1 | **Problem 1**

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$$
\begin{aligned}
KE_{total} &= \sum_{i=1}^{N} \frac{1}{2} m_i v_i^2 \\
&= \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{v}_i \cdot \vec{v}_i) \\
&= \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{V}_{CM}(t)^2 + 2\vec{V}_{CM}(t) \cdot \vec{v}'_i(t) + \vec{v}'_i(t)^2) \\
&= \sum_{i=1}^{N} \frac{1}{2} m_i (\vec{V}_{CM}(t)^2 + 2\vec{V}_{CM}(t) \cdot \vec{v}'_i(t)) + \sum_{i=1}^{N} \frac{1}{2} m_i (v_i'^2) \\
&= \frac{1}{2} M V_{CM}^2 + V_{CM} \cdot \sum_{i=1}^{N} m_i \vec{v}'_i + \sum_{i=1}^{N} \frac{1}{2} m_i (v_i'^2) \\
&= \frac{1}{2} M v_{CM}^2 + \frac{1}{2} \sum_{i=1}^{N} m_i (v_i'^2)
\end{aligned}
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We know that the equation of the center of mass is $\vec{R}_{CM} = \frac{1}{M}\sum_{i=0}^{N} m_i \vec{r}_i$. We can rewrite this in the reference frame of the center of mass, and we get $\vec{R}'_{CM} = \frac{1}{M}\sum_{i=0}^{N} m_i \vec{r}'_i = 0$. We can differentiate both sides to get $0 = \frac{d}{dt}\frac{1}{M}\sum_{i=0}^{N} m_i \vec{r}'_i = \frac{1}{M}\sum_{i=0}^{N} m_i \vec{v}'_i = \sum i = 0^N m_i \vec{v}'_i$. Therefore, we know that $\sum_{i=0}^{N} m_i \vec{v}'_i = 0$ and we can take it out of the equation in the solution above.

More intuitively, the distance between a point on a mass and its center of mass is always constant, so its change over time should be 0, making the term also equal 0.