

## 1 | Reading

### 1.1 | Definition of a Definite Integral

For each interval  $[x_i, x_{i+1}]$ , we choose  $x_i^*$  in the interval to be the position of the minimum (for lower bound) or maximum (upper bound) value.

## 2 | Problems

### 2.1 | exr1.3

Using the left edge: -8.4375

Summation notation for left edge approximation:

$$\sum_{i=0}^n \underbrace{\frac{b-a}{n}}_{\text{width}} \underbrace{f\left(a + \frac{b-a}{n}i\right)}_{\text{height}}$$

### 2.2 | exr1.4 (in class)

0.21875 using the left estimate

### 2.3 | exr1.5

#### 2.3.1 | left estimate

34.7 feet (add all except last number and divide by two, because we are stopping at 3.0 seconds in.)

#### 2.3.2 | right estimate

44.8 feet (add the last number and drop the zero from the beginning)

#### 2.3.3 | middle estimate

Not enough information to do it for  $\Delta x = 0.5$ , so I will use  $n = 3$  aka  $\Delta x = 1$

$$6.2 + 14.9 + 19.4 = 40.5 \text{ feet}$$

### 2.4 | exr1.6

$$2.4.1 \mid \int_0^1 \sqrt{x^2 + 1} dx$$

$\sqrt{x^2 + 1}$  is the length the hypotenuse of a triangle with leg-lengths 1 and  $x$ . Because  $x$  is continuous, this is like the area of a right triangle with leg-lengths 1 and 1, which is  $\boxed{\frac{1}{2}}$ .

1. **TODO** Wolfram Alpha doesn't agree though

Probably because as you take approximations, there will be overlap, so the actual value is bigger than I think it is.

I also don't know how to take the anti-chain-rule, so I don't know how to integrate the function symbolically.

$$2.4.2 \mid \int_0^3 (x-1) dx$$

Not sure area wise, but the anti-derivative is guess-able:

$$\frac{d}{dx} \left( \frac{x^2}{2} - x \right) = x - 1$$

$$\frac{3^2}{2} - 3 = 1.5$$

2.5 **| exr1.7**2.5.1 **| right endpoint approx for  $y = x^2$** 

$$\sum_{i=1}^{n+1} \Delta x f(i\Delta x) = \sum_{i=1}^{n+1} \frac{1}{n} \left( \frac{i}{n} \right)^2$$

where  $\Delta x = \frac{1}{n}$

2.5.2 **| general form for left-side riemann sum**

See exr1.3

2.6 **| exr1.11**

$$\int_{\pi}^{2\pi} \cos(x) dx = \lim_{n \rightarrow \infty} \sum_{i=0}^n \frac{\pi}{n} \cos \left( \pi + \frac{i\pi}{n} \right)$$

2.7 **| exr1.12**2.7.1 **| 1**

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \Delta x \sqrt{4 + (1 + k\Delta x)^2} \text{ where } \Delta x = \frac{2}{n}$$

2.7.2 | 2

$$\lim_{n \rightarrow \infty} \sum_{k=0}^n \Delta x (2 + k\Delta x)^2 + \frac{1}{2 + k\Delta x} \text{ where } \Delta x = \frac{3}{n}$$

2.8 | **exr1.13**

2.8.1 | 1

$$\int_0^1 \frac{e^x}{1+x} dx$$

2.8.2 | 2

$$\int_2^5 x \sqrt{1+x^3} dx$$

2.8.3 | 3

$$\int_1^3 \frac{x}{x^2+4} dx$$

2.9 | **exr1.14**

Graphically, it's the right triangle from origin to B minus the one from origin to A. Algebraically,

$$\begin{aligned} \frac{d}{dx} \frac{1}{2} x^2 &= x \\ \Rightarrow \int_a^b x dx &= \frac{1}{2} b^2 + c - \left( \frac{1}{2} a^2 + c \right) = \frac{b^2 - a^2}{2} \end{aligned}$$