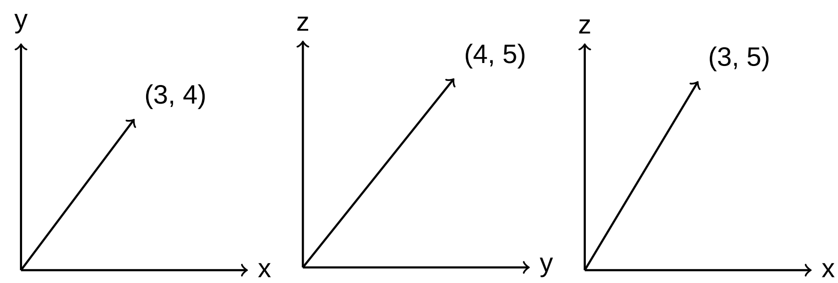
1 | projections or smt

1.1 | projections



1.2 | language of projections?

Let $\vec{\mathbf{c}} = \vec{B} - \vec{A}$. Our goal is to find the magnitude $|\vec{\mathbf{c}}|$.
Lets form a right triangle:

$$\vec{\mathbf{c}} = \vec{\mathbf{c}}_{x,y} + \vec{\mathbf{c}}_z$$

and thus,

$$|\vec{\mathbf{c}}| = \sqrt{\left|\vec{\mathbf{c}}_{x,y}\right|^2 + \left|\vec{\mathbf{c}}_z\right|^2} pro$$

The distance between two points can be found using the Pythagorean theorem.

$$\begin{aligned} |\vec{\mathbf{c}}| &= \sqrt{\left|\vec{\mathbf{c}}_{x,y}\right|^2 + \left|\vec{\mathbf{c}}_z\right|^2} \\ &= \sqrt{\sqrt{|\vec{\mathbf{c}}_x|^2 + |\vec{\mathbf{c}_y}|^2}^2 + \left|\vec{\mathbf{c}}_z\right|^2} \\ &= \sqrt{\left|\vec{\mathbf{c}}_x\right|^2 + \left|\vec{\mathbf{c}}_y\right|^2 + \left|\vec{\mathbf{c}}_z\right|^2} \end{aligned}$$

2 | vectors problems

2.1 | adding two vectors $\langle a_1, a_2, a_3\rangle$ and $\langle b_1, b_2, b_3\rangle$

2.1.1 | the coordinates of the sum

$$(a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

2.1.2 | adding vectors

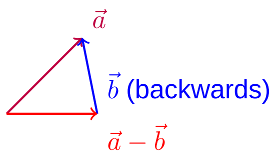
Geometrically, it is putting the vectors tip to tail. Follow one, then follow the other. Algebraically, it is adding each of the components. See the previous part

2.1.3 | **subtracting vectors**

We want to define $\vec{c} = \vec{a} - \vec{b}$ such that $\vec{b} + \vec{c} = \vec{a}$.

Geometrically, that means following $\vec{a}$, and then following $\vec{b}$ backwards (ie. we want to define a negative vector as the same vector backwards).

Algebracially, we see that it inherits the properties from addition/subtraction.



2.2 | **finding the vector between two points**

Take the points as vectors, and subtract them.

$$\langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$$

2.3 | **practice problems**

2.3.1 | **magnitude of** a

$$|\langle 4, 0, 3 \rangle| = \sqrt{4^2 + 3^2} = 5$$

2.3.2 | **magnitude of** b

$$|\langle -2, 1, 5 \rangle| = \sqrt{(-2)^2 + 1^2 + 5^2} = \sqrt{30} = 5.47722557505$$

2.3.3 | $\vec{a} + \vec{b}$

$$\langle 2, 1, 8 \rangle$$

2.3.4 | $\vec{a} - \vec{b}$

$$\langle 6, -1, -2 \rangle$$

2.3.5 | $3\vec{b}$

$$\langle -6, 3, 15 \rangle$$

2.3.6 | $2\vec{a} + 5\vec{b}$

$$\langle -2, 5, 31 \rangle$$

2.3.7 | $\hat{a}, \hat{b}$

$$\left\langle \frac{4}{5}, 0, \frac{3}{5} \right\rangle$$
$$\left\langle \frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right\rangle$$

2.3.8 | $\theta_{\vec{a}x}$

Lets make a right triangle in the plane that contains the tip and tail of the vector and the x-axis.
The height will be from the x-axis to the tail, so we'll take the diagonal in the yz plane

$$h = \sqrt{a_y^2 + a_z^2}$$

The base of the triangle will be along the x-axis. So, the base is just the x component a_x.
And so, we can find theta using the tangent

$$\tan\theta = \frac{\sqrt{a_y^2 + a_z^2}}{a_x}$$

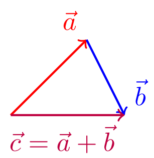
You could also do it with the cosine, as in dot product:

$$\cos\theta = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

2.3.9 | **actual numbers**

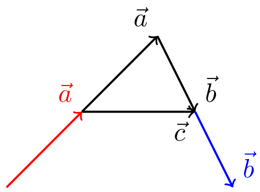
$$\begin{aligned} \cos\theta &= \frac{\vec{a}_x}{|\vec{a}|} \\ &= \frac{4}{5}\theta \quad = \cos^{-} \frac{4}{5} = \boxed{36.8} \end{aligned}$$

2.4 | **triangle proof**

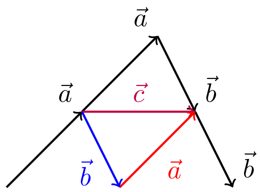


Lets let $\vec{a}, \vec{b}$ be the two sides and $\vec{c}$ be the middle side. This is the small triangle. Then, let's double each of the side lengths:

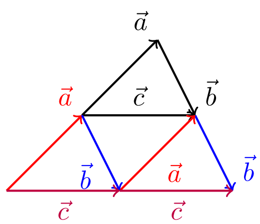
Now, we can double $\vec{a}$ and $\vec{b}$ to look at the impact on the larger triangle.



Because vector addition is commutative, $\vec{a} + \vec{b} = \vec{b} = \vec{a}$. Geometrically, this means



Using these new vectors, we can see that the bottom edge is equal to $2\vec{c}$



Algebraically:

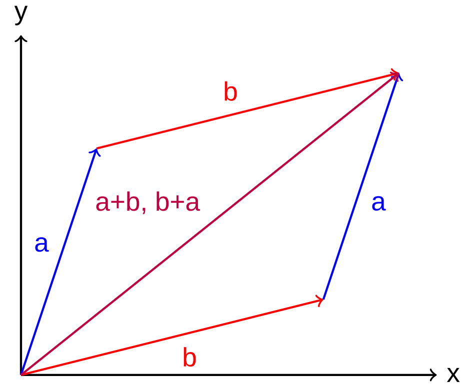
$$2\vec{a} + 2\vec{b} = 2(\vec{a} + \vec{b}) = 2\vec{c}$$

Thus, $\vec{c}$ the line segment that joins the midpoints of two sides of the triangle (sides $2\vec{a}$ and $2\vec{b}$. $\vec{c}$ is half the magnitude of the third side ($2\vec{c}$), and parallel because $2\vec{c}$ is a scalar multiple of $\vec{c}$.

3 | **proving vector properties**

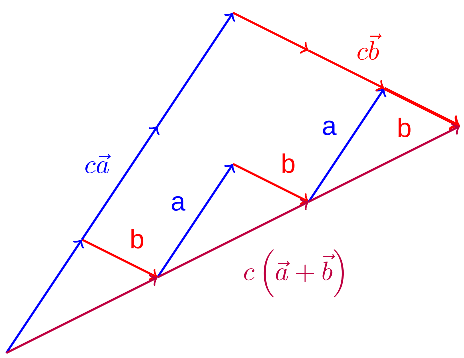
You are really stretching my LaTeX abilities here

3.1 | $\vec{a}+\vec{b}=\vec{b}+\vec{a}$



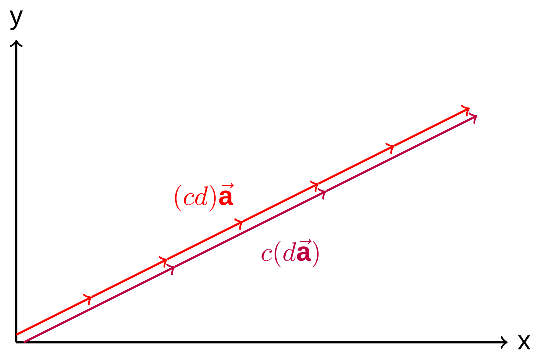
$$\begin{aligned}\vec{a}+\vec{b} &= \langle a_x+b_x, a_y+b_y\rangle \\ &= \langle b_x+a_x, b_y+a_y\rangle \\ &= \vec{b}+\vec{a}\end{aligned}$$

3.2 | $c(\mathbf{a}+\mathbf{b})=c\mathbf{a}+c\mathbf{b}$



$$\begin{aligned}c\left(\vec{a}+\vec{b}\right) &= \underbrace{\left(\vec{a}+\vec{b}\right)+\cdots+\left(\vec{a}+\vec{b}\right)}_{c\text{ times}} \\ &= \underbrace{\vec{a}+\cdots+\vec{a}}_{c\text{ times}}+\underbrace{\vec{b}+\cdots+\vec{b}}_{c\text{ times}} \\ &= c\vec{a}+c\vec{b}\end{aligned}$$

3.3 | $(cd)\mathbf{a} = c(d\mathbf{a})$



$$
\begin{aligned}
(cd)\vec{a} &= cd\langle a_x, a_y\rangle \\
&= c\langle da_x, da_y\rangle \\
&= c(d\vec{a})
\end{aligned}
$$