# 1 | Integral of ln(x)

$$\begin{aligned}\int \ln(x)\,dx &= \int 1\cdot\ln(x)\,dx\\ &= x\cdot\ln(x) - \int x\cdot\frac{1}{x}dx\\ &= x\cdot\ln(x) - \int 1dx\\ &= x\cdot\ln(x) - x + C\end{aligned}$$

# 2 | Integral of $x^5 \sin(x)$

$$\begin{aligned}\int x^5\sin(x)\,dx &= -x^5\cos(x) - \int -5x^4\cos(x)\,dx\\ &= -x^5\cos(x) + 5x^4\sin(x) + \int -20x^3\sin(x)\,dx\\ &= -x^5\cos(x) + 5x^4\sin(x) + 20x^3\cos(x) - \int 60x^2\cos(x)\,dx\\ &= -x^5\cos(x) + 5x^4\sin(x) + 20x^3\cos(x) - 60x^2\sin(x)\\ &\quad + \int 120x\sin(x)\,dx\\ &= -x^5\cos(x) + 5x^4\sin(x) + 20x^3\cos(x) - 60x^2\sin(x)\\ &\quad - 120x\cos(x) - \int -120\cos(x)\,dx\\ &= -x^5\cos(x) + 5x^4\sin(x) + 20x^3\cos(x) - 60x^2\sin(x)\\ &\quad - 120x\cos(x) + 120\sin(x)\end{aligned}$$

# 3 | Diff in High Dimensions

## 3.1 | 9)

$$\nabla f(x,y) = \begin{bmatrix}\tan(y)\\ \frac{x}{\sec^2(y)}\end{bmatrix}$$

## 3.2 | 12)

$$\nabla f(x,y,z) = \begin{bmatrix}2x\\ 7z\\ 7y\end{bmatrix}$$

# 4 | $e^x \cos(y)$

First, we find the derivative of $f(x,y)$:

$$\nabla f(x,y) = \begin{bmatrix} e^x \cos(y) \\ -e^x \sin(y) \end{bmatrix} \tag{1}$$

Then, we can find the derivative of $f(x,y)$ at point $(1, \frac{\pi}{4})$:

$$\nabla f(1, \frac{\pi}{4}) = \begin{bmatrix} e^{(1)} \cos(\frac{\pi}{4}) \\ -e^{(1)} \sin(\frac{\pi}{4}) \end{bmatrix} \quad = \begin{bmatrix} \frac{e}{\sqrt{2}} \\ -\frac{e}{\sqrt{2}} \end{bmatrix}$$

Armed with this knowledge, we can get the slope in a particular direction. The equation for this is

$$f'(\theta) = \frac{\partial}{\partial x} f \cdot \cos(\theta) + \frac{\partial}{\partial y} f \cdot \sin(\theta) \tag{2}$$

We know that $\theta = \frac{\pi}{6}$, so we can plug this in to $f'$ to get:

$$\begin{aligned} f'(\theta) = f'(\frac{\pi}{6}) &= \frac{\partial}{\partial x} f \cdot \cos\left(\frac{\pi}{6}\right) + \frac{\partial}{\partial y} f \cdot \sin\left(\frac{\pi}{6}\right) \\ &= \frac{e}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{e}{\sqrt{2}} \cdot \frac{1}{2} \\ &= \frac{\sqrt{3}e}{2\sqrt{2}} - \frac{e}{2\sqrt{2}} \\ &= \frac{(\sqrt{3}-1)}{2\sqrt{2}} e \end{aligned}$$