

We

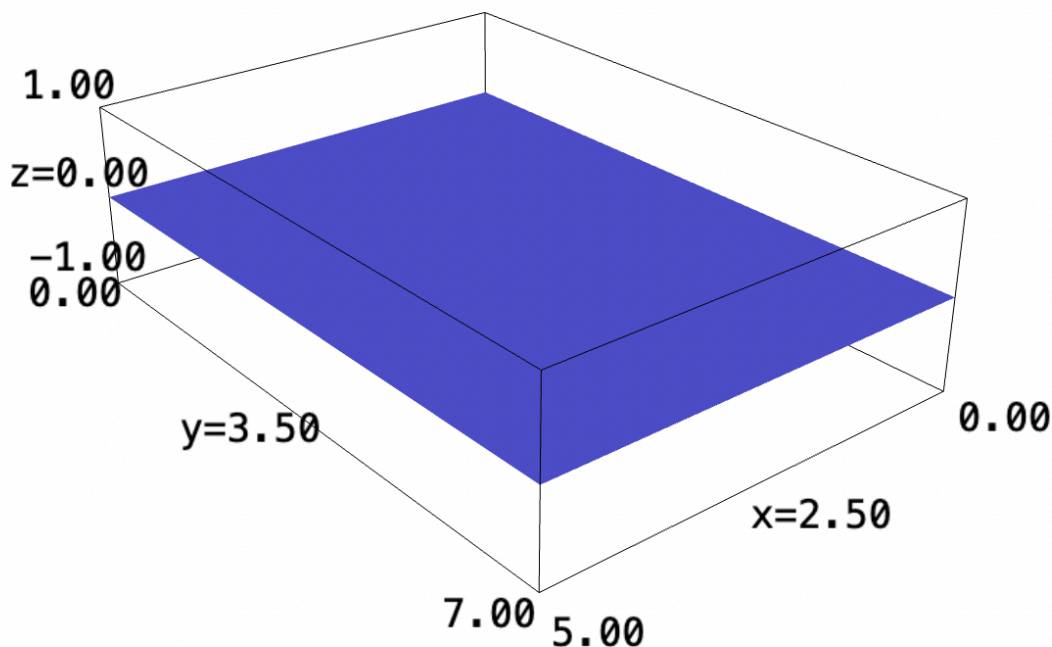
## 1 | Single Value Function

$$f_1 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (1)$$

$$f_1(x, y) = 0 \quad (2)$$

What's the area of this function?

```
f(x,y) = 0
plot3d(f, (x,0,5), (y,0,7))
```



We can take the area of the shape, essentially by taking the volume by height 1: that is, for a rectangle of  $l, w, h$ , its top-area is simply  $l \cdot w$ , also known as  $lw \cdot 1$ . Therefore:

$$\int_0^7 \int_0^5 1 dx dy = 35 \quad (3)$$

The area of the shape is therefore 35.

## 2 | Area of the Plane

We want to first figure the correction per every given slice  $dA = n dV$  to setup a surface integral. By pythagoras (i.e. projecting the changes to the parallelity of the surface), we have that:

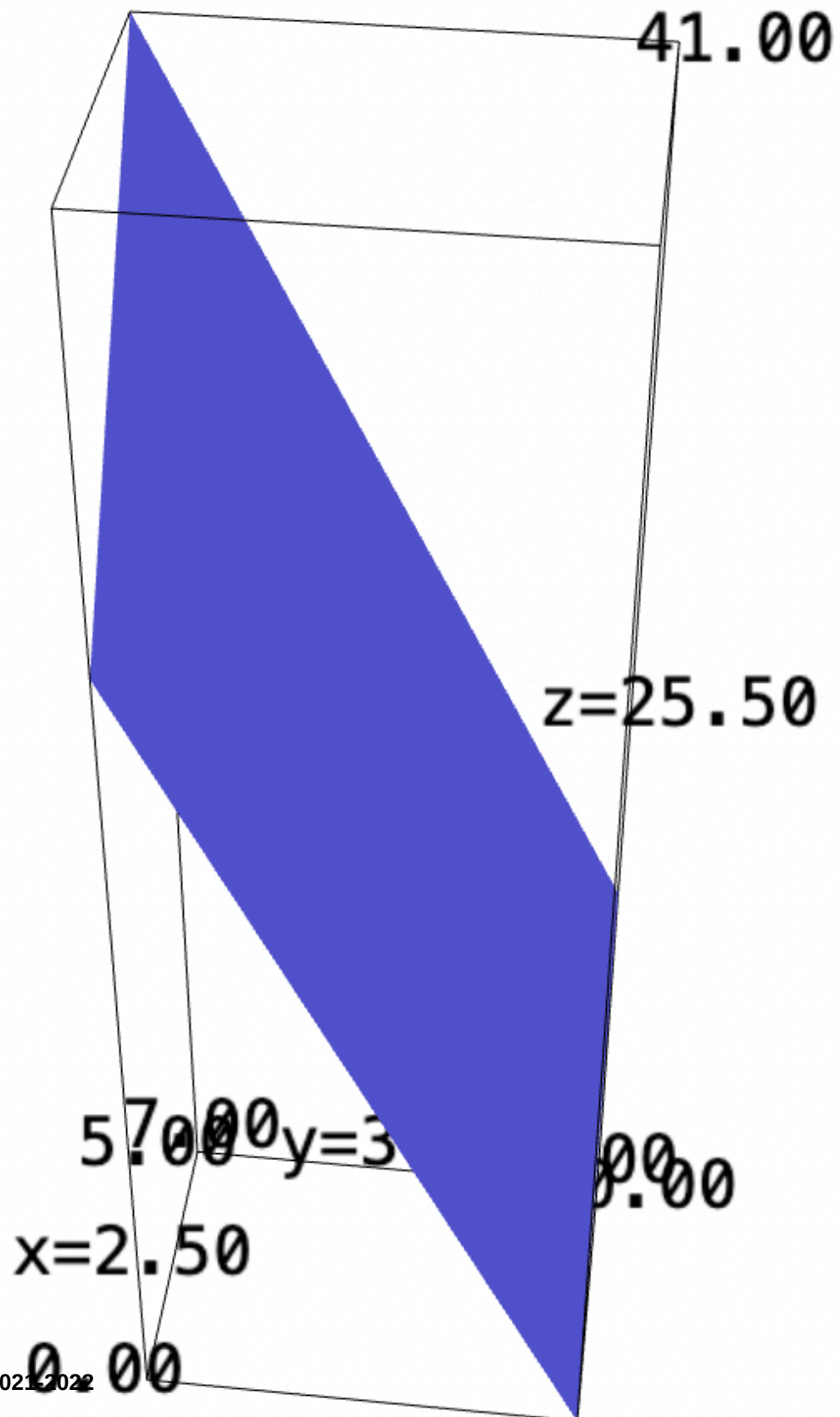
$$dA = \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2} dV \quad (4)$$

What's the area of the following function by (5, 7)?

$$f_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (5)$$

$$f_2(x, y) = 2x + 3y + 10 \quad (6)$$

```
f(x,y) = 2*x+3*y+10  
plot3d(f, (x,0,5), (y,0,7))
```



$$dA = \sqrt{1 + 4 + 9}dV = \sqrt{14} dV \quad (7)$$

Therefore, taking the integral:

$$\int_0^5 \int_0^7 \sqrt{14} dy dx \quad (8)$$

$$\Rightarrow 35\sqrt{14} \quad (9)$$

```
float(35*sqrt(14))
```

It appears that the surface area is about 130.958 units.

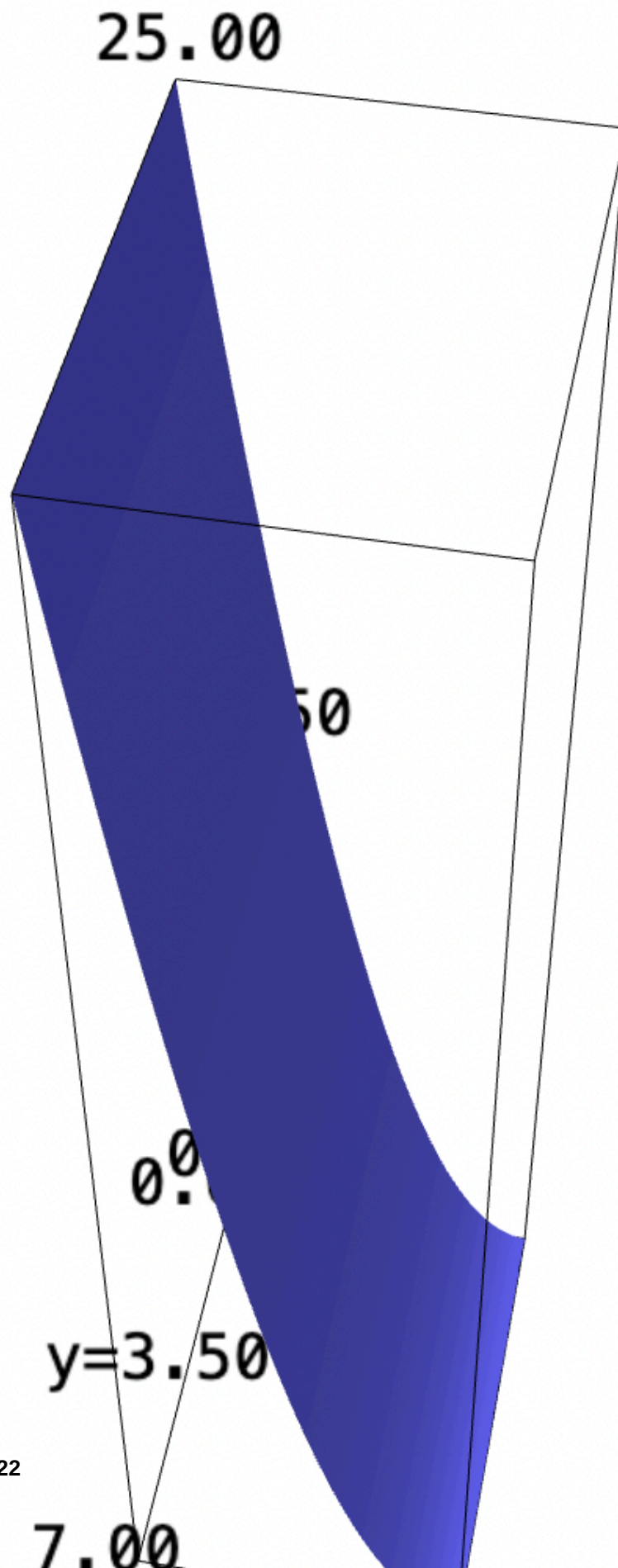
### 3 | Area of a Parabola

What's the area of the following function by (5, 7)?

$$f_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (10)$$

$$f_3(x, y) = x^2 \quad (11)$$

```
f(x,y) = x^2
plot3d(f, (x,0,5), (y,0,7))
```



We will again find the area correction factor:

$$dA = \sqrt{1 + 4x^2} dV \quad (12)$$

And therefore, taking the integral:

$$\int_0^5 \int_0^7 \sqrt{1 + 4x^2} dy dx \quad (13)$$

This problem is solvable by trig substitution followed by integration by parts. For now, however, we will leverage a calculator.

```
f(x,y) = sqrt(1+4*x^2)
f.integrate(y, 0,7).integrate(x,0,5)
float(f.integrate(y, 0,7).integrate(x,0,5))
```

Evidently, the surface area of the shape is about 181.1197 units.

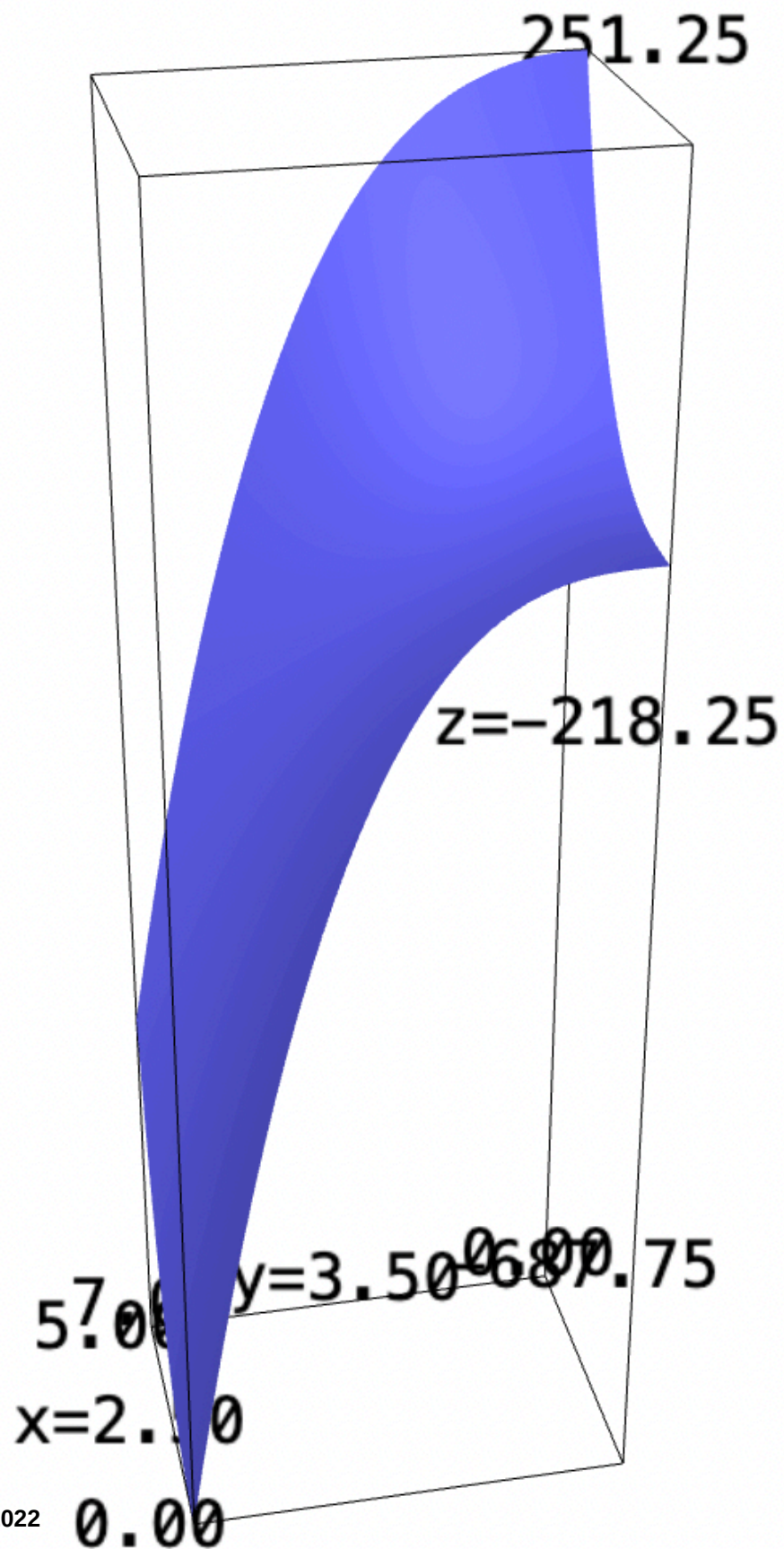
## 4 | Another Surface Area

What's the area of the following function by (5,7)?

$$f_3 : \mathbb{R}^2 \rightarrow \mathbb{R}^1 \quad (14)$$

$$f_3(x,y) = x^2 - y^2 \quad (15)$$

```
f(x,y) = (x^2-y^2)*sqrt(1+4*x^2+4*y^2)
plot3d(f, (x,0,5), (y,0,7))
```



Let's instead parameterize this function first to take its surface area. We will take the most basic parameterization.

$$\vec{v}(x, y) = x\hat{i} + y\hat{j} + (x^2 - y^2)\hat{k} \quad (16)$$

Taking, therefore, the partial derivatives:

$$\frac{\partial \vec{v}}{\partial x} = \hat{i} + 2x\hat{k} \quad (17)$$

$$\frac{\partial \vec{v}}{\partial y} = \hat{j} - 2y\hat{k} \quad (18)$$

Taking their cross product for the differential area, then:

$$(\hat{i} + 2x\hat{k}) \times (\hat{j} - 2y\hat{k}) \quad (19)$$

$$\Rightarrow (\hat{i}\hat{j} + 2x\hat{k}\hat{j} - \hat{i}2y\hat{k} - 2x\hat{k}2y\hat{k}) \quad (20)$$

$$\Rightarrow (\hat{k} - 2x\hat{i} + 2y\hat{j}) \quad (21)$$

We will take now the a

We will find the correction factor, again:

$$dA = \sqrt{1 + 4x^2 + 4y^2} dV \quad (22)$$

We will again take this integral, digitally this time:

$$\int_0^5 \int_0^7 (x^2 - y^2) \sqrt{1 + 4x^2 + 4y^2} dy dx \quad (23)$$

```
f(x,y) = (x^2-y^2)*sqrt(1+4*x^2+4*y^2)
f.integrate(y, 0,7).integrate(x,0,5)
float(f.integrate(y, 0,7).integrate(x,0,5))
```

The shape is largely underneath the x-axis during the area on the rectangle. Therefore, we have an negative area! It is about 3719.47 units.