

## 1 | Axler6.23 orthonormal

**def**

- A list of vectors is called *orthonormal* if each vector in the list has norm 1 and is orthogonal to all other vectors in the list
- in other words, a list  $e_1, \dots, e_m$  of vectors in  $V$  is orthonormal if

$$\langle e_j, e_k \rangle = \begin{cases} 1 & \text{if } j = k \\ 0 & \text{if } j \neq k \end{cases}$$

## 2 | results

### 2.1 | Axler6.25 norm of an orthonormal linear combination Pythagorean theorem

If  $e_1, \dots, e_m$  is an orthonormal list of vectors in  $V$  then

$$\|a_1 e_1 + \dots + a_m e_m\|^2 = |a_1|^2 + \dots + |a_m|^2$$

Pythagorean theorem

### 2.2 | Axler6.26 orthonormal lists are linearly independent

Any orthogonal list is linearly independent, so an orthonormal list must also be linearly independent.

Or you can use 6.25, set it to zero, and see that all the coefficients must be zero.