

## 1 | adjoint, $T^*$

def

Suppose  $T \in \mathcal{L}(V, W)$ . The *adjoint* of  $T$  is the function  $T^* : W \rightarrow V$  s.t.

$$\langle Tv, w \rangle = \langle v, T^*w \rangle$$

Apparently there's another meaning for 'adjoint' in linear algebra too, but it's not covered here.

This definition makes sense because of the Riesz Representation Theorem... :question:

Adjoint is kind of like complex conjugates, as seen in Axler 7.10

## 2 | results

### 2.1 | Useful technique: 'flip $T^*$ from one side of an inner product to become $T$ on the other side'

You can always do this by definition of adjoint.

### 2.2 | Axler 7.5 the adjoint is a linear map

If  $T \in \mathcal{L}(V, W)$ , then  $T^* \in \mathcal{L}(W, V)$ .

### 2.3 | Axler 7.6 Properties of the adjoint

2.3.1 |  $(S + T)^* = S^* + T^*$  for all  $S, T \in \mathcal{L}(V, W)$

2.3.2 |  $(\lambda T)^* = \bar{\lambda}T^*$  for all  $\lambda \in \mathbb{F}$  and  $T \in \mathcal{L}(V, W)$

2.3.3 |  $(T^*)^* = T$  for all  $T \in \mathcal{L}(V, W)$

2.3.4 |  $I^* = I$

2.3.5 |  $(ST)^* = T^*S^*$  for all  $T \in \mathcal{L}(V, W)$  and  $S \in \mathcal{L}(W, U)$  where  $U$  is an inner product space over  $\mathbb{F}$

### 2.4 | Axler 7.7 null space and range of $T^*$

Suppose  $T \in \mathcal{L}(V, W)$ . Then,

2.4.1 |  $\text{null } T^* = (\text{range } T)^\perp$

2.4.2 |  $\text{range } T^* = (\text{null } T)^\perp$

2.4.3 |  $\text{null } T = (\text{range } T^*)^\perp$

2.4.4 |  $\text{range } T = (\text{null } T^*)^\perp$