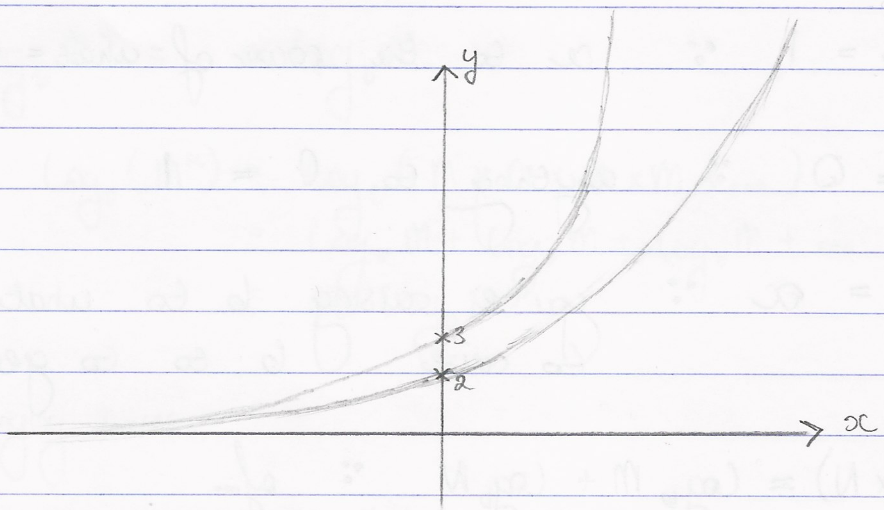
Exponents & Logarithms

Euler's number



Compound interest (100%):

Starting value (x)	End value (y)	Times a year (n)
$100	$200	1
$100	$225	2
$100	$244.14	4
"	$256.58	8
"	$263.74	16
"	$267.70	32
... "	...	...
"	$271.82	32768
... "	...	...
"	$y \rightarrow e \times 100$	$n \rightarrow \infty$

$$\Rightarrow e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$ ← instaneous rate of increase

Its derative:

If you plot $y = k^x$ & $y = \frac{d}{dx} k^x$, you will see an overlap of the graphs at $k = e$.

$$\Rightarrow \frac{d}{dx} e^{ax} = ae^{ax}$$

∴ the rate of change is a times greater

Logarithms

$\log_a a = 1$ ∵ a to the power of what = a

$\log_b 1 = 0$ ∵ anything to 0 = 1

$b^{\log_b x} = x$ ∵ you're raising b to what you have to raise b to to get x

$\log_b (M \times N) = \log_b M + \log_b N$ ∵ of...

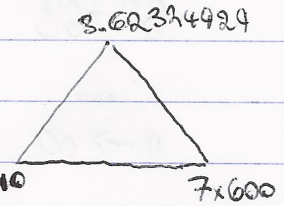
$$b^{\log_b M + N} = b^{\log_b M} \times b^{\log_b N} = M \times N$$

M+N

splitting x^{a+b} into $x^a x^b$

It splits multiply 2 no.'s into the addition of 2 no.'s:

$$\begin{aligned} \log(7 \times 600) &= \log 7 + \log 600 \\ &= 0.84509804 + 2.77815125 \\ &= 3.62324929 \end{aligned}$$

⇒ 

$\Rightarrow 10^{3.62324929} = \underline{4200}$

$\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$

$\therefore b^{\log_b\left(\frac{M}{N}\right)} = b^{\log_b M} - b^{\log_b N} = M - N$

M/N

$$\log_b m^x = x \log_b m \quad \because$$

$$\log_b(m^x) = \log_b(m \times m \times m \times m \times \ldots)$$
$$= \log_b m + \log_b m + \log_b m + \ldots$$
$$= x \log_b m$$

Changing bases...

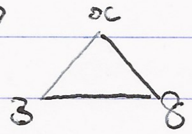
$$x = \log_3 8 \Rightarrow 3^x = 8$$

$$\log_{10} 3^x = \log_{10} 8$$

$$x \log_{10} 3 = \log_{10} 8$$

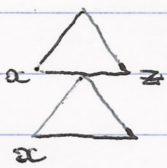
$$\Rightarrow x = \frac{\log_{10} 8}{\log_{10} 3} = 1.892789261\ldots$$

Triangle of Power

$3^x = 8 \Rightarrow$ 

& can solve for x by $\log_3 8$

hide what you want & you can see the eq. for it.

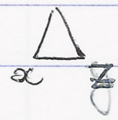


$= x^{\log_x z} = z$

$\because$ x to the x to the ~~what~~ gives z

OR $x^y =$ 

& $\sqrt[y]{z} =$ 

& $\log_x z =$ 

Alternate proof of division

Let $x^a = A \Rightarrow a = \log_x A$

$x^b = B \Rightarrow b = \log_x B$

& let $x^c = A \div B = x^a \div x^b = x^{a-b}$

$= x^{\log_x A - \log_x B}$

& $\therefore c = \log_x (A \div B)$ & $c = \log_x A - \log_x B$ (same base)

$\Rightarrow \log_x (A \div B) = \log_x A - \log_x B$