Homework 3
M1522.001800 Computer Vision (2019 Spring)

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1 Theory Questions [30 pts]

1.1 Question : Focal length (10 points)

By appying **"thin lens equation"**, the relation between effective focal length(d), focal length(f), and the distance(D) from object to lens is described as below. and magnification is described as d and D

$$\frac{1}{d} + \frac{1}{D} = \frac{1}{f} \qquad m = \frac{d}{D}$$

Using thses two equation, we can show the fact that

$$\begin{aligned} d &= (1+m)f && (1)\\ &= (1+\frac{d}{D})(\frac{1}{\frac{1}{d}+\frac{1}{D}}) && (2)\\ &= (\frac{D+d}{D})(\frac{dD}{D+d}) && (3)\\ &= d && (4) \end{aligned}$$

And also another fact that

$$\begin{aligned} d+D &= (m+2+\frac{1}{m})f && (5)\\ &= (m+1)f + (1+\frac{1}{m})f && (6)\\ &= d + (1+\frac{1}{m})f \qquad \text{(by above fact)} && (7)\\ &= d + (1+\frac{D}{d})(\frac{dD}{D+d}) && (8)\\ &= d + (\frac{D+d}{d})(\frac{dD}{D+d}) && (9)\\ &= d+D && (10) \end{aligned}$$

1.2 Question : Homography (20 points)

(a) 3×3 $\qquad \because$ homogeneous coordinates add one more coordinate for division
(b) 3. Even if z coordinate do not need after division, x and y are affected by z. So column space of S should be $\mathbb{R}^3$, which means the rank of S should be 3.

(c) when it comes to homography, it seems to have 9 unknowns. However, in fact, it has 8 unknowns because homography(S) is up to scale like below.

$$p = \begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} \equiv \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x_q \\ y_q \\ 1 \end{bmatrix} = Sq \equiv k(Sq) = (kS)q \qquad k \neq 0, k \in \mathbb{R}$$

$$p \longrightarrow (x_p, y_p), \qquad k(Sq) \longrightarrow (\frac{kx_p}{k}, \frac{ky_p}{k}) = (x_p, y_p)$$

$\because$ After converting to image coordinates from homogeneous coordinates, p and $k(Sq)$ is same.
So we can set k to arbitrary element of S which is not zero to reduce one unknown(say i). then

$$\begin{bmatrix} x_p \\ y_p \\ 1 \end{bmatrix} \equiv \begin{bmatrix} a/i & b/i & c/i \\ d/i & e/i & f/i \\ g/i & h/i & 1 \end{bmatrix} \begin{bmatrix} x_q \\ y_q \\ 1 \end{bmatrix}$$

Finally, we need at least 8 equations which means 4 point correspondences to estimate S by working with least square method.

(d) Under below definition,

$$p_1{}^i = \begin{bmatrix} x_1{}^i \\ y_1{}^i \\ 1 \end{bmatrix} \equiv \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_2{}^i \\ y_2{}^i \\ 1 \end{bmatrix} = Hp_2{}^i$$

we can describe $p_1{}^i$ as H and $p_2{}^i$

$$x_1{}^i = \frac{h_{00}x_2{}^i + h_{01}y_2{}^i + h_{02}}{h_{20}x_2{}^i + h_{21}y_2{}^i + h_{22}} \qquad y_1{}^i = \frac{h_{10}x_2{}^i + h_{11}y_2{}^i + h_{12}}{h_{20}x_2{}^i + h_{21}y_2{}^i + h_{22}}$$

multifly denominator each coordinates of $p_1{}^i$ and apply least square method to the difference of left term and right term. Keep this procedure for every point(N), we can derive the requirement.

$$\begin{gathered}(h_{20}x_2{}^i + h_{21}y_2{}^i + h_{22})x_1{}^i = h_{00}x_2{}^i + h_{01}y_2{}^i + h_{02} \\ \hookrightarrow h_{00}x_2{}^i + h_{01}y_2{}^i + h_{02} - (h_{20}x_2{}^i + h_{21}y_2{}^i + h_{22})x_1{}^i = 0 \\ (h_{20}x_2{}^i + h_{21}y_2{}^i + h_{22})y_1{}^i = h_{10}x_2{}^i + h_{11}y_2{}^i + h_{12} \\ \hookrightarrow h_{10}x_2{}^i + h_{11}y_2{}^i + h_{12} - (h_{20}x_2{}^i + h_{21}y_2{}^i + h_{22})y_1{}^i = 0\end{gathered}$$

$$\begin{bmatrix} x_2{}^1 & y_2{}^1 & 1 & 0 & 0 & 0 & -x_1{}^1x_2{}^1 & -x_1{}^1y_2{}^1 & x_1{}^1 \\ 0 & 0 & 0 & x_2{}^1 & y_2{}^1 & 1 & -y_1{}^1x_2{}^1 & -y_1{}^1y_2{}^1 & y_1{}^1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_2{}^N & y_2{}^N & 1 & 0 & 0 & 0 & -x_1{}^Nx_2{}^N & -x_1{}^Ny_2{}^N & x_1{}^N \\ 0 & 0 & 0 & x_2{}^N & y_2{}^N & 1 & -y_1{}^Nx_2{}^N & -y_1{}^Ny_2{}^N & y_1{}^N \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \dots \\ 0 \end{bmatrix}$$

To solve this matrix equation, we need just 4 correspondences, because there is unseen constraint $||h|| = 1$ which of value we can arbitrarily define. the reason why we can use this additional constraint is explained at (c)