

# 1 | Problem 1

## 1.1 | (1a)

$$PE = -W$$

\[  $W = \int_{R_e}^{\infty} F(r) dr$  \] We know that the force applied to a point mass  $m$  by the gravitational field of the earth (with mass  $M_e$ ) with distance  $x$  is modeled by

$$F(r) = \frac{GmM_e}{r^2}$$

$$\begin{aligned} W &= \int_{R_e}^{\infty} \frac{GmM_e}{r^2} dr \\ &= GmM_e \int_{R_e}^{\infty} \frac{1}{r^2} dr \end{aligned}$$

. Therefore, our work integral can be modified to be \[

$$\begin{aligned} &= GmM_e \left[ -\frac{1}{r} \right]_{R_e}^{\infty} \\ &= -\frac{GmM_e}{R_e} \\ PE &= \frac{GmM_e}{R_e} \end{aligned}$$

## 1.2 | (1b)

$$KE = \frac{1}{2}mv^2$$

$$KE = PE$$

$$\left[ \frac{1}{2}mv^2 = \frac{GmM_e}{R_e} \right]$$

$$v = \sqrt{\frac{2GM_e}{R_e}}$$

## 1.3 | (1c)

$$v = \sqrt{\frac{2GM_e}{R_e}}$$

$$\left[ = \sqrt{\frac{2 \cdot 6.674 \cdot 10^{-11} \cdot 5.972 \times 10^{24}}{6.371 \cdot 10^6}} \right]$$

$$= 11185.7 m/s$$

$$= 25020.1 mph$$

## 2 | Problem 2

$$\sum_{i=1}^n \vec{F}_{net,i} = \left( \sum_{i=1}^n m_i \right) \ddot{\vec{r}}_{CM}$$

$$\sum_{i=1}^n m_i \ddot{\vec{r}}_i = \left( \sum_{i=1}^n m_i \right) \ddot{\vec{r}}_{CM}$$

$$\int \int \sum_{i=1}^n m_i \ddot{\vec{r}}_i dt dt = \int \int \left( \sum_{i=1}^n m_i \right) \ddot{\vec{r}}_{CM} dt dt \quad \text{Both constants are the same constant on both sides}$$

$$\int \sum_{i=1}^n m_i \dot{\vec{r}}_i dt + C_1 = \int \left( \sum_{i=1}^n m_i \right) \dot{\vec{r}}_{CM} dt + C_1$$

$$\sum_{i=1}^n m_i \vec{r}_i + C_1 t + C_2 = \left( \sum_{i=1}^n m_i \right) \vec{r}_{CM} + C_1 t + C_2$$

of the equation so they will cancel out. The sum of all mass is just  $M$ .  $\left[ \vec{r}_{CM} = \frac{1}{M} \sum_{i=1}^n m_i \vec{r}_i \right]$

## 3 | Problem 3

Any force within a system will have an opposite force applied as well (Newton's 3rd law). Therefore, forces within a system will cancel out and will have no effect on the center of mass.

## 4 | Problem 4

$$\vec{v} = \frac{< 1, -4, 1 > + 2 < -3, -2, 6 > + 3 < 2, 5, -3 > + 4 < -2, 4, 6 >}{1 + 2 + 3 + 4}$$

$$=< -0.7, 2.3, 2.8 >$$

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