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Homework Assignment 9

Алгебра и геометрия, 1 семестр

2024.12.11

Задача 410(d).

Обратить матрицу: $\begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Решение: $\begin{pmatrix} 1 & 3 & -5 & 7 \\ 0 & 1 & 2 & -3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \sim \left(\begin{array}{cccc|cccc} 1 & 3 & -5 & 0 & 1 & 0 & 0 & -7 \\ 0 & 1 & 2 & 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 3 & 0 & 0 & 1 & 0 & 5 & -17 \\ 0 & 1 & 0 & 0 & 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \sim$

$\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1 & -3 & 11 & -38 \\ 0 & 1 & 0 & 0 & 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right)$

$\mathrm{A}^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -3 & 11 & -38 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Answer:

$A^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -3 & 11 & -38 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Задача 410(f).

Обратить матрицу: $\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

Решение:

$\mathrm{detA} = (\text{-}1)^{1+1}\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} + (-1)^{1+2}\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} + (-1)^{1+3}\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix}$

$+ (-1)^{1+4}\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = -4 - 4 - 4 - 4 = -16$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = -4$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -4$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -4$$

$$A_{14} = (-1)^{1+4} \begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = -4$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -4$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -4$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = 4$$

$$A_{24} = (-1)^{2+4} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = 4$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & -1 & 1 \end{vmatrix} = -4$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = 4$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -4$$

$$A_{34} = (-1)^{3+4} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 4$$

$$A_{41} = (-1)^{4+1} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 1 \end{vmatrix} = 4$$

$$A_{42} = (-1)^{4+2} \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 4$$

$$A_{43} = (-1)^{4+3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 4$$

$$A_{44} = (-1)^{4+4} \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = -4$$

$$A^{-1} = \tfrac{-1}{16} \begin{pmatrix} -4 & -4 & -4 & -4 \\ -4 & -4 & 4 & 4 \\ -4 & 4 & -4 & 4 \\ -4 & 4 & 4 & -4 \end{pmatrix} = \tfrac{-1}{4} \begin{pmatrix} -1 & -1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & 1 & 1 & -1 \end{pmatrix} = \tfrac{1}{4} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

$Answer:$

$$\frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

Задача 416.

$$\begin{pmatrix} 2 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 \end{pmatrix}^{-1}$$

Решение:

$$\left(\begin{array}{ccccc|ccccc} 2 & -1 & 0 & \dots & 0 & -1 & 0 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & 0 & 0 & 1 & 0 & \dots & 0 \\ 0 & -1 & 2 & \dots & 0 & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 & 0 & 0 & 0 & \dots & 1 \end{array}\right) \begin{array}{c} +s_2+s_3+...+s_n \\ +s_3+s_4+...+s_n \\ +s_4+...+s_n \\ \\ ... \end{array}$$

$$\left(\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \dots & 1 & 1 & 1 & 1 & \dots & 1 \\ -1 & 1 & 0 & \dots & 1 & 0 & 1 & 1 & \dots & 1 \\ 0 & -1 & 1 & \dots & 1 & 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 & 0 & 0 & 0 & \dots & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \dots & 1 & 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 0 & \dots & 2 & 1 & 2 & 2 & \dots & 2 \\ 0 & -1 & 1 & \dots & 1 & 0 & 0 & 1 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 & 0 & 0 & 0 & \dots & 1 \end{array}\right) \sim$$

$$\left(\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \dots & 1 & 1 & 1 & 1 & \dots & 1 \\ 0 & 1 & 0 & \dots & 2 & 1 & 2 & 2 & \dots & 2 \\ 0 & 0 & 1 & \dots & 3 & 1 & 2 & 3 & \dots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & n+1 & 1 & 2 & 3 & \dots & n \end{array}\right) \begin{array}{c} -\frac{1}{n+1}\cdot s_n \\ -\frac{2}{n+1}\cdot s_n \\ -\frac{3}{n+1}\cdot s_n \\ \\ ..... \\ \cdot\frac{1}{n+1} \end{array} \sim$$

$$\left(\begin{array}{ccccc|ccccc} 1 & 0 & 0 & \dots & 0 & \frac{n}{n+1} & \frac{n-1}{n+1} & \frac{n-2}{n+1} & \dots & \frac{1}{n+1} \\ 0 & 1 & 0 & \dots & 0 & \frac{n-1}{n+1} & \frac{2n-2}{n+1} & \frac{2n-2}{n+1} & \dots & \frac{2}{n+1} \\ 0 & 0 & 1 & \dots & 0 & \frac{n-2}{n+1} & \frac{2n-4}{n+1} & \frac{3n-6}{n+1} & \dots & \frac{3}{n+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 & \frac{1}{n+1} & \frac{2}{n+1} & \frac{3}{n+1} & \dots & \frac{n}{n+1} \end{array}\right)$$

$$=> \ \frac{1}{n+1}\begin{pmatrix} n & n-1 & n-2 & \dots & 1 \\ n-1 & 2n-2 & 2n-4 & \dots & 2 \\ n-2 & 2n-4 & 3n-6 & \dots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \dots & n \end{pmatrix}$$

$$Answer: \frac{1}{n+1}\begin{pmatrix} n & n-1 & n-2 & \dots & 1 \\ n-1 & 2(n-1) & 2(n-2) & \dots & 2 \\ n-2 & 2(n-2) & 3(n-2) & \dots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \dots & n \end{pmatrix}$$

Задача 442(b).

Найти ранг матрицы: $\begin{pmatrix} 2 & 1 & 11 & 2 \\ 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 2 & -1 & 5 & -6 \end{pmatrix}$

Решение:

$$\begin{pmatrix} 2 & 1 & 11 & 2 \\ 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 2 & -1 & 5 & -6 \end{pmatrix} \sim \begin{pmatrix} 0 & 2 & 6 & 8 \\ 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 0 & -2 & -6 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 0 & 2 & 6 & 8 \\ 0 & -2 & -6 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 0 & 2 & 6 & 8 \\ 0 & 2 & 6 & 8 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 4 & -1 \\ 11 & 4 & 56 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$=> RANK = 2$

Задача 442(c).

Найти ранг матрицы: $\begin{pmatrix} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 & 6 \\ 1 & 2 & 3 & 14 & 32 \\ 4 & 5 & 6 & 32 & 77 \end{pmatrix}$

Решение:

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 & 6 \\ 1 & 2 & 3 & 14 & 32 \\ 4 & 5 & 6 & 32 & 77 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 2 & 3 & 13 & 28 \\ 0 & 5 & 28 & 61 & 77 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 3 & 9 & 18 \\ 0 & 0 & 6 & 18 & 36 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1 & 4 \\ 0 & 1 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$=> RANK = 3$
