1 | Algebreic and Geometric Multiplicities

I missed the last ten minutes of class and had to look up what the algebreic and geometric multiplicities are. I used this source.

Also it says something about

> It is a fact that summing up the algebraic multiplicities of all the eigenvalues of an $n \times n$ matrix A gives exactly n.

Which reminds me of the fundamental theorem of algebra…

1.1 | $\begin{pmatrix} 4 & -12 \\ 2 & 0 \end{pmatrix}$

1.1.1 | Geometric multiplicity

The null space is span $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ which is dimension $\boxed{1}$.

1.1.2 | Algebraic multiplicity

The determinant of $\begin{pmatrix} 2 & -2 \\ 2 & -2 \end{pmatrix}$ is

$$-\lambda(4-\lambda)-(-4)=\lambda^2-4\lambda+4=(\lambda-2)^2$$

So the algebraic multiplicity is $\boxed{2}$

1.2 | $\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$

1.2.1 | Geometric

Null space of 1 ($(x, 0, 0)$) has dim 1. Null space of 3 ($\left(x, \frac{-2x}{3}, \frac{4x}{3}\right)$) has dim 1 as well.

1.2.2 | Algebraic

The determinant simplifies to one factored term:

$$(1-\lambda)^2(3-\lambda)$$

So 1 has a multiplicity 2 and 3 has multiplicity 1?

1.3 | $\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 3 \end{pmatrix}$

1.3.1 | Geometric

For $\lambda = 1$, null space is $(x, y, 0)$ so dim 2. For $\lambda = 3$, null space is $(x, \frac{-x}{2}, x)$ so dim 1.

1.3.2 | Algebraic

The determinant is the same as the previous matrix, so once again, 1 has multiplicity 2 and 3 has multiplicity 1.