For the sake of simplicity I will assume that the stick has 0 width, meaning that it effectively acts as a line.

1 | **Problem 1)**

We know that kinetic energy is conserved, so we know that $\Delta KE = 0$. As such, we know the following:

$$KE_{ball;0} + KE_{stick;0} = KE_{ball;f} + KE_{stick;f}$$

We know that the stick is initially at rest, and the ball is moving with velocity v_0. In addition, after the collision the ball has velocity v_f and the stick COM has a velocity of v_{CM} and the stick has angular velocity of ω.

Then, our equation becomes

$$(\frac{1}{2}m_1v_0^2) + (0) = (\frac{1}{2}m_1v_f^2) + (\frac{1}{2}m_2v_{CM}^2 + \frac{1}{2}I_0\omega^2)$$

In addition, linear momentum is also conserved. That is, $\Delta P = 0$. We know that angular velocity has no effect on linear momentum. Therefore, the following equation holds true:

$$\begin{aligned}
P_{ball;0} + P_{stick;0} &= P_{ball;f} + P_{stick;f} \\
(m_1v_0) + (0) &= (m_1v_f) + (m_2v_{CM}) \\
m_1(v_0 - v_f) &= m_2v_{CM} \\
v_0 - v_f &= \frac{m_2}{m_1}v_{CM} \\
v_f &= v_0 - \frac{m_2}{m_1}v_{CM}
\end{aligned}$$

We have three unknowns: v_f, v_{CM}, and ω.