D - Sequences and series

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<td>What are the 3 types of sequences?</td>
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Increasing - $u_{n+1} > u_n$
Decreasing - $u_{n+1} < u_n$
Periodic - $u_{n+a} = u_n$ (usually has some trig function)

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<td>What are the all arithmetic sequences formulae?</td>
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<tr><td>$u_n = a + (n-1)d$</td><td>nth term of the sequence</td></tr>
<tr><td>$S_n = \frac{n}{2}(a + l)$</td><td>Sum of first n terms using first and last term</td></tr>
<tr><td>$S_n = \frac{n}{2}(2a + (n-1)d)$</td><td>Sum of first n terms using first term and common difference</td></tr>
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Where a = first term, l = last term, d = difference.
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<td>What are the all geometric sequences formulae?</td>
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<tr><td>$u_n = ar^{n-1}$</td><td>nth term of the sequence</td></tr>
<tr><td>$S_n = \frac{a(1-r^n)}{1-r}\left(= \frac{a(r^n-1)}{r-1}\right)$</td><td>Sum of first n terms</td></tr>
<tr><td>$S_\infty = \frac{a}{1-r}, \lvert r \rvert < 1$</td><td>Sum to infinity</td></tr>
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Where a = first term, r = common ratio.
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<td>Limit of a sequence</td>
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As $n \to \infty$, $u_{n+1} = u_n = L$
This substitution can be used to find the limit.

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<td>Derivation of geometric sequence formula</td>
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$$S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} \quad [1]$$
$$\therefore S_n \times r = (a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1}) \times r$$
$$S_n \times r = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n \quad [2]$$
$$[2]-[1]:$$
$$(S_n \times r) - S_n = (ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n) - (a + ar + ar^2 + \ldots + ar^{n-1})$$
$$\therefore S_n(r-1) = ar^n - a \qquad \text{so } S_n = \frac{a(r^n-1)}{r-1} \qquad (\text{provided } r \neq 1)$$
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