

## 1 | Electric Charge

We are finally taking a surface integral! This is essentially multiplying the surface area of the shape of the function to the value of the function itself.

Firstly, taking the area  $dA$  by  $dV$ :

$$dA = \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \quad (1)$$

$$= \sqrt{1 + (3)^2 + (2)^2} \quad (2)$$

$$= \sqrt{14} \quad (3)$$

Supplying the value into the function:

$$\int_0^7 \int_0^{11} (3x + 2y + 7)\sqrt{14} \, dy \, dx \quad (4)$$

$$\Rightarrow \sqrt{14} \int_0^7 \int_0^{11} (3x + 2y + 7) \, dy \, dx \quad (5)$$

$$\Rightarrow \sqrt{14} \int_0^7 (3xy + y^2 + 7y) \Big|_0^{11} \, dy \, dx \quad (6)$$

$$\Rightarrow \sqrt{14} \left( \frac{33x^2}{2} + 198x \right) \Big|_0^7 \quad (7)$$

$$\Rightarrow \frac{4389\sqrt{14}}{2} \quad (8)$$

The charge, therefore, is proportional to  $\frac{4389\sqrt{14}}{2}\rho$ .

## 2 | Infinite wire

Recall first that a semicircle with radius 7 can be defined as:

$$y = \sqrt{7^2 - x^2} \quad (9)$$

$$= \sqrt{49 - x^2} \quad (10)$$

Let's first figure the value of this function  $dA$ :

$$dA = \sqrt{1 + \left(\frac{d}{dx} \sqrt{49 - x^2}\right)^2} \quad (11)$$

$$= \sqrt{1 + \left(\frac{d}{dx} \sqrt{49 - x^2}\right)^2} \quad (12)$$

$$= \sqrt{1 - \frac{x^2}{x^2 - 49}} \quad (13)$$

We will take the line integral of this function, and proceed to multiply by the value of  $xy$  at that point.

$$\int_0^7 \int_0^7 xy \sqrt{1 - \frac{x^2}{x^2 - 49}} dx dy \quad (14)$$

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f(x,y) = x*y*sqrt(1-x^2/(x^2-49))
f.integrate(x, 0,7).integrate(y,0,7)

2401/2
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Looks like the solution for the wire's weight is about  $\frac{2401}{2}$  grams.

### 3 | More Difficult Polar Coordinates

Recall that, to figure the unit sphere volume, we can convert an  $\mathbb{R}^2 \rightarrow \mathbb{R}^1$  result into circular coordinates.

That, by pythagoras,  $x^2 + y^2 = r^2$ . Therefore, the expression of:

$$f(x, y) = \frac{1}{(x^2 + y^2)^k} \Rightarrow f(r, \theta) = \frac{1}{r^{2k}} \quad (15)$$

We also note that, due to the correction factor,  $dA = r dr d\theta$ .

Taking the actual integral, therefore, will result in:

$$\int_0^{2\pi} \int_0^1 r^{-k} dr d\theta \quad (16)$$

$$\Rightarrow \int_0^{2\pi} \lim_{x \rightarrow 0} \left( \frac{1}{-k+1} - \frac{1}{x^{k-1} - k + 1} \right) d\theta \quad (17)$$

Evidently, when  $k \leq 1$ , the second term would become infinity large.

Now, we essentially want to take this idea and expand it to  $n$  dimensions, to figure the correct spherical coordinates.

Turns out, the naïve version of the  $n$  sphere integral is the same correction factor multiplied by  $\sin^{n-\{2\ldots(n-1)\}}$ . Therefore, the same logic from above actually holds for  $n$  volcano as well: that, by very high dimension Pythagoras,  $x_1^2 + x_2^2 + \dots + x_n^2 = r^2$ .

Therefore:

$$\frac{1}{(x_1^2 + x_2^2 + \dots + x_n^2)^k} = \frac{1}{r^{2k}} \quad (18)$$

We will again note than the correction factor:  $dA = r^{n-1} \sin^{n-2} \sin^{n-3} dr d\theta_1 \dots d\theta_n$ .

Therefore:

$$\int \dots \int_0^1 \frac{1}{r^{2k-n+1}} \sin^{n-2} \sin^{n-3} dr d\theta_1 \dots d\theta_n \quad (19)$$

$$\Rightarrow \int \dots \int_0^1 r^{-2k+n-1} \sin^{n-2} \sin^{n-3} dr d\theta_1 \dots d\theta_n \quad (20)$$

$$\Rightarrow \int \dots \int \frac{r^{-2k+n}}{-2k+n} \Big|_0^1 \sin^{n-2} \sin^{n-3} dr d\theta_1 \dots d\theta_n \quad (21)$$

At this point, we can analyze the solution. The first term will always be 1 over a certain value. The second term, however, is more interesting.

Case 1:  $-2k + n < 0$ . If the value of  $-2k + n < 0$ , the  $r$  would have to be transported below the fraction. Therefore, taking  $\lim_{r \rightarrow 0} \frac{1}{r^{2k-n}(-2k+n)}$  would be  $+\infty$ .

Case 2:  $-2k + n \geq 0$ . This would render no problem: the second term would be simply 0.

Therefore, if the terms that make up the infinite volcano  $-2k + n > 0$ , the results would be infinite. Otherwise, the results are finite.