1 | Series Convergence

1.1 | Geometric Series

In $\sum_{k=0}^{\infty} a(r^k)$, where $|r| < 1$, the series converges to $\sum_{k=0}^{\infty} a(r^k) = \frac{a}{1-r}$

In $\sum_{k=0}^{n} a(r^k)$, $\sum_{k=0}^{n} a(r^k) = \frac{a-ar^{n+1}}{1-r}$

1.2 | nth term divergence test

If $\lim_{n\to\infty} a_n$ is not zero, the series **will** diverge. The inverse is not necessarily true; that is, if this fails, use another test to test convergence.

1.3 | Intergral Test

If the intergral to infinity is convergent, the sequence is convergent as long as the sequence is continuous, positive, and decreasing. The inverse applies, too.

1.4 | Power Series

$\sum_{n=1}^{\infty} \frac{1}{n^p}$

If a p-series has a p > 1, the p-series will converge

If a p-series has a p <= 1, the p-series will diverge

1.5 | Comparison Test

Both provided that $a_n, b_n \geq 0$ & $a_n \leq b_n$

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Also, if $\lim_{n\to\infty} \frac{a_n}{b_n} = C\ (0 < c < \infty)$, the two series will either both converge or both diverge. So you only need to test one.

1.6 | Alternating Series Test

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1.7 | Ratio Test

In a geometric series, the common ratio is simply $r = \frac{r^{n+1}}{r^n}$.

If r is an real value, $|r| < 1$, then series converges. If $|r| \geq 1$, the series diverges.

As limit goes to infinity in the r, if the common ratio approaches <1, that means that the ratio will get smaller and smaller, just like if r were to be a real value and it was smaller than one. Meaning that the series **converges.**

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And so, formally.

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The inverse is true, too.

However, if the ratio is equal to one, the test is inconclusive.

Absolute Convergence => series who converge and whose absolute value converges

Conditional Convergence => series who converge and whose absolute value does not converge

1.8 | So what is the error of a talor series? (Lagrange Error)

The error at point x of a nth degree talor polynomial centered at a modeling a function with an absolute maximum value of M in its $n+1$th dervitave between a bound containing x and a:

$|E(x)| \leq \frac{M(x-a)^{n+1}}{(n+1)!}$

1.9 | Power Series

$f(x) = \sum_{n=0}^{\infty} a_n(x-c)^n = a_n(x-c)^0 + a_n(x-c)^1...$

For instance, a geometric series is a special power series…

$g(x) = \sum_{n=0}^{\infty} = ax^n$

This geometric series converges if $|x| < 1$, and so it has an interval of convergence of $-1 < x < 1$. If this converges, this function will converge to $\frac{a}{1-x}$

Interval of Convergence: at what values of x does the series converge?

Radius of Convergence: at what absolute distance from c (the "centering" of the series) will the series converge?

To figure the interval of convergence, simply use the ratio test and solve for x that makes the ratio < 1. Then, think about the inconclusive cases whereby ratio $= 1$ — then, use the comparison test, or intergral test.

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Derivatives, intergrals have the same radius of convergence as the parent function, but their interval may be different due to different behavior at endpoints