

1 | boatman problem

Target displacement: $\langle 3\text{km}, 2\text{km} \rangle$

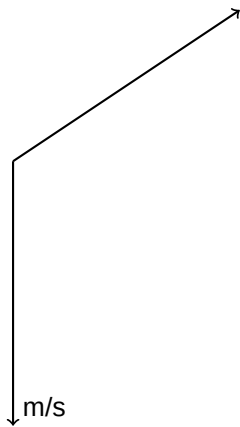
We are working with the velocities of the boat and the river. The velocity of the river is defined as $r = \langle 0, -3.5 \rangle$. We want to find vector $v = \langle v_x, v_y \rangle$ s.t.

$$\begin{aligned} |v| &= 13 \text{ km/h} \\ \lambda(v + r) &= \langle 3, 2 \rangle \end{aligned}$$

Where the trip will take λ hours

$$\begin{aligned} v_x^2 + v_y^2 &= 13^2 \\ \lambda(v_x + 0) &= 3 \\ \lambda(v_y + -3.5) &= 2 \\ v_x &= \frac{3}{\lambda} \\ v_y &= \frac{2}{\lambda} + 3.5 \\ \frac{3^2}{\lambda^2} + \left(\frac{2}{\lambda} + 3.5\right)^2 &= 13^2 \\ \frac{3^2}{\lambda^2} + \frac{2^2}{\lambda^2} + 3.5^2 + \frac{4(3.5)}{\lambda} &= 13^2 \\ \frac{3^2 + 2^2}{\lambda^2} + \frac{4(3.5)}{\lambda} &= 13^2 - 3.5^2 \\ 3^2 + 2^2 + 4(3.5)\lambda &= \lambda^2 (13^2 - 3.5^2) \\ 13 + 4(3.5)\lambda &= \lambda^2 (156.75) \\ -156.75\lambda^2 + 14^2 + 13 &= 0 \\ \frac{-14 \pm \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} & \\ \frac{-14 + \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} &= -0.24676847741 \\ \frac{-14 - \sqrt{14^2 + 4(13)156.75}}{-2(156.75)} &= 0.336082671987 \end{aligned}$$

Maybe it's time to do it geometrically



Let θ be the angle difference that you paddle at, and ϕ be the angle that you are aiming for.

$$3.5^2 \lambda^2 = 13 + 13\lambda - 2(13)13\lambda \cos \theta$$

$$\tan \phi = \frac{3}{2}$$

$$\sin(\theta + \phi) = \frac{3.5\lambda + 2}{13\lambda}$$

attempt 3: after getting help from leonard

$$\beta = \alpha + \frac{\pi}{2} = \tan^{-1} \frac{2}{3} = 2.158$$

$$\frac{\sin \beta}{|v|} = \frac{\sin \gamma}{3.5}$$

$$\frac{\sin(2.158)}{13} = \frac{\sin \gamma}{3.5}$$

$$3.5 \frac{\sin(2.158)}{13} = \sin \gamma$$

$$\gamma = 3.5 \frac{\sin(2.158)}{13} = 0.2241$$

$$\alpha + \gamma = 0.588 + 0.2241 = 0.8121 \text{ radians}$$

The speed

$$\frac{3}{13 \cos 0.812} = 0.3353 \text{ hours}$$

dang it i was actually right the first time. apparently math isn't a democracy.

2 | circular motion

$$\theta'(t) = \omega_0$$

$$\theta(t) = \int \omega_0 dt = \omega_0 t + \theta_0$$

2.1 | position

$$\begin{aligned}\vec{r}(t) &= (R \cos \theta(t), R \sin \theta(t)) \\ &= (R \cos(\omega_0 t + \theta_0), R \sin(\omega_0 t + \theta_0))\end{aligned}$$

2.2 | velocity

$$\vec{v}'(t) = (-\omega_0 R \sin(\omega_0 t + \theta_0), \omega_0 R \cos(\omega_0 t + \theta_0))$$

2.3 | acceleration

$$\begin{aligned}\vec{a}(t) &= \vec{v}'(t) \\ &= \frac{d}{dt} (-\omega_0 R \sin(\omega_0 t + \theta_0), \omega_0 R \cos(\omega_0 t + \theta_0)) \\ &= (-\omega_0^2 R \cos(\omega_0 t + \theta_0), -\omega_0^2 R \sin(\omega_0 t + \theta_0))\end{aligned}$$

2.4 | perpendicular

$$\begin{aligned}\text{pos: } & \frac{R \sin(\omega_0 t + \theta_0)}{R \cos(\omega_0 t + \theta_0)} \\ \text{vel: } & \frac{\omega_0 R \cos(\omega_0 t + \theta_0)}{-\omega_0 R \sin(\omega_0 t + \theta_0)}\end{aligned}$$

See? They are negative reciprocals of each other.

$$\begin{aligned}|\vec{v}| &= \sqrt{\omega_0^2 R^2 \sin^2(\omega_0 t + \theta_0) + \omega_0^2 R^2 \cos^2(\omega_0 t + \theta_0)} \\ &= \omega_0 R \sqrt{\sin^2(\omega_0 t + \theta_0) + \cos^2(\omega_0 t + \theta_0)} \\ &= \omega_0 R \sqrt{1} \\ &= \omega_0 R\end{aligned}$$

2.5 | acceleration

$$\begin{aligned}\text{accl} &= -\omega_0^2 R (\cos(\omega_0 t + \theta_0), \sin(\omega_0 t + \theta_0)) \\ \text{pos} &= R (\cos(\omega_0 t + \theta_0), \sin(\omega_0 t + \theta_0))\end{aligned}$$

The acceleration is a negative scalar multiple of the position, and thus points towards the center.

$$|\vec{a}| = -\omega_0^2 R \sqrt{1} = \frac{\omega_0^2 R^2}{R} = \frac{|\vec{v}|^2}{R}$$

2.6 | non uniform motion

$$\theta''(t) = a_0$$

$$\theta'(0) = 0$$

$$\theta(0) = 0$$

$$\theta'(t) = a_0 t + C_0 = a_0 t$$

2.7 | angular position

$$\theta(t) = \frac{1}{2}a_0 t^2 + C_1 = \frac{1}{2}a_0 t^2$$

2.8 | position

$$\begin{aligned}\vec{r}(t) &= (R \cos \theta(t), R \sin \theta(t)) \\ &= \left(R \cos \left(\frac{1}{2}a_0 t^2 \right), R \sin \left(\frac{1}{2}a_0 t^2 \right) \right)\end{aligned}$$

2.9 | velocity

$$\vec{v}(t) = R \left(-\sin \left(\frac{1}{2}a_0 t^2 \right) a_0 t, \cos \left(\frac{1}{2}a_0 t^2 \right) a_0 t \right)$$

2.10 | acceleration

$$\begin{aligned}\vec{a}(t) &= a_0 R \left(-\cos \left(\frac{1}{2}a_0 t^2 \right) a_0 t^2 - \sin \left(\frac{1}{2}a_0 t^2 \right), \right. \\ &\quad \left. -\sin \left(\frac{1}{2}a_0 t^2 \right) a_0 t^2 + \cos \left(\frac{1}{2}a_0 t^2 \right) \right)\end{aligned}$$

2.11 | velocity vector

Both magnitudes are non-zero, so we can use the dot product to show that the position and velocity are perpendicular.

$$\begin{aligned}\vec{r}(t) \cdot \vec{v}(t) &= R \left(-\cos \left(\frac{1}{2}a_0 t^2 \right) \sin \left(\frac{1}{2}a_0 t^2 \right) a_0 t + \sin \left(\frac{1}{2}a_0 t^2 \right) \cos \left(\frac{1}{2}a_0 t^2 \right) a_0 t \right) \\ &= R0 = 0\end{aligned}$$

Thus, the velocity is tangent to the circle.

Magnitude:

$$\begin{aligned}\vec{v}(t) &= Ra_0t \sqrt{\sin^2\left(\frac{1}{2}a_0t^2\right) + \cos^2\left(\frac{1}{2}a_0t^2\right)} \\ &= Ra_0t\sqrt{1} = Ra_0t = R\theta'(t)\end{aligned}$$

2.12 | acceleration

We can split it up because we know the $|\vec{v}|^2$ term is the one with $a_0^2t^2$

$$\begin{aligned}\vec{a}(t) &= R\left(-a_0^2t^2\left(\cos\left(\frac{1}{2}a_0t^2\right), \sin\left(\frac{1}{2}a_0t^2\right)\right) + a_0\left(-\sin\left(\frac{1}{2}a_0t^2\right), \cos\left(\frac{1}{2}a_0t^2\right)\right)\right) \\ &= R\left(-\frac{|\vec{v}|^2}{R^2}\right)\end{aligned}$$