

We are given that the object m_1 collides with the rod with velocity v_0 , and the rod is floating in free space. Given m_1 , v_0 , m_2 , I_0 , and r , we are to figure out the final velocity of m_1 after collision v_f , the velocity of m_2 after collision v_{CM} , and of course the rotation of the rod after collision ω .

We are assuming that this collision is elastic.

We have, then, for conservation of linear momentum:

$$m_1 v_0 = m_1 v_f + m_2 v_{CM} \quad (1)$$

Furthermore, we understand that kinetic energy is also conserved here; therefore:

$$\frac{1}{2} m_1 v_0^2 = \left(\frac{1}{2} m_1 v_f^2 \right) + \left(\frac{1}{2} m_2 v_{CM}^2 \right) + \left(\frac{1}{2} I_0 \omega^2 \right) \quad (2)$$

$$\Rightarrow m_1 v_0^2 = (m_1 v_f^2) + (m_2 v_{CM}^2) + (I_0 \omega^2) \quad (3)$$

as the point mass does not have any rotational inertia, and the rod is not rotating at the start.

Lastly, we understand that the angular momentum is conserved through a collision; letting the origin as the center of mass of the rod:

$$m_0 r^2 \left(\frac{v_0}{r} \right) = m_0 r^2 \left(\frac{v_f}{r} \right) + I_0 \omega \quad (4)$$

We now have a system of three equations that can be combined to solve for three unknowns v_f , v_{CM} , and ω .

Performing the actual solution,

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var("I m1 v0 m2 r vf vcm w")
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expand(solve([m1*v0 == m1*vf+m2*vcm, m1*v0^2==m1*vf^2+m2*vcm^2+I*w^2, vf==w*r], vf, vcm, w, to_poly_sol
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