CH - Hyperbolic functions

Derive arcsinh x	$$\begin{aligned} y &= \sinh^{-1} x \\ x &= \sinh y \\ x &= \frac{e^y - e^{-y}}{2} \\ 2x &= e^y - e^{-y} \\ 2xe^y &= e^{2y} - 1 \\ e^{2y} - 2xe^y - 1 &= 0 \\ e^y &= \frac{2x \pm \sqrt{(-2x)^2 + 4}}{2} \\ &= x \pm \sqrt{x^2 + 1} \\ y &= \ln\left(x \pm \sqrt{x^2 + 1}\right) \\ \sinh^{-1} x &= \ln\left(x + \sqrt{x^2 + 1}\right) \end{aligned}$$ We take the positive log at the end.
Derive arccosh x	We must restrict y ≥ 0 for a one-to-one mapping.

	$$\begin{aligned} y &= \cosh^{-1} x \\ x &= \cosh y \\ x &= \frac{e^y + e^{-y}}{2} \\ 2x &= e^y + e^{-y} \\ 2xe^y &= e^{2y} + 1 \\ e^{2y} - 2xe^y + 1 &= 0 \\ e^y &= \frac{2x \pm \sqrt{(-2x)^2 - 4}}{2} \\ &= x \pm \sqrt{x^2 - 1} \\ y &= \ln\left(x \pm \sqrt{x^2 - 1}\right) \\ \cosh^{-1} x &= \ln\left(x + \sqrt{x^2 - 1}\right) \end{aligned}$$ We take the solution with the positive as we said $y \geq 0$ so $e^y \geq 1$ for all y, and since x - \sqrt(x^2 - 1) fails to exceed this for some x (eg, -1), we discard it.

Derive arctanh x	$\begin{aligned} y &= \tanh^{-1} x \\ x &= \tanh y \\ x &= \frac{e^y - e^{-y}}{e^y + e^{-y}} \\ &= \frac{e^y - e^{-y}}{e^y + e^{-y}} \cdot \frac{e^y}{e^y} \\ &= \frac{e^{2y} - 1}{e^{2y} + 1} \\ xe^{2y} + x &= e^{2y} - 1 \\ e^{2y} - xe^{2y} &= 1 + x \\ e^{2y} &= \frac{1+x}{1-x} \\ 2y &= \ln\left(\frac{1+x}{1-x}\right) \\ y &= \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \end{aligned}$
Most of the graph work you can figure out using your calculator.	