DF - Game theory

<table>
<tr><td>What is the definition for zero-sum games?</td><td>For each pair of strategies (1) X’s gains + Y’s gains = 0 (1).</td></tr>
<tr><td>What is a play-safe strategy?</td><td>A strategy that gives best guaranteed outcome regardless of what the opponent does.</td></tr>
<tr><td>What is a pure strategy?</td><td>When both players play their play-safe strategy every time <u>THUS</u> minimising potential losses for each player.</td></tr>
<tr><td>What is the value of a game to a player?</td><td>The payoff to the player if they use their best strategy.

This best strategy may be their play-safe strategy or an optimal mixed strategy.</td></tr>
<tr><td>How can you show there is no stable solution? If so, what must be done?</td><td>
<table>
<tr><td></td><td></td><td colspan="3">B</td><td></td><td></td></tr>
<tr><td></td><td></td><td>K</td><td>Q</td><td>J</td><td>Row minimum</td><td></td></tr>
<tr><td rowspan="3">A</td><td>K</td><td>5</td><td>−4</td><td>2</td><td>-4</td><td></td></tr>
<tr><td>Q</td><td>3</td><td>1</td><td>4</td><td>1</td><td>← max = 1</td></tr>
<tr><td>J</td><td>2</td><td>3</td><td>−1</td><td>-1</td><td></td></tr>
<tr><td colspan="2">Column maximum</td><td>5</td><td>3</td><td>4</td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td>↑
min = 3</td><td></td><td></td><td></td></tr>
</table>

By showing that the minimax (worst case for A) ≠ maximin (worst case for B).
Since stable solution exists ⇔ row maximin = col minimax.
Use an optimal mixed strategy where you play each move with some probability.

This stable solution would mean that neither player can gain by changing their play-safe strategy.</td></tr>
<tr><td>What should you make sure of when finding an optimal mixed strategy using Simplex?</td><td>

All the entries are positive (as simplex requires all variables to be non-negative).
If they aren’t, add some constant to each entry then subtract this from the value once you’ve found it.

<u>This is demonstrated below:</u></td></tr>
</table>

		B	
		B_1	B_2
A	A_1	1	−2
	A_2	0	−1
	A_3	−1	0

Add 2 to each table entry.

		B	
		B_1	B_2
A	A_1	3	0
	A_2	2	1
	A_3	1	2

All entries must be non-negative.

Let A play A_1, A_2 and A_3 with probabilities p_1, p_2 and p_3

Maximise $P = v$

A's expected profit, P, is the value, v, of the game.

subject to
$$v \le 3p_1 + 2p_2 + p_3$$
$$v \le p_2 + 2p_3$$
$$p_1 + p_2 + p_3 \le 1$$
$$v, p_1, p_2, p_3 \ge 0$$

You replace $p_1 + p_2 + p_3 = 1$ with the inequality shown so that you can introduce a slack variable for the simplex tableau. The slack variable will be found to be zero.

$$P - v = 0$$
$$v - 3p_1 - 2p_2 - p_3 + s = 0$$
$$v - p_2 - 2p_3 + t = 0$$
$$p_1 + p_2 + p_3 + u = 1$$

Introduce slack variables.

Perform Simplex…

This gives $v = 1.5$ when $p_1 = \frac{1}{4}$, $p_2 = 0$ and $p_3 = \frac{3}{4}$

So the value of the game is −0.5

Subtract the 2 you added initially.

A's optimal strategy is to play A_1 and A_3 with probabilities 0.25, 0.75 and never play A_2

The value of the game to B is 0.5. Suppose B plays B_1 and B_2 with probabilities q_1 and q_2

If A plays A_1, the value to B is $-q_1 + 2q_2$, so $-q_1 + 2q_2 = 0.5$ [1]

Similarly, for A_2 $q_2 = 0.5$ [2]

and A_3 $q_1 = 0.5$ [3]

[1], [2] and [3] are all satisfied by $q_1 = 0.5$ and $q_2 = 0.5$, so B plays B_1 and B_2 with equal probability.

You may want to find the optimal mixed strategy for the other player too.

How should you set up a game theory problem involving Simplex?

1. Ensure the value you're maximising is less than all the probability constants (eg, $v \le p_2 + 2p_3$).
2. Ensure all probabilities sum to ≤ 1 (you can add a slack variable which will then equal zero).
3. All the variables are ≥ 0 as shown.

	Let A play A_1, A_2 and A_3 with probabilities p_1, p_2 and p_3 Maximise $P = v$ subject to $v \leq 3p_1 + 2p_2 + p_3$ $v \leq p_2 + 2p_3$ $p_1 + p_2 + p_3 \leq 1$ $v, p_1, p_2, p_3 \geq 0$ $P - v = 0$ $v - 3p_1 - 2p_2 - p_3 + s = 0$ $v - p_2 - 2p_3 + t = 0$ $p_1 + p_2 + p_3 + u = 1$ A's expected profit, P, is the value, v, of the game. You replace $p_1 + p_2 + p_3 = 1$ with the inequality shown so that you can introduce a slack variable for the simplex tableau. The slack variable will be found to be zero. Introduce slack variables. *This is because simplex requires us to be working in the positive region of a graph.*
How can you solve m x n zero-sum games?	1. Look for a stable solution. If none then mixed strategy. 2. Look for dominance and reduce size. 3. If m x 2 or 2 x n, sketch graphs. Otherwise use LP.