

1 | circumference

Start with the arc length formula

$$2 \int_0^a \sqrt{1 + f'(x)^2} dx$$

$$f(x) = \sqrt{a^2 - x^2} \text{ so } f'(x) = \frac{1}{2\sqrt{a^2 - x^2}}(-2x) = -\frac{x}{\sqrt{a^2 - x^2}}$$

$$2 \int_0^a \sqrt{1 + \frac{x^2}{a^2 - x^2}} dx = 2 \int_0^a \sqrt{\frac{a^2}{a^2 - x^2}} dx$$

Now we need to use trig substitution. Lets use $x = a \sin \theta$, with the limit assumptions $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. Then, $dx = a \cos \theta d\theta$

$$\begin{aligned} &= 2 \int \sqrt{\frac{a^2}{a^2 - a^2 \sin^2 \theta}} d\theta \\ &= 2 \int \sqrt{\frac{1}{1 - \sin^2 \theta}} d\theta \\ &= 2 \int \sqrt{\frac{1}{\cos^2 \theta}} d\theta \\ &= 2 \int \sqrt{\sec^2 \theta} d\theta \\ &= 2 \int \sec \theta d\theta \\ &= 2 \ln |\sec \theta + \tan \theta| \end{aligned}$$

Now for the triangle part! $\sec \theta = \frac{1}{\cos \theta} = \frac{\sqrt{a^2 - x^2}}{a}$, $\tan \theta = \frac{x}{\sqrt{a^2 - x^2}}$

$$= 2 \ln \left| \frac{a}{\sqrt{a^2 - x^2}} + \frac{x}{\sqrt{a^2 - x^2}} \right|$$

Seems like a lot of dividing by zero

2 | surface area

I don't really understand how we use arclength in the calculation of surface area. I tried just using the circumference, but that didn't work with the slicing assumption because the surface curves.