PS#19: Irradiative Flight

Multivariable Calculus

You are flying your Cessna 172 in a downwards-spiraling holding pattern, preparing to land at Fort Decadence Regional Airport, on the remote northern California coast. Your position in space, at some time t, can be described by the function:

$$\vec{p}(t \text{ minutes}) = \begin{bmatrix} 5\cos\left(\frac{\pi}{10}t\right) \text{ miles} \\ 7\sin\left(\frac{\pi}{10}t\right) \text{ miles} \\ -\frac{1}{60}t + 1 \text{ miles} \end{bmatrix}$$

(where the rows of the matrix/column vector are, from top to bottom, your x, y, z coordinates, in a coordinate system centered at the airport. Also note the units!)

To get some intuition for what your flight path looks like, graph it. **How long does it take to make a full rotation** of this elliptical helix/spiral? **How far do you descend vertically** during that full rotation? Also, **how fast are you going?**

You keep descending. Your altimeter is now reporting that your elevation is 528 feet. Gosh, the airport looks weird. OH NO!!!! You forgot to read your air charts (or whatever they're called) correctly—you haven't been descending towards the airport; you've been descending towards AN ABANDONED NUCLEAR DISASTER SITE of some sort!!! It's spewing radiation! Alpha particles, gamma particles, zeta particles—all of it! Helpfully, your navigation charts tell you that (in a coordinate system centered on the site), the amount of sieverts of radiation per minute you receive, as a function of your position (x, y, z), is:

$$r(x, y, z) = \frac{1}{x^2 + y^2 + z^2} \text{ sieverts per minute}$$

Immediately, you shove the throttle forward and yank back on the yoke to get out of there![1] The Cessna's max rate of (vertical) climb (under your current conditions) is about four meters per second, and its max groundspeed is (say) 200mph[2] **What's your direction vector?** And ***now*** **how fast are you going?** Give the vector both in standard vector notation, and also, since this is a navigational situation, give it as a heading (an angle along the xy plane, but with zero degrees as north) as well as an angle from the horizontal.)

Finally, and most importantly: **how much total radiation do you absorb over this perilous journey**, in sieverts? (Is that, like, a bad amount? Or a not-great-but-not-awful amount?)

(There's a lot going on in this problem—and I intentionally didn't number any of the individual questions—so when you write this up, be sure you explain your thought process and show your work thoroughly and clearly! And, as always, that means lots of lucid English prose, lots of mathematical symbols, lots of pictures, et cetera!)

[1] That probably wasn't the right thing to do. In the post-mortem that NTSB investigators will conduct later, they'll note that the right way to point the aircraft, once you realized you were descending towards a nuclear disaster site, was in the direction in which the amount of radiation you receive drops off the fastest. The exact vector was redacted from the public report. So, flash forward, and calculate it yourself. **What direction should you have steered the 172 in?**

[2] I got distracted down a really cool rabbit-hole on airplane rates of climb while writing this problem—suffice it to say that this is a very sad simplification! There are all sorts of cool relationships between plane groundspeed and plane vertical-rates-of-climbs, and what the engines can do, "flight envelopes," et cetera, et cetera!