Hypothesis Testing (under Poisson)

Essence of Hypothesis Testing

Four siblings have to decide who washes the dishes each night. Bill proposes that he draws a name from a hat each night and whoever is selected washes them.

null hypothesis

$\mathcal{H}_0$: Bill's picking is truly random (give him the benefit of the doubt)

alternate hypothesis $\mathcal{H}_1$: Bill's picking is rigged.

$P(\text{Bill not picked in 3 consecutive nights}) = \left(\frac{3}{4}\right)^3 = \frac{27}{64} \approx 0.42$

This seems alright, nothing too suspicious.

$P($" " " " $12 \text{ consecutive nights}) = \left(\frac{3}{4}\right)^{12} \approx 0.032 = 3.2\%$

His siblings would get suspicious as this observation is SO unlikely to happen if H_0 is true that you begin to question it.

As this is $<$ 5% conventional critical level, you would reject $\mathcal{H}_0$ in favour of $\mathcal{H}_1$.

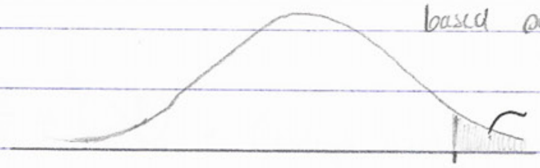
Essentially... if the evidence against the null hypothesis is strong enough (above a certain significance level), we can reject the null hypothesis in favour of the alternate one.

Caution • Hypothesis should be made about parameters, NOT statistics. E.g., $H_0: \lambda = \frac{1}{4}$ or $H_1: \lambda > \frac{1}{4}$

p-value • It is a measure of evidence against H_0.
• It is the probability of getting $\leq$ or $\geq$ a value given the null hypothesis is true. OR the probability the data is at least extreme as observed.

poisson distribution

based on H_0



p-value (area to the right of the observation)

observed value

$\Rightarrow$ the further to the right, the smaller the p-value, the greater the evidence against H_0

$\Rightarrow$ reject H_0 if p-value $\leq \alpha$ (some significance level)

(It should be noted: the greater the sample size, the greater the ~~[illegible] [illegible] the p-value greater the power of~~ sample size, the greater the power of the p-value)

Errors Type I: rejecting H_0 when in fact it is true.
" II: failing to reject H_0 when in fact false.

$\Rightarrow$

Table of Outcomes	H_0 false	H_0 true
Reject H_0	Correct	Type I error
Don't reject H_0	Type II error	Correct

underlying truth

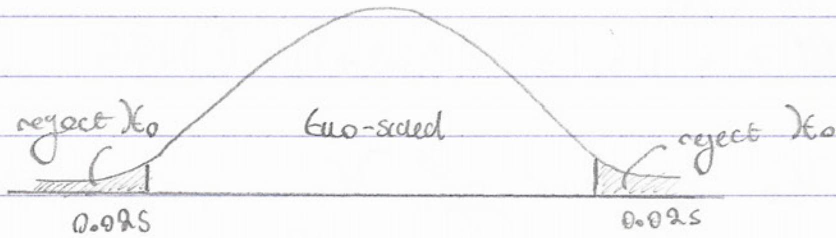
$P(\text{Type I Error} \mid H_0 = \text{true}) = \alpha$ (significance level)

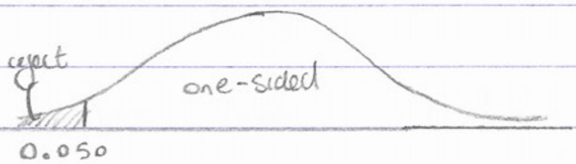
YET if α decreases $P(\text{Type II Error} \mid ")$ increases

One-sided & Two-sided Tests

One-sided H: $\mathcal{H}_0 : \lambda = 2$
$\mathcal{H}_1 : \lambda > 2$

Two-sided H: $\mathcal{H}_0 : \lambda = 7.25$
$\mathcal{H}_1 : \lambda \neq 7.25$





(you have more power here YET lose the ability to detect the other side)

Extend Example

A poisson distribution has an assumed parameter of 8.74 and is being tested to see if it is an over-estimate.

$\mathcal{H}_0 : \lambda_X = 8.74$ where $X \sim Po(8.74)$
$\mathcal{H}_1 : \lambda_X < 8.74$

A sample of size 16 is taken to test at a 5% significance level (α). Find any critical values.

Let $Y \sim Po(139.84)$ 16×8.74 we multiply to increase the accuracy, otherwise we would be assuming a sample size of 1

$\because <$ used in $\mathcal{H}_0$

$P(Y \leq 120) = 0.0483$
$P(Y \leq 121) = 0.0579$

Critical values = 120

The sample size of 16 size has an avg. of 7.2.

This is the test statistic, the total no. of events in a sample size of n

$7.2 \times 16 = 115.2$
& $115.2 < 120$

<u>OR</u> $7.2 \times 16 \approx 115$
& $P(Z \leq 115 \mid Z \sim Po(139.84)) = 0.0175$ which is < 0.05

Conclusions are important!

$\Rightarrow$ Reject H_0 in favour of H_1 and conclude that there is sufficient evidence @ the 5% significance level to suggest $\lambda_X < 8.74$

A sample of size 25 is taken and has a total 196.5 (5% level).

25×8.74

can find critical values of let $W \sim Po(218.5)$

$P(W \leq 196) = 0.06642$ & $P(W \leq 197) = 0.07594$

$\because$ both > 0.05, accept H_0 and conclude there is insufficient evidence at the 5% level to suggest $\lambda_X < 8.74$