$$\begin{aligned}\Delta\mathsf{PE} &= \mathsf{PE}_H - \mathsf{PE}_{h_0}\\ \mathsf{PE}_H &= mgH\\ \mathsf{PE}_{h_0} &= mgh_0\\ \Delta\mathsf{PE} &= mg(H-h_0)\end{aligned}$$

$$\begin{aligned}\Delta\mathsf{PE} &= \mathsf{KE}\\ mg(H-h_0) &= \frac{1}{2}mv^2\\ g(H-h_0) &= \frac{1}{2}v^2\\ \sqrt{2g(H-h_0)} &= v\end{aligned}$$

$\begin{cases} x(t_f) = x_f = v_0\cos\theta t_f = \sqrt{2g(H-h_0)}\cos\theta t_f\\ y(t_f) = 0 = v_0\sin\theta t_f - \frac{1}{2}gt_f^2 + h_0 = \sqrt{2g(H-h_0)}\sin\theta t_f - \frac{1}{2}gt_f^2 + h_0\end{cases}$

$\frac{x_f}{v_0\cos\theta} = t_f$

We can then plug this in and apply implicit differentiation to get $\frac{dx_f}{d\theta}$:

$$\begin{aligned}0 &= -\frac{1}{2}g\left(\frac{x_f}{v_0\cos\theta}\right)^2 + v_0\sin\theta\frac{x_f}{v_0\cos\theta} + h_0\\ \frac{d}{d\theta}0 &= \frac{d}{d\theta}\left(-\frac{1}{2}g\left(\frac{x_f}{v_0\cos\theta}\right)^2\right) + \frac{d}{d\theta}v_0\sin\theta\frac{x_f}{v_0\cos\theta} + \frac{d}{d\theta}h_0\\ 0 &= \frac{d}{d\theta}\left(-\frac{1}{2}g\left(\frac{x_f}{v_0\cos\theta}\right)^2\right) + \frac{d}{d\theta}\tan\theta x_f + 0\\ 0 &= \frac{d}{d\theta}\left(-\frac{1}{2}g\frac{x_f^2}{v_0^2\cos^2\theta}\right) + \frac{d}{d\theta}\tan\theta x_f + 0\\ 0 &= \frac{d}{d\theta}\left(-\frac{1}{2v_0^2}gx_f^2\frac{1}{\cos^2\theta}\right) + \frac{d}{d\theta}\tan\theta x_f\\ 0 &= -\frac{g}{2v_0^2}\frac{d}{d\theta}\left(x_f^2\frac{1}{\cos^2\theta}\right) + \frac{d}{d\theta}\tan\theta x_f\\ 0 &= -\frac{g}{2v_0^2}\left(2x_f\frac{dx_f}{d\theta}\frac{1}{\cos^2\theta} + x_f^2 2\tan\theta\sec^2\theta\right)\frac{1}{\cos^2\theta}\sec^2\theta x_f + \tan\theta\frac{dx_f}{d\theta}\end{aligned}$$

$$
\begin{aligned}
0 &= -\frac{g}{2v_0^2}2x_f\frac{dx_f}{d\theta}\frac{1}{\cos^2\theta} - \frac{g}{2v_0^2}x_f^2 2\tan\theta\sec^2\theta + \sec^2\theta x_f + \tan\theta\frac{dx_f}{d\theta} \\
\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta - \sec^2\theta x_f &= -\frac{g}{2v_0^2}2x_f\frac{dx_f}{d\theta}\frac{1}{\cos^2\theta} + v_0\tan\theta\frac{dx_f}{d\theta} \\
\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta - \sec^2\theta x_f &= \frac{dx_f}{d\theta}\left(v_0\sec^2\theta x_f - \frac{g}{2v_0^2}2x_f\frac{1}{\cos^2\theta}\right) \\
\frac{\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta - \sec^2\theta x_f}{v_0\sec^2\theta x_f - \frac{g}{2v_0^2}2x_f\frac{1}{\cos^2\theta}} &= \frac{dx_f}{d\theta}
\end{aligned}
$$

We can now optimize this monstrosity:

$$
\frac{\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta - \sec^2\theta x_f}{v_0\sec^2\theta x_f - \frac{g}{2v_0^2}2x_f\frac{1}{\cos^2\theta}} = 0
$$

$$
\begin{aligned}
\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta - \sec^2\theta x_f &= 0 \\
\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta - \sec^2\theta x_f &= 0 \\
\frac{g}{2v_0^2}2x_f^2\tan\theta\sec^2\theta &= \sec^2\theta x_f \\
\frac{g}{v_0^2}x_f\tan\theta &= 1 \\
\tan\theta &= \frac{1}{x_f}\frac{v_0^2}{g} \\
\tan\theta &= \frac{v_0^2}{gx_f}
\end{aligned}
$$

$$
\begin{aligned}
0 &= -\frac{1}{2}g\left(\frac{x_f}{v_0\cos\theta}\right)^2 + v_0\sin\theta\frac{x_f}{v_0\cos\theta} + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2}\frac{1}{\cos^2\theta} + x_f\tan\theta + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2}(1+\tan^2\theta) + x_f\tan\theta + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2}\left(1+\frac{v_0^4}{g^2x_f^2}\right) + x_f\frac{v_0^2}{gx_f} + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2} - \frac{1}{2}g\frac{x_f^2}{v_0^2}\frac{v_0^4}{g^2x_f^2} + \frac{v_0^2}{g} + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2} - \frac{1}{2}\frac{v_0^2}{g} + \frac{v_0^2}{g} + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2} - \frac{v_0^2}{2g} + \frac{v_0^2}{g} + h_0 \\
0 &= -\frac{1}{2}g\frac{x_f^2}{v_0^2} - \frac{v_0^2}{2g} + \frac{v_0^2}{g} + h_0
\end{aligned}
$$

$$
\begin{aligned}
v_0 &= \sqrt{2g(H-h_0)} \\
v_0^2 &= 2g(H-h_0) \\
0 = -\frac{1}{2}g\frac{x_f^2}{v_0^2} &- \frac{v_0^2}{2g} + \frac{v_0^2}{g} + h_0
\end{aligned}
$$