

#flo #inclass

# 1 | probability

## 1.1 | intro

given a sample space, a **probability map**  $P$  is a function from subsets of  $\Omega$  to  $[0,1]$  where  $P(\Omega) = 1$   
 can imagine a bunch of disjoint sets,  $A_1, A_2, A_3$ , ect. then the prob

$$P(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} P(A_i)$$

where all  $A_j$  are disjoint.

note:  $\Omega$  and the empty set are disjoint  $P(A^c)$  means a complement, or every outcome not in  $A$ , is just  $1 - P(A)$ .

## 1.2 | inclusion / exclusion

overlapping sets,  $A$  and  $B$  counting formula,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$  #extract

if we have three,  $P(a \cup b \cup c) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$   
 demotmot's problem? de montmort.

$A_i = i^{th}$  card has the number  $i$  on it  $P(\text{winning}) = P(A_1 \cup A_2 \cup \dots \cup A_n) = \sum P(A_i) - \sum P(A_i \cap A_j) + \sum P(A_i \cap A_j \cap A_k) \dots$  ?? goes to  $1 - 1/e$

this is called a derangement > In combinatorial mathematics, a derangement is a permutation of the elements of a set, such that no element appears in its original position. In other words, a derangement is a permutation that has no fixed points. -wiki

## 1.3 | independence

if we flip a coin and then roll a die,  $P(2H) = P(H) P(2) = 1/2 * 1/6 = 1/12$

2 events  $A$  and  $B$  are independent if  $P(a \cap B) = P(A) * P(B)$

conditional probability! def  $P(A|B) = \frac{P(A \cap B)}{P(B)}$   $P(A|B)P(B) = P(B|A)P(A) = P(A \text{ and } B)$

this is basically bayes theorem:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

which is also written as:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B \text{ and } A) + P(B \text{ and } A^c)} = \frac{P(B|A)P(A)}{P(B|A) * P(A) + P(B|A^c) * A^c}$$

but don't memorize it in this way.

## 1.4 | example

- disease, occurs 1 in a 1000 people
  - 98% right when person has the disease
  - 90% of being right when they don't have it
  - but, we have 100% accuracy if we do an unpleasant test, which we wan't to avoid
- screen test comes back positive. what is the prob that you have the disease?
  - $P(\text{disease} \mid \text{positive test})$

	positive	negative
disease	.98/1000	0.02/1000
no disease	.1 * 999/1000	.9 * 999/1000

as marginal is 1/1000 -> 999/1000

so,  $P(d|p) = \frac{P(\text{dispease and positive})}{P(\text{positive})}$

we have two ways to test positive, so we can add them. this yields around 1%.

this is called, switching conditioning. so don't panic about positive tests until you know what is going on about them!