$$1 \mid \int_{1}^{2} \frac{e^{\frac{1}{x}}}{x^{2}} dx$$

$$\int -e^{u} du = -e^{u} + C$$

$$= -e^{\frac{1}{2}} + e^{\frac{1}{1}}$$

$$= e - e^{\frac{1}{2}}$$

$$2 \mid \int_{0}^{1} r e^{\frac{r}{2}} dr$$

$$\int_{0}^{1} re^{\frac{r}{2}} dx \implies r2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr$$

$$= 2re^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr$$

$$= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}}$$

$$= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}}$$

$$\implies 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4)$$

$$= 4 - 2e^{\frac{1}{2}}$$

$$3 \mid \int_4^9 \frac{\ln y}{\sqrt{y}} dy$$

$$\begin{split} \int \frac{\ln y}{\sqrt{y}} dy &= 2 \ln y \sqrt{y} - \int 2 \frac{1}{y} \sqrt{y} dy \\ &= 2 \ln y \sqrt{y} - \int 2 \frac{1}{\sqrt{y}} dy \\ &= 2 \ln y \sqrt{y} - 2 \int y^{-\frac{1}{2}} dy \\ &= 2 \ln y \sqrt{y} - 4 \sqrt{y} + C \\ &= 2 \sqrt{y} (\ln y - 2) + C \\ \Longrightarrow & 6 (\ln 9 - 2) - 4 (\ln 4 - 2) \end{split}$$

Page 2

4 | 
$$\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$$

Let  $u = \sqrt{x}$ ,  $du = \frac{1}{2\sqrt{x}}dx$ , dx = 2udu

$$\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx = 2 \int u \cos u du$$

$$= 2u \sin u - 2 \int \sin u du$$

$$= 2u \sin u + 2 \cos u$$

$$= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x}$$

$$\Rightarrow 2\pi^{\frac{1}{4}} \sin \pi^{\frac{1}{4}} + 2 \cos \pi^{\frac{1}{4}} - 2$$

## $5 \mid \int_1^e \sin \ln x dx$

$$\begin{split} \int_1^e \sin\ln x dx &= x \sin\ln x - \int x \frac{1}{x} \cos\ln x dx \\ &= x \sin\ln x - \int \cos\ln x dx \\ &= x \sin\ln x - \left(x \cos\ln x + \int x \frac{1}{x} \sin\ln x dx\right) \\ &= x \sin\ln x - x \cos\ln x - \int \sin\ln x dx \\ 2\int \sin\ln x dx &= x \sin\ln x - x \cos\ln x \\ \int \sin\ln x dx &= \frac{1}{2} x (\sin\ln x - \cos\ln x) \\ &\Rightarrow \quad \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0) \\ &= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2} \end{split}$$

6 | 
$$\int_0^1 \frac{x^3}{\sqrt{4+x}} dx$$

Let  $u = 4 + x^2$ , du = 2xdx

$$\int \frac{x^3}{\sqrt{4+x}} dx = \frac{1}{2} \int \frac{(u-4)}{\sqrt{u}} du$$
$$= \frac{1}{2} \int \sqrt{u} dx - 2 \int \frac{1}{\sqrt{u}} dx$$

## 7 | (additional problems)

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7.1 |  $\int \sin^2 x dx$ 

$$\begin{split} \int \sin^2 x dx &= -\sin x \cos x + \int \cos^2 x dx \\ &= -\sin x \cos x + \int 1 dx - \int \sin^2 x dx \\ 2 \int \sin^2 x dx &= -\sin x \cos x + x \\ \int \sin^2 x dx &= \frac{1}{2} (x - \sin x \cos x) \end{split}$$

7.2 |  $\int \cos^2 x dx$ 

$$\int \cos^2 x dx = \cos x \sin x + \int \sin^2 x dx$$

$$= \cos x \sin x + \int 1 dx - \int \cos^2 x dx$$

$$2 \int \cos^2 x dx = \cos x \sin x + x$$

$$\int \cos^2 x dx = \frac{1}{2} \cos x \sin x + \frac{x}{2} + C$$

7.3 |  $\int \sin^2 x \cos^2 x dx$ 

$$\begin{split} \sin 2x &= 2\sin x \cos x \\ &\int \sin^2 x \cos^2 x dx = \frac{1}{4} \int \sin^2 2x dx \\ &\text{Let } u = 2x, du = 2dx \\ &= \frac{1}{8} \int \sin^2 u du \\ &= \frac{1}{8} \frac{1}{2} (u - \sin u \cos u) \\ &= \frac{1}{16} (2x - \sin 2x \cos 2x) + C \end{split}$$

7.4 |  $\int \sin^3 x dx$ 

$$\int \sin^3 x dx = \int \sin x (1 - \cos^2 x) dx$$

$$= \int \sin x dx - \int \cos^2 x \sin x dx$$
Let  $u = \cos x$ ,  $du = -\sin x dx$ 

$$= \int \sin x dx + \int u^2 du$$

$$= -\cos x + \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}\cos^3 x - \cos x + C$$

7.5 |  $\int \cos^3 x dx$ 

$$\int \cos^3 x dx = \int \cos x (1 - \sin^2 x) dx$$

$$= \int \cos x - \sin^2 x \cos x dx$$

$$= \sin x - \int u^2 du$$

$$= \sin x - \frac{1}{3}u^3 + C$$

$$= \sin x - \frac{1}{3}\sin^3 x + C$$