1 | **shaded area problems**

choices. Lets do sea shell:

$$\begin{aligned}
r &= \sqrt{\theta} \\
A &= \int_0^{2\pi} \frac{1}{2}\sqrt{\theta}d\theta \\
&= \frac{1}{2}\int_0^{2\pi} \theta^{\frac{1}{2}}d\theta \\
&= \frac{1}{\cancel{2}}\cancel{2}\theta^{\frac{3}{2}} \\
&= \boxed{\theta^{\frac{3}{2}}}
\end{aligned}$$

Okay, now for the petal: OH WAIT I FORGOT TO DO THE one-half r-squared

$$\begin{aligned}
r &= \sin 2\theta \\
A &= \int_0^{\frac{\pi}{2}} \sin 2\theta d\theta \\
A &= 2\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta \\
A &= 2\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta \\
A &= 2\left(\sin^2\theta - \int_0^{\frac{\pi}{2}} \cos\theta\sin\theta d\theta\right) && \text{Integration by parts} \\
2\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta &= 2\left(\sin^2\theta - \int_0^{\frac{\pi}{2}} \cos\theta\sin\theta d\theta\right) \\
2\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta &= \sin^2\theta
\end{aligned}$$

???? i don't know how to integrate by parts
Anyway, Wolfram Alpha says that the area is $-\frac{1}{2}\cos 2\theta$

2 | **intersection of two circles**

$$\begin{aligned}
r &= 1 \\
r &= 2\cos\theta
\end{aligned}$$

The intersection

$$\begin{aligned}
1 &= 2\cos\theta \\
\frac{1}{2} &= \cos\theta \\
\theta &= \frac{\pi}{3}, -\frac{\pi}{3}
\end{aligned}$$

So, lets take the integral between those, then subtract the isocelese triangle:

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}1^2 d\theta = \frac{1}{2}\frac{\pi}{3} - -\frac{1}{2}\frac{\pi}{3} \quad = \frac{\pi}{3}$$

Now, lets find the area of the triangle to subtract:
the base:

$$y_1 = \sin\frac{\pi}{3} \quad = \frac{\sqrt{3}}{2}$$
$$y_2 = -\frac{\sqrt{3}}{2}$$
$$b = \sqrt{3}$$

the height of the triangle:

$$h = \cos\frac{\pi}{3} \quad = \frac{1}{2}$$

So the area of the triangle is $\frac{1}{2}bh = \frac{\sqrt{3}}{4}$
and thus,

$$A = 2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) \quad = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

3 | overlap between $r = 1$ and $1 + 2\cos 2\theta$

The plan: do the part outside the circle, then subtract it from the full big petal. Add the inner petal, and multiply by two.

3.1 | the bounds

Lets start with finding the points where things cross the origin

$$1 + 2\cos 2\theta = 0$$
$$2\cos 2\theta = -1$$
$$\cos 2\theta = -\frac{1}{2}$$
$$2\theta = \frac{2\pi}{3}, -\frac{2\pi}{3}$$
$$\theta = \frac{\pi}{3}, -\frac{\pi}{3}$$

The bounds for the outer portion of the big petal is

$$1 + 2\cos 2\theta = 1$$
$$\cos 2\theta = 0$$
$$2\theta = \frac{\pi}{2}, -\frac{\pi}{2}$$
$$\theta = \frac{\pi}{4}, -\frac{\pi}{4}$$

3.2 | large petal area

$$A_p = \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \frac{1}{2}(1 + 2\cos 2\theta)^2 d\theta$$

3.3 | large petal outer area

$$A_o = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{2}\left((1 + 2\cos 2\theta)^2 - 1\right) d\theta$$

3.4 | small petals

$$A_s = \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \frac{1}{2}(1 + 2\cos 2\theta)^2 d\theta$$

3.5 | total area

$$A = 2(A_p - A_o + A_s) = 2\pi - 4$$