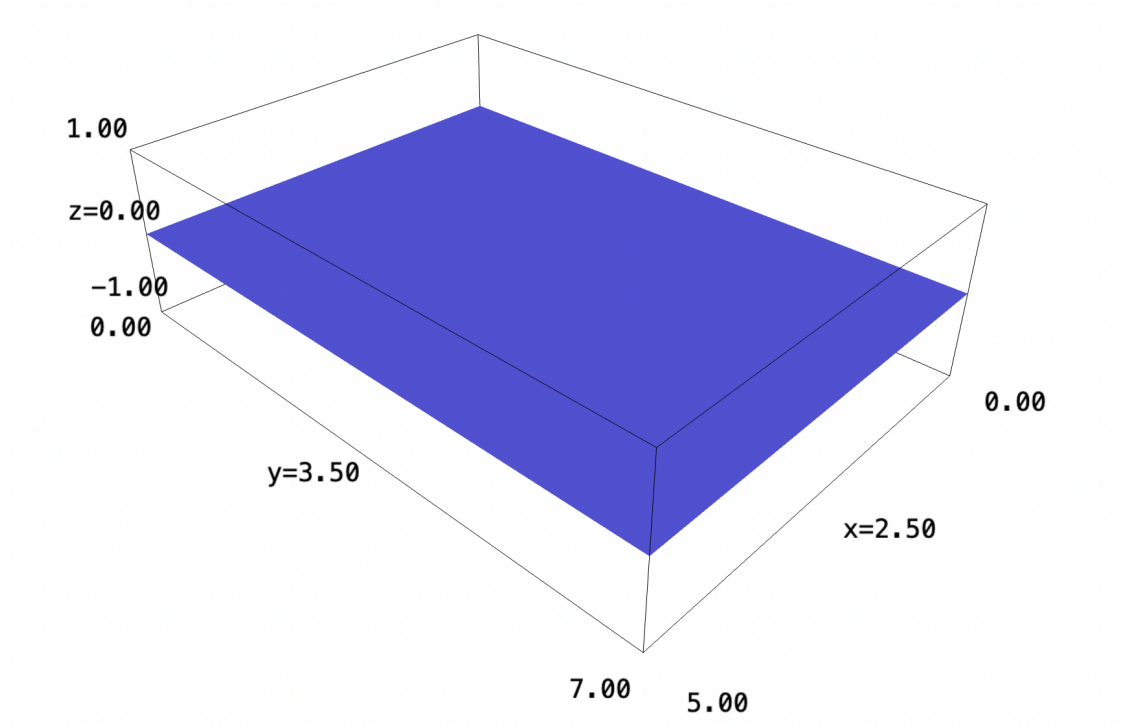


1 | Rectangle

```
f(x) = 0
plot3d(f, (x,0,5), (y,0,7))
```



$$dA = \sqrt{1} = 1 \quad (1)$$

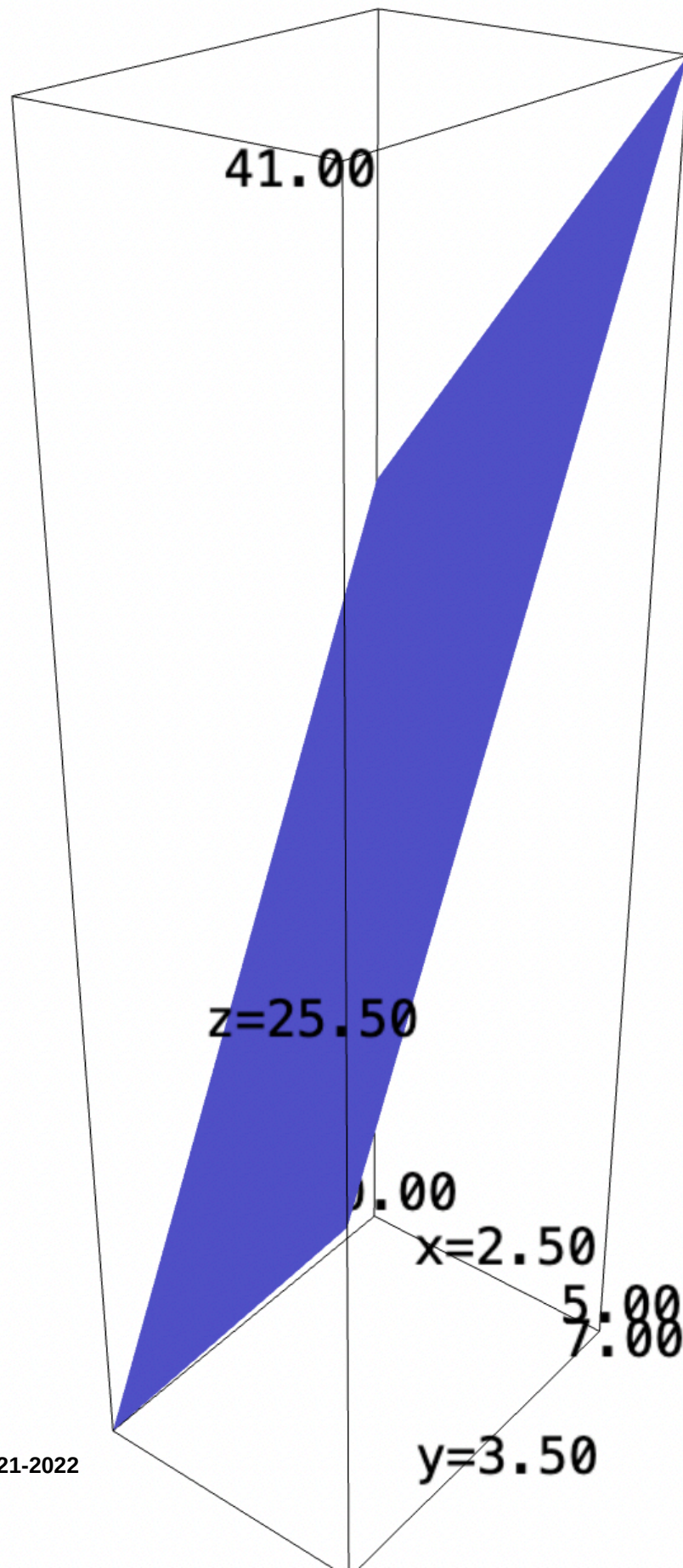
$$\iint_V 1 dV \quad (2)$$

$$\Rightarrow \int_0^5 \int_0^7 1 dy dx \quad (3)$$

$$\Rightarrow 35 \quad (4)$$

2 | Area of a Plane

```
f(x,y) = 2*x+3*y+10
plot3d(f, (x,0,5), (y,0,7))
```



$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dV \quad (5)$$

$$= \sqrt{1 + 2^2 + 3^2} dV \quad (6)$$

$$= \sqrt{1 + 4 + 9} dV \quad (7)$$

$$= \sqrt{14} dV \quad (8)$$

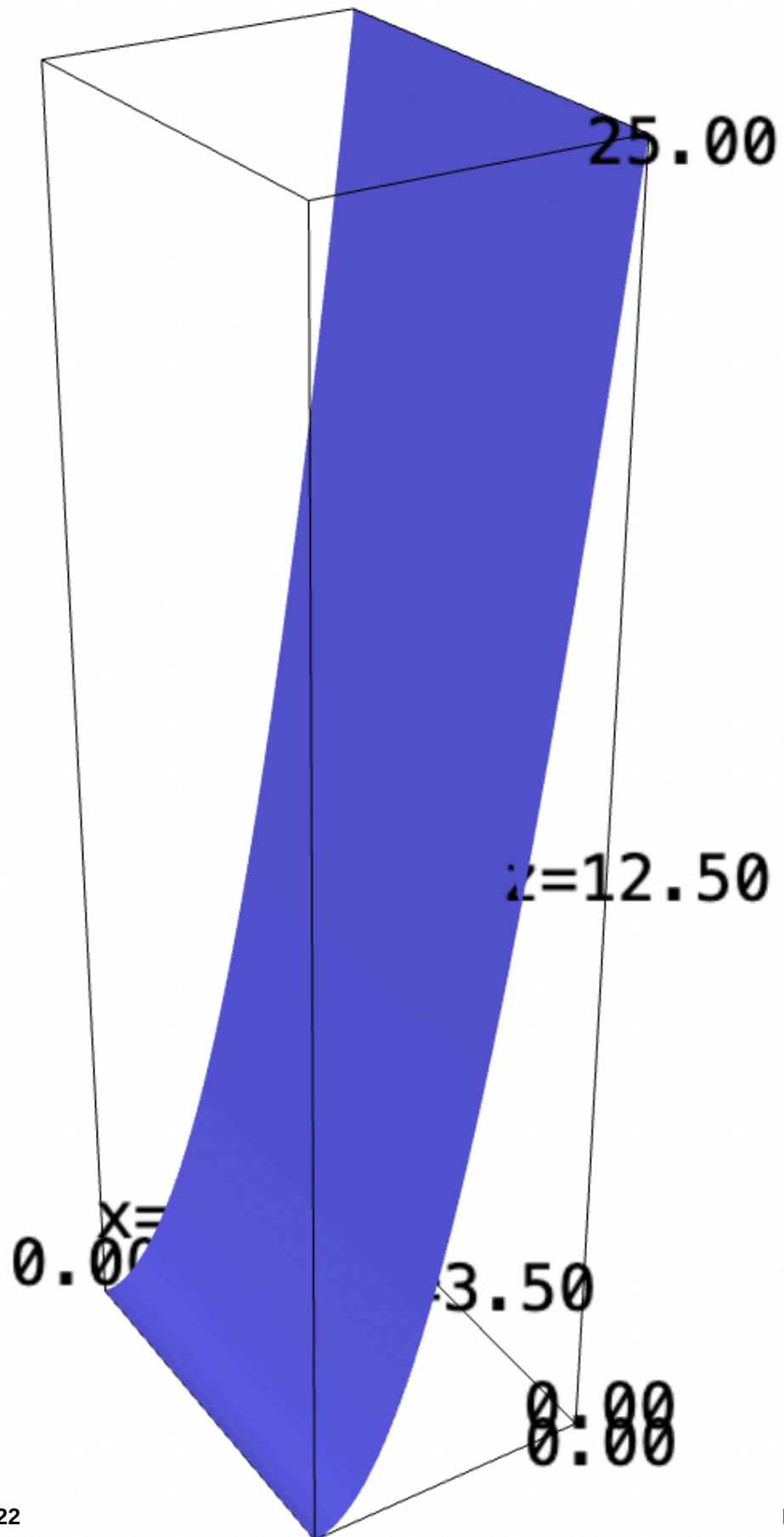
$$\iint_V \sqrt{14} dV \quad (9)$$

$$\Rightarrow \int_0^5 \int_0^7 \sqrt{14} dy \, dx \quad (10)$$

$$\Rightarrow 35\sqrt{14} \quad (11)$$

3 | Parabola

```
f(x,y) = x^2
plot3d(f, (x,0,5), (y,0,7))
```



$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dV \quad (12)$$

$$= \sqrt{1 + (2x)^2} dV \quad (13)$$

$$= \sqrt{1 + 4x^2} dV \quad (14)$$

$$\iint_V \sqrt{1 + 4x^2} dV \quad (15)$$

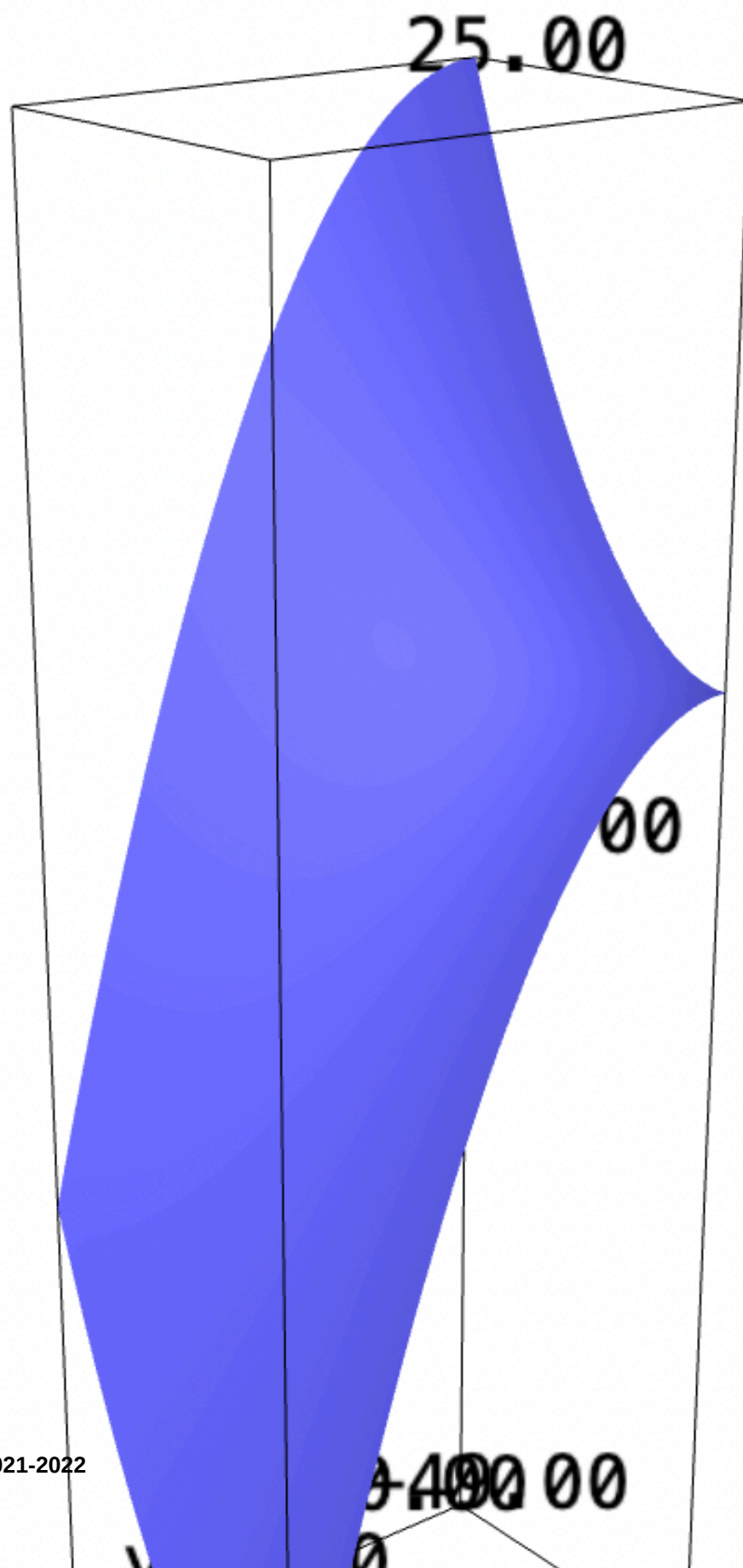
$$\Rightarrow \int_0^5 \int_0^7 \sqrt{1 + 4x^2} dy dx \quad (16)$$

$$\Rightarrow \frac{35}{2} \sqrt{101} + \frac{7}{4} \operatorname{arcsinh}(10) \quad (17)$$

```
f(x,y) = sqrt(1+4*x^2)
f.integrate(y, 0,7).integrate(x, 0,5)
```

4 | Hyperbolic Parabaloid

```
f(x,y) = x^2-y^2
plot3d(f, (x,0,5), (y,0,7))
```



$$\vec{v}(x, y) = x\hat{i} + y\hat{j} + (x^2 - y^2)\hat{k} \quad (18)$$

$$\frac{\partial \vec{v}}{\partial x} = \hat{i} + 2x\hat{k} \quad (19)$$

$$\frac{\partial \vec{v}}{\partial y} = \hat{j} - 2y\hat{k} \quad (20)$$

$$\frac{\partial \vec{v}}{\partial x} \times \frac{\partial \vec{v}}{\partial y} \quad (21)$$

$$\Rightarrow (\hat{i} + 2x\hat{k}) \times (\hat{j} - 2y\hat{k}) \quad (22)$$

$$\Rightarrow (\hat{i}\hat{j} + \hat{k}2x\hat{j} - \hat{i}2y\hat{k} - 2x\hat{k}2y\hat{k}) \quad (23)$$

$$\Rightarrow (\hat{i}\hat{j} + \hat{k}2x\hat{j} - \hat{i}2y\hat{k}) \quad (24)$$

$$\Rightarrow (\hat{k} - 2x\hat{i} + 2y\hat{j}) \quad (25)$$

$$\begin{pmatrix} -2x \\ 2y \\ 1 \end{pmatrix} \quad (26)$$

$$dA = \sqrt{1 + 4x^2 + 4y^2} \, dx \, dy \quad (27)$$

$$\iint_V \sqrt{1 + 4x^2 + 4y^2} dV \quad (28)$$

$$\Rightarrow \int_0^5 \int_0^7 \sqrt{1 + 4x^2 + 4y^2} \, dy \, dx \quad (29)$$

$$\approx 326.54 \quad (30)$$

```
f(x,y) = sqrt(1+4*x^2 + 4*y^2)
float(f.integrate(y, 0, 7).integrate(x,0,5))
```