\int_{-30}^{30}\int_{-20\sqrt{1-\left(\frac{x}{30}\right)^2}}^{20\sqrt{1-\left(\frac{x}{30}\right)^2}} 1dy\ dx \tag{8}$$

We will endeavor now to use technology.

```
var("x y")
f(x,y) = 1
f.integrate(y, -20*sqrt(1-(x/30)^2), 20*sqrt(1-(x/30)^2)).integrate(x, -30,30)
```

It appears that the area of the floor is 600π.

We can do this something for the function of the roof. We will first figure correction factor dA, then take the integral as prescribed.

$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} \tag{9}$$

At this point, we realize that the actual function will turn to be much too complicated to integrate by hand at this moment; therefore, we will create the expression digitally.

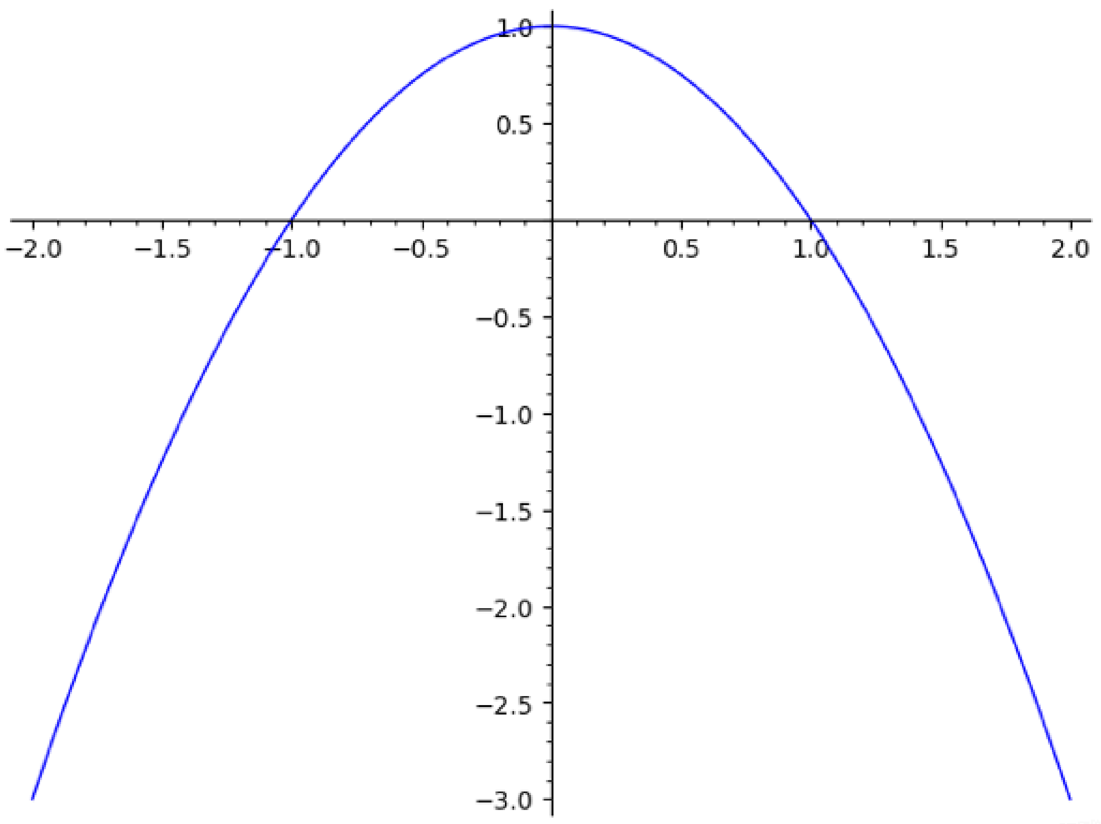
```
r(x,y) = (1/400)*(sqrt(3)*x-y)^2 - (1/400)*(sqrt(3)*y+x)^2 + 10
da = sqrt(1+r.diff(x)^2+r.diff(y)^2)
monte_carlo_integral(da.integrate(y, -20*sqrt(1-(x/30)^2), 20*sqrt(1-(x/30)^2)), [-30], [30], 10000000)
```

Looks like the result is converting to about 2002.2 for this shape.

2 | Three Dimensional Region!

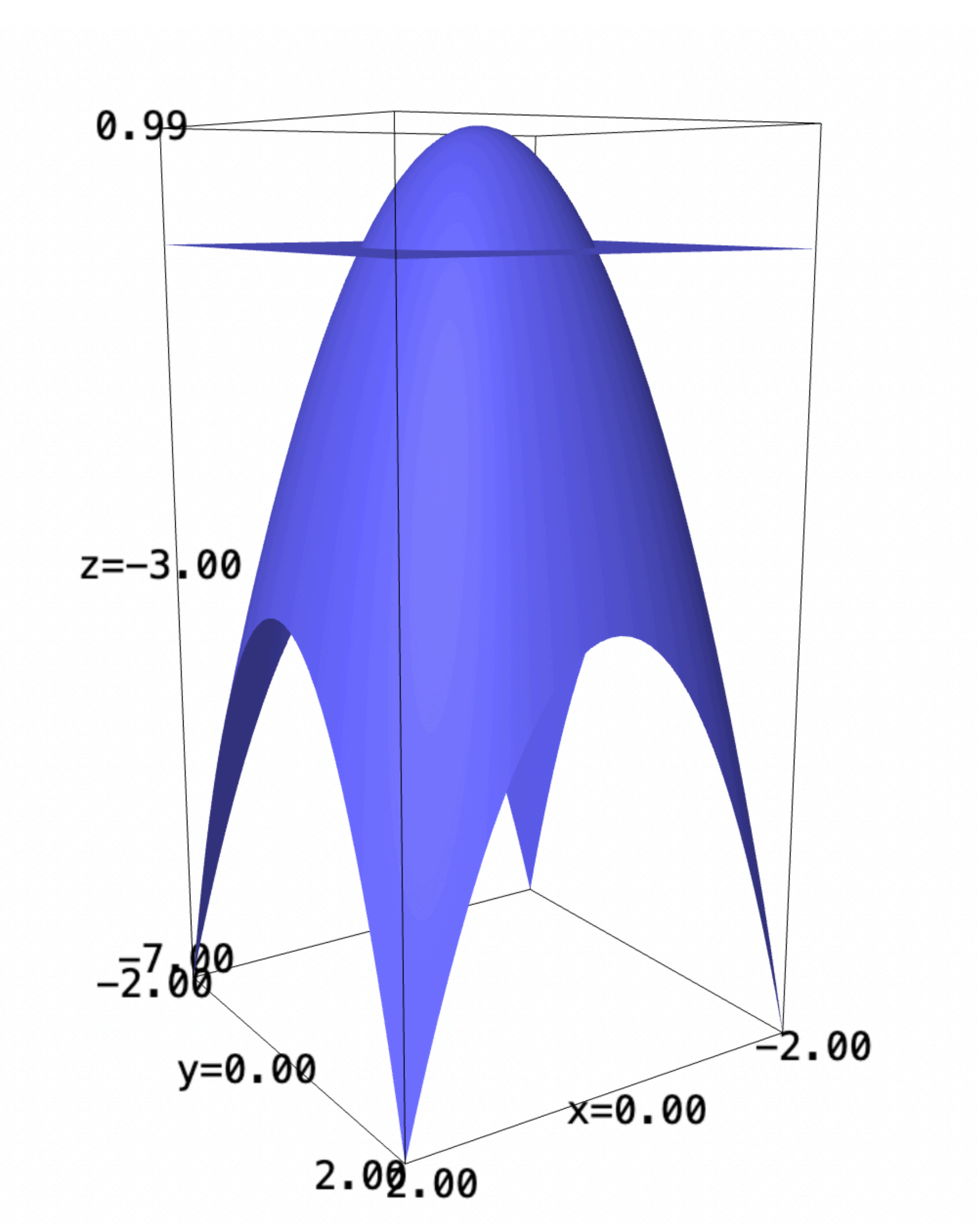
Slowly adding up the arguments to this figure reveals its general shape:

```
f(x) = 1-x^2
plot(f, -2, 2)
```



And, adding up the y component:

```
f(x,y) = 1-x^2-y^2
plot3d(f, (x,-2,2), (y,-2,2)) + plot3d(0, (x,-2,2), (y,-2,2))
```



We can see that the positive component exists at $x = [-1, 1]$, $y = [-1, 1]$. If the same pattern holds, then, the maximum volume would be over areas:

$$x = [-1, 1], y = [-1, 1], z = [-1, 1] \tag{10}$$

Taking the actual integral:

```
a(x,y,z) = 1-x^2-y^2-z^2
a.integrate(x,-1,1).integrate(y,-1,1).integrate(z,-1,1)
```

Shockingly, the resulting integral is 0. However, we can actually move the bounds to see that all other manifestations about this point would actually result in even more negative values.

```
a(x,y,z) = 1-x^2-y^2-z^2
a.integrate(x,-1,1).integrate(y,-1,1).integrate(z,-1,1)
```

Taking the Jacobian matrix of this expression would reveal that:

$$\nabla a = \begin{bmatrix} -2x \\ -2y \\ -2z \end{bmatrix} \tag{11}$$

We see that there is no other point but $(0,0,0)$ is the maxima, and—by the pattern of the function—holding any two variables constant and squaring the remaining one will approach 0 (the smallest non-negative value) at bounds $[-1,1]$. Therefore, it makes sense that the bounds prescribed would be the actual bounds desired.

3 | Spinning around the origin

I suppose intuitively the function needs to be axially symmetric to actually perform this trick. This means that, for exponential functions, it has to have a square (or a square of a square, etc.) term on top.