1 | Why be a king, when you can be a God?

We are going to take a definite integral across all of infinity by three dimensions.

$$e(x, y, z) = \frac{1}{7^{(x^2+y^2+z^2)^{\frac{3}{2}}}} \tag{1}$$

Given that universes are spheres and its far easier to take the spherical units here, we will take this integral with spherical coordinates.
We realize that $x^2 + y^2 + z^2 = \rho^2$ in spherical space by pythagoras. Hence:

$$e(\rho, \theta, \phi) = \frac{1}{7^{\rho^{2\frac{3}{2}}}} \tag{2}$$

$$= \frac{1}{7^{\rho^3}} \tag{3}$$

We note that, to calculate dV, we have $dV = \rho^2\ sin\theta\ d\phi\ d\theta\ d\rho$.
Taking the actual dV, then:

$$\iiint_V \frac{\rho^2\ \sin\theta}{7^{\rho^3}}\ d\phi\ d\theta\ d\rho \tag{4}$$

Evidently, its time for u sub:

$$u = \rho^3 \tag{5}$$

$$\frac{du}{d\rho} = 3\rho^2 \tag{6}$$

$$du = 3\rho^2\ d\rho \tag{7}$$

And hence:

$$\iiint_V \frac{\sin\theta}{7^u}\ d\phi\ d\theta\ du \tag{8}$$

We will take bounds by $[0, \pi]$, first against ϕ:

$$\int_0^\pi \frac{\sin\theta}{7^u}\ d\phi \tag{9}$$

$$\Rightarrow \frac{\pi \sin\theta}{7^u} \tag{10}$$

Great, we will now take the integral by θ, again by bounds $[0, \pi]$:

$$\int_0^\pi \frac{\sin\theta}{7^u}\ d\theta \tag{11}$$

$$\Rightarrow \frac{-\pi \cos\pi}{7^u} \tag{12}$$

$$\Rightarrow \frac{\pi}{7^u} \tag{13}$$

And finally, we take the integral du from infinity:

$$\int_0^\infty \frac{\pi}{7^u}\ du \tag{14}$$
$$\Rightarrow \pi \int_0^\infty \frac{1}{7^u}\ du \tag{15}$$
$$\Rightarrow \pi \int_0^\infty 7^{-u}\ du \tag{16}$$
$$\Rightarrow -\pi\ \frac{1}{7^x\ ln(7)}\bigg|_0^\infty \tag{17}$$
$$\Rightarrow \frac{\pi}{ln(7)} \tag{18}$$

Multiplying the spherical result by 2 (we got half the circle), we get that the final energy is:

$$\frac{2\pi}{ln(7)} \tag{19}$$

The actual universe has net energy of 0 because its an infinitely large system with no external energy sources (as far as we know).

2 | Area of a Circle

We can essentially leverage the polar expression:

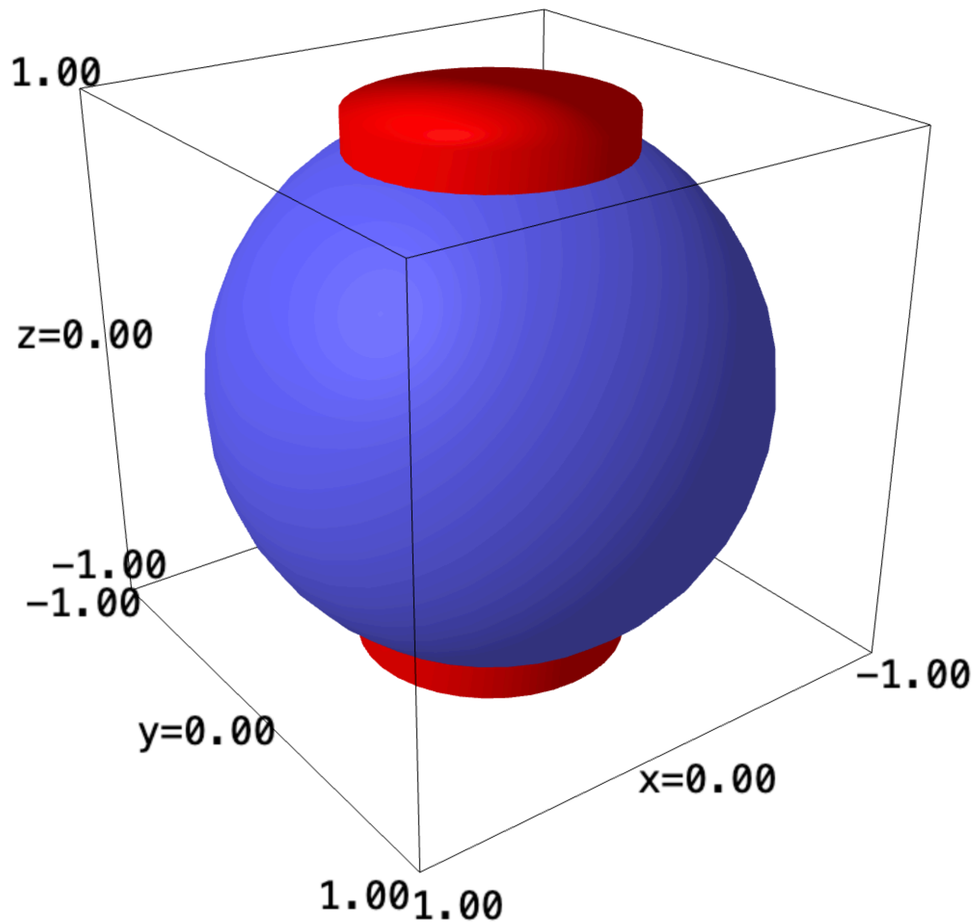
$$\iint_A\ dA \tag{20}$$

In practice, this looks like:

$$\int_0^r \int_0^{2\pi} r\ d\theta\ dr \tag{21}$$
$$\Rightarrow \int_0^r 2\pi r\ dr \tag{22}$$
$$\Rightarrow 2\pi \frac{r^2}{2} \tag{23}$$
$$\Rightarrow \pi r^2\ \blacksquare \tag{24}$$

3 | Sphere sticking out a cylinder

```
from sage.plot.plot3d.shapes import Sphere, Cylinder
Sphere(1) + Cylinder(1/2, 2, color='red').translate(0,0,-1)
```



It looks like a little lantern!

We will first convert all of the system into a cylindrical system:

the sphere:

$$r^2 + z^2 \leq 1 \tag{25}$$

and, the cylinder:

$$r = \frac{1}{2} \tag{26}$$

We can see that our sphere is between $[-1, 1]$ in all directions. Furthermore, our cylinder is cut in the middle.

For every slice, it contains an area of:

$$\pi \left(r(h)\right)^2 \, dh \tag{27}$$

where, $r(h)$ is a function in h which maps the radius. We will note now that a large sphere is made of small concentric spheres. At every disk, a sphere's h (radius by height) would be the same as its r (radius by width).

Hence, we have:

$$\pi h^2\ dh \tag{28}$$

for every disk.

We furthermore have a tiny cut-out in the middle of the shape, of a cylinder of height dh and area $\pi r^2 = \frac{\pi}{4}$.
For every disk, with the cut-out, then:

$$\pi h^2\ dh - \frac{\pi}{4}\ dh \tag{29}$$

We finally perform the integration:

$$\int_{-1}^{1} \pi h^2\ dh - \frac{\pi}{4}\ dh \tag{30}$$

$$\Rightarrow \left(\frac{h^3\pi}{3} - \frac{h\pi}{4}\right)\Bigg|_{-1}^{1} \tag{31}$$

$$\Rightarrow \frac{2\pi}{3} - \frac{2\pi}{4} \tag{32}$$

4 | Volcano Volume

4.1 | General volcano

To figure the volume of the shape $f(x,y) = \frac{1}{(x^2+y^2)^k}$.

We will convert this system into polar form again for the ease of computation. Recall again that, by pythagoras, $x^2 + y^2 = r^2$.

Hence:

$$f(r,\theta) = \frac{1}{r^{2k}} \tag{33}$$

We will again take the integral, with ranges $r = [0,1], \theta = [0, 2\pi]$ from before, and $dA = r\ dr\ d\theta$:

$$\int_0^{2\pi} \int_0^1 r^{-k}\ dr\ d\theta \tag{34}$$

$$\Rightarrow \int_0^{2\pi} \lim_{x\to 0} \left(\frac{1}{-k+1} - \frac{1}{x^{k-1}}\frac{1}{-k+1}\right)\ d\theta \tag{35}$$

Evidently, when $k \leq 1$, the second term would become infinity large.

4.2 | 1D Volcano

We will take the same integral again, but in 1-d. We realize that $\frac{1}{x} = ln(x)$, so, we have two cases:

Case 1: $k \neq 1$

$$\int_{-1}^{1} x^{-k} dx \tag{36}$$

$$\Rightarrow \frac{1}{-k+1} - \frac{-1^{-k+1}}{-k+1} \tag{37}$$

where the value shifts between a real value for odd negative k, and even for even negative k.
Case 2: $k = 1$

$$\int_{-1}^{1} x^{-1} dx \tag{38}$$

$$\Rightarrow ln(x)|_{-1}^{1} \tag{39}$$

Where, the second value would tend towards $+\infty$.
Hence, when $k = 1$, the second term would become infinity large.