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Sensitivity to geometry in humans and other animals

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QUESTION

Are humans and other animals sensitive to geometry when categorizing objects and if they are, what intuitions could underlie this sensitivity?

ABSTRACT

Geometry can be defined as the mathematical formalization of space and of objects with spatial content (i.e., objects defined by angles, lengths, parallel lines, etc.). Amongst all theories of geometry, Euclidean geometry is considered the most intuitive of all for humans. The reason for this, it has been argued, is that humans and maybe other animals, evolved intuitions coherent with Euclidean principles that helped them better interact with the physical world. The physical world, however, is not Euclidean in all of its aspects. Objects' mass and their interaction with the gravitational field are not considered in Euclidean geometry, although objects' geometrical characteristics can determine their physical properties. This association between geometry and physics could influence how animal minds categorize geometrical objects. In this paper, I briefly review the evidence suggesting that humans and other animals are sensitive to differences in objects' geometrical characteristics. I further address the question whether Euclidean or physics intuitions underlie humans' and other animals' sensitivity to geometry and conclude that although physics intuitions might better explain how animals, including humans, categorize objects in terms of basic geometrical characteristics such as their shape, size, orientation, sense, and position, humans are special in their ability to reason about more abstract Euclidean concepts.

Keywords: cognition, geometry intuitions, intuitive physics, Euclidean intuitions, object categorization, invariant transformations

INTRODUCTION

Geometry is one of the oldest branches of mathematics. Its earliest records go back to ancient Mesopotamia and Egypt [1], where its use was mostly practical; amongst other things, it helped kings and pharaohs build mighty palaces and gigantic pyramids. With the advent of Euclid and his Elements, concepts of geometry were explained in such an intuitive and rigorous way that they crossed centuries without alterations [2]. Euclidean geometry was geometry, and its axioms were thought to accurately reflect the geometry of the physical world. Today, although several non-Euclidean theories exist, Euclidean geometry remains the most famous theory of geometry and the one that is primarily taught in schools [3].

Geometry can be defined as the science of space [4]. As such, it comprises concepts like distance, shape, angle, size, and relative position of objects. Euclidean plane geometry, for example, is based on a set of axioms about points, lines, and distances (e.g., "Given two points, it is possible to draw a straight line joining them") from which are derived concepts like angles, circles, triangles and other polygons. One of the endeavors of geometry is the search for invariants [5][6]. Invariants are properties of space that remain unaltered after a transformation. For example, in Euclidean plane geometry, the angle and relative distance between different segments of objects are considered fundamental properties [5]. Two objects that share these properties (i.e., that have the same relative shape and size) are considered to be equal or congruent. These fundamental properties are invariant under rotation (the movement of an object around a point), translation (the movement of an object along a line), reflection (the creation of the mirror image of an object on the other side of a line), and homothecy (the change of an objects' global size); these transformations are thus called invariant transformations. Moreover, two objects whose metric proportions are conserved but whose global size is different are considered similar. In contrast, in Euclidean geometry of solid objects, which is part of Euclid's full theory, global size and sense are also fundamental properties of objects, so that the invariant transformations in this variant of Euclidean geometry are only rotation and translation [5]. In summary, in Euclidean geometry, changes in position and orientation can always be ignored when categorizing objects, whereas variability in the metric proportions of objects can never be overlooked.

Because animals interact with objects that vary in terms of their geometrical characteristics, it seems only natural to ask if animal minds are sensitive to objects' geometrical characteristics and whether they take them into account when categorizing objects. The idea that animals might be sensitive to geometry is not new. Philosophers, mathematicians, and psychologists have argued that animal minds should incorporate Euclidean principles because these principles reflect the physical world. This, in turn, would explain why Euclidean concepts seem so intuitive to the human mind [3][5][7][8][9][10][11] [12]. This is equivalent to saying that analyzing space and objects in terms of Euclidean principles gave an advantage to animals (at least to humans), so that over time, minds embracing Euclidean principles reproduced more successfully and survived longer.

The present paper pursues two goals. The first is to give a brief overview of the research that has investigated whether or not animal minds categorize objects based on their geometrical characteristics. The second goal is to shed light on the underlying intuitions best explaining the documented behavior. A particular focus here will be on the question to which extent those findings actually support the hypothesis that animals have indeed Euclidean intuitions. To foreshadow, the literature review will reveal that some findings seem to speak against this hypothesis. I will then present an alternative view that I and my colleagues have proposed recently [13]. The core idea of the new theory is that animals, including humans, do not spontaneously categorize geometric objects based on Euclidean properties per se but rather based on the physical properties that are determined by objects' geometrical characteristics.

SENSITIVITY TO GEOMETRY IN HUMANS

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Are human minds sensitive to objects' geometrical characteristics? Decades of research have been devoted to this question, with different studies investigating slightly different facets of geometrical cognition. The following overview will be structured according to the geometrical characteristics that have been tested in different studies.

To make following along easier, Figure 1 depicts a two-dimensional and a three-dimensional frame of reference that helps illustrate relevant geometrical transformations. Whenever I will say something about the “orientation" of an object, I either speak, in the 2D context, of the degree of that object's rotation around a point in the 2D plane created by the X- and Y-axis (left reference frame in Figure 1) or, in the 3D context, about its degree of rotation around the Z-axis (right reference frame in Figure 1). Objects that have different “senses" are objects that are mirror images or reflections of each other with respect to a line in the 2D plane or with respect to a plane in the 3D context. In Euclidean geometry, the whereabouts of the reflection line or plane are irrelevant. I will, however, distinguish between objects reflected with respect to a vertical or a horizontal line or plane, i.e., objects with different vertical and horizontal senses (see Figure 2) because this distinction is important in environments with gravity. We will see below that this distinction might also matter for animals. When I refer to the position of objects, I refer to their position with respect to the X- and Y-axes in the 2D plane and to the X-, Y-, and Z-axes in the 3D context. Again, Euclidean geometry does not differentiate between vertical or horizontal differences in position. In environments with gravity, this is relevant, though. I will, therefore, make a distinction between vertical and horizontal positions.

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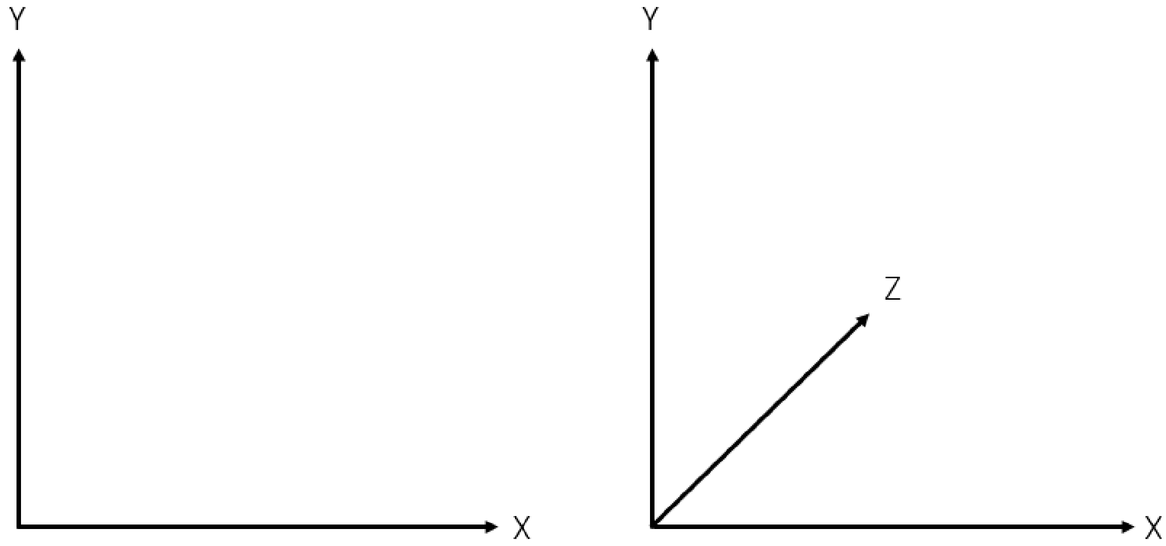

*Figure 1 - **2D and 3D frames of reference** - The left reference frame represents a two-dimensional context, where the X-axis is the horizontal axis and the Y-axis is the vertical axis. The right reference frame represents a three-dimensional context, where the X- and Z- axes are the horizontal axes and the Y-axis is the vertical axis.*

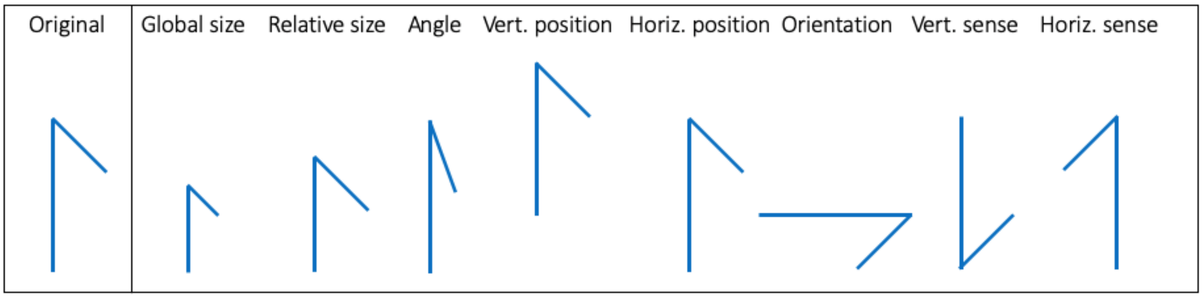

*Figure 2 - **Geometrical characteristics of objects** - From left to right are shown nine objects that differ in terms of their geometrical characteristics. The first L-shaped object, called the "original" object, is followed by eight objects that vary in terms of their global size, relative size, angle, vertical position, horizontal position, orientation, vertical sense, and horizontal sense.*

Size and shape

Numerous studies have tested if humans are sensitive to objects' size and shape. These studies have revealed that humans of different age groups and from different cultures are sensitive to objects' global and relative size and shape. For example, educated adults were shown and asked to compare multiple lines with varying lengths [14]. It was found that participants performed very well in that task. However, the same study also found that accuracy drops when lines are presented in a three-dimensional instead of two-dimensional environment.

A classic task used in many studies to test reasoners' sensitivity to geometry is the deviant detection task (see Figure 3). In that task, subjects are presented with an

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array of geometrical shapes (say two rows with 3 shapes) and asked to pick the one that they think is different from all the others. Using such deviant detection tasks it has been found that not only educated adults from western cultures, so-called WEIRD people, but also Mundurucu adults who had never received any formal (Western) education in geometry, and children of different ages (i.e., 3-12 years-olds) can easily identify the one amongst six 2D objects that had a different size [3][5][15][13]. Furthermore, as has been revealed by looking-time experiments, even infants seem to discriminate between objects of different sizes. In such looking time experiments, it was observed that infants tend to look longer at a novel object if that object is larger or smaller than previously presented objects [16]. However, it was also found that this ability of size discrimination appears to depend on the presence of a size referent [17]. Another line of investigation found that infants can also use size information to make predictions about dynamic visual scenarios. For instance, in a study by Baillargeon and Graber [18], 5.5-months-old infants were presented with a short and a tall object. Both objects then disappeared behind an occluder with holes at different heights. Infants were found to be surprised (i.e., to look longer) when they saw that the short object appeared behind the higher hole or when they saw that the tall object did not appear behind the higher hole. By contrast, they were not surprised in the plausible trials, i.e, when the short object appeared in the lower hole or the tall object appeared in the higher.

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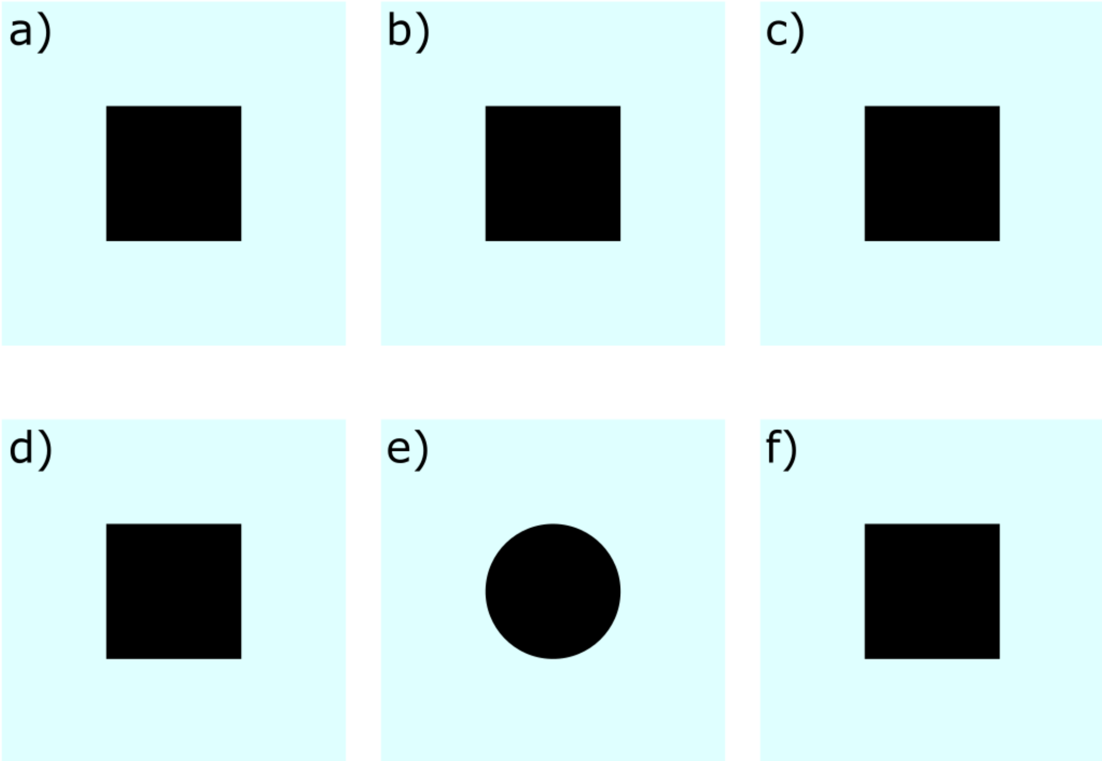

*Figure 3 - **Example of a deviant detection task** - In a deviant detection task, participants are typically presented with an array of six objects, five of which share the same characteristics that a sixth object, the deviant (pannel e), does not share. Participants are asked to identify the deviant.*

Similarly, in a deviant detection task testing different L-shaped objects in WEIRD adults, in Mundurucu adults, and in children of different age groups it was found that all these groups easily identified the deviant shape whose angle between its two parts was different from five other objects [3][5][13][15]. It has also been found that adults easily detect changes in the aspect ratio of different geometrical figures [19]. Even infants were found to perceive differences in the shape of 2D figures [20][21], although sensitivity to angle can be overridden by sensitivity to orientation. For example, infants looked longer at L-shaped objects whose relative size changed (i.e., the size relation between their two segments) and whose angle between their two segments changed. However, in the latter case, they only did so when the objects' orientation did not change much [22] or when they were first familiarized with objects that had different orientations [23][24].

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Orientation, sense, and position

Humans are sensitive to objects' degree of orientation. For example, in a deviant detection task, WEIRD adults, Munducuru adults, and preschoolers easily identified the deviant 2D object with a different orientation [11][13][25]. Sensitivity to orientation has also been found to affect object categorization in adults, children, and infants [21][22][23][24][26][27][28][29]. For example, one intriguing finding was that adults and infants alike seem to consider squares and diamonds (squares that are rotated at 45°) as different objects [21]. Both adults and children can learn, however, to ignore objects' orientation if they are familiarized with stimuli that have different orientations [24][29].

Educated adults, Mundurucu adults, and preschoolers also easily identified the deviant 2D object whose vertical or horizontal sense differed from those of five other objects [11][13][25], demonstrating that humans are sensitive to objects' sense. Performance was found to drop, however, when the objects were presented at different orientations [5][15][25]. This performance drop was particularly strong in children and in adults who never received formal education. Orientation also was found to influence the identification of the symmetrical corners of isosceles triangles (triangles with two sides of the same length) in infants, especially when differences in orientation between objects are larger than 180° [30]. Interestingly, children and adults alike seem to differentiate between horizontal (or left/right) mirror images and vertical (or up/down) mirror images. Vertical mirror images seem easier to detect than horizontal ones [31][32].

Finally, in a deviant detection task, WEIRD adults were found to easily detect the deviant 2D object amongst six objects that had a different vertical or horizontal position relative to a presented 2D frame [13]. This finding demonstrates that humans are sensitive to objects' positions. Infants are also sensitive to the position of objects. For example, 5- to 24-months-old infants tend to look longer at objects that change location [33][34][35]. However, it is not clear whether this sensitivity reflects their object categorization tendencies or their navigation abilities, which seem to be based on different core cognitive systems [3].

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Lines and angles

Geometrical objects can also be categorized based on more abstract concepts, e.g., parallelism, and right angles, or based on whether they are triangles or other geometric shapes. Humans seem to be sensitive to these abstract concepts. For instance, participants of different age groups and cultures differentiated parallel from non-parallel lines and right angles from other angles [11][25][36][37][38]. They were also able to reason about whether lines with different angular relationships would cross or not, whether different lines could cross two or three different dots on a plane or on a spherical surface, and they were able, except for young US children, to infer the size of the third angle of a triangle when shown its two other angles [38].

Polygons

Depending on the geometrical characteristics of objects, they will be more or less regular in shape. In geometry, objects that are made of connected straight segments that form a closed space are called polygons. Polygons can be more or less regular, depending on whether their segments have the same length (i.e., they are equilateral) and on whether their vertices have the same angle (i.e., they are equiangular). Humans have given names to polygons that present regularities, such as equilateral triangles (equilateral and equiangular), squares (equilateral and equiangular), and rectangles (equiangular), to name only a few. Note that sensitivity to such regularities also requires minds to have higher-order concepts such as parallelism, right angles, equal angles, and lengths, so that objects that do not necessarily have the same overall shape, size, orientation, sense, and position can still be classified in the same category. For example, different objects can be characterized as parallelograms (i.e., geometrical figures that have two sets of opposed parallel lines of the same length) even if they do not have the same overall size, shape, orientation, sense, or position (see Figure 4). Humans are sensitive to these regularities. For example, in a deviant detection task, Munducuru adults and preschoolers easily detected deviant polygons amongst sets of six polygons that had different orientations [11][25]. Furthermore, adults with or without formal education in geometry and kindergartners identified more easily deviant quadrilaterals amongst regular than amongst irregular quadrilaterals

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[39]. This suggests that they possess intuitive categories for geometrical regularities that help them sort geometrical objects. Infants also seem to discriminate between polygons, at least between rectangles and squares. For example, 11-weeks-old infants that habituated to a rectangle acted surprised when they were suddenly shown a square [21]. However, this effect could also have been due to differences in objects' areas. Furthermore, infants of the same age did not have a novelty preference for a circle when familiarized with a triangle, or vice versa [40], while they had a novelty preference for circles with different topological characteristics, suggesting that sensitivity to geometric shapes might develop with age.

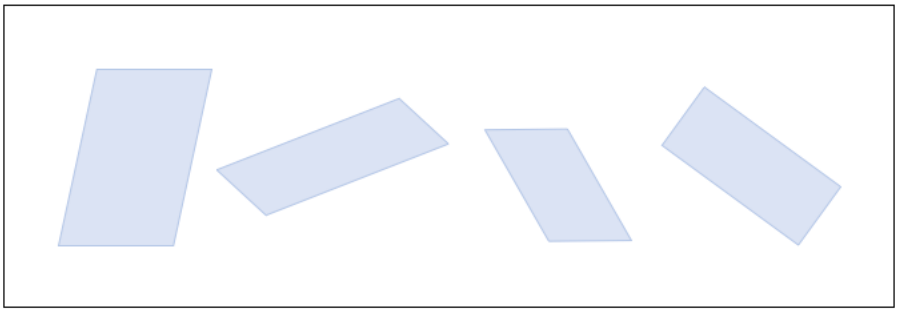

*Figure 4 - **Examples of parallelograms** - Four parallelograms that differ in terms of their size, shape, orientation, sense, and position.*

SENSITIVITY TO GEMOETRY IN OTHER ANIMALS

Size and shape

There is evidence that several non-human animal species discriminate objects based on their size [41][42][43][44][45][46][47]. For example, apes and monkeys are able to discriminate cubes that only slightly vary in terms of their volume and side length [44]. Carrion crows easily discriminate between lines of different lengths [43]. Bees can learn to associate disks' size with a reward when other cues are not available [45]. Similarly, several animal species also have been found to discriminate between objects' shapes [41][42][47][48][49][50][51][52][53]. For example, rhesus monkeys and goldfish can learn to discriminate between geometric forms such as circles and squares [48][53], pigeons can learn to discriminate between different letters of the

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alphabet [42], and young domestic chicks can learn to distinguish between sequences of objects of different shapes [54].

Orientation, position, and shape

Like humans, other animals are also sensitive to the orientation of objects. For example, like humans, goldfish seem to consider squares and diamonds as different objects [48][55]. Rats can also learn to discriminate between letters that have different degrees of rotation. Pigeons do not treat letters or geometric shapes that have different degrees of rotation as equivalent [42][56]. On the other hand, they were found to treat letters that have different horizontal positions as equivalent [42], suggesting that horizontal position is not relevant for object categorization (at least in pigeons). Similar to adults and children, different animals such as pigeons [42], rats [57], octopuses [58], cichlids [59], cats, and rhesus monkeys [60] were found to be sensitive to objects' sense and to find it more difficult to discriminate horizontal than vertical mirror images.

Polygons

Contrary to humans, other animals do not seem to have intuitive categories for regular geometric shapes. For example, in tasks similar to those used with humans, guinea baboons did not identify more easily deviant shapes among regular than irregular shapes [39]. It also seems that rats have no intuitive categories for triangles and squares, as they do not spontaneously learn to discriminate them based on their holistic properties (i.e., their aspect ratio). When no other alternatives are available, they can learn to use aspect ratio to discriminate between polygons, but their performance remains poor [61]. Pigeons do not seem to have complete concepts of "triangularity", as they do not transfer knowledge between triangles that have different orientations [56]. However, they do show transfer between triangles that have different sizes. Jungle crows, on the other hand, generalize between triangles of different shapes but not between triangles of different areas [62].

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UNDERLYING INTUITIONS

It seems clear from the evidence reviewed above that humans and other animals' sensitivity to geometry is not purely Euclidean. In fact, if humans and other animals were intuitive Euclidean reasoners, they should be highly sensitive to changes in objects' metric proportions but ignore changes in objects' orientation and position, as those are invariant transformations in Euclidean geometry. Furthermore, as in Euclidean geometry, there is no difference between a vertical and horizontal reflection, intuitive Euclidean thinkers should not differentiate between vertical and horizontal mirror images. The evidence reviewed above, however, has shown us that animals do not spontaneously ignore orientation when categorizing objects, nor do they seem to treat vertical and horizontal mirror images as equivalent.

One explanation for this deviation from Euclidean principles is that animals do not categorize geometric objects based on their geometrical, and therefore perceptual properties, but based on whether objects are assumed to share some “hidden", “distal" properties. Object categorization does in fact not solely depend on objects' perceptual characteristics because the solution to the question of whether two objects are equal or not is relative to the level of specificity of the question [63]. Reasoners might want to categorize objects depending on their geometrical characteristics but they might also want to categorize objects depending on whether they share physical or biological properties. These are related questions. What differs between these questions are the properties that are considered fundamental in objects and that should, therefore, not be altered by transformations, and the characteristics that are considered as irrelevant and that can be overlooked.

As mentioned above, in Euclidean geometry, rotation and translation are considered invariant transformations. Recently, my colleagues and I [13] suggested that these Euclidean transformations are not invariant in the physical world that animals experience. In the 3D environments in which minds evolved, 3D objects have a mass and interact with the gravitational field. Importantly, different orientations and

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positions with respect to the gravity vector, the vector pointing to the Earth's center, will provide objects with different levels of gravitational potential energy. Gravitational potential energy is a physical property of objects that depends on their mass and on the height of their center of mass. Furthermore, gravitational potential energy can be converted into kinetic energy, which in turn can be transferred from one object to another. Objects whose levels of gravitational potential energy differ thus have different potentials to create causal chains of events. For example, an object that has a higher vertical position than another object has more gravitational potential energy and can thus transfer more kinetic energy to another object if it came to fall, creating a potentially different causal chain of events than the lower object. Geometrical transformations that are invariant in Euclidean geometry, such as rotation and translation, are thus not necessarily invariant in terms of objects' physical properties. This is directly relevant to the question of whether animals intuitively categorize objects in terms of their geometrical properties or in terms of their physical properties. Depending on the level of specificity at which they categorize objects, different geometrical characteristics will be relevant.

My colleagues and I [13] argue that natural selection favored minds that were more sensitive to differences in geometrical characteristics that determine differences in objects' gravitational potential energy (i.e., differences in size, shape, orientation, vertical senses, and vertical position) than to geometrical differences that do not determine differences in objects' gravitational potential energy (i.e., differences in horizontal sense and horizontal position). The idea is that such sensitivity could help minds better track objects' potential to transfer energy to other objects, and therefore, better predict some aspects of their causal environment.

Furthermore, we argue that those differences in objects' geometrical characteristics that are associated with differences in objects' gravitational potential energy are also easier to track because they can be related to the gravity vector, an objective and invariant referent, whereas the less informative differences (i.e., differences in horizontal sense and horizontal position) have to be related to arbitrary referents. In other words, although rotation and translation are invariant transformations in Euclidean geometry, they are not invariant transformations in

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physics. More particularly, rotation, vertical translation, and vertical reflection are not invariant transformations in physics, while horizontal reflection and horizontal translations are. This is because the latter but not the former conserve objects' gravitational potential energy. Hence, objects that are similar in terms of their geometrical “kind" can be different in terms of their physical “kind".

This new theory has so far been tested in WEIRD human adults. In different experiments, we presented adults with a fictitious scenario in which astronauts sent to an alien planet encountered several objects that were described to emit “alpha rays" (see Figure 5). Crucially, not all objects emitted the same intensity of alpha rays. When participants were asked to select one out of two novel objects that they think emits a different intensity of alpha rays than the first object they were shown previously, they tended to select the new object that differed from the first in terms of its level of gravitational potential energy. For example, when participants were presented with two novel objects, one of which was a horizontal mirror-image of the first object encountered (and had, therefore, the same gravitational potential energy), and the other was a vertical mirror-image of the first object encountered (and had, therefore, a different gravitational potential energy), they tended to infer that the latter was producing a different intensity of alpha rays. More generally, in line with the novel theory, participants considered differences in geometrical characteristics associated with differences in objects' gravitational potential energy (i.e., differences in objects' size, shape, orientation, vertical sense, and vertical position) to be more informative than differences in geometrical characteristics that do not reflect changes in gravitational potential energy (i.e, differences in horizontal sense and horizontal position).

This behavior was not due to perceptual differences between objects but seem to really reflect differences in their physical properties. In fact, in a different experiment in which the objects were described as 2D objects painted on a wall and, therefore, deprived of mass and gravitational potential energy, participants did for example no longer think that vertical mirror-images were more informative than their horizontal counterparts.

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Participants even took into account the reason for the differences in gravitational potential energy between objects. For example, in a further experiment in which differences in objects' geometrical characteristics were not due to intrinsic differences between objects but to external entities (i.e., objects were no longer standing on their own but instead were manipulated by robot arms), participants seem to consider differences in object's gravitational potential energy as less informative. This seems to indicate that they categorize objects based on their physical "kind", i.e., based on intrinsic physical properties rather than extrinsic differences.

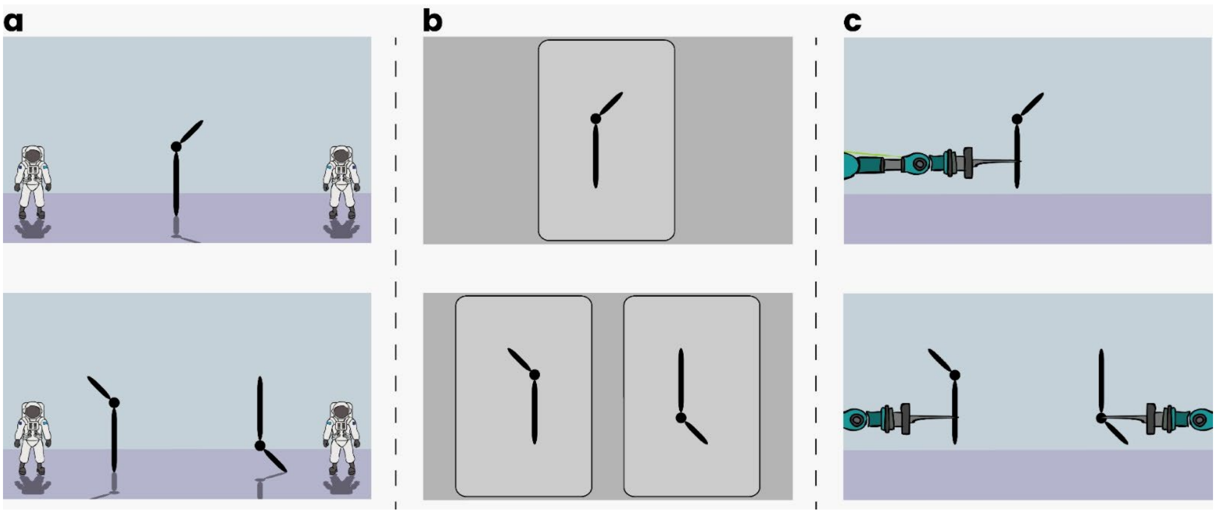

*Figure 5 - **Examples of stimuli used in Placì et al. [13]** - Participants saw a first object and learn that it emitted alpha rays. They were then presented with two new objects and were told that one of them emitted a different intensity of alpha rays than the first object. Participants had to choose the object they thought emitted a different intensity of alpha rays. The context in which the objects were presented differed. For example, in one experiment, the objects were 3D objects standing on their own (a); in other experiments, they were 2D objects painted on a wall (b), or 3D objects manipulated by robotic arms (c).*

Crucially, participants' categorization patterns between scenarios (i.e., 3D environment without manipulation, 2D environment, and 3D environment with manipulation) cannot be explained by perceptual differences between objects, because in all three scenarios, the perceptual differences between objects were kept constant.

These findings support the theory that it is people's intuitions about physics rather than their Euclidean intuitions that determine how they spontaneously categorize geometrical objects in different contexts. These categorization patterns seem not to be due to discrimination issues, as in general human adults with and

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without formal education in geometry, as well as children seem able to discriminate between objects' size, shape, orientation, sense, and position [11][13][25].

One could argue that our findings reflect humans' categorization patterns in very specific contexts in which it is made explicit that objects have different physical properties (i.e., different potentials to emit alpha rays). This, however, seem not to be the case. Even in contexts in which humans know nothing about objects' physical properties, they still seem to consider differences in objects' gravitational potential energy (or at least the energy objects would have if they were 3D objects). For example, intuitive physics can better explain why adults and children do not consider squares and diamonds as similar geometrical figures [21]. Indeed, although their shape and their size are the same, their orientation differs, which would reflect differences in their level of gravitational potential energy if they were 3D objects. Intuitive physics can also explain why adults and children are more sensitive to differences in vertical than differences in horizontal sense [31][32]: again, differences in vertical sense are informative about differences in objects' levels of gravitational potential energy, while differences in horizontal sense are not.

Although more research is needed to verify the presence of similar intuitions in other animals, the evidence reviewed above suggests that animals are more sensitive to differences in objects' geometrical characteristics that are associated with differences in objects' levels of gravitational potential energy. As reviewed above, several species, including goldfish, rats, pigeons, octopuses, cichlids, cats, and rhesus monkeys appear to be more sensitive to differences in objects' size, shape, orientation, and vertical sense. By contrast, animals seem to consider horizontal sense and horizontal position, two geometrical characteristics that are not associated with changes in gravitational potential energy, as less relevant [42][48][55][56][58][59][60][64].

Humans with and without formal education in geometry do, however, not only categorize geometric objects according to their size, shape, orientation, sense, and position. They also intuitively categorize objects based on more abstract concepts such as whether they are regular shapes characterized by parallel lines, equilateral and

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equiangular segments [11][25][38][39]. To do so, humans have to abstract objects' absolute global size, orientation, sense, and position, and only take into account segments' length and angular relationships. Such sensitivity to abstract geometrical regularities is better explained by Euclidean intuitions and has not (yet) been described in other animals (see for example [39][56][61][62] for example in baboons, rats, pigeons, and crows). This sensitivity in humans also seems to be independent of sensorimotor experiences, as it encompasses concepts about infinity and impossibility (e.g., the concept that two parallel lines can never cross each other). However, as reviewed above, part of this sensitivity and intuitions about more abstract geometrical concepts seem to depend on some kind of experience, as younger children do not show to have the same sensitivity and intuitions as older children and adults [38][40].

OPEN QUESTIONS AND DIRECTIONS FOR FUTURE STUDIES

It seems that humans are Euclideans in some aspects of their everyday reasoning. This seems to be the case when humans reason about objects in terms of more abstract geometrical structures instead of categorizing objects in terms of their more basic characteristics such as their absolute shape, size, their directional and positional characteristics. Sensitivity to more abstract concepts such as parallelism, right angles, equal length, and equal angles could have been advantageous for manufacturers who manipulated and constructed new structures in which the weight of different elements had to be distributed in a given way in order to withstand external forces such as muscular forces, gravity, or wind. In this sense, geometry could be seen as a mathematical language of thought specific to humans, acquired to help them combine weights of different objects and arrange them in a way that leads to stable structures. Although several other animals species are known to use tools (see [65] for a review), the extent to which they combine objects to manufacture new tools seems limited (see for example [66][67]), which could in part explain why their minds do not incorporate these more abstract geometrical concepts. Humans' sensitivity to geometrical regularities is old [68] and even seems to transcend the history of Homo

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sapiens, as records of geometric patterns have been found in carvings from Homo erectus [69]. At a certain moment during the evolution of the genus Homo, some individuals might have been able to abstract the structure of objects from their physical frame, so as to only consider the geometrical relationships connecting different segments, independently from their relation to gravity. The formalization of these relationships might have led to the development of a symbolic mathematical language [70] that prehistoric humans expressed in their art and applied to manufacture tools and create architecture, and that eventually led to Euclid's Elements. Of course, these are speculations that need further investigations.

Several other lines of research are needed if we want to come to a better understanding of the intuitions underlying sensitivity to geometry, and of the differences between humans and other species in this regard.

A first promising line of research could investigate sensitivity to geometry in newborn animals deprived of any specific experience with geometric objects. In fact, although intuitions about objects' physical properties could be underlying animals' sensitivity to geometry, it is not clear whether such intuitions are acquired through experience or whether they are already present at birth. Sensitivity to geometry for navigational purposes seems, for example, to be independent of any specific experience, as newly-hatched chicks already use geometric cues to reorient [71][72][73].

Another line of research could investigate whether non-human animals' categorization of geometrical objects also depends on whether objects were manipulated or not. It would be interesting to learn whether a difference can be found between animals who manipulate objects in their everyday life (e.g., tool users) and animals who do not. It would also be interesting to investigate whether proficient tool users (e.g., chimpanzees or New Caledonian crows) are sensitive to more abstract geometrical regularities than non-tool users.

Finally, further studies about the ontogeny of humans' sensitivity to geometry, and how its development is associated with the development of other abilities, such

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as tool manipulation and manufacturing, could help us better understand the factors that lead to the emergence of more abstract geometric concepts.

CONCLUSION

Sensitivity to geometry seems to be widespread in the animal kingdom. Animals, including humans and other mammals, birds, fish, and invertebrates, seem to consider some of the geometrical characteristics of objects when categorizing them. However, contrary to what has been long suggested, it seems that intuitive physics rather than Euclidean intuitions underlie sensitivity to geometry. Several species, including humans, seem indeed to be more sensitive to geometrical transformations that affect the physical properties of objects rather than their Euclidean properties. Humans, however, seem to be special in their ability to reason about more abstract geometry concepts, such as whether objects are characterized by parallel lines, right angles, equal-length segments, or equal angles. In this respect, it seems right to say that humans are intuitive Euclidean thinkers who developed a symbolic mathematical language that formalizes the geometrical regularities they observe or manufacture themselves.

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