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Sensitivity to geometry in humans and other animals

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QUESTION

Are humans and other animals sensitive to geometry when categorizing objects and if they are, what intuitions could underline this sensitivity?

ABSTRACT

Geometry can be defined as the mathematical formalization of space. Amongst all theories of geometry, Euclidean geometry is considered the most intuitive of all for humans. The reason for this, it has been argued, is that humans (and maybe other animals) spontaneously categorize geometric objects based on their Euclidean properties. In this paper, I briefly review the evidence suggesting that humans and other animals are sensitive to objects' geometric features. I further address the question whether Euclidean principles underlie humans' and other animals' sensitivity to geometry and if not, whether other principles can better account for animals' categorization of geometric objects. The conclusion of the paper is that animals, including humans, do not spontaneously categorize geometric objects based on their Euclidean properties; however they can learn or be probed to do so. When this is not the case, other non-Euclidean properties, e.g., objects' topological properties or physical properties, seem to be relevant both for human and nonhuman animals. Humans, however, seem to be special in their ability to analyse objects in terms of more abstract Euclidean concepts.

Keywords: cognition, geometry intuitions, intuitive physics, Euclidean intuitions, object categorization, invariant transformations

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INTRODUCTION

Geometry is one of the oldest branches of mathematics. Its earliest records go back to ancient Mesopotamia and Egypt [1] where its use was mostly practical; amongst other things, it helped kings and pharaohs build mighty palaces and gigantic pyramids. With the advent of Euclid and his Elements [2], concepts of geometry were explained in such an intuitive and rigorous way that they crossed centuries without alterations [3]. Euclidean geometry was geometry, and its axioms were thought to accurately reflect the geometry of the natural world. Today, although several non-Euclidean theories exist, Euclidean geometry remains the most famous theory of geometry and the one that is primarily taught in schools [4].

Geometry can be defined as the science of space [5]. As such, it comprises concepts like distance, angle, shape, size, and relative position of objects. One of the endeavours of geometry is the search for invariants [6] [7]. Invariants are properties of space that remain unaltered after a certain type of allowed transformations occurred. For example, in Euclidean geometry, objects are defined in terms of their angle and distance relationships [6]. Two objects that have the same absolute shape and size are considered equivalent. These fundamental properties are invariant under rotation (the movement of an object around a point or a line), translation (the linear movement of an object along a line), and reflection (the creation of the mirror image of an object over a line or a plane); these transformations are the invariant transformations of Euclidean geometry. In other words, in Euclidean geometry, differences in objects' position (see *Figure 1C*), orientation (see *Figure 1D*), and sense (see *Figure 1E*) are irrelevant, whereas variability in objects' shape and size (see *Figure 1A and B*) is relevant.

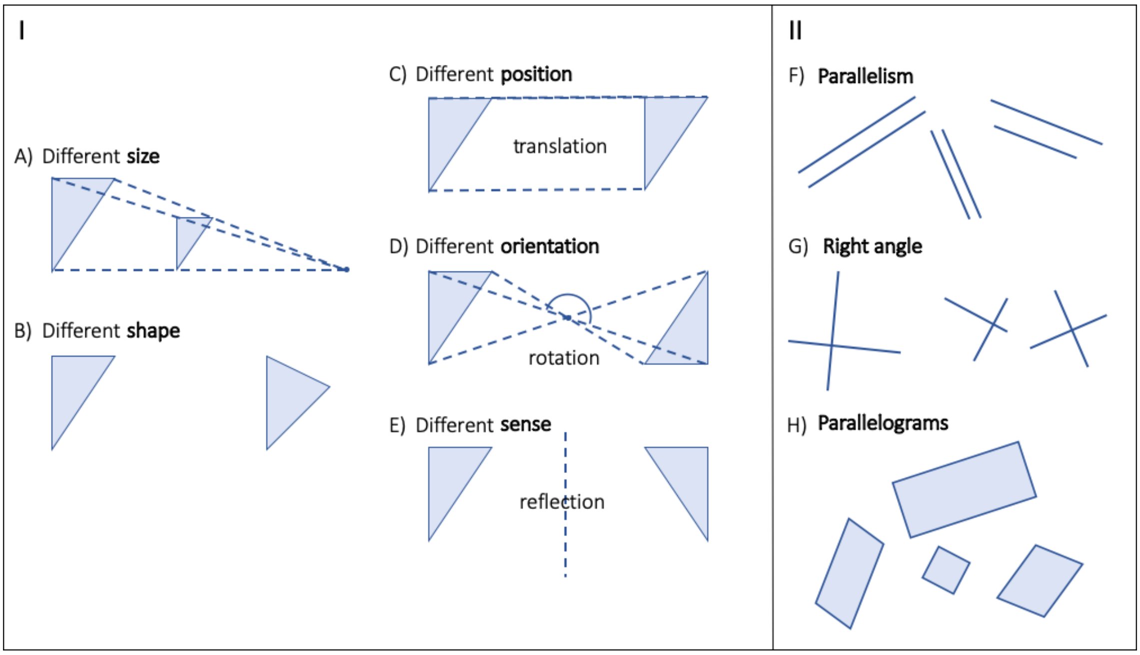

Figure 1 - Different geometric concepts *- In panel I are depicted concepts such as size (A), shape (B), position (C), orientation (D), and sense (E). In panel II are depicted more abstract concepts such as parallelism (F), right angle (G), and parallelograms (H).*

Other examples of non-Euclidean geometries are affine geometry, projective geometry, and topology or rubber-sheet geometry. According to Felix Klein's Erlangen program [6] [7] [8], all theories of geometry can be classified in terms of their invariants and in terms of their invariant transformations into a hierarchy of geometries with increasing levels of abstraction. For example, invariant properties of affine geometry are parallel lines, which are also invariant in Euclidean geometry; however other invariants of Euclidean geometry, e.g., angle and distance relationships, are not invariant in affine geometry, putting affine geometry at a higher degree of abstraction than Euclidean geometry. In topology, the allowed transformations include continuous deformation of objects such as bending, twisting or stretching. Examples of topological invariants under continuous deformation are connectedness, i.e., whether two objects are connected, compactness, i.e., whether an object has holes, or dimension, whether an object has one (e.g., a line), two (e.g., a compact square), or several dimensions (a cube). These invariants remain unaltered under the transformations of projective, affine or Euclidean geometry (but not vice versa), making it one of the most abstract forms of geometry [8] [9] [10] [11] [12].

Because animals interact with different objects that vary in terms of their geometric features, it seems only natural to ask if animal minds are sensitive to objects' geometric features, i.e., whether they consider them for categorization. The idea that animals might be sensitive to geometry is not new. Philosophers and psychologists have argued that animal minds should incorporate Euclidean principles when analysing objects because these principles reflect the geometry of the natural world [13] [14] [15] [16] [17]. This is equivalent to saying that analysing objects in terms of Euclidean principles gave an advantage to animals so that over time, minds embracing Euclidean principles reproduced more successfully and survived longer. The idea that Euclidean principles might underlie how animal minds, especially human minds, might represent objects led recently to several studies [6] [18] [19] [20] [21] [22].

The present paper pursues two goals. The first is to give a brief overview of the research that has investigated whether or not animal minds are sensitive to objects' geometric features. The review will mainly discuss studies about the categorization of simple 2D objects and not studies about how animals recognize 3D objects under different viewpoints in depth. For this purpose, the literature review will focus on studies that used simple 2D stimuli deprived of depth cues.

The second goal of the present paper is to elucidate the underlying principles best explaining the documented behaviour. A particular focus here will be on the extent to which Euclidean principles can account for animals' sensitivity to geometry. To foreshadow, the literature review will reveal that although some animals, especially humans, can categorize objects in a way that agrees with Euclidean principles, animals' spontaneous categorization tendencies seem not to be Euclidean. Other alternatives that could underlie animals' sensitivity to geometry will be discussed.

EUCLIDIAN AND NON-EUCLIDIAN CATEGORIZATION

What would it entail for a mind to categorize objects according to Euclidean principles? As in Euclidean geometry, angle and distance are fundamental properties that have to remain invariant for objects to be equivalent, categorizing objects according to Euclidean principles should imply a higher sensitivity to objects' size and shape and a higher disregard for objects' position, orientation, and sense.

Euclidean geometry has other important concepts such as points, lines, straight lines, parallel lines (see *Figure 1F*), right angles (see *Figure 1G*), circles, and regular geometric shapes (e.g., equilateral triangles, squares, rectangles, rhombus, parallelograms, etc.) [2] [18]. Categorizing objects in line with Euclidean principles should also imply categorizing objects according to these concepts. Note that these concepts are more abstract because they are not determined by local values (i.e., absolute distances and angles) but rather by the geometric regularities that characterize objects. For example, for the categorization of objects as parallelograms, i.e., quadrilaterals characterised by two opposite sets of parallel lines (see *Figure 1H*), absolute differences in shape, size, orientation, position, and sense do not matter.

Categorizing objects in disagreement with Euclidean principles could be any reliance on non-Euclidean properties such as position, orientation, sense, or topological properties. Other non-Euclidean properties such as nonaccidental properties (i.e., properties that are invariant over orientation in depth, see [23] [24] [25]) or complex structural properties (e.g., skeletal descriptors, see [26]), will not be discussed in this review because they pertain to the study of object recognition under different viewpoints.

SENSITIVITY TO GEOMETRY IN HUMANS

Are human minds sensitive to objects' geometric features? Decades of research have been devoted to this question, with different studies investigating slightly different facets of geometric cognition. The following overview will be structured according to the geometric features that have been tested in different studies.

Euclidean properties – size and shape

Numerous studies have tested whether humans are sensitive to objects' size and shape. These studies have revealed that humans of different age groups and cultures are sensitive to objects' size and shape. For example, adults were shown and asked to compare multiple lines with varying lengths [27], which they managed easily as long as lines were not presented in a 3D environment.

A task used in many studies to test reasoners' ability to discriminate objects based on their geometric features is the deviant detection task. In that task, subjects are presented with an array of geometric figures (see *Figure 2*) and asked to pick the one that is different from all the others (i.e., the deviant). Educated adults from western cultures, so-called WEIRD people, but also Mundurucu adults who have never received any formal (Western) education in geometry, and children of different ages (i.e., 3-12-years-olds) can identify the size deviant amongst six geometric figures [6] [19] [22] [28] [29]. However, if size merely interferes with the task (i.e., all objects have a different size so that size is not a diagnostic feature of the deviant) human participants, at least WEIRD adults, can also disregard size to identify the deviant [29].

New-borns can also discriminate between objects of different sizes. For example, in a looking time experiments, new-borns looked longer at a novel object if that object was larger or smaller than previously presented objects [30]. However, infants' and toddlers' ability to discriminate objects' size appears to depend on the presence of a size referent [31].

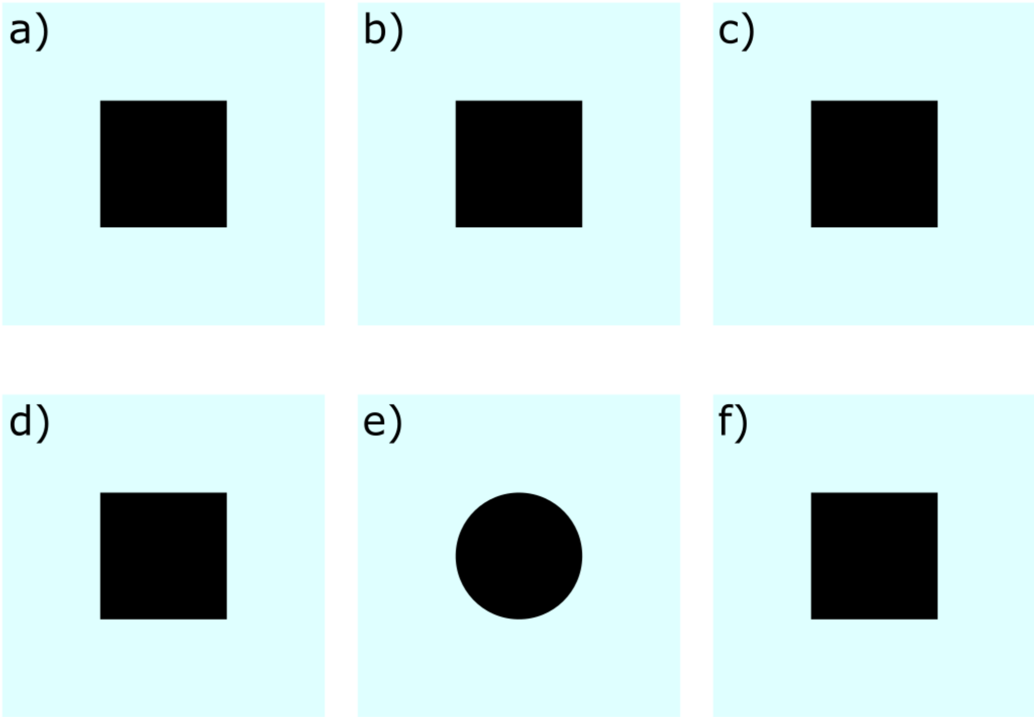

Figure 2 - Example of a deviant detection task *- In a deviant detection task, participants are typically presented with an array of six objects, five of which share the same features that a sixth object, the deviant (pannel e), does not share. Participants are asked to identify the deviant.*

Similarly, WEIRD and Munduruku adults, and children of different age groups (3-12-years-olds), easily identified the deviant shape whose angle between its two parts was different from five other objects [6] [19] [22] [28] [29]. As with size, if shape merely interferes with the task, WEIRD adults can also disregard it to identify the deviant [29]. Human adults also detect changes in the aspect ratio of different geometric figures [32]. Even infants (2-10-months-olds) perceive differences in the shape of 2D stimuli [33] [34] [35], although objects' orientation can interfere with the detection of shape differences. For example, infants looked longer at L-shaped objects whose relative size (i.e., the size relation between their two segments) and shape changed. However, in the latter case, they only did so when the objects' orientation did not vary much [35] or when infants were first familiarized with objects that had different orientations [36] [37]. Infants can also discriminate between more complex geometric shapes, e.g., circles, triangles, and squares, an ability that seems to develop with age. For example, 11-weeks-old infants that habituated to a rectangle showed a novelty preference for a square [34]. However, this effect could also have been due to differences in objects' areas. Infants of the same age did not have a novelty preference for a circle when familiarized with a triangle, or vice versa [11]. By three months, infants become better at discriminating geometric shapes [11] [38]. For example, 3- and

4-month-olds (but not new-borns) who were habituated with different exemplars of triangles (or squares, circles, and crosses) had a novelty preference for an object of a different category when presented simultaneously with a novel object of the familiar category and a novel object of a new category [38]. Three- and 4-month-olds who were habituated with distorted squares had a novelty preference for a triangle or a diamond when presented simultaneously with a square and either a diamond or a triangle [39].

Abstract Euclidean concepts

Lines and angles

Geometric objects can also be analysed in terms of more abstract concepts, e.g., parallelism, right angles, or triangularity. Humans seem to be sensitive to these abstract concepts. For instance, preschoolers (3-5-years-olds), Mundurucu adults who did not receive any formal education in geometry, and American adults discriminated parallel from non-parallel lines and right angles from other angles [18] [19] [40] [41]. American and Mundurucu children (7-13-years-olds) were further able to reason about whether lines with different angular relationships would cross or not, whether different lines could cross two or three different dots on a plane or on a spherical surface, and they were able, except for young US children (5-6-years-olds), to infer the size of the third angle of a triangle when shown its two other angles [20].

Polygons

Geometric objects can be more or less regular in shape. Objects made of connected straight segments that form a closed space are called polygons. Polygons can be more or less regular, depending on whether their segments have the same length (i.e., they are equilateral) and on whether their vertices have the same angle (i.e., they are equiangular). Humans named polygons that present regularities, such as equilateral triangles (equilateral and equiangular), squares (equilateral and equiangular), and rectangles (equiangular), to name only a few. As a reminder, sensitivity to such regularities also requires minds to have higher-order concepts to classify objects that do not necessarily have the same absolute shape, size, orientation, sense, and position in the same category. Humans are sensitive to regular geometric shapes. For example, in a deviant detection task, Mundurucu adults and preschoolers (3-5-years-olds) detected deviant polygons amongst sets of six polygons [18] [19]. Furthermore, adults with or without formal education in geometry, and French kindergartners (5-6-years-olds) were better at identifying deviant quadrilaterals among regular than irregular quadrilaterals [21]. This suggests that humans possess categories for geometric regularities that are independent from culture and formal education and that help them sort geometric objects.

Non-Euclidean properties: orientation, sense, and position

Humans are sensitive to variations in objects' orientation [34] [35] [36] [37] [42] [43] [44] [45] [46]. For example, in a deviant detection task, WEIRD adults, Mundurucu adults, and preschoolers easily identified the 2D object that had a different orientation than other objects

[18] [19] [29]. Furthermore, adults and infants alike do not consider squares and diamonds (squares that are rotated at 45°) to be equivalent [34]. Similarly, when infants are habituated to a 2D shape at a constant orientation, they show dishabituation when the form is presented at a novel orientation [46]. However, humans can also learn to disregard orientation [6] [18] [19] [21] [22] [28] [29] [35] [37] [45] [43]. For example, when 6-9-months-old infants are first familiarized with objects that have different orientations, they do not dishabituate when presented with an object that has a novel orientation [46]. When orientation merely interferes in a deviant detection task, American and Mundurucu adults, as well as children (3-12-years-olds) can disregard orientation to find shape and size deviants [6] [18] [19] [21] [22] [28] [29]. Disregarding orientation does not necessarily mean that orientation can be ignored. In some cases, e.g., when participants are asked to match objects based on their sense, the ability to mentally rotate objects is required [44] [45].

WEIRD adults, Mundurucu adults, and preschoolers (3-5-years-olds) also easily identified the 2D object whose sense differed from the sense of five other objects [6] [18] [19] [22] [29], demonstrating that humans can discriminate objects' sense. Performance was found to drop, however, when the objects were presented at different orientations [6] [18] [19] [22] [29]. This performance drop was particularly strong in children and in Mundurucu adults who never received formal education in geometry. As for other geometric features, sense can also easily be disregarded in discrimination tasks [29] [47]. Orientation also was found to influence the identification of the symmetrical corners of isosceles triangles (triangles with two sides of the same length) in infants (5-month-olds), especially when differences in orientation between objects are larger than 180° [48]. Interestingly, children and adults alike seem to differentiate between horizontal (left-right) mirror images and vertical (up-down) mirror images. Vertical mirror images seem easier to discriminate than horizontal ones [49] [50] [51].

Finally, in a deviant detection task, WEIRD adults easily identified the 2D object amongst six objects that had a different vertical or horizontal position relative to a frame [29]. Infants are also sensitive to the position of objects. For example, 5- to 24-months-old infants tend to look longer at objects that change location [52] [53] [54]. Objects' position can, however, also easily be ignored. For example, object position does not interfere in how quickly human adults name objects [47] and at least horizontal position seem not to matter when human adults generalize the properties objects [29].

Other non-Euclidean properties

Humans are sensitive to objects' topological features already a few days after birth [11] [38] [55], and in certain circumstances, even more so than to objects' geometric shape. For example, in a study in which adults were presented at visual threshold with a circle, a ring, a square, and a triangle, they were more likely to say that the circle and the ring were different than the circle and any of the other geometric shapes [8]. Infants as young as 1.5-months were able to discriminate objects (e.g., a circle and a ring) based on their topological features, while they failed to discriminate objects based on their geometric shape (e.g., a circle and a triangle) [11].

SENSITIVITY TO GEOMETRY IN OTHER ANIMALS

Euclidean properties: size and shape

There is evidence that several non-human animals discriminate objects based on their size [56] [57] [58] [59] [60] [61][62]. For example, carrion crows discriminate between lines of different lengths [58], and apes and monkeys are able to discriminate cubes that only slightly vary in terms of their volume and side length [59]. Bees can learn to associate disks' size with a reward when other cues are not available [60]. Several animal species can also discriminate objects based on their shape [56] [57] [61] [62] [63] [64] [65] [66] [67] [68]. For example, rhesus monkeys and goldfish can learn to discriminate geometric forms such as circles and squares [63] [67], pigeons can learn to discriminate different letters of the alphabet [57], and young domestic chicks can learn to discriminate sequences of objects of different shapes [68].

Abstract Euclidean concepts: polygons

Contrary to humans, other animals do not seem to have categories for regular geometric figures. For example, in tasks similar to those used with humans, guinea baboons were not better at detecting deviant quadrilaterals among regular than irregular figures [21]. It also seems that rats have no categories for triangles and squares, as they do not spontaneously learn to discriminate them based on their holistic properties (i.e., their aspect ratio). When no other alternatives are available, they can learn to use aspect ratio to discriminate different polygons, but their performance remains poor [69]. Pigeons do not seem to have complete concepts of “triangularity'', as they do not transfer knowledge between triangles that vary in orientation [70]. However, they do show transfer between triangles that have different sizes. Jungle crows, on the other hand, generalize between triangles of different shapes but not between triangles of different areas [71].

Non-Euclidean properties: orientation, position, and sense

Like humans, other animals are also sensitive to the orientation of objects. For example, goldfish also treat squares and diamonds as different objects [63] [72]. Rats can learn to discriminate letters that have different degrees of rotation [73]. Pigeons do not spontaneously generalize between geometric shapes that have different degrees of rotation [57] [70]. However other animals, e.g., pigeons [74] [75] baboons [76] [77] macaques [78] [79], and dolphins [80], can learn to match objects in match-to-sample tasks based on their shape and their sense, irrespective of their orientation.

That nonhuman animals can discriminate the position of objects is well illustrated by their ability to retrieve food in different locations [81] [82] [83]. However, when categorizing objects, animals can also disregard position. Pigeons, for example, spontaneously generalize between letters that have different horizontal positions [57]. Nonhuman animals such as pigeons [84], rats [73], octopuses [85], cichlids [86], cats [87], and rhesus monkeys [88] can also learn to discriminate objects' sense. The discrimination of mirror-images, however, seems to be harder than the discrimination of non-mirror images, especially the discrimination of horizontal mirror-image, and therefore, of horizontal sense.

Other non-Euclidean properties

Non-human animals are also sensitive to objects' topological features. For example, pigeons [12] can rapidly learn to discriminate objects with different topological properties (holes vs. no holes). Similarly, bees [89] can discriminate objects based on their topological features but not based on their geometric shape.

UNDERLYING PRINCIPLES

It seems clear from the evidence reviewed above that humans' and other animals' sensitivity to geometry is not purely Euclidean insofar as they do not seem to disregard objects' orientation when categorizing objects [34] [46] [57] [63] [70] [72] [73]. Furthermore, as in Euclidean geometry, there is no difference between a vertical and horizontal reflection, Euclidean thinkers should not differentiate between vertical and horizontal mirror images. The evidence reviewed above, however, has shown that animals, including humans, do not treat vertical and horizontal mirror images as equivalent [49] [50] [51] [73] [84] [85] [86] [87] [88]. The findings reviewed above further suggest that non-Euclidean properties, e.g., topological properties, might be considered in some circumstances more important than objects' Euclidean features for categorization [8] [11] [12] [38] [55] [89].

There are several non-mutually exclusive explanations for this deviation from Euclidean principles. For one, a higher sensitivity to objects' topological properties than to their shape has been explained by theories of visual perception asserting that topological properties are the primitives of visual representation, and as such, they are needed at the starting point of object perception [8] [9] [10] [11] [12]. The rationale is that topological properties, e.g., the number of holes in an object, are more stable across transformations than other geometric properties (e.g., Euclidean properties) because they are invariant under continuous deformations, whereas other properties, e.g., angle, distance, parallelism, are not. The first step a visual system should undertake to discriminate objects' features from their background and from other objects should be, according to this theory, to parse visual scenes according to topological features [8] [9] [10] [11] [12]. Note that this theory does not negate that animals can categorize objects according to other geometric features; it only asserts that categorization follows a hierarchical process whose first step relies on topological properties. The evidence reviewed above seems in line with this theory: sensitivity to differences in objects' topological properties seems to arise earlier during human infancy than sensitivity to Euclidean properties [11] [38] [55]. Even during adulthood, the visual system seems to extract information about objects' topology before it extracts information about objects' shape [8]. Other animals, such as pigeons [12] and bees [82] seem to be sensitive to objects' topological features more than to their geometric shape, suggesting that this is an evolutionary ancient feature of visual perception.

Another deviation from Euclidean principles in how human and other animals categorize geometric objects is their tendency to confuse horizontal (left-right) mirror images more than vertical (up-down) mirror images [49] [50] [51] [73] [84] [85] [86] [87] [88]. One explanation for this phenomenon is that the left-right confusion stems from evolutionary adaptations to an environment that has no absolute left and right [51] [84]. Gravity provides the environment with an absolute up and down, the down being the direction of the gravity vector [29] [90]. The most natural reflection in the environment is, therefore, the reflection over the gravity vector. Discriminating left from right did not bring any advantage to animals because all objects have a left and a right profile.

Another explanation for the left-right confusion, but also for animals' sensitivity to orientation, is that animals might not only categorize geometric objects based on their geometric features but also in terms of objects' "hidden'' or "distal'' properties [29]. These are related questions that only differ in terms of their specificity [91]. More particularly, what differs between these questions are the properties that are considered fundamental and that should not be altered by transformations, and the features that are considered irrelevant and that can be overlooked.

In the 3D environments in which animals evolved, objects have a mass and interact with the gravitational field. Importantly, different orientations, vertical positions, and vertical (up-down) senses with respect to the gravity vector will provide objects with different levels of gravitational potential energy. From this follows that rotation, vertical reflection, and vertical translation are not invariant transformations in terms of objects' physical properties. Gravitational potential energy is a physical property of objects that depends on their mass and on the height of their centre of mass and that can be converted into kinetic energy. Kinetic energy can in turn be transferred from one object to another. Objects whose levels of gravitational potential energy differ thus have different potentials to create causal chains of events. For example, an object that has a higher vertical position than another object has more gravitational potential energy and can thus transfer more kinetic energy to another object if it came to fall, creating a potentially different causal chain of events than the lower object. My colleagues and I [29] hypothesised that natural selection shaped animal minds to be more sensitive to differences in geometric features that predict differences in objects' gravitational potential energy (i.e., differences in size, shape, orientation, vertical sense, and vertical position) than to geometric differences that do not predict differences in objects' gravitational potential energy (i.e., differences in horizontal sense and horizontal position). Such sensitivity could, in the absence of any additional information about the objects, help animals better track objects' potential to transfer energy to other objects, and therefore, better predict some aspects of animals' causal environment.

So far, we tested this theory only in WEIRD human adults [29]. In different experiments, adults were presented with a fictitious scenario in which astronauts sent to an alien planet encountered several objects that were described to emit "alpha rays'' (see *Figure 3*). Crucially, not all objects emitted the same intensity of alpha rays. When participants were asked to select one out of two novel objects that they thought emitted a different intensity of alpha rays than the first object they were shown, they tended to select the new object that differed from the first in terms of its level of gravitational potential energy. For example, when participants were presented with two novel objects, one of which was a horizontal mirror-image of the first object encountered (and had, therefore, the same gravitational potential energy) and the other a vertical mirror-image of the first object encountered (and had, therefore, a different gravitational potential energy), they tended to infer that the latter was producing a different intensity of alpha rays. More generally, participants considered differences in geometric features associated with differences in objects' gravitational potential energy (i.e., differences in objects' size, shape, orientation, vertical sense, and vertical position) to be more informative about potentials to produce alpha rays than differences in geometric features that do not reflect changes in gravitational potential energy (i.e., differences in horizontal sense and horizontal position).

This behaviour was not modulated by geometric differences between objects per se but seem to be influenced by differences in objects' physical properties. In fact, in a different experiment in which the same objects were described as 2D objects painted on a wall and, therefore, deprived of mass and gravitational potential energy, participants did, for example, no longer consider vertical mirror-images more informative than their horizontal counterparts.

Similarly, when the differences in gravitational potential energy between objects were determined by external entities (i.e., by robots manipulating the objects), participants seem to find them less informative about objects' potential to produce alpha rays. This indicates that participants categorized objects based on their physical "kind", i.e., based on intrinsic physical properties rather than extrinsic differences.

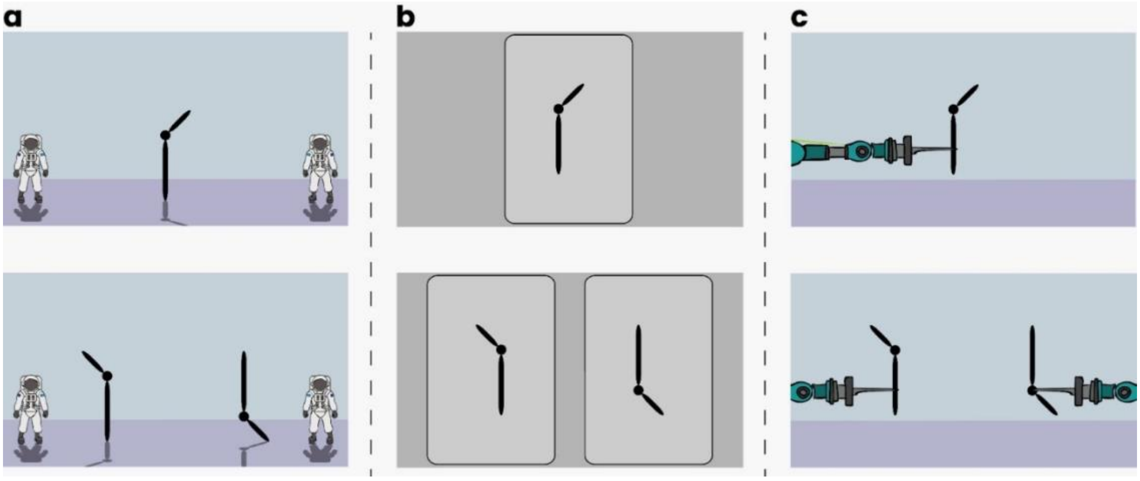

Figure 3 - Examples of stimuli used in Placì et al. *[13] - Participants saw a first object and learn that it emitted alpha rays. They were then presented with two new objects and were told that one of them emitted a different intensity of alpha rays than the first object. Participants had to choose the object they thought emitted a different intensity of alpha rays. The context in which the objects were presented differed. For example, in one experiment, the objects were 3D objects standing on their own (a); in other experiments, they were 2D objects painted on a wall (b), or 3D objects manipulated by robotic arms (c).*

These findings provide initial evidence supporting the theory that people's intuitions about physics can determine how they categorize geometric objects in different contexts. It is possible, however, that it was participants' knowledge about physical processes that drove their decisions so that applying similar methods to children or adults who did not receive any formal education in physics would yield different results. At the moment, the only evidence suggesting that these results might be generalizable across cultures, age categories, and even species, are the above-mentioned findings that individuals of several species, including human adults and children, do not seem to consider orientation as irrelevant when categorizing objects [34] [46] [57] [63] [70] [72] [73] and that individuals are more sensitive to differences in vertical than differences in horizontal sense [49] [50] [51] [73] [84] [85] [86] [87] [88].

At least two prerequisites must be met to attribute these categorization patterns to physics intuitions. Given that objects' centre of mass can only be derived from objects' global features, the first prerequisite is that geometric objects should be categorized based on their global features rather than their local features. Several species, including chimpanzees [92], baboons [93], capuchin monkeys [94], and pigeons [95] were shown to prefer categorizing objects based on their local cues rather than their global ones, whereas humans were shown to prefer categorizing objects based on their global cues [92] [93]. However, the stimuli presented in the studies investigating these preferences were constituted of discontinuous elements, which was not the case in the studies investigating animals' sensitivity to objects' geometric features. Also, pigeons [12] were shown to spontaneously categorize objects based on their topological, and therefore global, properties, showing that preference for local vs. global cues can vary within the same species. It remains unclear, therefore, whether, in the studies investigating animals' sensitivity to objects' geometric features, animals categorized objects based on their global cues (but see [69] and [57] for examples in which rats, and maybe pigeons, categorized geometric objects based on local cues).

Another prerequisite for a categorization based on differences in objects' level of gravitational potential energy is that animals should consider 2D stimuli as physical objects, i.e., as objects with mass. Although most shapes animals encounter in their lives stem from 3D objects with mass, some shapes, e.g., the shapes displayed on butterflies, fish, or tigers, are actual 2D shapes, providing a good reason for animals to differentiate between 2D and 3D objects. A recent study with dogs [96] found that dogs did not wholly confuse 2D pictures of objects as the real 3D objects from which the pictures were taken, however, they might have confused the 2D stimuli for another type of 3D stimuli. Another study showed that infants, before the age of 19 months, seem to confuse pictures for real physical objects, as evidenced by the infants trying to grasp the objects in the pictures [97]. In both studies, however, the 2D stimuli were pictures of real objects, whereas the 2D stimuli in the studies reviewed above were abstract shapes. It remains unclear, therefore, whether animals consider abstract 2D stimuli as physical objects or not.

Are humans and other animals Euclidean thinkers?

Do humans and other animals at all categorize objects based on their Euclidean properties? This seems to be the case, at least for humans. As reviewed above, although humans are sensitive to objects' orientation, sense, and position, they can also categorize objects irrespective of these features [6] [18] [19] [21] [22] [28] [29] [35] [37] [45] [46] [47]. The ability to disregard orientation, sense, or position seems to be modulated by whether human participants are first familiarized with objects that vary in terms of these features [46], whether these features are made irrelevant in the task [6] [18] [19] [21] [22] [28], or whether participants are given cues that the objects to be categorized have been manipulated [29]. In addition, the ability to disregard sense (especially horizontal sense) and position seems also to stem from the lower saliency of both features in the human visual system, which makes them less likely to be discriminated or to interfere in categorization and recognition tasks [47] [49] [50] [51]. Not only can humans disregard directional and positional features when the conditions are met, but they can also disregard absolute shape and size, which further enables them to categorize objects according to more abstract Euclidean concepts [6] [18] [19] [20] [21] [40] [41].

Although more comprehensive data is needed for each species separately, pooled together the evidence about other animals suggests that at least some animal species can disregard orientation and position and that they are in general not very good at discriminating sense [57] [74] [75] [76] [77] [78] [79] [80] [84] [85] [86] [87] [88], which in turn, makes sense easy to disregard. This evidence could speak in favour of an object categorization based on Euclidean properties. However, sensitivity to more abstract geometric features has not (yet) been described in other animals.

It is possible that sensitivity to geometry, particularly in the context of object categorization, follows a hierarchical structure. Animals, including humans, use topological properties to parse visual scenes into objects and backgrounds. They then consider other geometric features, e.g., size, shape, orientation, sense, particularly vertical sense, to categorize objects, possibly because these geometric features are informative about objects' physical properties (i.e., their level of gravitational potential energy). Some animals can in addition learn to disregard some geometric features, e.g., orientation, position, and sense. This ability could help them better cope with the fact that directional and positional features are not necessarily informative about objects' intrinsic properties, as these features can be determined by external forces, e.g., manipulation by animals, wind, or gravity. Animals who can disregard orientation, sense, and position could be considered limited Euclidean thinkers to the extent to which they seem able to abstract the structure of objects from their physical frame, creating a Euclidean space where objects can be analysed independently from their relation to gravity.

Humans, however, seem special in their ability to analyse objects in terms of more abstract Euclidean concepts. One reason for this could be their propensity to not only manipulate objects but to also assemble objects into more complex structures. Sensitivity to more abstract concepts such as parallelism, right angles, equilateralism, and equiangularism could have been advantageous for manufacturers who manipulated and constructed new structures in which the weight of different elements had to be distributed in a specific way to withstand external forces (e.g., muscular forces, gravity, or wind). In this sense, geometry could be seen as a mathematical language of thought that humans acquired to help them combine the weights of different objects and arrange them in an efficient way that leads to stable structures. Although several other animal species are known to use tools (see [98] for a review), the extent to which they combine objects to manufacture new tools seems limited (see for example [99] [100]). Humans' sensitivity to geometric regularities is old [101] and even seems to transcend the history of Homo sapiens, as records of geometric patterns have been found in carvings from Homo erectus [102]. At a certain moment during the evolution of the genus Homo, some individuals might have not only been able to abstract the structure of objects from their physical frame but might also have started to consider objects as abstract structures that can be analysed in terms of their geometric relationships. The formalization of these relationships might have led to the development of a symbolic mathematical language [103] that prehistoric humans expressed in their art, applied to manufacture tools and create architecture, and that eventually led to Euclid's Elements. Of course, these are speculations that need further investigation.

Several other lines of research are needed to reach a better understanding of the intuitions underlying sensitivity to geometry. A first promising line of research could investigate sensitivity to geometry in new-born animals deprived of any specific experience with geometric objects. Sensitivity to geometry for navigational purposes seems, for example, to be independent of any specific experience, as newly-hatched chicks already use geometric cues to reorient [104] [105] [106].

Another line of research could investigate whether non-human animals' categorization of geometric objects depends on whether objects were/are being manipulated or not. It would be interesting to learn whether a difference can be found between animals who manipulate objects in their everyday life (e.g., tool users) and animals who do not.

Finally, future research could also study the development of abstract Euclidean concepts during human ontogeny. Although infants are sensitive to some Euclidean properties such as size and shape [30] [33] [34] [35], little is known about when they start analysing geometric objects in terms of more abstract Euclidean concepts. Further investigating whether this ability is associated with the development of other abilities, such as object manipulation and manufacturing, could help us better understand the factors that lead to the emergence of more abstract geometric concepts.

CONCLUSION

Sensitivity to geometry is widespread in the animal kingdom. Animals, including humans and other mammals, birds, fish, and invertebrates, can categorize objects according to their geometric features. As a first step, animals seem to rely on topological properties to parse visual scenes into objects and backgrounds. As a second step, animals seem to rely on other geometric features such as size, shape, orientation, and sense (at least vertical sense) to discriminate and generalize between objects, possibly because these features predict differences in objects' physical properties. Humans and some other animals can, however, also disregard orientation, sense, and position, when categorizing objects, an ability that is in line with a representation of objects based on Euclidean properties. Humans can further analyse objects in terms of more abstract Euclidean concepts, an ability that might have led to the development of a symbolic mathematical language.

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