HW12

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May 20, 2019

1 QUESTION

1. Prove the case that the cut-level is greater than zero and the cut-formula is not principal in the second premise of the proof of cut-elimination in **G3**[**mic**].
2. Prove the case that the cut-level is greater than zero and the cut-formula is introduced by (L∃) and (R∃) of the proof of cut-elimination in **G3**[**mi**].

2 ANSWER

1.

in G3c Every (Cut) in **G3c** which satisfying the condition must have the following form:

$$\dfrac{\begin{array}{c}\vdots\ \delta_0\\ \vdash_a \Gamma \Rightarrow \Delta, \varphi\end{array} \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{10}\\ \vdash_{b-1} \varphi, \Theta \Rightarrow \Lambda\end{array} \quad \left[\begin{array}{c}\vdots\ \delta_{11}\\ \vdash_{b-1} \varphi, \Sigma \Rightarrow \Pi\end{array}\right]}{\vdash_b \varphi, \Gamma \Rightarrow \Delta}\,\mathrm{J}}{\vdash \Gamma \Rightarrow \Delta}\,\mathrm{Cut}^{\varphi}_{a+b}$$

We can transform into the following proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\ \delta_0\\ \vdash_a \Gamma \Rightarrow \Delta, \varphi\end{array}}{\vdash_a \Gamma, \Theta \Rightarrow \Delta, \Lambda, \varphi}\,\mathrm{W} \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{10}\\ \vdash_{b-1} \varphi, \Theta \Rightarrow \Lambda\end{array}}{\vdash_{b-1} \varphi, \Gamma, \Theta \Rightarrow \Delta, \Lambda}\,\mathrm{W}}{\Gamma, \Theta \Rightarrow \Delta, \Lambda}\,\mathrm{Cut}^{\varphi}_{a+b-1} \quad \left[\dfrac{\dfrac{\begin{array}{c}\vdots\ \delta_0\\ \vdash_a \Gamma \Rightarrow \Delta, \varphi\end{array}}{\vdash_a \Gamma, \Sigma \Rightarrow \Delta, \Pi, \varphi}\,\mathrm{W} \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{11}\\ \vdash_{b-1} \varphi, \Sigma \Rightarrow \Pi\end{array}}{\vdash_{b-1} \varphi, \Gamma, \Sigma \Rightarrow \Delta, \Pi}\,\mathrm{W}}{\Gamma, \Sigma \Rightarrow \Delta, \Pi}\,\mathrm{Cut}^{\varphi}_{a+b-1}\right]}{\Gamma, \Gamma \Rightarrow \Delta, \Delta}\,\mathrm{J}}{\Gamma \Rightarrow \Delta}\,\mathrm{C}$$

in G3[**mi**] Every (Cut) in **G3**[**mi**] which satisfying the condition must have the following form:

$$\dfrac{\begin{array}{c}\vdots\ \delta_0\\ \vdash_a \Gamma \Rightarrow \varphi\end{array} \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{10}\\ \vdash_{b-1} \varphi, \Theta \Rightarrow (\theta)\end{array} \quad \left[\begin{array}{c}\vdots\ \delta_{11}\\ \vdash_{b-1} \varphi, \Sigma \Rightarrow (\eta)\end{array}\right]}{\vdash_b \varphi, \Gamma \Rightarrow (\delta)}\,\mathrm{J}}{\vdash \Gamma \Rightarrow (\delta)}\,\mathrm{Cut}^{\varphi}_{a+b}$$

We can transform into the following proof:

$$\dfrac{\dfrac{\dfrac{\dfrac{\begin{array}{c}\vdots\ \delta_0\\ \vdash_a \Gamma \Rightarrow \varphi\end{array}}{\vdash_a \Gamma, \Theta \Rightarrow \varphi}\,\mathrm{LW} \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{10}\\ \vdash_{b-1} \varphi, \Theta \Rightarrow (\theta)\end{array}}{\vdash_{b-1} \varphi, \Gamma, \Theta \Rightarrow (\theta)}\,\mathrm{LW}}{\Gamma, \Theta \Rightarrow (\theta)}\,\mathrm{Cut}^{\varphi}_{a+b-1} \quad \left[\dfrac{\dfrac{\begin{array}{c}\vdots\ \delta_0\\ \vdash_a \Gamma \Rightarrow \varphi\end{array}}{\vdash_a \Gamma, \Sigma \Rightarrow \varphi}\,\mathrm{LW} \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{11}\\ \vdash_{b-1} \varphi, \Sigma \Rightarrow (\eta)\end{array}}{\vdash_{b-1} \varphi, \Gamma, \Sigma \Rightarrow (\eta)}\,\mathrm{LW}}{\Gamma, \Sigma \Rightarrow (\eta)}\,\mathrm{Cut}^{\varphi}_{a+b-1}\right]}{\Gamma, \Gamma \Rightarrow (\delta)}\,\mathrm{J}}{\Gamma \Rightarrow (\delta)}\,\mathrm{LC}$$

2. Every (Cut) in **G3**[**mi**] which satisfying the condition must have the following form:

$$\dfrac{\dfrac{\begin{array}{c}\vdots\ \delta_{00}\\ \vdash_{a-1} \Gamma \Rightarrow \varphi^x_t\end{array}}{\vdash_a \Gamma \Rightarrow \exists x\varphi}\,\mathrm{R}\exists \qquad \dfrac{\begin{array}{c}\vdots\ \delta_{10}\\ \vdash_{b-1} \varphi^x_y, \Gamma \Rightarrow (\delta)\end{array}}{\vdash_b \exists x\varphi, \Gamma \Rightarrow (\delta)}\,\mathrm{L}\exists}{\Gamma \Rightarrow (\delta)}\,\mathrm{Cut}^{\exists x\varphi}_{a+b}$$

By lemma 3.5.2(β), we can obtain $\vdash_{b-1} [(\varphi^x_y)^y_t, \Gamma^y_t \Rightarrow (\delta^y_t)]^*$. And this is exactly $\vdash_{b-1} [\varphi^x_t, \Gamma \Rightarrow (\delta)]^*$ since y is free in $\Gamma, \exists x\varphi(, \delta)$.

Then we can transform the original proof to the following one:

$$\dfrac{\begin{array}{c}\vdots\ \delta_{00}, 3.5.2(\alpha)\\ \vdash_{a-1} [\Gamma \Rightarrow \varphi^x_t]^*\end{array} \qquad \begin{array}{c}\vdots\ \delta_{10}, 3.5.2(\beta)\\ \vdash_{b-1} [\varphi^x_t, \Gamma \Rightarrow (\delta)]^*\end{array}}{[\Gamma \Rightarrow (\delta)]^{**}}\,\mathrm{Cut}^{\varphi^x_t}_{a+b-2}$$