A - Proof

How can you prove a number is prime by exhaustion?	Prove 97 is a prime. 97 / 2 = 48.5, 97 / 3 = 32.333, 97 / 5 = 19.4, 97 / 7 = 13.86. We don't need to go beyond 7 since 7 < sqrt(97) < 10. If there is a factor above 10, there must be one below 10.
Conjecture	The claim you're testing.
Consequence and equivalence	● Consequence: A ⇒ B means if A is true then B is also true. ● Equivalence: A ⇔ B means A implies B and B implies A. ● Eg, $x = -1 \Rightarrow x^3 = x$ yet not ⇔ since $x^3 = x$ can also have x = 0, 1.
Prove that sqrt(2) is irrational	Suppose $\sqrt{2}$ were rational $\rightarrow \sqrt{2} = \frac{n}{m}$, reduced $\rightarrow \left(\frac{n}{m}\right)^2 = 2 \quad \rightarrow n^2 = 2m^2$ $\rightarrow n^2$ is even $\quad \rightarrow n$ is even $\rightarrow n^2$ is divisible by 4 $\rightarrow m^2$ is even $\quad \rightarrow m$ is even $\rightarrow \frac{n}{m}$ is not reduced $\rightarrow \sqrt{2}$ is not rational
Proving infinitely many primes	Suppose there are only finitely many primes, let's say n of them. We denote them by $p_1, p_2, \cdots, p_n$. Now construct a new number $$p = p_1 \times p_2 \times p_3 \times \cdots \times p_n + 1.$$ Clearly, p is larger than any of the primes, so it doesn't equal one of them. Since $p_1, p_2, \cdots, p_n$ constitute *all* primes p can't be prime. Thus it must be divisible by at least one of our finitely many primes, say p_m (with $1 \leq m \leq n$). But when we divide p by p_m we get a remainder 1. That's a contradiction, so our original assumption that there are finitely many primes must be false. Thus there are infinitely many primes.