6 | Deriving arclength forumlas

$$S = \int_C ds = \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2}$$

6.1 | if $y = f(x)$

$$\begin{aligned}
\int_C ds &= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\left(\frac{\Delta x^2}{\Delta x^2} + \frac{\Delta y^2}{\Delta x^2}\right) \Delta x^2} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \Delta x \\
&= \int \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx
\end{aligned}$$

6.2 | if $y = y(t)$ and $x = x(t)$

$$\begin{aligned}
S &= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\Delta x_i^2 + \Delta y_i^2} \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{\left(\frac{\Delta x_i^2}{\Delta t_i^2} + \frac{\Delta y_i^2}{\Delta t_i^2}\right) \Delta t_i^2} \\
&= \int \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt
\end{aligned}$$

7 | applying arclength formulas

Lets use the curve from problem four (see assignment 3).

$$r = 1 + 2\cos\theta \frac{dr}{d\theta} \quad = -2\sin\theta$$

Then, applying the arclength formula for polar equations:

$$\begin{aligned}
S &= \int \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta \\
&= \int \sqrt{r^2 + (-2\sin\theta)^2} d\theta \\
&= \int \sqrt{r^2 + 4\sin^2\theta} d\theta
\end{aligned}$$

Check out the desmos.