Курганов Владислав Андреевич
st132903
st132903@student.spbu.ru

Homework Assignment 7

Алгебра и геометрия, 1 семестр

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Задача 276.

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix}$$

Решение:

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 1 & 3 & 6 & 10 \\ 1 & 4 & 10 & 20 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 9 \\ 0 & 3 & 9 & 19 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 3 \\ 0 & 2 & 5 & 9 \\ 0 & 3 & 9 & 19 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 \\ 0 & 2 & 5 & -1 \\ 0 & 3 & 9 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1$$

Answer: 1

Задача 295.

$$\begin{vmatrix} 1 & 2 & 2 & \dots & 2 \\ 2 & 2 & 2 & \dots & 2 \\ 2 & 2 & 3 & \dots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & \dots & n \end{vmatrix}$$

Решение:

$$\begin{vmatrix} 1 & 2 & 2 & \dots & 2 \\ 2 & 2 & 2 & \dots & 2 \\ 2 & 2 & 3 & \dots & 2 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 2 & 2 & 2 & \dots & n \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 & \dots & 2 \\ 1 & 0 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & (n-2) \end{vmatrix} = \begin{vmatrix} a_0 & 2 & 2 & \dots & 2 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (n-2) \end{vmatrix} = - \begin{vmatrix} 2 & a_0 & 2 & \dots & 2 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & (n-2) \end{vmatrix}$$

$$= -2 * (1 * 1 * 2 * 3 * ... * (n-2)) = -2 * (n-2)!$$

$Answer: -2 * (n-2)!$

Задача 280.

$$\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix}$$

Решение:

$$\begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 1 \\ 1 & 1 & 4 & 1 & 1 \\ 1 & 1 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 6 \end{vmatrix} = \begin{vmatrix} 2 & 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 0 & 0 \\ -1 & 0 & 3 & 0 & 0 \\ -1 & 0 & 0 & 4 & 0 \\ -1 & 0 & 0 & 0 & 5 \end{vmatrix} = \begin{vmatrix} a_0 & 1 & 1 & 1 & 1 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{vmatrix}$$

$a_0 = (2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5})$

$= (2 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}) * 2 * 3 * 4 * 5 = 394$

$Answer: 394$

Задача 306.
$$\begin{vmatrix} \alpha & \alpha\beta & 0 & \dots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix}$$

Решение:

$$\begin{vmatrix} \alpha & \alpha\beta & 0 & \dots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix} = \alpha \begin{vmatrix} 1 & \beta & 0 & \dots & 0 & 0 \\ 1 & \alpha+\beta & \alpha\beta & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix} =$$

$$\alpha \begin{vmatrix} 1 & \beta & 0 & \dots & 0 & 0 \\ 0 & \alpha & \alpha\beta & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix} = \alpha^2 \begin{vmatrix} 1 & \beta & 0 & \dots & 0 & 0 \\ 0 & 1 & \beta & \dots & 0 & 0 \\ 0 & 1 & \alpha+\beta & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 & \alpha+\beta \end{vmatrix} =$$

$$\alpha^n \begin{vmatrix} 1 & \beta & 0 & \dots & 0 & 0 \\ 0 & 1 & \beta & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 & 1 \end{vmatrix} = \alpha^n * 1 = \alpha^n$$

$Answer: \alpha^n$
