True or false (and give a proof of your answer): The exists $T \in \mathcal{L}(\mathbb{R}^3)$ s.t. T is not self-adjoint but there is a basis of $\mathbb{R}^3$ consisting of eigenvectors of T.

The real spectral theorem suggests that if a map is self-adjoint, it must have an orthogonal basis of eigenvectors. Thus, we can choose a map which does not have an orthogonal basis of eigenvectors. For instance, choose the non-orthogonal eigenpairs

$$\begin{aligned} T(1,0,0) &= (1,0,0) \\ T(0,1,0) &= (0,2,0) \\ T(1,1,1) &= (3,3,3) \end{aligned}$$