SA - DRVs

What is expected value?	The theoretical mean of a random variable. *Hence it's not exactly based on sample data. The more tests you perform, the closer the mean of all your outcome will become to the expected value.*
What is a probability mass function?	A function giving exact values for discrete random variables.
How is the expected value calculated under DRVs?	$$E(X) = \sum_{\forall x} (x \cdot P(X = x))$$ ● Multiply each value by its probability of occurring. $$E[g(X)] = \sum_{\forall x} (g(x) \cdot P(X = g(x)))$$ *This is why $E(X^2)$ or $E(X^{-1})$ works*
What is the expectation of… aX ± b?	E(aX + b) = aE(X) ± b
What is the expectation of… X ± Y? When is this the case?	E(X ± Y) = E(X) ± E(Y) This is as long as they're independent.
What is the formula for variance (σ^2)?	$$\sigma^2 = E(X^2) - (E(X))^2$$ *"The mean of the squares minus the square of the means."*
What is the variance of… aX+b?	a^2Var(X) Adding a constant doesn't affect the variability of X.
What is the variance of… X ± Y? When is this the case?	Var(X) + Var(Y) This is as long as they're independent.

What is the equation for the expectation of a discrete uniform distribution?	$$E(T) = \frac{n+1}{2}$$
What is the equation for the variance of a discrete uniform distribution?	$$Var(T) = \frac{n^2-1}{12}$$
Prove the equation for the expectation of discrete uniform distribution	$$\begin{aligned} E(T) &= \left(1 \cdot \frac{1}{n}\right) + \left(2 \cdot \frac{1}{n}\right) + \cdots + \left(n \cdot \frac{1}{n}\right) \\ &= \frac{1}{n}(1 + 2 + \cdots + n) \\ &= \frac{1}{n}\sum_{k=1}^{n} k = \frac{1}{n} \cdot \frac{n(n+1)}{2} \\ &= \frac{n+1}{2} \end{aligned}$$
Prove the equation for the variance of discrete uniform distribution	$$\begin{aligned} Var(T) &= E(T^2) - \left(E(T)\right)^2 \\ &= \frac{1}{n} \cdot \sum_{k=1}^{n} k^2 - \left(\frac{1}{n} \cdot \sum_{k=1}^{n} k\right)^2 \\ &= \frac{1}{n} \cdot \frac{n(n+1)(2n+1)}{6} - \left(\frac{1}{2} \cdot \frac{n(n+1)}{2}\right)^2 \\ &= \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 \\ &= (n+1)\left[\frac{2n+1}{6} - \frac{n+1}{4}\right] \\ &= (n+1)\left[\frac{n-1}{12}\right] = \frac{n^2-1}{12} \end{aligned}$$