1 | projections or smt

1.1 | figma

1.2 | language of projections?

2 | vectors problems

2.1 | adding two vectors $\langle a_1, a_2, a_3\rangle$ and $\langle b_1, b_2, b_3\rangle$

2.1.1 | the coordinates of the sum

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

2.1.2 | adding vectors

Geometrically, it is putting the vectors tip to tail. Follow one, then follow the other. Algebraically, it is adding each of the components. See the previous part

2.1.3 | subtracting vectors

We want to define $\vec{c} = \vec{a} - \vec{b}$ such that $\vec{b} + \vec{c} = \vec{a}$.

Geometrically, that means following $\vec{a}$, and then following $\vec{b}$ backwards (ie. we want to define a negative vector as the same vector backwards). Algebracially, we see that it inherits the properties from addition/subtraction.

2.2 | finding the vector between two points

Take the points as vectors, and subtract them.

$$\langle x_1 - x_0, y_1 - y_0, z_1 - z_0\rangle$$

2.3 | practice problems

2.3.1 | magnitude of a

$$|\langle 4, 0, 3\rangle| = \sqrt{4^2 + 3^2} = 5$$

2.3.2 | magnitude of b

$$|\langle -2, 1, 5\rangle| = \sqrt{(-2)^2 + 1^2 + 5^2} = \sqrt{30} = 5.47722557505$$

2.3.3 | $\vec{a}+\vec{b}$

$$\langle 2,1,8\rangle$$

2.3.4 | $\vec{a}-\vec{b}$

$$\langle 6,-1,-2\rangle$$

2.3.5 | $3\vec{b}$

$$\langle -6,3,15\rangle$$

2.3.6 | $2\vec{a}+5\vec{b}$

$$\langle -2,5,31\rangle$$

2.3.7 | $\hat{a},\hat{b}$

$$\left\langle \frac{4}{5},0,\frac{3}{5}\right\rangle$$
$$\left\langle \frac{-2}{\sqrt{30}},\frac{1}{\sqrt{30}},\frac{5}{\sqrt{30}}\right\rangle$$

2.3.8 | $\theta_{\vec{a}x}$

Lets make a right triangle in the plane that contains the tip and tail of the vector and the x-axis.
The height will be from the x-axis to the tail, so we'll take the diagonal in the yz plane

$$h=\sqrt{a_y^2+a_z^2}$$

The base of the triangle will be along the x-axis. So, the base is just the x component a_x.
And so, we can find theta using the tangent

$$\tan\theta=\frac{\sqrt{a_y^2+a_z^2}}{a_x}$$

You could also do it with the cosine, as in dot product:

$$\cos\theta=\frac{a_x}{\sqrt{a_x^2+a_y^2+a_z^2}}$$

2.4 | **triangle proof**

Lets let $\vec{a}, \vec{b}$ be the two sides and $\vec{c}$ be the middle side. This is the small triangle. Then, let's double each of the side lengths:

$$2\vec{a} + 2\vec{b} = 2(\vec{a} + \vec{b}) = 2\vec{c}$$

Thus, the middle line is half the magnitude of the longer third side.

3 | **proving vector properties**

You are really stretching my LaTeX abilities here

3.1 | $a + b = b + a$

\[[thick,->] (0,0) – (4.5,0) node[anchor=north west] {x axis}; [thick,->] (0,0) – (0,4.5) node[anchor=south east] {y axis}; [red,thick]]