1 | adjoint, T^* def

Suppose $T \in L(V, W)$. The *adjoint* of T is the function $T^* : W \to V$ s.t.

$$\langle Tv, w \rangle = \langle v, T^*w \rangle$$

Apparently there's another meaning for 'adjoint' in linear algebra too, but it's not covered here.

This definition makes sense because of the Riesz Representation Theorem...:question:

Adjoints are kind of like complex conjugates, as seen in Axler 7.10

2 | results

2.1 | Useful technique: 'flip T^* from one side of an inner product to become T on the other side'

You can always do this by definition of adjoint.

2.2 | Axler7.5 the adjoint is a linear map

If
$$T \in \mathcal{L}(V, W)$$
, then $T^* \in \mathcal{L}(W, V)$.

2.3 | Axler7.6 Properties of the adjoint

2.3.1
$$|(S+T)^* = S^* + T^*$$
 for all $S, T \in \mathcal{L}(V, W)$

2.3.2
$$|(\lambda T)^* = \overline{\lambda} T^*$$
 for all $\lambda \in \mathbb{F}$ and $T \in \mathcal{L}(V, W)$

2.3.3
$$|(T^*)^* = T$$
 for all $T \in L(V, W)$

2.3.4
$$|I^* = I|$$

2.3.5 $|(ST)^* = T * S*$ for all $T \in \mathcal{L}(V, W)$ and $S \in \mathcal{L}(W, U)$ where U is an inner product space over \mathbb{F}

2.4 | Axler7.7 null space and range of T^*

Suppose $T \in \mathcal{L}(V, W)$. Then,

2.4.1
$$T^* = (T)^{\perp}$$

2.4.2
$$|T^* = (T)^{\perp}$$

2.4.3 |
$$T=(T^*)^\perp$$

2.4.4 |
$$T = (T^*)^{\perp}$$

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