

Задача 102.

Вычислить $(x - 1 - i)(x - 1 + i)(x + 1 + i)(x + 1 - i)$.

Решение:

$$(x-1-i)(x-1+i)(x+1+i)(x+1-i) = (x^2 - 2x + 2)(x^2 + 2x + 2)$$

$$\alpha * \bar{\alpha} = (x - 1)^2 + 1 = x^2 - 2x + 2$$

$$\beta * \bar{\beta} = x^2 + 2x + 2 = (x + 1)^2 + 1$$

$$(x^2 - 2x + 2)(x^2 + 2x + 2) = x^4 2x^3 + 2x^2 - 2x^3 - 4x^2 - 4x + 2x^2 + 4x + 4 = x^4 - 4x^2 + 4 + 4x^2 = x^4 + 4$$

Answer : $x^4 + 4$

Задача 105(с).

Вычислить $(1 + 2i)^5 - (1 - 2i)^5$.

Решение:

$$(1+2i)^5 - (1-2i)^5$$

$$1)(1+2i)^5 = (1+2i)^2(1+2i)^2(1+2i) = (1+4i+4i^2)(1+4i+4i^2)(1+2i) = (-3+4i)(-3+4i)(1+2i) = (9-12i-12i+16i^2)(1+2i) = (9-24i-16)(1+2i) = (-24i-7)(1+2i) = -24i-48i^2-7-14i = -38i+41$$

$$(1-2i)^5 = 38i+41$$

$$(1+2i)^5 - (1-2i)^5 = -38i+41 - 38i-41 = -76i$$

Answer : $-76i$

Задача 107(d).

Вычислить $\frac{(1-i)^5 - 1}{(1+i)^5 + 1}$.

Решение:

$$(1-i)^5 = (1-i)^2 * (1-i)^2 * (1-i) = 4i^2 * (1-i) = 4i^2 - 4i^3 = 4i - 4$$

$$(1-i)^2 = 1 - 2i - 1 = -2i$$

$$\frac{(4i - 4 - 1)}{(-4i - 4 + 1)} = \frac{(-5 + 4i)}{(-3 - 4i)} = \frac{(-5 + 4i)}{(-3 - 4i)} * \frac{(-3 + 4i)}{(-3 + 4i)} = \frac{(-3 + 4i)}{(-3 + 4i)} = \frac{-12i + 16i^2 + 15 - 20i}{(12i - 16i^2 + 9 - 12i)} = \frac{-1 - 32i}{25}$$

$$\text{Answer : } \frac{-1 - 32i}{25}$$

Задача 108(b).

Решить систему уравнений

$$\begin{cases} (2+i)x + (2-i)y = 6, \\ (3+2i)x + (3-2i)y = 8. \end{cases}$$

Решение:

$$\begin{cases} (2+i)x + (2-i)y = 6 \\ (3+2i)x + (3-2i)y = 8 \end{cases}$$

$$\begin{cases} (3+2i)(2+i)x + (2-i)(3+2i)y = 6(3+2i) \\ (3+2i)(2+i)x + (3-2i)(2+i)y = 8(2+i) \end{cases}$$

$$\begin{aligned} (2-i)(3+2i)y - (3-2i)(2+i)y &= 6(3+2i) - 8(2+i) \\ (6+4i-3i+2)y - (6+3i-4i+2)y &= 18+12i-16-8 \\ y(2i) &= 2+4i \end{aligned}$$

$$y = \frac{2+4i}{2i} = \frac{1+2i}{i} = \frac{i(2-i)}{i} = 2-i$$

$$\begin{aligned} (2+i)x + (2-i)(2-i) &= 6 \\ 2x + ix + 4 - 4i - 1 &= 6 \\ x(2+i) &= 4i + 3 \\ x(2+i) &= 4i + 3 \end{aligned}$$

$$x = \frac{4i+3}{2+i} * \frac{2-i}{2-i} = \frac{8i+4+6-3i}{5} = \frac{5i+10}{5} = i+2$$

Answer : $x = i + 2; y = 2 - i$ **Задача 112(b, g).**Вычислить: b) $\sqrt{-8i}$, g) $\sqrt{2-3i}$.*Решение:*

b) $\sqrt{-8i} = a + bi$

$$-8i = (a + bi)^2$$

$$-8i = a^2 + 2abi + b^2 * i^2$$

$$-8i = a^2 + 2abi - b^2$$

$$\begin{cases} a^2 - b^2 = 0 \\ -8 = 2ab \end{cases} \Rightarrow \begin{cases} a^2 = b^2 \\ -4 = ab \end{cases} \Rightarrow \begin{cases} a^2 = b^2 \\ a = \frac{-4}{b} \end{cases} \Rightarrow \begin{cases} (\frac{-4}{b})^2 = b^2 \\ \frac{16}{b^2} = b^2 \end{cases} \Rightarrow \begin{cases} \frac{16}{b^2} = b^2 \\ a = \frac{-4}{b} \end{cases} \Rightarrow$$

$$\begin{cases} b = 2 \\ a = -2 \end{cases} \text{ and } \begin{cases} b = -2 \\ a = 2 \end{cases}$$

Answer : $(2 - 2i), (-2 + 2i)$

$$\begin{aligned}
&g) \sqrt{2-3i} = a + bi \\
&(2-3i) = (a+bi)^2 \\
&2-3i = a^2 + 2abi + b^2 * i^2 \\
&2-3i = a^2 + 2abi - b^2
\end{aligned}$$

$$\begin{cases} a^2 - b^2 = 2 \\ 2ab = -3 \end{cases} \Rightarrow \begin{cases} a^2 - b^2 = 2 \\ b = \frac{-3}{2a} \end{cases} \Rightarrow \begin{cases} a^2 - (\frac{-3}{2a})^2 = 2 \\ b = \frac{-3}{2a} \end{cases}$$

$$\begin{aligned}
&a^2 - \frac{9}{4a^2} = 2 \\
&4a^4 - 8a^2 - 9 = 0 \\
&t = a^2, t \geq 0 \\
&4t^2 - 8t - 9 = 0
\end{aligned}$$

$$\begin{aligned}
&D = 64 + 4 * 4 * 9 = 4\sqrt{13} \\
&t_1 = \frac{8 + 4\sqrt{13}}{8} = \frac{2 + \sqrt{13}}{2} \\
&t_2 = \frac{8 - 4\sqrt{13}}{8} = \frac{2 - \sqrt{13}}{2}, t < 0!
\end{aligned}$$

$$a^2 = \frac{2 + \sqrt{13}}{2} \Rightarrow a_1 = \sqrt{\frac{2 + \sqrt{13}}{2}}, a_2 = -\sqrt{\frac{2 + \sqrt{13}}{2}}$$

$$b_1 = \frac{-3}{2\sqrt{\frac{2 + \sqrt{13}}{2}}} = \frac{-3}{2 * \frac{\sqrt{2 + \sqrt{13}}}{\sqrt{2}}} = \frac{-3\sqrt{2}}{2\sqrt{2 + \sqrt{13}}} = \frac{-\sqrt{18}}{\sqrt{8 + 4\sqrt{13}}}$$

$$b_2 = \frac{3}{2\sqrt{\frac{2 + \sqrt{13}}{2}}} = \frac{3}{2 * \frac{\sqrt{2 + \sqrt{13}}}{\sqrt{2}}} = \frac{3\sqrt{2}}{2\sqrt{2 + \sqrt{13}}} = \frac{\sqrt{18}}{\sqrt{8 + 4\sqrt{13}}}$$

$$\text{Answer: } (\sqrt{\frac{2 + \sqrt{13}}{2}} - i * \frac{-\sqrt{18}}{\sqrt{8 + 4\sqrt{13}}}); -(\sqrt{\frac{2 + \sqrt{13}}{2}} - i * \frac{-\sqrt{18}}{\sqrt{8 + 4\sqrt{13}}})$$

Задача 113(b).

Решить уравнение $x^2 - (3 - 2i)x + (5 - 5i) = 0$.

$$\text{Решение: } x^2 - (3-2i)x + (5-5i) = 0$$

$$a=1 \quad b=3-2i \quad c=5-5i$$

$$D = (3 - 2i)^2 - 4(5 - 5i) = 9 - 12i - 4 - 20 + 20i = 8i - 15$$

$$x_1 = \frac{-(3 - 2i) + \sqrt{8i - 15}}{2} = \frac{-3 + 2i + \sqrt{8i - 15}}{2}$$

$$x_2 = \frac{-3 + 2i - \sqrt{8i - 15}}{2}$$

$$\begin{aligned}
&\sqrt{8i - 15} = ai + b \\
&8i - 15 = a^2 + 2iab - b^2
\end{aligned}$$

$$\begin{aligned}
\begin{cases} -15 = a^2 - b^2 \\ 8i = 2iab \end{cases} &\Rightarrow \begin{cases} a^2 - b^2 + 15 = 0 \\ 4 = ab \end{cases} \Rightarrow \begin{cases} \frac{16}{b^2} - b^2 + 15 = 0 \\ a = \frac{4}{b} \end{cases} \Rightarrow \begin{cases} 16 - b^4 + 15b^2 = 0 \\ a = \frac{4}{b} \end{cases} \Rightarrow \begin{cases} t^2 - 15t^2 - 16 = 0 \\ a = \frac{4}{b} \end{cases} \\
\Rightarrow \begin{cases} t_1 = -1 \\ t_2 = 16 \\ a = \frac{4}{b} \end{cases} &\Rightarrow \begin{cases} b_1^2 = -1 \\ b_2^2 = 16 \\ a = \frac{4}{b} \end{cases} \Rightarrow \begin{cases} b_1 = \pm\sqrt{-1} \\ b_2 = \pm 4 \\ a = \frac{4}{b} \end{cases} \Rightarrow \begin{cases} b_1^2 = -1 \\ b_2^2 = 16 \\ a = \frac{4}{b} \end{cases} \Rightarrow \begin{cases} b_1 = 4 \\ b_2 = -4 \\ a_1 = 1 \\ a_2 = -1 \end{cases}
\end{aligned}$$

$$\sqrt{8i - 15} = 1 + 4i$$

$$\sqrt{8i - 15} = -1 - 4i$$

$$x_1 = \frac{-(3 - 2i)(+)(1 + 4i)}{2}$$

$$x_2 = \frac{-(3 - 2i)(-)(-1 - 4i)}{2}$$

$$x_1 = \frac{-3 + 2i + 1 + 4i}{2} = \frac{-2 + 6i}{2} = -1 + 3i$$

$$x_2 = \frac{-3 + 2i - 1 - 4i}{2} = \frac{-4 - 2i}{2} = -2 - i$$

$$\text{Answer : } (-1 + 3i); (-2 - i)$$
