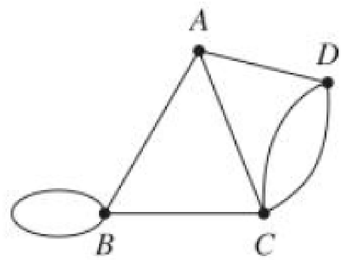
DA - Graphs

Define edge/arc	A line connecting two vertices.
Define network/weighted graph	A graph with weighted edges (i.e., edges with numbers).
Define order/degree of vertex	The number of edges starting and finishing at the vertex.
Define loop	An edge starting and finishing at the same vertex.
Define weight	A real world value assigned to an edge (e.g., metres).
Define cycle	A closed path.
Define Hamiltonian cycle (with example)	A route that... ● Visits every vertex exactly once ● Returns to the starting vertex ● Without repeating any edges Example: ABCDEA for a pentagonal graph
When is and isn't a graph Hamiltonian-connected?	● Is when there exists a Hamiltonian cycle for every vertex. ● Isn't when you're forced through one vertex twice or thrice. *If there exists a cycle for one then there must exist a cycle for them all (as you can merely reorder the letters).*
No. of Hamiltonian Cycles	● For distinct cycles in which you consider ABCA to equal ACBA, it is: $$\frac{(n-1)!}{2}$$ ● So for non-distinct, it is: $$(n-1)!$$ *This is because if you had 4 vertices (in ABCDA), you would have 3! ways of arranging (A**<u>BCD</u>**A)**.** However, for any start it would be 4!.*

	Both ABCDA and ADCBA have the same shape but opposite directions. If you treat them as equal, you'll get half as many cycles.
Define trial	A route in which no edge is repeated (e.g., ABCEA).
Define Eulerian trail	A route which visits every edge EXACTLY ONCE.
Define Eulerian cycle	A trial which starts and ends at the same vertex **AND** visits every edge **EXACTLY ONCE**.
TEPEV	<u>**T**</u>rial <u>**E**</u>dge <u>**P**</u>ath <u>**E**</u>dge <u>**V**</u>ertex.
Define connected graph	Every node is connected to the graph. *Therefore, it is possible to get from any node to any other node (however not necessarily directly).*
Define simple graph	A graph with no loops and no repeated edges.
Define complete graph (K_n)	A graph where every vertex is connected to every other vertex directly.
What is the no. of edges on a complete graph?	The number of edges on a complete graph is defined by: $$\frac{n(n-1)}{2}$$ *A complete graph has n - 1 outgoing edges and n edges in total thus you'd be tempted to say there are n(n - 1) edges. However, every edge is counted twice because every edge going out a vertex is going into another thus you divide by 2.*
Define Eulerian graph	A **CONNECTED** graph with **ONLY EVEN VERTICES**. *This means you can start and end at the same point (because every vertex has an edge going into and out of it).*
Define semi-Eulerian graph	A **CONNECTED** graph with **EXACTLY TWO ODD VERTICES**. *This means if you start at an odd degree vertex, you can visit every vertex yet you'll end up at the other odd degree vertex.*
Define non-Eulerian graph	A **CONNECTED** graph with **MORE THAN TWO ODD VERTICES**.

<table>
<tr><td>Define digraph</td><td>A graph with one or more directed edges.</td></tr>
<tr><td>Define bipartite graph</td><td>Two sets of vertices with edges that can only connect from one set to the other.</td></tr>
<tr><td>Define adjacency/incidence matrix</td><td>A representation of the direct routes between the vertices of a graph in a matrix.

It is routes because…


<table>
<tr><th></th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>A</td><td>0</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>B</td><td>1</td><td>2</td><td>1</td><td>0</td></tr>
<tr><td>C</td><td>1</td><td>1</td><td>0</td><td>2</td></tr>
<tr><td>D</td><td>1</td><td>0</td><td>2</td><td>0</td></tr>
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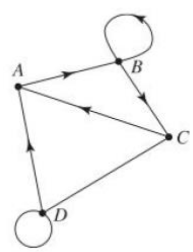
Has 2 direct routes of getting from B to B.
Whereas…


<table>
<tr><th></th><th colspan="5">To</th></tr>
<tr><td rowspan="5">From</td><td></td><td>A</td><td>B</td><td>C</td><td>D</td></tr>
<tr><td>A</td><td>0</td><td>1</td><td>0</td><td>0</td></tr>
<tr><td>B</td><td>0</td><td>1</td><td>1</td><td>0</td></tr>
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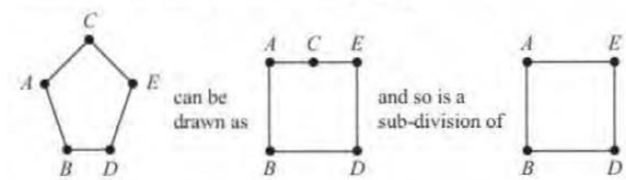
<tr><td>D</td><td>1</td><td>0</td><td>1</td><td>2</td></tr>
</table>
Only has 1 direct route of getting from B to B.</td></tr>
<tr><td>Define tree and a spanning tree</td><td>A tree is a simple graph with no cycles.
A spanning tree is a connected tree.</td></tr>
<tr><td>How many edges are there in a spanning tree of n vertices and why?</td><td>n - 1 edges.
For each new vertex you add to the spanning tree, you increase the original tree (of 2 vertices and 1 edge) by 1 edge.</td></tr>
<tr><td>Define minimum spanning tree</td><td>A spanning tree of minimum weight.</td></tr>
</table>

Define planar graph	A graph that can be (re)drawn so that none of its arcs cross. *This can be useful for insulated wires on microchips.*
Define isomorphic graph	A graph that has the same number of vertices connected in the same way as another.
Define complement of a graph	A graph of all missing edges required to make another graph complete.
Define subgraph	Part of a graph.
Define subdivision	Where you split an edge into two edges by adding an extra vertex. 
What is Euler's Formula and when does it hold true?	Holds true for any **CONNECTED PLANAR** graph (e.g., many 3D solids). $V + F - E = 2$ *This can be remembered by using the vertices, faces, and edge of a cube.*
What is Kuratowski's Theorem?	A graph is planar if and only if it does not contain a subgraph that is a subdivision of K_5 or of $K_{3,3}$.
What is the triangle inequality?	**Key point** The triangle inequality states that the sum of the lengths of any two sides of a triangle cannot be less than the length of the third side. $AB + BC \geq AC$