

3 | $\int \ln x dx$

$$\begin{aligned}
 \int \ln x dx &= \int 1 \ln x dx \\
 &= x \ln x - \int x \frac{1}{x} dx \\
 &= x \ln x - \int 1 dx \\
 &= \boxed{x \ln x - x}
 \end{aligned}$$

4 | $\int \tan^{-1} x dx$

$$\begin{aligned}
 \int \tan^{-1} x dx &= x \tan^{-1} x - \int x \frac{1}{x^2 + 1} dx \\
 &= x \tan^{-1} x - \frac{1}{2} \int \frac{du}{u} \\
 &= x \tan^{-1} x - \frac{1}{2} \ln u + C \\
 &= x \tan^{-1} x - \frac{1}{2} \ln(x^2 + 1) + C
 \end{aligned}$$

5 | $\int x \sec^2 x dx$

$$\begin{aligned}
 \int x \sec^2 x dx &= x \tan x - \int \tan x dx \\
 &= x \tan x + \ln |\cos x| + C
 \end{aligned}$$

6 | $\int x^2 e^{5x} dx$

$$\begin{aligned}
 \int x^2 e^{5x} dx &= x^2 \frac{1}{5} e^{5x} - \int 2x \frac{1}{5} e^{5x} dx \\
 &= x^2 \frac{1}{5} e^{5x} - 2x \frac{1}{25} e^{5x} + \int 2 \frac{1}{25} e^{5x} dx \\
 &= \frac{1}{5} e^{5x} \left(x^2 - \frac{2}{5} x + \frac{2}{25} \right) + C
 \end{aligned}$$

7 | $\int x^2 \cos x dx = f(x) - \int 2x \sin x dx$ Find $f(x)$

$$f(x) = x^2 \sin x$$

8 | $\int x \cos x dx$

$$\begin{aligned}\int x \cos x dx &= x \sin x - \int \sin x dx \\ &= x \sin x + \cos x + C\end{aligned}$$

9 | $\int x^2 \sin x dx$

$$\begin{aligned}\int x^2 \sin x dx &= -x \cos x - \int -\cos x dx \\ &= -x \cos x - \sin x + C\end{aligned}$$

10 | $\int x^3 e^{x^2} dx$

Let $u = x^2$

$$\begin{aligned}\int x^3 e^{x^2} dx &= \int x^2 x e^{x^2} dx \\ &= \int u \frac{1}{2} du e^u \\ &= \frac{1}{2} \int u e^u du \\ &= \frac{1}{2} u e^u - \frac{1}{2} \int e^u du \\ &= \frac{1}{2} u e^u - \frac{1}{2} e^u + C \\ &= \frac{1}{2} x^2 e^{x^2} - \frac{1}{2} e^{x^2}\end{aligned}$$

11 | $\int x^2 \ln x dx$

$$\begin{aligned}\int x^2 \ln x dx &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^3 \frac{1}{x} dx \\ &= \frac{1}{3} x^3 \ln x - \int \frac{1}{3} x^2 dx \\ &= \frac{1}{3} x^3 \ln x - \frac{1}{9} x^3 + C\end{aligned}$$

12 | $\int \cos \ln x dx$

$$\begin{aligned}
 \int 1 \cos \ln x dx &= x \cos \ln x + \int \sin \ln x dx \\
 &= x \cos \ln x + x \sin \ln x - \int \cos \ln x dx \\
 2 \int \cos \ln x dx &= x \cos \ln x - x \sin \ln x \\
 \int \cos \ln x dx &= \frac{1}{2} (x \cos \ln x - x \sin \ln x) + C
 \end{aligned}$$

Or you could use $u = \ln x$, apparently.

13 | multiple parts

13.1 | e

13.2 | $\int e^{2x} \cos 3x dx$

$$\begin{aligned}
 \int e^{2x} \cos 3x dx &= \cos 3x \frac{1}{2} e^{2x} + \int 3 \sin 3x \frac{1}{2} e^{2x} dx \\
 &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \int 3 \cos 3x \frac{1}{4} e^{2x} dx \\
 &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} - \frac{9}{4} \int e^{2x} \cos 3x dx \\
 \frac{13}{4} \int e^{2x} \cos 3x dx &= \cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} + C \\
 \int e^{2x} \cos 3x dx &= \frac{4}{13} \left(\cos 3x \frac{1}{2} e^{2x} + 3 \sin 3x \frac{1}{4} e^{2x} \right) + C \\
 &= \frac{2}{13} e^{2x} \left(\cos 3x + \frac{3}{2} \sin 3x \right) + C
 \end{aligned}$$

13.3 | evaluate previous from $[0, \frac{\pi}{6}]$

$$\frac{3}{13} e^{\frac{\pi}{3}} - \frac{2}{13}$$

14 | $\int \sec^3 x dx$

$$\begin{aligned}\int \sec^3 x dx &= \int \sec x \sec^2 x dx \\&= \sec x \tan x - \int \sec x \tan^2 x dx \\&= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx \\&= \sec x \tan x - \int \sec^3 x dx - \int \sec x dx \\2 \int \sec^3 x dx &= \sec x \tan x - \int \sec x dx \\&= \sec x \tan x - \ln |\sec x + \tan x| + C \quad \int \sec^3 x dx = \frac{1}{2} (\sec x \tan x - \ln |\sec x + \tan x|) + C\end{aligned}$$