PS#23: More 3D integrals!

Nueva Multivariable Calculus

(Read the solution set to the last pset, obviously! Also read my lengthy conceptual notes on multiple integration.)

1. On the last problem set, we calculated the volume of a sphere that had been Phineas Gage'd through the center by a cylinder. As you read on the solution set, the integral I used to calculate that volume, and the answer I got, was:

$$\begin{array}{c}\text{volume of this sphere}\\ \text{after being i-Lab drill-pressed}\\ \text{/Phineas Gage'd}\end{array} = \int_{z=-1}^{z=+1}\int_{\theta=0}^{\theta=2\pi}\int_{r=1/2}^{r=\sqrt{1-z^2}} 1\, r\, dr\, d\theta\, dz = \frac{5}{6}\pi$$

   This is wrong. I know it's wrong because Ella got a different answer, and I was all excited about putting *her* wrong answer on this problem set (anonymized, of course), and asking each of you to explain what *she* did wrong... but then I mistakenly showed it all to a physics PhD friend who is one of the most nerd-snipeable people I know, and she got obsessed and worked it all out and then pointed out that Ella was in fact right, and I was wrong. (I said many not-school-appropriate words in response. She sent me a picture of several pages of integrals and pictures that she had eagerly and immediately sketched and calculated.)

   So. What did I do wrong? (The actual answer should be $\frac{\sqrt{3}}{2}\pi$, which is close to what I got, numerically, but different.) Read through my solution, think about what I did, and explain what I did wrong! And if you didn't get $\frac{\sqrt{3}}{2}\pi$ over the weekend, do the problem again, correctly.)

   (Seniors, if you've spent too much time in my classes, and if you're salty about me locking you out, this is your chance to get back at me, on my own turf!!! Go wild.)

   (To be clear, I now know what I did wrong, although I'm certainly not above legitimately asking any of you for help!)

2. Using a triple integral, find the volume of a sphere!!! (Show and explain all your work, obviously! We all know how this story ends, so it's all about the *telling*.)

3. Find the volume of the shape given by the upper half of the sphere $z^2 = 4 - x^2 - y^2$ with the cone $z^2 = 3x^2 + 3y^2$ removed. (And do all this using a *single* integral!)

   (Pictures, explanation, etc.; I don't need to tell you to do that...)

4. Gah, guys, I'm frustrated that I set up that integral in the last problem set wrong and got the wrong answer. MVC is so hard! Let's go back to 1VC! Much easier; much more fun, You might know that the **bell curve** or **Gaussian distribution** from statistics is given (more or less) by:

$$f(x) = e^{-x^2}$$

   What's the total area under this function, from $x = -\infty$ to $x = +\infty$? Work out this integral, by hand, without using technology, or looking anything up. (Seriously!)

   (It's not 1—to do actual statistics with this function, people usually **normalize** it first, dividing it by the total area so that we get a probability distribution that adds up to just 1, i.e., 100%.)