1 | $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2} dx$

$$\begin{aligned}\int -e^u du &= -e^u + C \\ &= -e^{\frac{1}{2}} + e^{\frac{1}{1}} \\ &= e - e^{\frac{1}{2}}\end{aligned}$$

2 | $\int_0^1 re^{\frac{r}{2}} dr$

$$\begin{aligned}\int_0^1 re^{\frac{r}{2}} dx &\implies r2e^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2re^{\frac{r}{2}} - \int 2e^{\frac{r}{2}} dr \\ &= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &= 2re^{\frac{r}{2}} - 4e^{\frac{r}{2}} \\ &\implies 2e^{\frac{1}{2}} - 4e^{\frac{1}{2}} - (-4) \\ &= 4 - 2e^{\frac{1}{2}}\end{aligned}$$

3 | $\int_4^9 \frac{\ln y}{\sqrt{y}} dy$

$$\begin{aligned}\int \frac{\ln y}{\sqrt{y}} dy &= 2\ln y\sqrt{y} - \int 2\frac{1}{y}\sqrt{y} dy \\ &= 2\ln y\sqrt{y} - \int 2\frac{1}{\sqrt{y}} dy \\ &= 2\ln y\sqrt{y} - 2\int y^{-\frac{1}{2}} dy \\ &= 2\ln y\sqrt{y} - 4\sqrt{y} + C \\ &= 2\sqrt{y}(\ln y - 2) + C \\ \implies &\quad 6(\ln 9 - 2) - 4(\ln 4 - 2)\end{aligned}$$

4 | $\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx$

Let $u = \sqrt{x}$, $du = \frac{1}{2\sqrt{x}} dx$, $dx = 2udu$

$$\begin{aligned}
\int_0^{\sqrt{\pi}} \cos \sqrt{x} dx &= 2 \int u \cos u du \\
&= 2u \sin u - 2 \int \sin u du \\
&= 2u \sin u + 2 \cos u \\
&= 2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} \\
\implies \quad & 2\pi^{\frac{1}{4}} \sin \pi^{\frac{1}{4}} + 2 \cos \pi^{\frac{1}{4}} - 2
\end{aligned}$$

5 | $\int_1^e \sin \ln x dx$

$$\begin{aligned}
\int_1^e \sin \ln x dx &= x \sin \ln x - \int x \frac{1}{x} \cos \ln x dx \\
&= x \sin \ln x - \int \cos \ln x dx \\
&= x \sin \ln x - \left(x \cos \ln x + \int x \frac{1}{x} \sin \ln x dx \right) \\
&= x \sin \ln x - x \cos \ln x - \int \sin \ln x dx \\
2 \int \sin \ln x dx &= x \sin \ln x - x \cos \ln x \\
\int \sin \ln x dx &= \frac{1}{2} x (\sin \ln x - \cos \ln x) \\
\implies \quad & \frac{1}{2} e (\sin 1 - \cos 1) - \frac{1}{2} (\sin 0 - \cos 0) \\
&= \frac{e}{2} (\sin 1 - \cos 1) + \frac{1}{2}
\end{aligned}$$

6 | $\int_0^1 \frac{x^3}{\sqrt{4+x}} dx$

Let $u = 4 + x^2$, $du = 2xdx$

$$\begin{aligned}
\int \frac{x^3}{\sqrt{4+x}} dx &= \frac{1}{2} \int \frac{(u-4)}{\sqrt{u}} du \\
&= \frac{1}{2} \int \sqrt{u} dx - 2 \int \frac{1}{\sqrt{u}} dx
\end{aligned}$$

7 | **(additional problems)**

7.1 | $\int \sin^2 x dx$

$$\begin{aligned}
\int \sin^2 x dx &= -\sin x \cos x + \int \cos^2 x dx \\
&= -\sin x \cos x + \int 1 dx - \int \sin^2 x dx \\
2\int \sin^2 x dx &= -\sin x \cos x + x \\
\int \sin^2 x dx &= \frac{1}{2}(x - \sin x \cos x)
\end{aligned}$$

7.2 | $\int \cos^2 x dx$

$$\begin{aligned}
\int \cos^2 x dx &= \cos x \sin x + \int \sin^2 x dx \\
&= \cos x \sin x + \int 1 dx - \int \cos^2 x dx \\
2\int \cos^2 x dx &= \cos x \sin x + x \\
\int \cos^2 x dx &= \frac{1}{2}\cos x \sin x + \frac{x}{2} + C
\end{aligned}$$

7.3 | $\int \sin^2 x \cos^2 x dx$

$$\begin{aligned}
\sin 2x &= 2 \sin x \cos x \\
\int \sin^2 x \cos^2 x dx &= \frac{1}{4}\int \sin^2 2x dx \\
\text{Let } u &= 2x, du = 2dx \\
&= \frac{1}{8}\int \sin^2 u du \\
&= \frac{1}{8}\frac{1}{2}(u - \sin u \cos u) \\
&= \frac{1}{16}(2x - \sin 2x \cos 2x) + C
\end{aligned}$$

7.4 | $\int \sin^3 x dx$

$$
\begin{aligned}
\int \sin^3 x dx &= \int \sin x(1-\cos^2 x)dx \\
&= \int \sin x dx - \int \cos^2 x \sin x dx \\
\text{Let } u = \cos x, du &= -\sin x dx \\
&= \int \sin x dx + \int u^2 du \\
&= -\cos x + \frac{1}{3}u^3 + C \\
&= \frac{1}{3}\cos^3 x - \cos x + C
\end{aligned}
$$

7.5 | $\int \cos^3 x dx$

$$
\begin{aligned}
\int \cos^3 x dx &= \int \cos x(1-\sin^2 x)dx \\
&= \int \cos x - \sin^2 x \cos x dx \\
&= \sin x - \int u^2 du \\
&= \sin x - \frac{1}{3}u^3 + C \\
&= \sin x - \frac{1}{3}\sin^3 x + C
\end{aligned}
$$