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# State of Charge and Health Estimation For Lithium-Ion Batteries Using Recursive Least Squares

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*Abstract*— **The state of charge (SOC) and state of health (SOH) of lithium-ion batteries (LIB) are two major indices of the battery management system (BMS) for system monitoring, health prognosis, and optimal usage. This paper presents a new SOC and SOH estimation method based on two recursive least squares (RLS) algorithms. First, a second-order equivalent circuit model is used to describe battery dynamics. Second, an open-circuit-voltage (OCV) and internal resistance estimation method is proposed based on the RLS algorithm. Instead of using SOC as a state in a state space model, the OCV is estimated from a linear regression directly. The battery total capacity is then estimated by a combination of the estimated OCV and another RLS algorithm. The accurate battery SOC and SOH can be obtained without a priori knowledge of battery parameters. Simulation results highlight the accuracy and robustness of the proposed method.**

## I. Introduction

Lithium-ion batteries (LIB) have been widely selected as the main energy storage for many different applications, such as electric vehicles (EV), smart-grids, consumer electronics, aircrafts, and satellites [1]. LIBs are the leading candidate for commercial use due to their high energy density, longevity, and low self-discharging rate. However, practical energy storage systems are composed of many individual cells to provide high output voltage, power, and enough stored energy. To handle these and achieve optimal energy utilization and minimization of aging, a battery management system (BMS)is needed, in which the state of charge (SOC) and state of health (SOH) estimation are two of the most important tasks. The estimated SOC and SOH can be used to gauge the battery remaining energy and useful life. However, inaccurate SOC and SOH estimation can result in over-charging/over-discharging the battery and reducing its lifespan. Now, the SOC and SOH are coupled through electrochemical reactions, and they also change on different time-scale, which lead to great challenges for simultaneous SOC and SOH estimation. Therefore, developing advanced estimation algorithms for these states using measurable variables, e.g., current and voltage signals, is therefore an important field of research for BMSs.

The existing SOC estimation methods can be roughly categorized into three classes: conventional method, model-free, and model-based algorithms. The coulomb-counting and open circuit voltage (OCV) table look-up methods are two of the most commonly used approaches for SOC estimation, because they are simple and easy to implement [2]. However, the coulomb-counting method suffers from initialization deviation and accumulated errors caused by noise and biased measurements. The OCV table look-up method is relatively accurate, though only when the battery reaches an equilibrium state. The model-free methods establish diverse models by simulating the nonlinear relationship between SOC and its influencing factors using data-driven toolboxes, with artificial neural networks and support vector regression. Although these methods are flexible and straightforward, their performance heavily depend on quantity and quality of training datasets. The model-based SOC estimation methods are more popular because they are generally closed-loop, adaptive, and can be easy to implement online [3]. These methods first use electrochemical or equivalent circuit models (ECM) where SOC is a state describing battery dynamics [4]. Then, diverse observers can be developed to estimate battery SOC, for example, the Luenberger observer, Kalman filter, particle filter as well as their variations [5].

The battery SOH is often represented by battery total capacity and internal resistance, which are critical parameters in model-based SOC estimation. In this regard, the estimation of SOC and SOH over a batterys lifetime often requires collaborative efforts. However, battery SOC and health indicators are implicit states and parameters that coupled with sophisticated electrochemical processes and also coupled with each other. This can be transferred to a simultaneous parameter identification and state estimation problem. A plethora of approaches have been reported in the literature to solve this problem. The joint-filtering and dual filtering have gained the most attention. Joint-filters directly estimate an augmented parameter-state vector, and thus have the disadvantage of large matrix operations. Dual-filters employ two parallel estimators to individually estimate parameters and states, such as recursive least square method (RLS)-Kalman filter [6] and dual Kalman filters [7]. Although diverse dual and joint filters can achieve co-estimation of SOC and SOH, the existence of model mismatch and measurement noises is still challenging the robustness, stability and convergency of these methods. Besides, the stability and convergence of these methods are sensitive to the initial values of the system parameters.

This paper proposes a new SOC and SOH estimation method based on RLS algorithms. An OCV estimator is firstly derived by combining a second-order ECM and an RLS algorithm. Then, the battery total capacity is filtered

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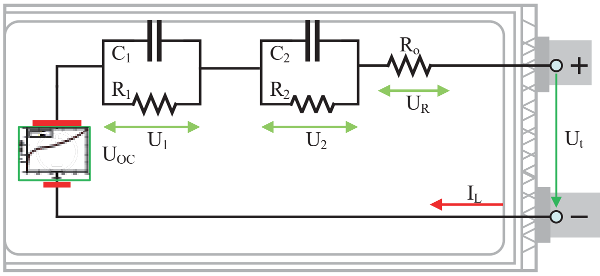


Fig. 1. The second-order Thevenin equivalent circuit model.

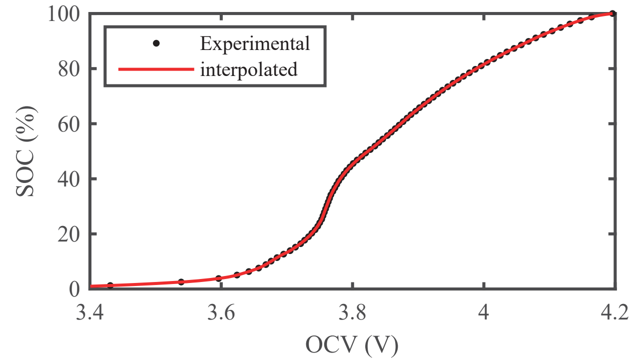


Fig. 2. The second-order Thevenin equivalent circuit model.

based on the estimated OCV and an extra RLS. Unlike the above referred methods, the battery SOC and SOH can be obtained with a reasonable accuracy without a priori knowledge of battery parameters.

## II. Battery Modeling

### A. Equivalent Circuit Model

The second-order ECM is shown in Fig. 1, where $U_{oc}$ is the OCV, $R_o$ is the ohmic internal resistance, $R_l$ and $C_l, l = 1,2$ are parallel connected polarization resistance and capacitance, $I_L$ and $U_t$ are battery load current and voltage. The dynamics of this model can be expressed as [2]

$$\dot{U}_l = -U_l/_l + I_L C_l, l = 1,2 \tag{1}$$

$$U_t = U_o c - I_L R_o - U_1 - U_2 \tag{2}$$

where $\tau_l = R_l C_l, l = 1,2$ are the time constants. $U_l$ are the terminal voltages of the RC networks. The battery parameters are assumed to be slowly time-varying. Substituting (1) into (2), we can obtain:

$$\begin{aligned} U_1 = &\frac{1}{\tau_1 - \tau_2}[\tau_1\tau_2\dot{U}_{oc} + \tau_1 U_{oc} - \tau_1\tau_2\dot{U}_t - \tau_1 U_t \\ &- I_L(\tau_2 R_1 + \tau_1 R_o + \tau_1 R_2) - \tau_1\tau_2 R_o I_L] \end{aligned} \tag{3}$$

where $\tau_l > \tau_2$. Differentiating (3) and substituting (1) and (3) into the derived equation, give:

$$\begin{aligned} \ddot{U}_t = &-\frac{U_t}{\tau_1\tau_2} - (\frac{1}{\tau_1} + \frac{1}{\tau_2})\dot{U}_t + (\frac{1}{\tau_1} + \frac{1}{\tau_2})\dot{U}_{oc} \\ &+ \ddot{U}_{oc} + \frac{U_{oc}}{\tau_1\tau_2} - \ddot{I}_L R_o - \frac{I_L}{\tau_1\tau_2}(R_o + R_1 + R_2) \\ &- \dot{I}_L(\frac{R_1}{\tau_1} + \frac{R_o}{\tau_2} + \frac{R_o}{\tau_1} + \frac{R_2}{\tau_2}) \end{aligned} \tag{4}$$

### B. On-line Parameter Identification

Discretizing (4) gives:

$$\begin{aligned} U_{t,k} = &\alpha_1 U_{t,k-1} + \alpha_2 U_{t,k-2} + U_{oc,k} - \alpha_1 U_{oc,k-1} \\ &- \alpha_2 U_{oc,k-2} + \alpha_3 I_{L,k} + \alpha_4 I_{L,k-1} + \alpha_5 I_{L,k-2} \end{aligned} \tag{5}$$

where the differentiation is approximated by backward difference, denoting the sampling rate by $\Delta t$. $\alpha_i, i = 1,2,,5$ can be formulated as

$$\alpha_1 = 1 + \frac{\tau_1\tau_2 - \Delta t^2}{(\tau_1 + \Delta t)(\tau_2 + \Delta t)}$$

$$\alpha_2 = -\frac{\tau_1\tau_2}{(\tau_1 + \Delta t)(\tau_2 + \Delta t)}$$

$$\alpha_3 = -R_o - \frac{\Delta t R_1}{\tau_1 + \Delta t} - \frac{\Delta t R_2}{\tau_2 + \Delta t}$$

$$\alpha_4 = \alpha_1 R_o + \frac{\tau_2 \Delta t R_1 + \tau_1 \Delta t R_2}{(\tau_1 + \Delta t)(\tau_2 + \Delta t)}$$

$$\alpha_5 = -\frac{\tau_1\tau_2 R_o}{(\tau_1 + \Delta t)(\tau_2 + \Delta t)}$$

Let $\gamma_k = U_{oc,k} - \alpha_1 U_{oc,k-1} - \alpha_2 U_{oc,k-2}$, then (5) can be rewritten as:

$$y_k = \theta_k^{\mathrm{T}} \varphi_k + \upsilon_k \tag{6}$$

where

$$\begin{aligned} y_k &= U_{t,k} \\ \theta_k &= [\alpha_1, \alpha_2, \gamma_k, \alpha_3, \alpha_4, \alpha_5]^{\mathrm{T}} \\ \varphi_k &= [U_{t,k-1}, U_{t,k-2}, 1, I_{L,k}, I_{L,k-1}, I_{L,k-2}]^{\mathrm{T}}, \end{aligned}$$

where $\upsilon_k$ represents additive noise with the model (6) on regression form, the battery parameter vector $\theta_k$ can be estimated by RLS, where $\gamma_k$ is regarded as a slow time-varying parameter.

## III. The Proposed SOC and SOH Estimator

### A. The Proposed OCV Estimator

The first step of the OCV estimation is to estimate the parameters $\theta$, which can be regarded as a constant vector due to the slowly time-varying characteristics of model parameters. At each time $k$, the estimated parameter vector is referred as $\hat{\theta}_k$. The estimate is good if the difference between the predicted output $\hat{y}_k$ and the measured output is small in magnitude in a least squares sense. Defining the predicted error as $e_k = y_k - \hat{y}_k$, the weighted least squares function can be formulated as

$$J(\hat{\theta}_k) = \sum_{i=0}^{k} \lambda^(k-i) e_i^2 \tag{7}$$

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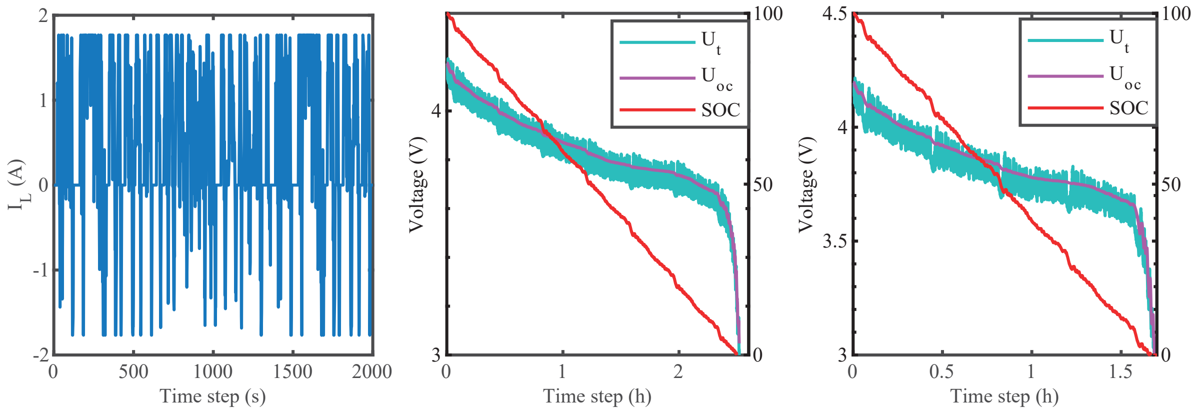


Fig. 3. (a) Current of the FUDS profile, (b) data of 1# cell, (c) data of 2# cell.

where $0 < \lambda \leq 1$ denotes the so-called forgetting factor. Recursive estimation of the parameter vector is desirable for on-line implementation, i.e.,

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \Delta\hat{\theta}_{k-1} \tag{8}$$

where $\Delta\hat{\theta}_{k-1}$ denotes the correction item from $k-1$ to $k$, which can be updated in a recursive manner. The RLS algorithm minimizing (7) can be summarized in two steps [5]:

**Algorithm 1: RLS-based parameter identification**

**Step.** 1 Initialization, for $k = 0$,
$\hat{\theta}_0 = 0; \mathbf{P}_0 = \delta\mathbf{I}$, where $\mathbf{I}$ is a $6\times 6$ identity matrix.

**Step.** 2 Computation, For $k = 1, 2, \cdots$
Generate data vector $\varphi_k$, where the current and voltage at time $k = -1, 0$ have been stored in advance.

$$L_k = \mathbf{P}_{k-1}\varphi_k/(\lambda + \varphi_k^{\mathrm{T}}\mathbf{P}_{k-1}\varphi_k)$$
$$\mathbf{P}_k = (\mathbf{P}_{k-1} - L_k\varphi_k^{\mathrm{T}}\mathbf{P}_{k-1})/\lambda$$
$$\hat{\theta}_k = \hat{\theta}_{k-1} + L_k(y_k - \hat{\theta}_{k-1}^{\mathrm{T}}\varphi_k)$$

### *B. The Calculation of SOC*

Based on the RLS algorithm, we can estimate the OCV related parameter $\gamma_k$. Thus, the estimated OCV can be expressed as:

$$\hat{U}_{oc,k} \approx \hat{\gamma}_k + \hat{\alpha}_{1,k}\hat{U}_{oc,k-1} + \hat{\alpha}_{2,k}\hat{U}_{oc,k-2} \tag{9}$$

where $\hat{\gamma}_k$, $\hat{\alpha}_{1,k}$, and $\hat{\alpha}_{2,k}$ are taken from $\hat{\theta}_k$. Eventually, the SOC can be estimated based on the inversed OCV table look-up method using $\hat{U}_{oc,k}$. The relationship between SOC and OCV is shown in Fig. 2.

### *C. The SOH Estimator*

*1) Calculation of Internal Resistance:* The SOH estimator should not only estimate battery internal resistance, but also estimate battery capacity. From the definition of the model parameters below (5), we can obtain

TABLE I

MODEL PARAMETERS IN THE SIMULATION STUDY

| No. | 1# | 2# |
|---|---|---|
| $Q$(Ah) | 1.2 | 0.8 |
| $R_o$ (mΩ) | 25 | 35 |
| $R_1$ (mΩ) | 20 | 26 |
| $R_2$ (mΩ) | 9 | 17 |
| $C_1$ (F) | 1200 | 1100 |
| $C_2$ (F) | 400 | 600 |

$$R_o = \alpha_5/\alpha_2 \tag{10}$$

$$(\tau_1 + \Delta t)(\tau_2 + \Delta t) = \frac{\Delta t^2}{1 - \alpha_2 - \alpha_1} \tag{11}$$

$$\tau_1\tau_2 = -\frac{\alpha_2\Delta t^2}{1 - \alpha_2 - \alpha_1} \tag{12}$$

From (11) and (12), we can solve for $\tau_1$ and $\tau_2$. $R_1$ and $R_2$ can then be solved according to the definition of $\alpha_3$ and $\alpha_4$.

*2) Estimation of Battery Capacity:* From the definition of $\hat{\gamma}_k$, we can obtain the OCV signal (9). The estimated OCV, $\hat{U}_{oc,k}$, from the RLS can be quite noisy and therefore the OCV estimates are filtered use moving average filter before calculating the battery capacity. Let $z_{k_1}$ and $z_{k_2}$ denote the resulting SOC estimates at time $k_1$ and $k_2$. The battery capacity can then be determined from

$$\sum_{i=k_1}^{k_2} I_{L,i}\Delta t = Q(z_{k_2} - z_{k_1}) \tag{13}$$

where $Q$ is the battery capacity. It can be clearly seen from (14) that calculating $Q$ can be transferred to a linear fitting problem, which can be solved by the RLS algorithm.

## IV. SIMULATION RESULTS

A simulation study is performed to illustrate the proposed RLS-based SOC and SOH estimation algorithm. The model parameters are shown in Table 1. The current, voltage, and



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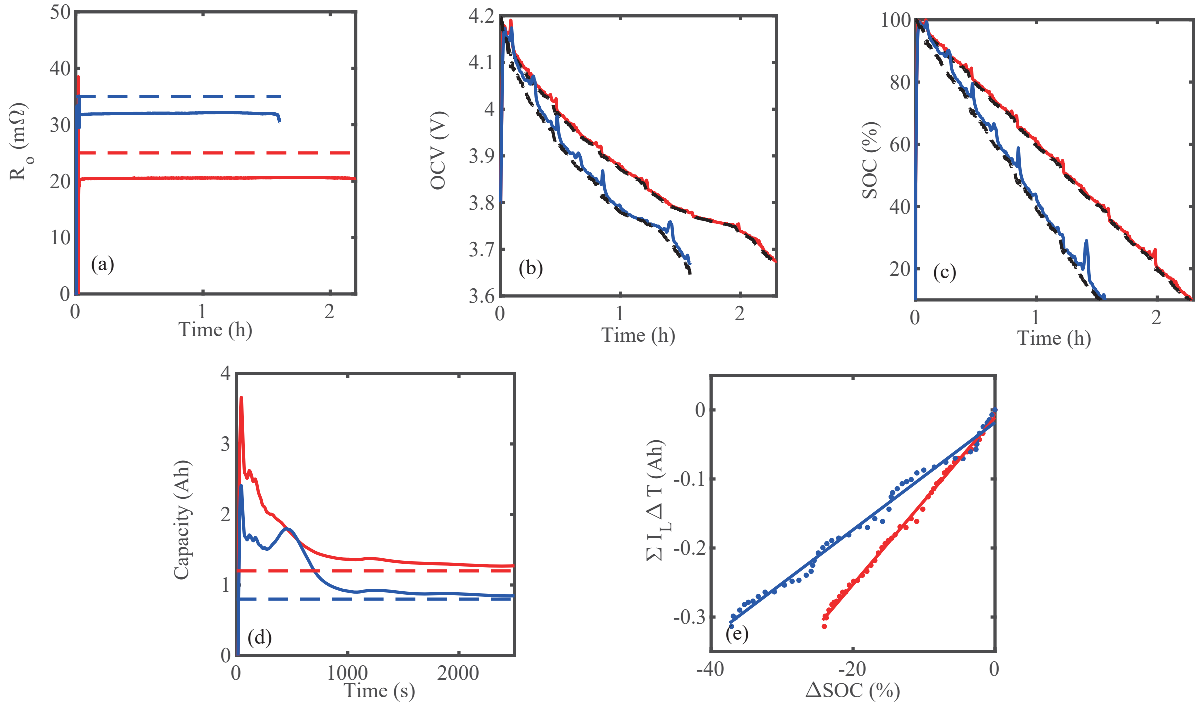


Fig. 4. (a) RLS-based estimated (a) resistance, (b) OCV, (c) SOC, (d) Capacity; (e) off-line fitted capacity. (Red and blue solid lines denote estimated values of 1# and 2# cells, respectively, dash lines denote reference value.

reference SOC are sampled at 1Hz. The noises of voltage and current sensors are subject to white, with zero mean and the variance are $1 \times 10^{-6}\text{V}^2$ and $1 \times 10^{-6}\text{A}^2$, respectively. The corresponding data of 1# and 2# cells are plotted in Fig. 3. The experimental results are plotted in Fig. 4. Fig. 4(a) shows the estimated $R_o$, where the bias of 1# and 2# are 3.84mΩ and 4.15mΩ, respectively. The root mean square errors (RMSE) of the estimated OCV are 22.3mV and 33.1mV for 1# and 2# cell, respectively. The RMSE of the estimated SOC of Cell 1# and 2# are 3.04% and 4.87%, respectively. The RLS-based estimated capacity of cell 1# and 2# are 1.27Ah and 0.84Ah, respectively, which are close to their actual value. In addition, the off-line fitted capacity (the slop of dash lines in Fig. 4(e)) of cell 1# and 2# are 1.21Ah and 0.78Ah. These results indicate good performance of the proposed SOC and SOH estimator. The reasons for the OCV estimation spikes in Fig. 4(b) mainly lie in the inaccurate discretization method, we can use the first order hold method or Tustins discretization method to obtain the discretized battery model in (5).

## V. Conclusion

This paper proposed a SOC and SOH estimation method based on two RLS algorithms. The proposed method can estimate battery SOC and SOH based on terminal voltage and current, without a priori knowledge of battery parameters. The simulation results confirmed the effectiveness and accuracy of the proposed method. In future works, the estimation bias of the identified model parameters should be further reduced.

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