1 | Review Sheet

1.1 | Problem 1

1.1.1 | (e)

\[
$$\begin{aligned} f(x) &= x(x^2+2) - \sin(x^4 - x^{90}) + e^{\sin(x)} + \ln\cos(x^2) \\ f'(x) &= 3x^2 + 2 - (4x^3 - 90x^{89})\cos(x^4 - x^9 0) + \cos(x)e^{\sin(x)} + -\frac{2x\sin(x^2)}{\cos(x^2)} \end{aligned}$$
\]

1.1.2 | (f)

\[
$$\begin{aligned} y &= \frac{x^5 + x^{25}}{\sin(x)} + x^5\sin(x) + x^3\sin(x)e^{5x} \\ \frac{d}{dx}[y] &= \frac{\sin(x)(5x^4 + 25x^{24}) - \cos(x)(x^4 + x^2 5)}{\sin^2(x)} + (5x^4\sin(x) + x^5\cos(x)) + ((3x^2\sin(x) + x^3\cos(x))e^{5x} + 5x^4 \sin \end{aligned}$$
\]

1.2 | Problem 4

1.2.1 | (a)

\[
$$\begin{aligned} V &= 24.0Lmol^{-1} \\ V(t) &= 24t \\ R(t) &= \sqrt[3]{\frac{3}{4}V(t)} \\ &= \sqrt[3]{18t} \\ t &= 3 \\ V(3) &= 72L \\ R(3) &= 3\sqrt[3]{2} * 10cm \qquad\qquad = 30\sqrt[3]{2}cm \\ V'(t) &= 24 \\ R'(t) &= \frac{18}{\sqrt[3]{18t}^2} \\ V'(3) &= 24Ls^{-1} \\ R'(3) &= \frac{18}{\sqrt[3]{18(3)}^2} \\ &= \frac{18}{6\sqrt[3]{2}} * 10cms^{-1} \quad = \frac{30}{\sqrt[3]{2}}cms^{-1} \end{aligned}$$
\]

1.2.2 | (b)

Assuming that the question is asking how much time would have passed when the radius is 3m: \[

$$\begin{aligned} 3m &= 30 * 10cm \\ R(t) &= 30 \\ \sqrt[3]{18t} &= 30 \\ 18t &= 30^3 \\ t &= \frac{30^3}{18} \\ &= 1500 \end{aligned}$$

\]

1.3 | Problem 5

1.3.1 | (e)

\[

$$\begin{aligned} &\int (4y - y^2 + 4y^3 + 1)^{-2/3}(12y^2 - 2y + 4)\,dy \\ &= 3\int \frac{1}{3}(4y - y^2 + 4y^3 + 1)^{-2/3}(12y^2 - 2y + 4)\,dy \\ &= 3(4y - 4^2 + 4y^3 + 1)^{1/3} + C \end{aligned}$$

\]

1.3.2 | (f)

\[

$$\begin{aligned} \int 2x \cos(x)\,dx &= 2x \sin(x) - \int 2 \sin(x)\,dx \\ &= 2x \sin(x) - 2\int \sin(x)\,dx \\ &= 2x \sin(x) + 2\cos(x) \end{aligned}$$

\]

2 | Arc Length

2.1 | Problem 2

$$f(x) = \frac{x^2}{8} - \ln x$$
$$f'(x) = \frac{1}{4}x - \frac{1}{x}$$
$$L = \int_1^2 \sqrt{1 + f'(x)^2}\, dx$$

\[$$= \int_1^2 \sqrt{1 + (\frac{1}{16}x^2 - \frac{1}{2} + \frac{1}{x^2})}\, dx$$ \]

$$= \int_1^2 \sqrt{\frac{1}{16}x^2 + \frac{1}{2} + \frac{1}{x^2}}\, dx$$
$$= [\frac{\sqrt{\frac{(x^2+4)^2}{x^2}}(x^3 + 8x \log(x))}{8(x^2+4)}]_1^2$$
$$= \frac{3}{8} + \log 2$$

2.2 | Problem 8

$$f(x) = x^2$$
$$f'(x) = 2x$$

\[$f'(x) = 6$ \]

$$2x = 6$$
$$x = 3$$