#flo #hw

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1 | 3C!

matricies as values of T_{vj} in terms of a a basis of W?

```
title: matrix, $A_{j,k}$
Let $m$ and $n$ denote possitive integers. An $m-\text{by}-n$ matrix A is a rectangular array of elemen

$$
A = \begin{bmatrix}
 A_{1,1} \dots & A_{1,n} \\

 \vdots & \vdots \\
 A_{m,1} \dots & A_{m,n}
 \end{bmatrix}.
$$
The notation $A-{j,k}$ denotes the entry in row $j$, column $k$ of $A$. In other words, the first index
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just defining a matrix simply. #question, what is a non-rectangular array? REMEBER :: this is 1-indexed! not 0-indexed!

key definition

```
title: matrix of a linear map, $M(T)$
Suppose $T \in L(V,W)$ and $v_1, \dots, v_n$ is a basis of $V$ and $w_1, \dots, w_n$ is a basis of $W$.

$$
T_{vk} = A_{1,k} w_1 + \dots + A_{m,k} w_m.
$$
If the bases are not clear from the context, then the notation
$M(T, (v_1, \dots, v_n), (w_1, \dots, w_n))$ is used.
```

the matrix which represents the linear map?

constructing the matrix: Screen Shot 2021-11-17 at 1.17.40 PM.png||300

if it maps from n-dim to m-dim, then the matrix is m-by-n.

1.0.1 | addition and SCAMUL of matricies

assume that V and W are finite-dim!

```
title: matrix addition
The sum of two matricies of the same size is the amtric obtained by adding corresponding entries in the
{the latex}.
In other words, $(A+C)_{j,k} = A_{j,k} + C_{j,k}$.
```

assuming that all the same bases are used for all three linear maps, $S + T, S, T$,

```
title: the matrix of the sum of linear maps
Suppose $S,T \in L(V,W).$ Then $M(S+T) = M(S)+M(T)$.
```

and also,

```
title: SCAMUL of a matrix
The product of a scalar and a matrix is the matrix obtained by multiplying each entry in the matrix by t
{the latex}

In other words, $(\lambda A)_{j,k} = \lambda(A_{j,k})$
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```
title: The matrix of a scalar times a linear map
Suppose $\lambda \in F$ and $T \in L(V,W)$. Then $M(\lambda T) = \lambda M(T).$
```

and, ofc, more vector spaces

```
title: $F^{m,n}$
For $m$ and $n$ positive integers, the set of all $m$-by-$n$ matrices with intries in $F$ is denoted by
```

sick.

```
title: $\dim F^{m,n} = mn$
suppose $m$ and $n$ are positive integers. With addition and SCAMUL defined as above, $F^{m,n}$ is a ve
```

1.0.2 | matrix multiplication

wait we are just getting to this? goddamn.

'makes sense' means having the operations defined.

this part doesn't make sense to me.

we have a desired equation, $M(ST) = M(S)M(T)$ and we want to define matrix multiplication as such that it holds. thus, we get,

```
title: matrix multiplication
Suppose A is an $m$-by-$n$ matrix and $C$ is an $n$-by-$p$ matrix. Then $AC$ is defined to be the $m$-b

$$
(AC)_{j,k} = \sum^{n}_{r=1}A_{j,r} C _{r,k}.
$$
In other words, the entry n row $j$, column $k$, of $AC$ is computed by taking row $j$ of A and column $
```

formally defining what we already know how to do. #cool!

remember, this only works when the columns of the first matrix equals the number of rows of the second matrix.

this is the motivation for the definition of matrix multiplication that we have been taught.

intresting how cyclic this type of understanding is.

anyways, It's not commutative!

```
title: the matrix of the product of linear maps
If $T\in L(U,V)$ and $S \in L(V,W)$, then $M(ST) = M(S)M(T)$.
```

vertically centered dot is a placeholder?

```
title: $A_{j,.}, A_{.,k}$
Suppose A is an $M$-by-$n$ matrix.

- If 1 <= j <= m, then $A_{j,.}$ denotes the 1-by-$n$ matrix consisting of row j of A.
- If 1 <= k <= n, then $A_{.,k}$ denotes the m-by-$1$ matrix consisting of column k of A.
```

what if it's less than one? does that mean the notation isn't defined? or is it like an index out of range err? #question

another way to think about matrix multiplication, - entry in row j column k of AC = row j of A * column k of C - ooh, that is alot cleaner.

```
title: Entry of matrix product equals row times column
Suppose A is an $m$-by-$n$ matrix and C is an $n$-by-$p$ matrix. Then
$$(AC)_{j,k} = A_{j,.}C_{.,k}$$
for 1 <= j <= n and 1 <= k <= p.
```

wait, another one? the column of k of AC equals A times column k of C.

```
title: column of matrix product equals matrix times column
Suppose A is an $m$-by-$n$ matric and C is an $n$-by-$p$ matrix. Then
$$(AC)_{.,k} = AC_{.,k}$$
for 1 <= k <= p.
```

final one, as a linear combination.

$$\begin{bmatrix} 7 \\ 19 \\ 31 \end{bmatrix} = 5 \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}.$$

what? this doesnt make sense.

```
title: Linear combination of columns
Suppose A is an $m$-by-$n$ matrix and $c = \begin{bmatrix}
 c_1 \\
 \vdots \\
 c_n
 \end{bmatrix}$ is an n-by-1 matrix.
 Then
 $$
 Ac = c_1 A_{.,1} + \dots + c_n A_{.,n}
 $$

In other words, $Ac$ is a linear combination of the columns of A, with the sclars that multiply the colu
```

wait but does that returns a matrix? #question

haha, two more ways are given by exrs. 10 & 11. amazing.

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this chapter was mostly about differnt ways we can think about, and thus define, matrix multiplication.
intrestng how we needed all this info which we understood through using matrix multiplication to underst
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