Ok so we know that:

$$|\vec{\tau}| = |\vec{F_\perp}||\vec{r}| \tag{1}$$

Different forces on different parts of a bar.

$$F\Delta x = F_1|r| = \frac{F_1}{2} \cdot |2\vec{r}| \tag{2}$$

Applying a parallel force, instead of a perpendicular force, at an angle, only the perpendicular component is causing rotation.

So really, the above statement is:

$$|\vec{\tau}| = |\vec{F}| \sin\theta |\vec{r}| \tag{3}$$

The lever arm, of course, can be looked at as:

$$|\vec{\tau}| = |\vec{F}||\vec{r}| \sin\theta \tag{4}$$

And therefore, $|\vec{r}_{\perp \vec{F}}|$ is the length of the lever arm.

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What's the direction of torque?

$$|A \times B| = |\vec{A}||\vec{B}| \sin\theta \tag{5}$$

Direction:

Use the right hand rule rotating 1st vector into the 2nd vector. (i.e. rotating $\vec{A}$ into $\vec{B}$. Fingers curl into the direction of rotation.

# 1 | **Rotational vs. Linear Dynamics**

| Thing | Linear | Rotational |
|---|---|---|
| Force | ma | Ialpha |
| Acceleration | a | alpha |
| Inertia | m | I |
| Force | dp/dt | dL/dt |
| Momentum | p=mV | L=r x mV |

# 2 | **Angular Momentum**

$$\tau = \vec{r} \times \vec{F} \tag{6}$$

$$= \vec{r} \times M\frac{d\vec{v}}{dt} \tag{7}$$

$$= \frac{dL}{dt} \tag{8}$$

Let's check that:

$$\vec{L} = \vec{r} \times m\vec{V} \tag{9}$$

Well, we can take the derivative on both sides:

$$\frac{d\vec{L}}{dt} = m\left(\frac{d\vec{r}}{dt} \times \vec{V} + \vec{r} \times \frac{d\vec{V}}{dt}\right) \tag{10}$$

We know that $\vec{V}$ is simply $\frac{d\vec{r}}{dt}$, and so:

$$\frac{d\vec{L}}{dt} = m\left(\vec{V} \times \vec{V} + \vec{r} \times \frac{d\vec{V}}{dt}\right) \tag{11}$$

$$\frac{d\vec{L}}{dt} = m\left(0 + \vec{r} \times \frac{d\vec{V}}{dt}\right) \tag{12}$$

$$\frac{d\vec{L}}{dt} = m\vec{r} \times \frac{d\vec{V}}{dt} \tag{13}$$

And lastly

$$\frac{d\vec{L}}{dt} = m\vec{r} \times \vec{F} \tag{14}$$

:tada:

So $\frac{dL}{dt} = \tau$.

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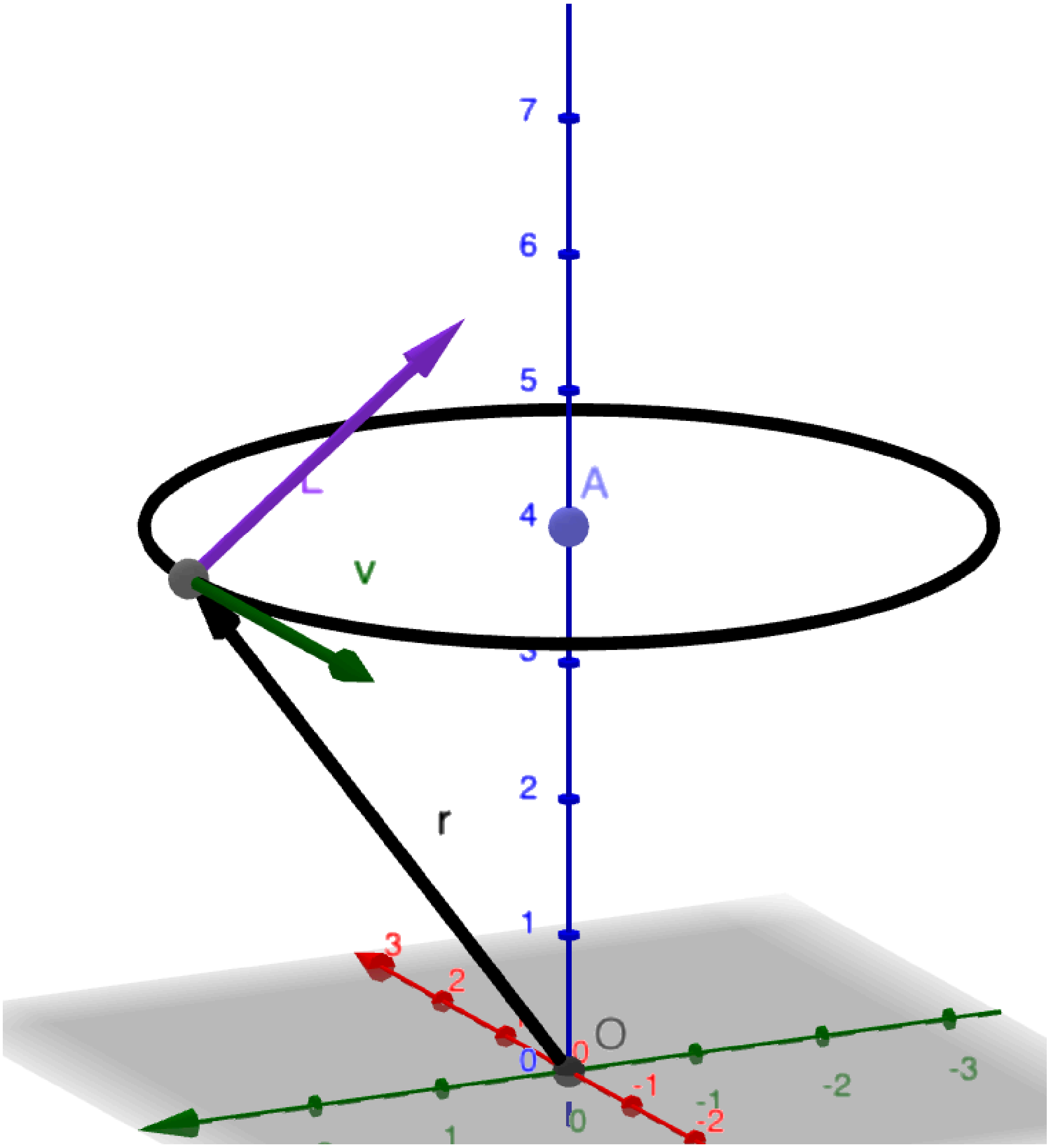
Remember to use the same reference frame for the content.

# 3 | A bit of Uniform Circular Motion




$$|\vec{V}| = \omega R \tag{15}$$

And really, the vector $V$ is represented as:

$$\vec{V} = \omega \times \vec{R} \tag{16}$$

$\omega$ would be defined in a direction such that the above statement makes sense.

To maintain uniform circular motion, we need to balance a system onto both directions

# 4 | Relationship between Torque and Angular Momentum

$$\vec{\tau} = \frac{d\vec{L}}{dt} \tag{17}$$

and, for:

$$\vec{L} \triangleq \frac{d\vec{L}_{syst}}{dt} \tag{18}$$

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$$\vec{L} \triangleq \vec{r} \times \vec{p} \tag{19}$$

$$\vec{\tau} \triangleq \vec{r} \times \vec{F} \tag{20}$$

and so, finally:

$$\vec{\tau} = \frac{d\vec{L}}{dt} \tag{21}$$

for a point mass.