

#flo #hw

## 1 | Linear Maps

no one gets excited about vector spaces -axler

the interesting part: linear maps!

```
title: learning objectives
- fundamentals theorem of linear maps
- matrix of linear map w.r.t. given bases
- isomorphic vec spaces
- product spaces
- quotient spaces
- duals spaces
  - vector space
  - linear map
```

## 2 | The vector space of linear maps

**key definition!**

```
title: linear map
aka *linear transformation.*
```

a *linear map* from  $V$  to  $W$  is a function  $T: V \rightarrow W$  with the following properties:

**\*\*additivity\*\***

$T(u+v) = Tu + Tv$  for all  $u, v \in V$ ;

**\*\*homogeneity\*\***

$T(\lambda v) = \lambda(Tv)$  for all  $\lambda \in F$  and  $v \in V$ .

the functional notation  $T(V)$  is the same as the notation  $Tv$  when talking about linear maps.

```
title: notation --  $L(V, W)$ 
```

the set of all linear maps from  $V$  to  $W$ .

### 2.0.1 | examples of linear maps

- 0?
  - 0 is the func that takes each ele from some vec space to the additive iden of another vec space.
    - \*  $0v = 0$
    - \* left: func from  $V$  to  $W$ , right: additive iden in  $W$
    - \* #question what does it mean for it to be a function from  $V$  to  $W$ ?
- identity, denoted  $I$

- $Iv = v$
- maps each element to itself linear transformation like a `.map`?
- differentiation and integration!
- multiplication by  $x^2$  (on polynomials)
- shifts! defined as,  $T(x_1, x_2, x_3, \dots) = (x_2, x_3, \dots)$ 
  - #question this is an example, but how do we define it as a transformation? or is giving an example in the general case the same thing as defining a transformation?
- from  $R^3 \rightarrow R^2$  ? #question what? how does this work?
- #review how this dimension shift works..

title: linear maps and basis of domain

Suppose  $v_1, \dots, v_n$  is a basis of  $V$  and  $w_1, \dots, w_n \in W$ . Then there exists a unique linear map  $T$  such that  $Tv_j = w_j$  for each  $j=1, \dots, n$ .

we can uniquely map between the basis of a subspace and a list of equal len in a diff subspace?

#question wait how does the uniqueness proof work here at the end?

## 2.0.2 | algebraic operations on $L(V, W)$

title: addition and SCAMUL

Suppose  $S, T \in L(V, W)$  and  $\lambda \in F$ . The *sum* of  $S+T$  and the *product*  $\lambda T$  are the linear maps defined by  $(S+T)(v) = Sv + Tv$  and  $(\lambda T)(v) = \lambda(Tv)$  for all  $v \in V$ .

oh jeez..

title:  $L(V, W)$  is a vector space!

with the operations of addition and SCAMUL as defined above,  $L(V, W)$  is a [[file:KBe20math530refVectorSpace.tex|vector space]].

and another one.

title: product of linear maps

if  $T \in L(U, V)$  and  $S \in L(V, W)$ , then the *product*  $ST \in L(U, W)$  is defined by  $(ST)(u) = S(Tu)$  for all  $u \in U$ .

$S \cdot T$ ?? what is this symbol?

title: algebraic props of products of linear maps

- associative
- identity
- distributive properties

multiplication of linear maps is not commutative! ie.  $ST = TS$  isn't always true.

title: linear maps take 0 to 0

suppose  $T$  is a linear map from  $V$  to  $W$ . Then  $T(0) = 0$

#review this chapter...

basically all just result blocks and nothing else

i don't have an intuitive understanding of the concept of a map. perhaps look into 3b1b vid on linear t.