

1 | Arc length

$$\begin{aligned}
 dr &= \sqrt{dx^2 + dy^2} \\
 &= \sqrt{dx^2 + dy^2} \cdot \frac{dx}{dx} \\
 &= \sqrt{\frac{dx^2 + dy^2}{dx^2}} \cdot dx \\
 &= \sqrt{1 + \frac{dy^2}{dx^2}} dx \\
 &= \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} dx
 \end{aligned}$$

Next, we will integrate to find the arc length as a whole. $L(x_0, x_1) = \int_{x_0}^{x_1} dr$

$$\begin{aligned}
 &= \int_{x_0}^{x_1} \sqrt{\left(\frac{dy}{dx}\right)^2 + 1} dx
 \end{aligned}$$

Given that the arc length is of function f :

$$= \int_{x_0}^{x_1} \sqrt{f'(x)^2 + 1} dx$$

In the case of the arc length of $f(x) = x^2$ from 5 to 20:

$$\begin{aligned}
 L(5, 20) &= \int_5^{20} \sqrt{4x^2 + 1} dx \\
 &= \left[\frac{1}{2} \sqrt{4x^2 + 1} x + \sinh^{-1} 2x \right]_5^{20} \\
 &= 375.346
 \end{aligned}$$