

#flo

# 1 | Talking about the reading (vector spaces)

## 1.1 | Vector space

### 1.1.1 | Identity

- It would be the additive identity, because the multiplicative one doesn't count because multiply doesn't take two elements from the same field ##### Operations
- Scalar multiplication
  - Not a multiplication on  $V$
  - We need another field of scalars
  - Fundamental difference: **operates on different objects** (only happens on scalar multiplications)
- addition ##### Linearity
- Something that's linear means "things work for addition and scalar multiplication"
- Take  $-2x + 1y = 3$ 
  - Multiplying by scalars
  - adding them
  - similar to a line in standard form—slope stays constant
- Take  $2x - 3y + 1z = 2$ 
  - a plane in 3d
  - if you pick a direction, the slope stays the same
  - thus, a plane is linear ##### Vector
- Something in a vector space
- infinite lists
  - It's like decimals, except you can choose any number instead of just [0-9]
  - base infinity basically
- Most common vector space
  - $\mathbb{F}^n$ , like  $\mathbb{R}^3$  (might also be  $\mathbb{C}^2$  or something, although that's hard to visualize)
  - #definition canonical
    - \* something "standard", basically everyone should know what you are talking about
    - \* canonical vector space is  $\mathbb{R}^2$  ##### Distributive property
- Important to tie operations together

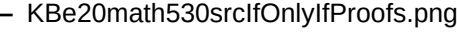
### 1.1.2 | Vector Space as a Set of Functions

- like  $\mathbb{R}^{[0,1]}$ : the functions from  $[0, 1]$  that end up as real numbers
  - Identity =  $f(x) = 0$  ##### Subspaces
- A subspace of this has to be a group on it's own
- Conditions for a subspace
  - See 1.34
  - Just check
    - \* additive identity
    - \* closed under addition
    - \* closed under scalar multiplication
- What other subspaces of this vector space are there that also have a domain from  $[0, 1]$ ?
  - Like continuous functions from zero to one
  - functions whose derivatives are continuous or constant or zero
  - even functions are also a subspace KBe20math530srcEvenFunctionsAreSubspacesOfFtotheS.png
- Subspaces of  $\mathbb{F}^3$ 
  - Most contain infinite vectors (except  $\{0\}$ )
  - $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$  is a subspace with infinite vectors ##### Notation
- #note  $\mathbb{F}^2$  is almost always either  $\mathbb{R}^2$  or  $\mathbb{C}^2$ , mostly  $\mathbb{R}^2$

### 1.2 | Direct sums

- Something that isn't a direct sum
  - in  $\mathbb{R}^3$ ,  $\begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$ 
    - \* Two ways to write 0:
 
$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \quad \text{### } \mathbb{F}^\infty$$
- Functions from naturals to your field, (assign an element to each natural)
  - that would be the same as ordering the elements in your field?
  - Tons of functions, any one is an infinite vector??

## 2 | If and Only If proofs (iff)

- You have to take the proof in both directions
  - **Assumption:** "now suppose the only way to write 0 as a sum of  $u_1 + \dots + u_m$ , where each  $u_j$  is in  $U_j$ , is by taking each  $u_j$  equal to 0"
    - Assume the red part, then show the green part. Then, assume the green and show it gets the red.
    - 
  - #future geometrical interpretation of determinants
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