

Suppose  $T \in \mathcal{L}(V)$  and  $\lambda \in \mathbb{F}$ . Prove that  $\lambda$  is an eigenvalue of  $T$  iff  $\bar{\lambda}$  is an eigenvalue of  $T^*$ .

Given  $\lambda$  is an eigenvalue of  $T$ , show that  $\bar{\lambda}$  is an eigenvalue of  $T^*$ . This will imply both directions, since  $\lambda = \overline{\bar{\lambda}}$  and  $T = T^{**}$ .

Suppose  $\mathcal{M}(T)$  is the matrix of  $T$  wrt some orthonormal basis. Then, the matrix  $\mathcal{M}(T^*)$  of  $T^*$  wrt the same orthonormal basis will equal the conjugate transpose of  $\mathcal{M}(T)$ .

Eigenvalues lie on the diagonal of a matrix, so the conjugate transpose will have the effect of conjugating each eigenvalue. Thus, the eigenvalues of  $\mathcal{M}(T)$  are conjugates of the eigenvalues of  $\mathcal{M}(T^*)$ .

$$\langle T - \lambda I v, v \rangle = \langle v, (T - \lambda I)^* \rangle = \langle v, T^* - \bar{\lambda} I v \rangle$$

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There exists some  $v$  s.t.

$$Tv = \lambda v$$

$$\langle \lambda v, w \rangle = \langle Tv, w \rangle = \langle v, T^* w \rangle$$