PS#19: A fun one!!! Three different, fun problems!

Nueva Multivariable Calculus

USE MATTHEW GILL'S BEAUTIFUL NUEVA LaTeX TEMPLATE AND MAKE IT LOOK REALLY, REALLY PRETTY. I REALLY LIKE EACH OF THE THREE PROBLEMS IN THIS PSET AND I REALLY LIKE THE THREE OF THEM TOGETHER—THEY'RE ALL INDIVIDUALLY VERY GOOD SONGS AND IN THE AGGREGATE A GREAT ALBUM—SO MAKE YOUR WRITEUP REFLECT THAT!!!!

- Roofs #2! (The off-axis shed roof at the paleontology museum!) (Note that I put up my own writeup to the first roofs problem on Canvas.)
- Do a 2D Taylor series! OK, the formula is basically a drag, and one of these things that becomes computationally unpleasant... but we probably should compute at least one. Consider the function:

  $$f : \mathbb{R}^2 \to \mathbb{R}^1$$

  $$f(x, y) = \cos(x)\cos(3y)$$

  Graph it! (Include a graph in your writeup!) What does it look like? Come up with a jaunty description. Then, grow a Taylor series of this function, around $(x = 0, y = 0)$, out to the cubic/third-order terms. Show your work and the answer, et cetera, explained clearly. Also, include graphs of the Taylor series at each stage (i.e, watch/show it growing!)
- Consider the function:

  $$g : \mathbb{R}^2 \to \mathbb{R}^1$$

  $$g(x, y) = x^2 - 6xy + y^2$$

  and the point on that function given by the (x, y)-coordinates $(0, 0)$ and the corresponding third/z/g/vertical coordinate.

  At $(0, 0)$, both the partial derivatives of this function are zero (*work this out and show me the details*), and both the second partial derivatives are positive (*work this out and show the details, too*). So then that point is a minimum, right? Right!?! That's how it'd work in 1D, yeah?!?

  Either convince me that it is, or convince me that it isn't.

  And, just to be clear: I'm a grouchy, old-school mathematician—I don't trust pictures. I trust algebraic, symbolic, logical mathematics. Pictures can deceive us. (And they only exist in two or three dimensions—there's no such thing as a picture of an eight-dimensional shape.) So you need to make an argument that somehow doesn't rely on "I graphed it in Geogebra, and it did/didn't look like a minimum." (To be clear, you *should* graph it, so you can get a feeling for what's going on—but, if you get a sense of what's going on, can you make a more mathematical argument, one that doesn't rely on pictures?)

  Don't look anything up about how to find maxima and minima of 2D functions—that would spoil the fun! Try thinking about this only with our existing knowledge of surfaces, differentiation in higher dimensions, optimization in one dimension, etc.