

## 1 | Motivation: Fibonacci series

Each number is the sum of two previous numbers, we all know it:

$$1, 1, 2, 3, 5, 8, 13, 21, \dots \quad (1)$$

How many digits does the *billionth* number have? We obviously can't just compute it.

## 2 | Motivation: Bernie Sanders Memes???

- $T(n) = 3$
- $T(n) = T(n-1) + 2T(n-1) - 2T(n-2)$

That, the first  $T(n-1)$  are the meme makers of the day,  $2T(n-1)$  are the newly anointed meme makers, and  $2T(n-2)$  are the tired meme-makers.

Let's, instead, guess that there is some kind of exponential relationship, in perspective to a variable  $r$  with a few degrees of freedom:

$$T(n) = T(n-1) + 2T(n-1) - 2T(n-2) \quad (2)$$

$$T(n) - T(n-1) - 2T(n-1) + 2T(n-2) = 0 \quad (3)$$

$$r^n - r^{n-1} - 2r^{n-1} + 2r^{n-2} = 0 \quad (4)$$

$$r^n - 3r^{n-1} + 2r^{n-2} = 0 \quad (5)$$

$$r^{n-2}(r-2)(r-1) = 0 \quad (6)$$

$$r = \{0, 1, 2\} \quad (7)$$

So there must be some kind of exponential relationship:

$$T(n) = a(1^n) + b(2^n) + c(0^n) \quad (8)$$

We can finally solve for  $a$  and  $b$  given the base cases: then we know that  $b = 3$ ,  $a = -3$ .