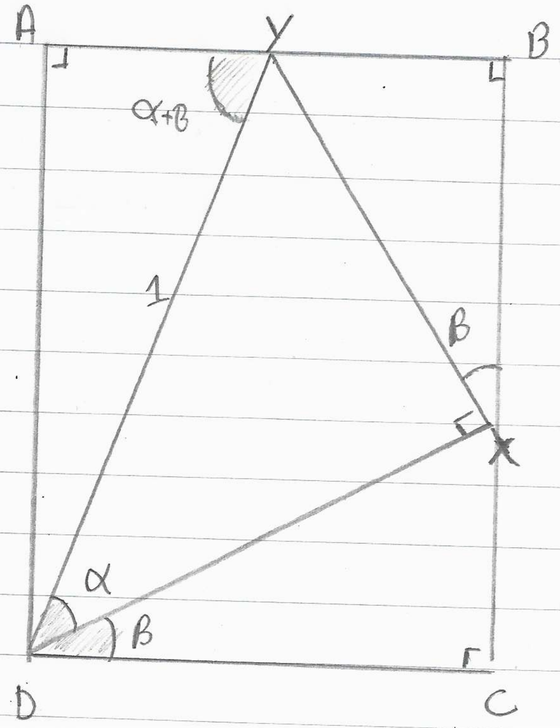
Single Maths HWK



$$\sin\alpha = \frac{O}{H} = \frac{XY}{DY} = XY$$

$$\sin\beta = \frac{O}{H} = \frac{CX}{DX} = \frac{BY}{XY}$$

$$\cos\alpha = \frac{A}{H} = \frac{DX}{DY} = DX$$

$$\cos\beta = \frac{A}{H} = \frac{CD}{DX} = \frac{BX}{XY}$$ similar triangle

$$\sin(\alpha+\beta) = \frac{O}{H} = AD = CX + BX$$

$$\sin\beta\cos\alpha = \frac{CX}{DX} \times DX = CX$$

$\frac{XY}{\cancel{BX}} = \cancel{DX}$ by ~~similar~~ congruent triangles

So $\sin\alpha\cos\beta = \frac{\cancel{XY}}{\cancel{BX}} \cancel{\times}$ ~~CAD~~ $XY \times \frac{BX}{XY} = BX$

So $\sin(\alpha+\beta) = BX + CX$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta.$

& $\cos(\alpha+\beta) = \frac{AY}{DY} = AY = CD - BY$

$\cos\alpha\cos\beta = \frac{CD}{\cancel{DX}} \times \cancel{DX} = CD$ & $\sin\alpha\sin\beta = \cancel{XY} \times \frac{BY}{\cancel{XY}}$

So $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

A2 Trigonometry

Compound Angles Extended.

- $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$

$\cos(\alpha - \beta) = \cos(\alpha)\cos(-\beta) - \sin(\alpha)\sin(-\beta)$

$= \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$

- $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$

$\sin(\alpha - \beta) = \sin(\alpha)\cos(-\beta) + \cos(\alpha)\sin(-\beta)$

$= \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

- Draw a triangle & say when an angle is acute.

Writing sine & cosine in terms of sine or cosine

$\sqrt{3}\sin\theta - \cos\theta$ can be written as $R\sin(\theta - \alpha)$

$\Rightarrow R\sin(\theta - \alpha) = R[\sin\theta\cos\alpha - \cos\theta\sin\alpha]$

$= R\sin\theta\cos\alpha - R\cos\theta\sin\alpha$

$\therefore R\sin\theta\cos\alpha = \sqrt{3}\sin\theta \Rightarrow R\cos\alpha = \sqrt{3}$

$-R\cos\theta\sin\alpha = -\cos\theta \Rightarrow +R\sin\alpha = 1$

$\frac{+R\sin\alpha}{R\cos\alpha} = \frac{1}{\sqrt{3}} \Rightarrow \tan\alpha = \frac{1}{\sqrt{3}}, \quad \alpha = \pi/6$

$R = 1/\sin\alpha = 1/\sin(\frac{\pi}{6}) = 2$

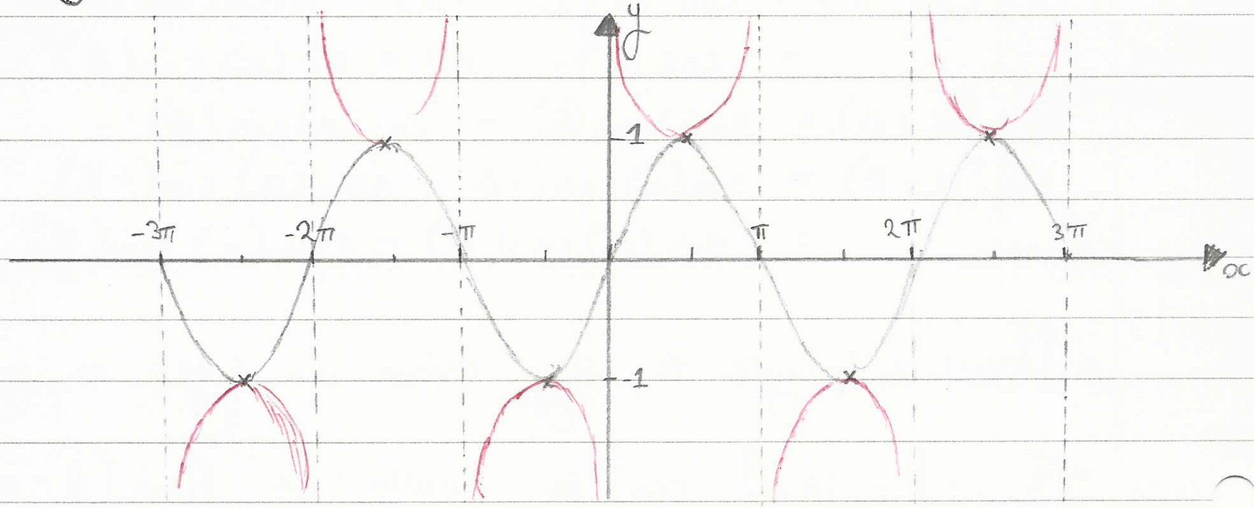
$\therefore \sqrt{3}\sin\theta - \cos\theta = 2\sin(\theta - \frac{\pi}{6})$

You can equate to:
$R\sin(\theta + \alpha)$
$R\sin(\theta - \alpha)$
$R\cos(\theta + \alpha)$
$R\cos(\theta - \alpha)$

(ignore those which give you α negative $\because$ it makes it difficult to equate)

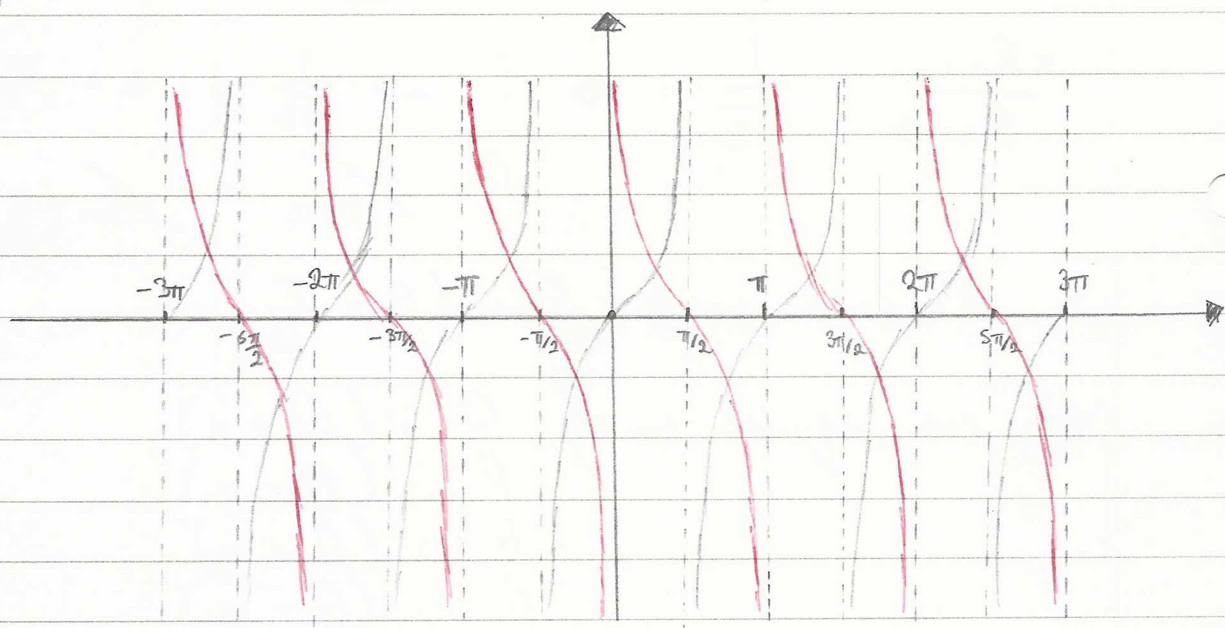
Reciprocal trigonometric functions

$\therefore \ y = \operatorname{cosec} x = 1/\sin x$



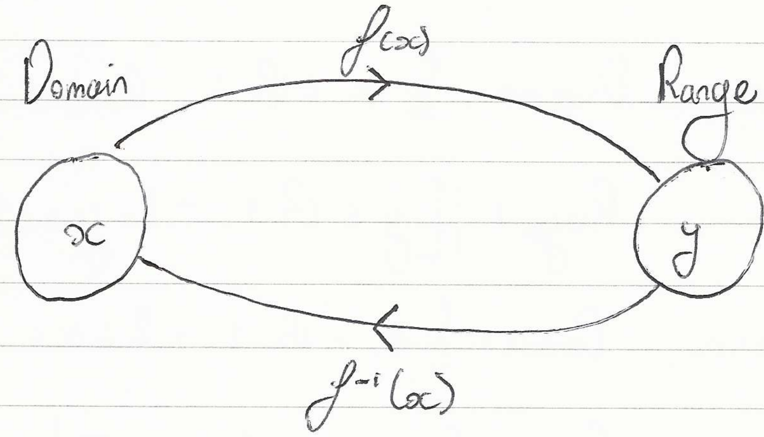
since $y = \cos(90° - x) = \sin x \Rightarrow y = \cos x = \sin(90° - x)$
$\therefore$ flip $y = \sin x$ in y-axis & move $90°$ right for both this & the inverse

$y = \cot x = 1/\tan x$



Inverse functions & inverse trigonometric functions

Values x can take



Set of values the domain maps to

You can have a many-to-one mapping e.g. $f(x) = x^2 + 4$

$\therefore f(2) = 8$

$f(-2) = 8$ YET $f^{-1}(8) = \pm 2$

& you cannot have both

$\therefore$ you need to restrict the domain e.g. $x > 0$

SO for $y = \sin x$, use $\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$ as the domain

$\therefore$ range $\Rightarrow -1 \leq y \leq 1$

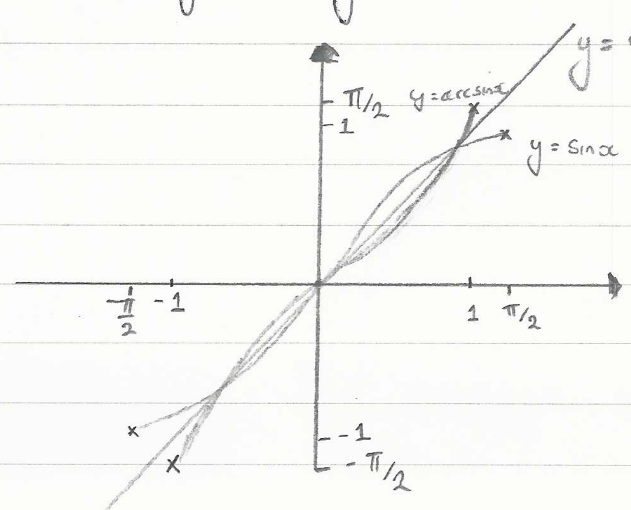
Graphs

$y = \sin x$, Domain: $\{x \in \mathbb{R} : \frac{-\pi}{2} \leq x \leq \frac{\pi}{2}\}$

Range: $\{y \in \mathbb{R} : -1 \leq y \leq 1\}$

$y = \arcsin x$, Domain: $\{x \in \mathbb{R} : -1 \leq x \leq 1\}$

Range: $\{y \in \mathbb{R} : \frac{-\pi}{2} \leq y \leq \frac{\pi}{2}\}$

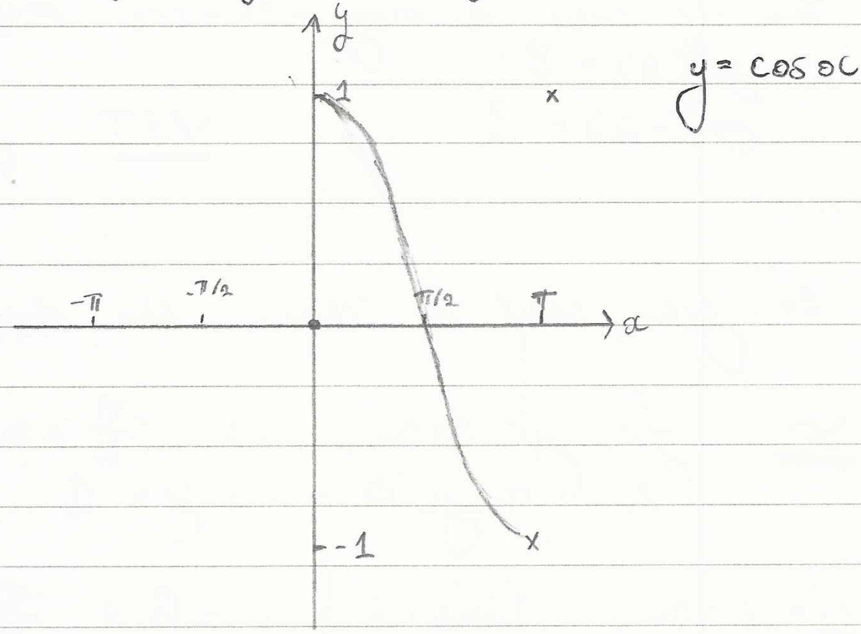


$y = \cos x$, Domain: $\{x \in \mathbb{R} : 0 \leq x \leq \pi\}$

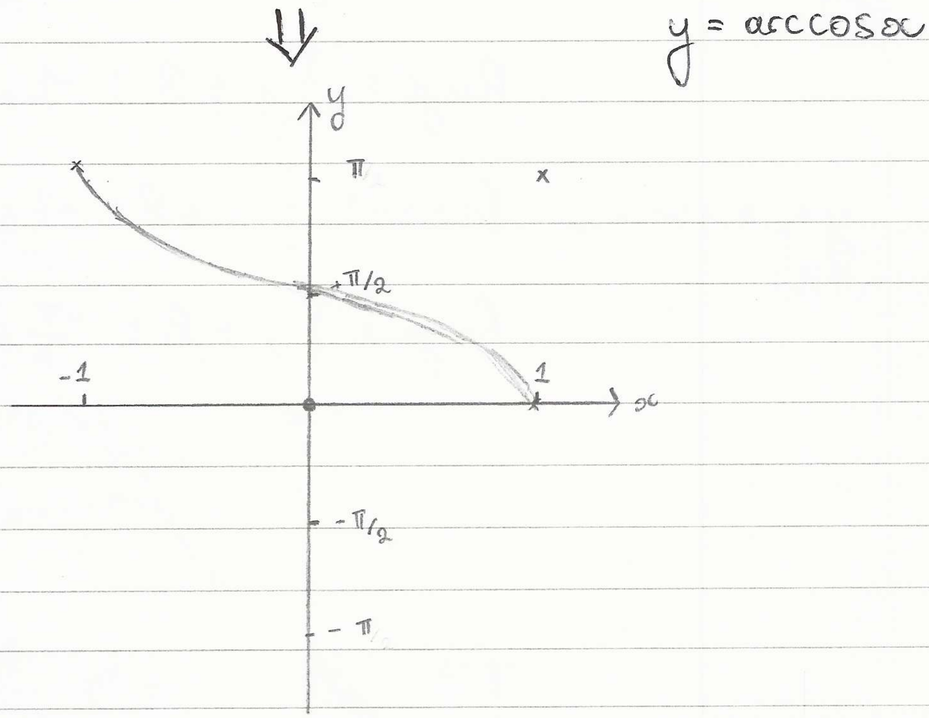
Range: $\{y \in \mathbb{R} : -1 \leq y \leq 1\}$

$y = \arccos x$, Domain: $\{x \in \mathbb{R} : -1 \leq x \leq 1\}$

Range: $\{y \in \mathbb{R} : 0 \leq y \leq \pi\}$

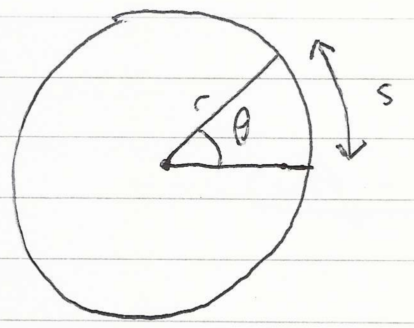


$\Downarrow$



Very similar approach for $y = \tan x$

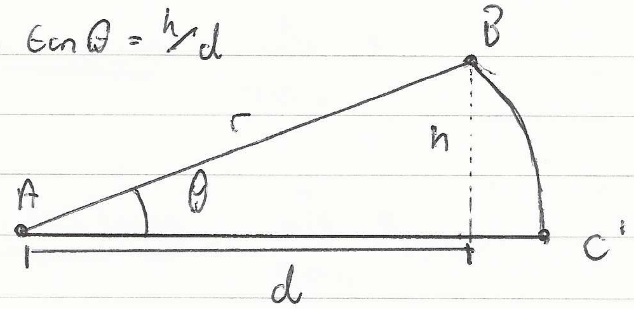
Radians & Small Angle ~~Aproxm~~ Aproximation



$$s = \frac{\theta}{2\pi} \times 2\pi r$$
$$= \theta r \quad \text{(radians)}$$

$$\text{Area} = \pi r^2 \times \frac{\theta}{2\pi}$$
$$= \frac{\theta r^2}{2} \quad \text{(radians}$$

$\sin\theta = h/r$, $\cos\theta = d/r$, $\tan\theta = h/d$



$\Rightarrow h = \sin\theta r$, $d = r\cos\theta$
$h = d\tan\theta$

$\therefore \quad h = \sin\theta r = d\tan\theta$

as $\theta \to 0$, $h \to r\theta$ & $d \to r$

$\therefore \quad h \approx \cancel{r}\theta \approx \cancel{r}\sin\theta \approx \cancel{r}\tan\theta$
$\Rightarrow \quad \theta \approx \sin\theta \approx \tan\theta$ } Small angle for sin, tan

$$\cos 2\alpha = 1 - 2\sin^2\alpha$$
$$\cos\theta = 1 - 2\sin^2\theta/2$$
$$= 1 - 2\left(\theta/2\right)^2$$
$$= 1 - \frac{\theta^2}{2}$$

Differentiating trig from first principles

~~glves)~~ knows that $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = \lim_{\theta\to 0} \frac{\theta}{\theta} = \lim_{\theta \to 0} 1 = 1$

& $\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = \lim_{\theta\to 0} \frac{1/2\,\theta^2}{\theta} = \lim_{\theta \to 0} \frac{1}{2}\theta = 0$

$$f(x) = \cos x$$

$$\Rightarrow f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cos x \cos h - \sin x \sin h - \cos x}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1) - \sin x \sin h}{h}$$

$$= \lim_{h \to 0} \frac{\cos x(\cos h - 1)}{h} - \lim_{h \to 0} \frac{\sin x \sin h}{h}$$

$$= \cancel{-\cos x \lim_{h \to 0} \frac{1 - \cos h}{h} - \lim_{h \to 0}}$$

$$= -\cos x \lim_{h \to 0} \frac{1 - \cos h}{h} - \sin x \lim_{h \to 0} \frac{\sin h}{h}$$

$$= -\cos x(0) - \sin x(1)$$

$$= -\sin x$$