

## 1 | Axler6.27 orthonormal basis

def

An *orthonormal basis* of  $V$  is an orthonormal list of vectors in  $V$  that is also a basis of  $V$ .

Pretty self explanatory.

How do we find an orthonormal basis? see the

## 2 | results

### 2.1 | Axler6.28 orthonormal list of the right length is a basis

Because it's linearly independent, and linearly independent lists of the right length are bases (Axler2.39).

### 2.2 | Axler6.30 vector as a linear combo of orthonormal basis

Suppose  $e_1, \dots, e_m$  is an orthonormal basis of  $V$  and  $v \in V$ . Then,

$$v = \langle v, e_1 \rangle e_1 + \dots + \langle v, e_n \rangle e_n$$

and

$$\|v\|^2 = |\langle v, e_1 \rangle|^2 + \dots + |\langle v, e_n \rangle|^2$$

By taking the inner product of both sides of the equation

$$v = a_1 e_1 + \dots + a_n e_n$$

with  $e_j$  for each  $e_j$ . And also the Pythagorean theorem

## 3 | see also

### 3.1 | orthonormal