PS#31: The Master Formula, Part II!

Nueva Multivariable Calculus

1. Using the cool fancy determinant-of-the-derivative-matrix method, derive/prove the correction factor for a triple integral in *cylindrical* coordinates!

2. You are Dr. Frankenstein, in the bowels of your basement laboratory, cooking up a devilish new coordinate system by bolting together disjointed parts of the other coordinate systems you have lying around! Your new coordinate system, **Frankenstein coordinates**, describes any point on $\mathbb{R}^2$ using:

   - the point's distance up from the x-axis (i.e., its y-coordinate)
   - the point's angle from the y-axis (i.e., its angle ϕ, measured clockwise).

   It's not rectangular—it's not polar—it's not *not* rectangular—it's not *not* polar—it's Frankenstein coordinates!!!

   (PICTURE REALLY SHOULD GO HERE. SORRY!)

   So, for example, the point given by the rectangular coordinates:

   $$\left(x = \frac{1}{2}, \, y = \frac{\sqrt{3}}{2}\right)$$

   is given by the Frankenstein coordinates:

   $$\left(\phi = \frac{\pi}{6}, \, y = \frac{\sqrt{3}}{2}\right)$$

   (a) Draw the point $(\phi = \pi/4, y = 378)$. (How would you describe it in rectangular?)

   (b) Draw the point $(\phi = 11\pi/12, y = -7)$. (How would you describe it in rectangular?)

   (c) Here's an example of a (nameless) function in Frankenstein coordinates, giving a y-coordinate in terms of an angle ϕ:

   $$y(\phi) = 5$$

   Without converting it to rectangular, can you sketch this?

   (d) Here's another:

   $$y(\phi) = 2\phi$$

   Sketch?

   (e) Sketch the region bounded by $y = 5$, $y = 7$, $\phi = 0$, and $\phi = \frac{\pi}{3}$ (described in Frankenstein coordinates). Then, using a double integral in Frankenstein coordinates, find the area of the region! (Do it using a double integral in Frankenstein coordinates—not using a double integral in rectangular or polar, and not using basic geometry.

   (f) Sketch the region bounded by $y = 0$, $y = 2\phi$, $\phi = 0$, and $\phi = \pi/4$. Again, using a double içtegral in Frankenstein coordinates, find its area!

3. This whole taking-the-determinant-of-the-derivative-matrix is a great strategy for finding correction factors of things when we're not changing dimensionality—e.g., when we're going from one 3D representation of a function (say, in rectangular) to another 3D representation (say, in spherical). But what if we need a correction factor for something where the dimensionality changes? For instance, can we use this method to find the curvy-arc-length and curvy-surface-area correction factors? This gets way out of stuff I'm comfortable with, but here's a very compelling Math StackExchange post asking the same question and proposing a method/

4. Google "**parabolic coordinates**," read about them, and tell me what you learn!

5. Google "**loxodromic coordinates**," read about them, and tell me what you learn! These are even more exotic than parabolic coordinates—I only know about them from sitting in on Jana's complex analysis class two years ago! (Elianna K.'s older sister taught us the cool Greek etymology of the word!)