

#source Axler2.A

## 1 | #definition span

The set of all linear combinations of a list of vectors  $v_1, \dots, v_m$  in  $V$  is called the span of  $v_1, \dots, v_m$ , denoted  $\text{span}(v_1, \dots, v_m)$ :

$$\text{span}(v_1, \dots, v_m) = \{a_1 v_1 + \dots + a_m v_m \mid a_1, \dots, a_m \in F\}$$

And the span of an empty list  $()$  is  $\{0\}$  - This is just to make Axler2.C work out nicely (KBeRefLinAlgDimension)

## 2 | Properties

- The span is the smallest containing subspace
  - The span of a list of vectors in  $V$  is the smallest subspace of  $V$  containing all the vectors in the list.

### 2.1 | #definition spans

If  $\text{span}(v_1, \dots, v_m) = V$ , then  $v_1, \dots, v_m$  **spans**  $V$

## 3 | Examples

### 3.1 | Axler 2.9

Suppose  $n$  is a positive integer. Show that  $(1, 0, \dots, 0), (0, 1, 0, \dots, 0), \dots, (0, \dots, 0, 1)$  spans  $F^n$ . - Basically, if a list of vectors spans a vector space then linear combinations of those vectors (almost like colloquial polynomials of those vectors) can form each vector in the space. - In this case, the vector space  $F^n$  is a list of vectors in  $F$ , and having the 1 in each slot is enough to, when scalar multiplied with  $a \in F$ , get all possibilities of  $F^n$ . - I need to wrap my head around this some more.