# U6 - Further Mechanics

## Circular Motion

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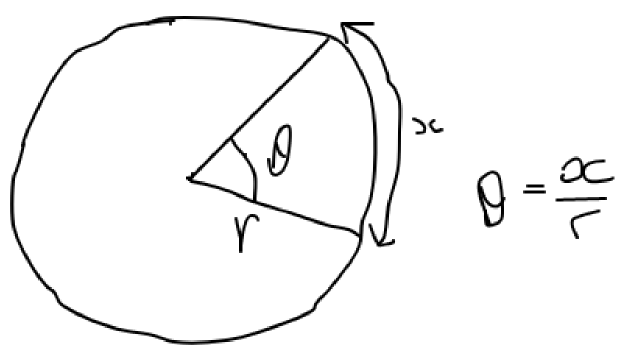
| **What is the basis of circular motion?** | A body pulled inwards is moving so fast sideways that the force pulling it inwards changes the direction of motion rather than the magnitude of velocity. |
| **What is centripetal force and what would happen without it?** | ● A resultant force (e.g., weight, friction, gravity) acting towards the centre of a circle providing **CONSTANT** acceleration.<br>● Without it, by Newton’s First Law, the object would fly off tangentially.<br><br>*Since it’s not a force itself, it should never be drawn on a free body diagram.* |
| **What is non-uniform circular motion?** | Having tangential acceleration as well as perpendicular acceleration. The net force is a combination of the centripetal force and the $F_{net}$ = ma of the tangential acceleration.<br><br>*This can be found when gravity is pulling a bucket down when spinning vertically AND when something speeds up when it begins spinning. With this, you have angular acceleration (the rate of change of angular speed).* |
| **How is a radian calculated, what is it defined as, and how can it be converted to degrees?** | ● Dividing the arc length by the radius.<br><br>*Hence, 360°* $=\frac{2\pi r}{r} = 2\pi$. |

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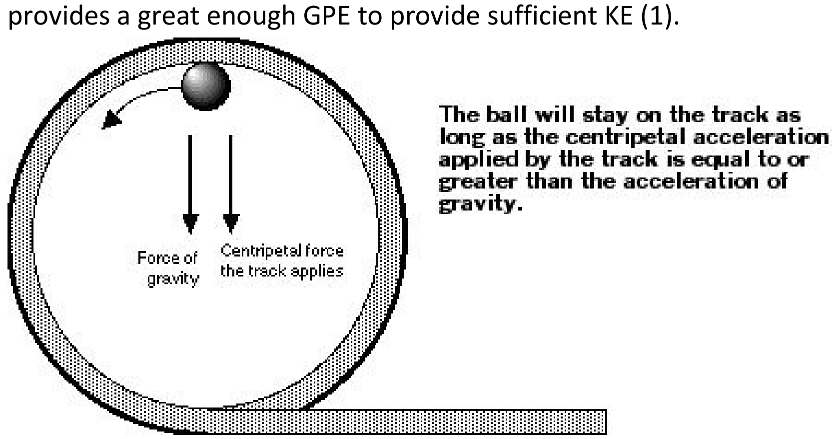
| | ● Thus, one radian (when x = r) is defined as "the angle subtended at the centre of the circle by an arc equal in length to the radius".<br>● Since 360° = $2\pi^c$, $1^c = \frac{360}{2\pi}$. To convert from radians to degrees, multiply by this. |
| **What is angular speed (ω)?** | The angle (θ, measured in radians) an objects moves through divided by the time taken to move through that angle.<br>$$\omega = \frac{\theta}{t} = \frac{v}{r} = 2\pi f$$<br>*To get from the first to the second, you would multiply both sides by the r to get speed then divide both sides by r. To get from the first to the last, you would replace ϑ with 2π and replace t (now the time period) with 1 / f.* |
| **What is centripetal acceleration equal to?** | $$a = \frac{v^2}{r} = \omega^2 r$$ |
| **What is necessary turning and how do banked tracks help?** | ● The centripetal force (provided by friction) to be sufficiently great. The frictional force on an icy surface is low ∴ you have to turn at a low speed.<br>● Banked tracks provide both friction and a reaction force ∴ turns can be made at higher speeds. |
| **How can a roller coaster go upside without falling off?** | It's moving fast enough (1) such that the **centripetal force is greater than or equal to its weight** (so $F_N = F_{CP} - mg$) (1) as the starting height provides a great enough GPE to provide sufficient KE (1).<br><br> |

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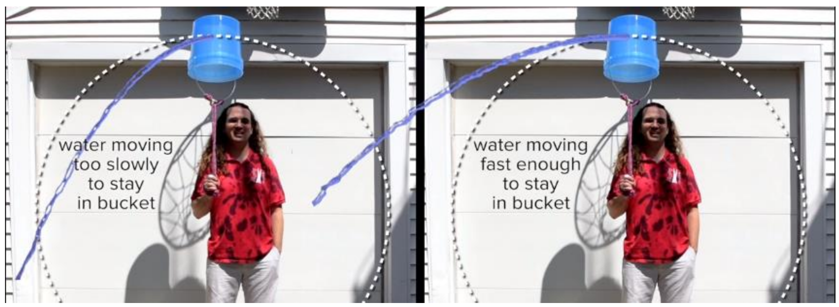
| | *Travelling at a sufficiently high speed, will increases the reaction force applied from the track.*<br><br>*As the centripetal force is sufficiently large at the top (the point of falling), it rotates in a circle rather than failing directly done.* |
| **What keeps water in a spinning bucket and how can the minimum velocity for which it can stay inside be found?** | ● The inertia of the water (its tendency to main its current state of motion).<br>● Water's speed = bucket's speed ∴ if the bucket is moving fast enough, the water will stay inside because its arc of projectile motion has a greater curvature than the circle in which its spinning.<br> <br>*The water wants to keep moving in a straight line yet the bucket keeps getting in the way.*<br><br>● The minimum angular velocity required for it to stay in the bucket is when the weight of the water is the only force providing the centripetal force downwards…<br>$mg = mr\omega^2$<br>$\omega = \sqrt{\frac{g}{r}}$ |
| **Define 'centrifugal force' with 2 examples** | ● An apparent 'force' due to the inertia of a body.<br>● Holding on to the railings on a merry-go-round, you'll feel pushing outwards (centre-feeling force).<br>● Water droplets flying off when a propeller spins too fast.<br>● In both cases, its the inertia of the body trying to continue in a straight line. For the former, it is pulled back inwards. For the latter, the friction isn't great enough (so it cannot provide a sufficiently great centripetal force). |

# Simple Harmonic Motion and Resonance

<table>
<tr><td>What is SHM and what is required of an object to undergo it?</td><td>

- A type of oscillation.
- Its acceleration must be ∝ to its displacement from equilibrium position.
- Its acceleration must be directed towards equilibrium position.

So...<br>
$a = -\omega^2 x$ from $a \propto -x$

*Thus, the same is expected of the restoring forces.*
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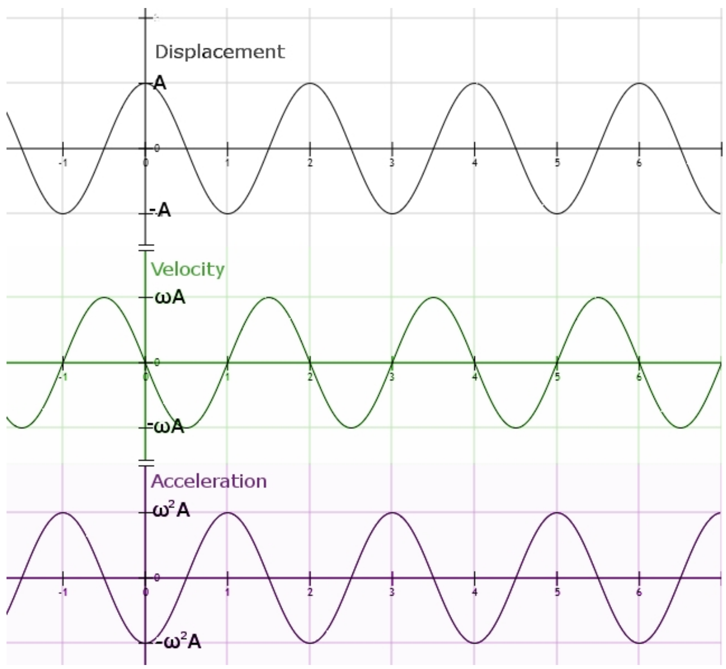
<tr><td>What do the displacement, velocity, and acceleration graphs for SHM look like?</td><td>




*Note that all these graphs have different scales*
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<tr><td>Draw a graph of the energies under SHM</td><td>

- The total energy is supplied by the initial displacement.
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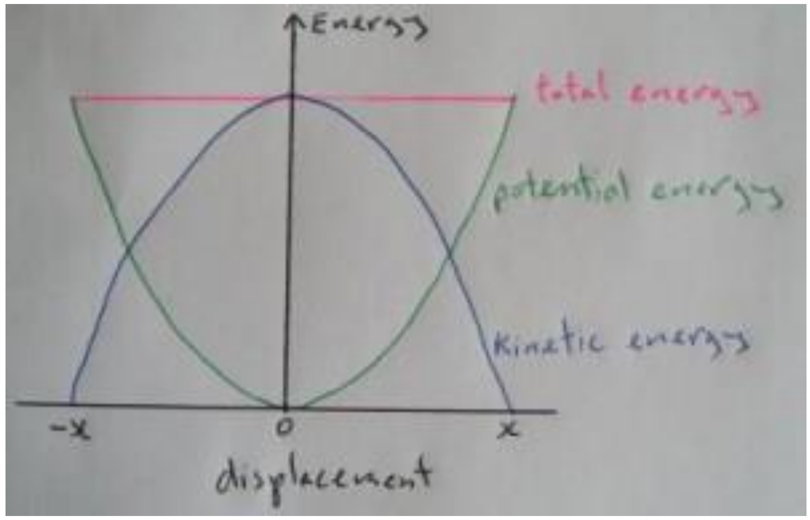
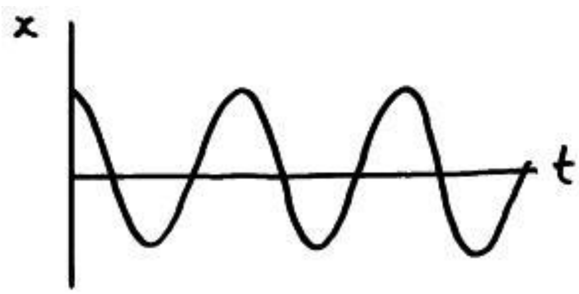
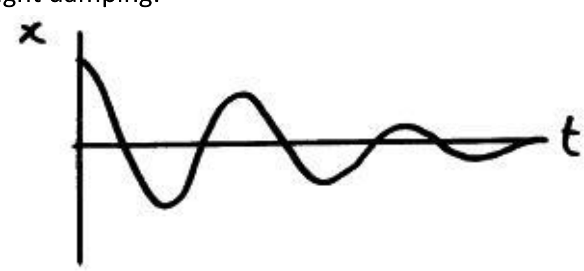
| | <br>*The potential energy is the same for a horizontally placed mass spring system.* |
| **What are free oscillators?** | Oscillators with **NO** periodic driving force acting on it:<br> |
| **What are force oscillators and what is their frequency called?** | ● Oscillators with a periodic driving force acting on it.<br>● Its frequency is the frequency of the driving force.<br><br>*Newton's Cradle as a whole is a free oscillator but the bobs themselves are forced oscillators.* |
| **What is required of the damping force?** | Its magnitude to be directly proportional to the frequency of the oscillator. |
| **What are the 3 types of damping with their associated graphs?** | ● Light damping:<br><br>*Reduces by the same fraction each cycle, more of less.* |

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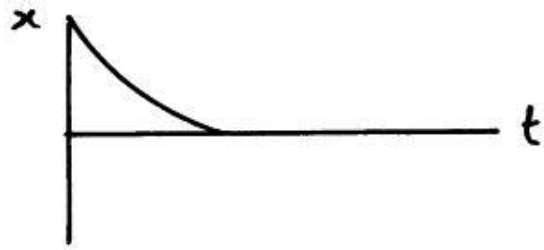
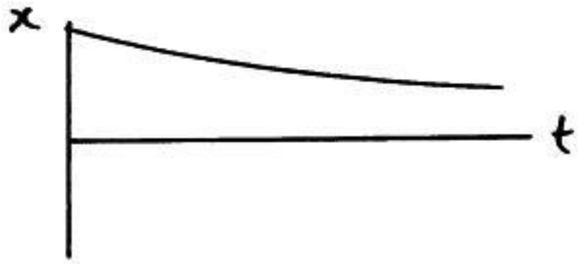
| | ● Critical damping:<br><br><br>*Used more often as it's more comfortable (eg vehicle suspension systems).*<br>● Overdamping:<br><br><br>*The damping force is so strong that the displaced object will return to equilibrium much more slowly (imagine it going in the opposite direction too.)* |
| **What is natural frequency and driving frequency?** | ● Natural frequency is the frequency at which the molecules of an object vibrate at naturally.<br>● Driving frequency is the frequency of the periodic force. |
| **What happens when the natural frequency equals and doesn't equal the natural frequency?** | When it equals, resonance occurs. |

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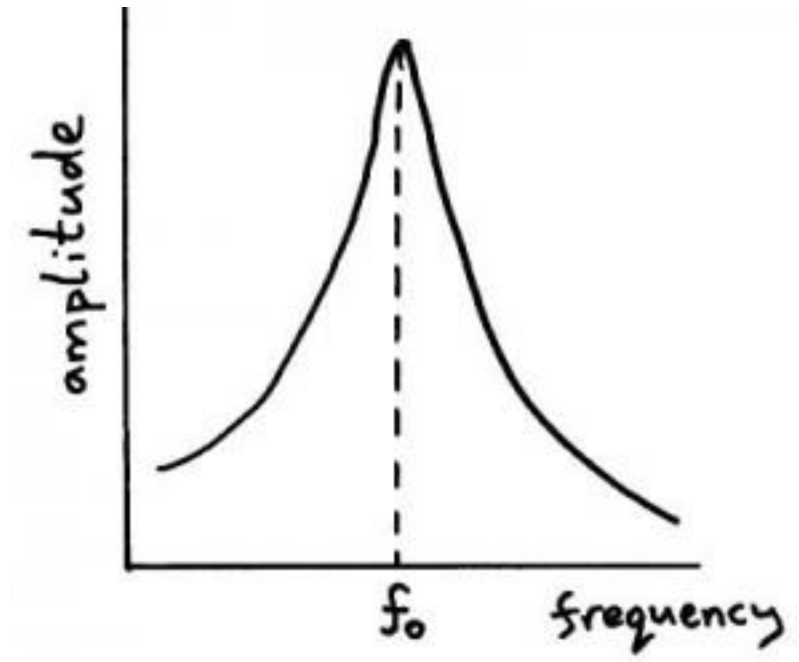
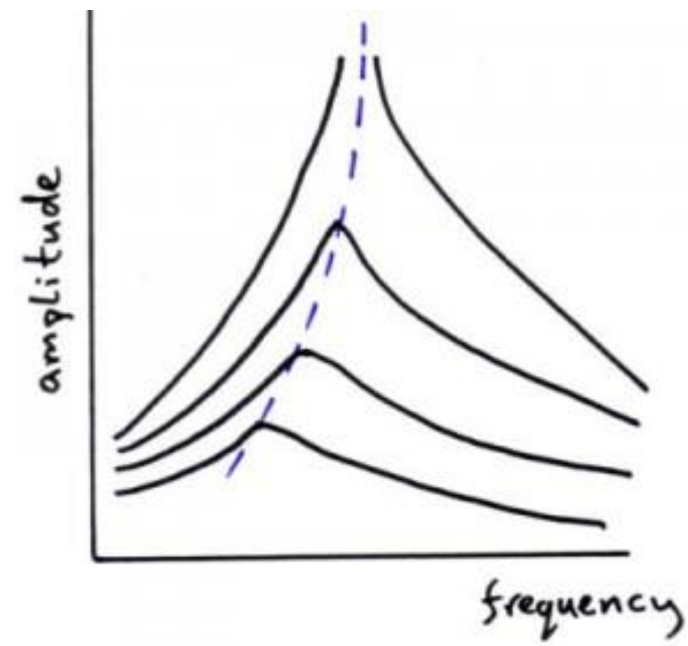
| | <br>*The driving frequency of the oscillator is plotted on the x-axis.* |
| **What is the effect of damping on resonance?** | <br>The lighter the damping, the closer to the resonant frequency is to the natural frequency of the object and the greater the amplitude. |

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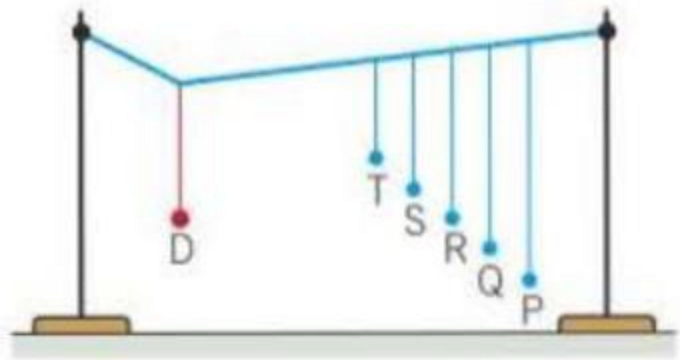
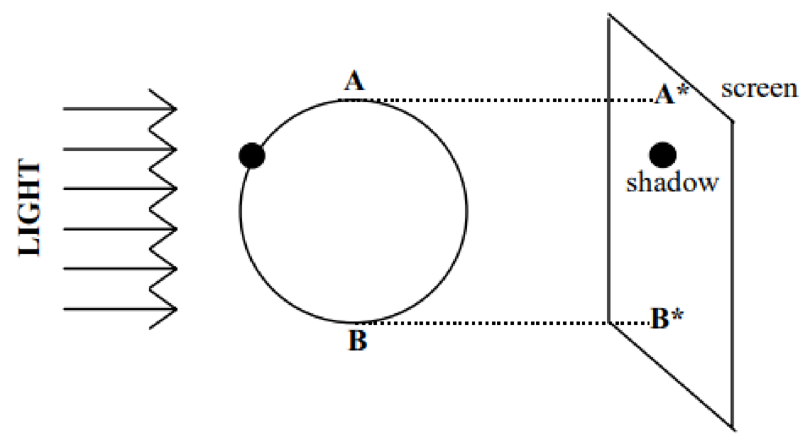
| **What are Barton's pendulums?** | <br>A set up where D is a forced oscillator to which R responds the most by having amplitude. |
| **Why is SHM only a good approximation for a pendulum at small angles?** | As the displacement (straight line path) is close enough to the curved path (until about 10°). |
| **What is the link between SHM and circular motion?** |  |