- · Vector spaces and fields are like groups
 - With 2 operations
- Vector
 - direction and magnitude
 - numbers with an order
 - * list = ordered set
 - * Nx1 matrix
 - A vector is not just an Nx1 matrix. A vector exists in a vector space
 - * might be full of physics vectors, matrices, or polynomials
- Field
 - Addition and multiplication might be different
 - * might be related to normal addition/multiplication
 - * might by any binary operation
 - * Addition is "primary" operation, multiplication is "secondary"
 - · addition is really good (more group like)
 - · multiplication needs to exclude the additive identity (because it can't have an inverse)
 - * questions
 - · multiplication is repeated addition?
 - · not necessarily
 - · binary expressions?
 - · associative?
 - · both yes
 - * 1.3 demonstrates that the complex numbers are a field
 - · commutativity
 - · associativity
 - · identities
 - · additive inverse
 - · multiplicative inverse except additive identity
 - · distributive
 - usually means Reals or Complex
 - * integers mod 3 are a field
 - \cdot #bonushw show integers mod 3 are a field
 - higher dimensions
 - * \mathbb{R}^2 is a Cartesian plane, \mathbb{R}^4 is a space
 - operations
 - * addition is really nice (element wise)
 - * scalar multiplication is easy enough

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- * vector vector multiplication is hard to find
- two square roots of i
 - fundamental theorem of algebra
 - * (important)
 - So, i should have two square roots
 - Powers of i go in a circle (90 degrees rotation)
 - * Complex number rotation gives a preferred direction
 - * So that's why the quadrants are numbered in that direction
 - One can be found geometrically 20math530srcSquareRootI.png
 - We could also try it algebraically

*
$$(a+bi)^2 = i = a^2 - b^2 + 2abi$$

* so
$$a^2 - b^2 = 0$$
 and $2ab = 1$

- · dot product
 - How much of \vec{A} is in the direction of \vec{B} multiplied by the magnitude of \vec{B}

-
$$\vec{A} \cdot \vec{B} = |A||B|cos\theta$$

* #bonushw prove that ^^

- Identity:
$$\frac{\vec{A} \cdot \vec{B}}{|A||B|} = cos\theta$$

- · cross product
 - only works on 3x1 matrices
 - steps
 - * determinant
 - \star *i*, *j*, *k* are the unit vectors

*

$$\begin{bmatrix} 2\\1\\0\\0\end{bmatrix} \begin{bmatrix} 1\\2\\-1 \end{bmatrix} = \left| \begin{bmatrix} i & j & k\\2 & 1 & 0\\1 & 2 & -1 \end{bmatrix} \right| = i \left| \begin{bmatrix} 1 & 0\\2 & -1 \end{bmatrix} \right| - j \left| \begin{bmatrix} 2 & 0\\1 & -1 \end{bmatrix} \right| + k \left| \begin{bmatrix} 2 & 1\\1 & 2 \end{bmatrix} \right| = \begin{bmatrix} -1\\2\\3 \end{bmatrix}$$

- dropping zero: $0a = (0+0)a = 0a + 0a \Rightarrow 0a = 0$, so the additive identity can't have a multiplicative inverse (everything multiplied it will just be the additive identity)
 - 20math530srcFieldsMultiplyCannotBeGroup.png
- · determinant
 - measures the "size" of a matrix, denoted absolute value (relevant to inverse of a matrix multiplication)
- #todo #exr0n #future prove identities are unique