

## 1 | Problem 1

$$KE_{\text{total}} = \sum_{i=1}^N \frac{1}{2} m_i (v_i \cdot v_i)$$

$$KE_{\text{total}} = \sum_{i=1}^N \frac{1}{2} m_i (\vec{V}_{\text{CM}} + \vec{v}_i')^2$$

$$KE_{\text{total}} = \sum_{i=1}^N \frac{1}{2} m_i (\vec{V}_{\text{CM}}^2 + 2\vec{V}_{\text{CM}} \vec{v}_i' + (\vec{v}_i')^2)$$

$$KE_{\text{total}} = \sum_{i=1}^N \left( \frac{1}{2} m_i \vec{V}_{\text{CM}}^2 + m_i \vec{V}_{\text{CM}} \vec{v}_i' + \frac{1}{2} m_i (\vec{v}_i')^2 \right)$$

$$KE_{\text{total}} = \sum_{i=1}^N \frac{1}{2} m_i \vec{V}_{\text{CM}}^2 + \sum_{i=1}^N m_i \vec{V}_{\text{CM}} \vec{v}_i' + \sum_{i=1}^N \frac{1}{2} m_i (\vec{v}_i')^2$$

$$KE_{\text{total}} = \frac{1}{2} \vec{V}_{\text{CM}}^2 \sum_{i=1}^N m_i + \vec{V}_{\text{CM}} \sum_{i=1}^N m_i \vec{v}_i' + \sum_{i=1}^N \frac{1}{2} m_i (\vec{v}_i')^2$$

$$\text{Define } M = \sum_{i=1}^N m_i.$$

$$\vec{r}_{\text{CM}}' = \frac{1}{M} \sum_i m_i \vec{r}_i'$$

$\vec{r}_{\text{CM}}' = 0$  by definition (it is relative to itself).

$$0 = \frac{1}{M} \sum_i m_i \vec{r}_i'$$

Differentiate with respect to time.

$$0 = \frac{1}{M} \sum_i m_i \vec{v}_i'$$

$$0 = \sum_i m_i \vec{v}_i'$$

Eliminate the middle term  $\vec{V}_{\text{CM}} \sum_{i=1}^N m_i \vec{v}_i'$  as it is equal to 0.

$$KE_{\text{total}} = \frac{1}{2} \vec{V}_{\text{CM}}^2 \sum_{i=1}^N m_i + \sum_{i=1}^N \frac{1}{2} m_i (\vec{v}_i')^2$$

$$KE_{\text{total}} = \frac{1}{2} M \vec{V}_{\text{CM}}^2 + \sum_{i=1}^N \frac{1}{2} m_i (\vec{v}_i')^2$$