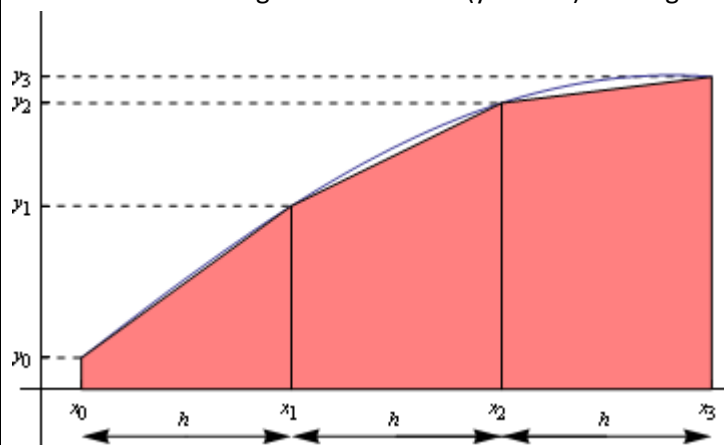


CJ - Numerical methods

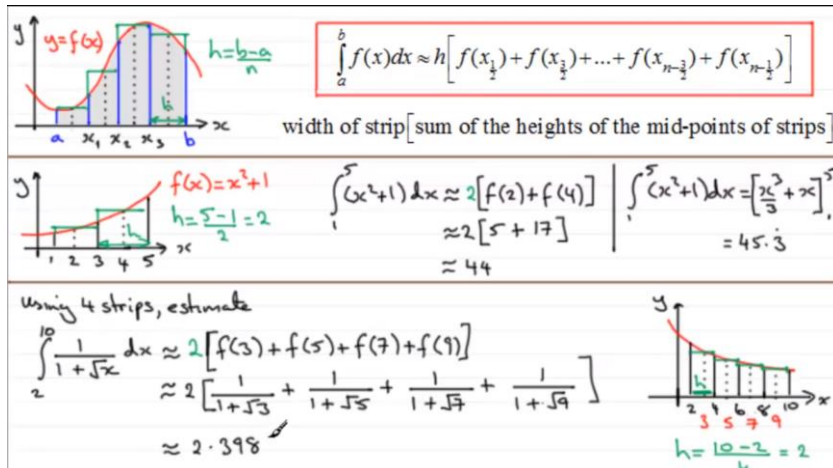
All of this is in the formula book.

What is an ordinate? (with example)

- A y-value.
- The following has 4 ordinates (y-values) starting from y_0 to y_3 .



How does the mid-ordinate rule work?



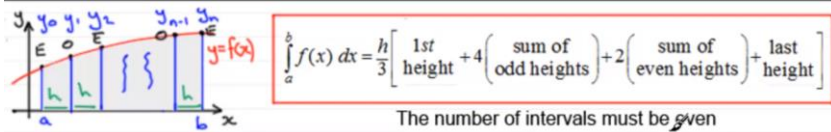
This comes from the formula:

$$\int_a^b y dx \approx h \left(y_1 + y_3 + \dots + y_{n-3} + y_{n-1} \right), \text{ where } h = \frac{b-a}{n}$$

How is Simpson's Rule better than the Midordinate rule?

By minimising the differences at the top and bottom of each rectangle by replacing the mid ordinates with a quadratic curve.

How does Simpson's rule work?



$$\int_0^2 \sqrt{1+x^2} dx \approx \frac{3}{3} \left[1 + 4(\sqrt{10} + \sqrt{82}) + 2(\sqrt{37}) + \sqrt{145} \right] \approx 74.077...$$

Estimate $\int_0^{\pi} \frac{1}{x + \cos x} dx$ using 2 intervals

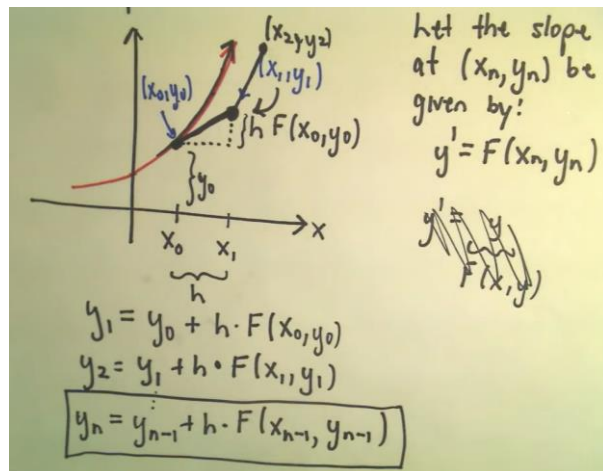
$$\int_0^{\pi} \frac{1}{x + \cos x} dx \approx \frac{\pi/2}{3} \left[\frac{1}{0 + \cos(0)} + 4 \left(\frac{1}{\frac{\pi}{2} + \cos(\frac{\pi}{2})} \right) + \frac{1}{\pi + \cos(\pi)} \right] \approx 2.101...$$

If the area required is divided into n strips of equal thickness, where n is even, **Key point**
then $\int_{x_0}^{x_n} f(x) dx \approx \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + y_5 + \dots + y_{n-1}) + 2(y_2 + y_4 + y_6 + \dots + y_{n-2})]$

This means you require 5 ordinates.

When is Euler's Method usually used? How does it work geometrically? What should you note?

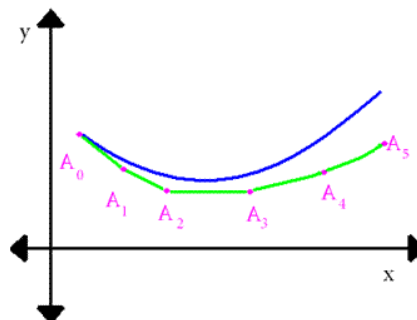
- To solve nonlinear differential equations (eg, $dy/dx = x^2 + y^2$).



- As shown, reducing step size increases accuracy.

When will Euler's Method be an underestimate or an overestimate?

Underestimate when convex:



Underestimate when concave:

