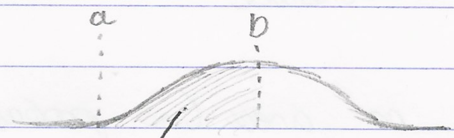
Continuous Random Variables

Fundamentals

- They can take on an infinite no. of possible values between an interval.
- Modelled by a curve, $f(x)$, called the probability density function (p.d.f).
- The probability is the area under the curve.



$$P(a < X < b) = \int_a^b f(x)\,dx$$

$(P(X = a) = 0 \because$ it is an infinitely small area$)$

- Rules:

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad , \quad f(x) \underset{\forall x}{\geq} 0 \because \text{probability}$$

Cumulative distribution function (c.d.f)

- Has the symbol $F(x)$ where $F(a) = P(X \leq a)$ $= \int_{-\infty}^{a} f(x)\,dx \Rightarrow$.

$$\underset{f(x)}{\text{p.d.f}} \underset{\text{differentiate}}{\overset{\text{integrate}}{\rightleftarrows}} \underset{F(x)}{\text{c.d.f}}$$

- Useful for medians, $F(x) = 0.5$, & quartiles, $F(x) = 0.25$.

Example

$$f(x) = \begin{cases} 1/4, & 0 \le x < 1 \\ x^3/5, & 1 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow F(x) = \begin{cases} \frac{1}{4}x + c_1 & 0 \le x < 1 \\ x^4/20 + c_2 & 1 \le x \le 2 \\ 1 & \text{otherwise} \end{cases}$$

REMEMBER cumulative!

when $F(0) = 0$, you have no area (no probability so $f(x)$)

$\Rightarrow \quad \frac{1}{4}(0) + c_1 \quad \Rightarrow \quad c_1 = 0$

when $x = 1$, $\quad \frac{1}{4}x = x^4/20 + c_2 \Rightarrow \frac{1}{4}x + \frac{1}{20} + c_2$

$\Rightarrow \quad c_2 = 1/5$

∴ remember, it is cumulative

OR

$$\frac{x^4}{20} + c_2 = 1 \text{ when } x = 2 \Rightarrow c_1 = 4/20 = 1/5$$

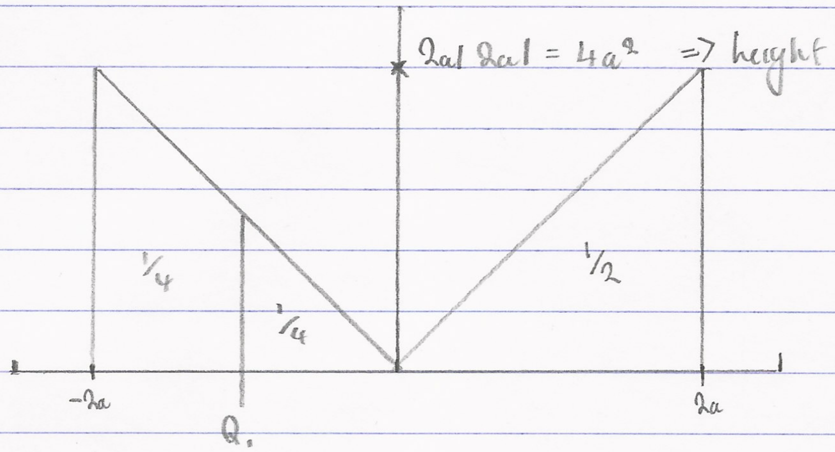
Expectation, variance, a function

- $E(X) = \mu = \int x f(x)\, dx$ — mean — each value · its probability
- $Var(X) = \sigma^2$
  $= E(X^2) - \mu^2$ — $\int x^2 f(x)\, dx - \mu^2$
- For a function $E(g(X)) \Rightarrow \int g(x) f(x)\, dx$

Example 1

Sketch the pdf: $\quad f(x) = \begin{cases} 2a\,|x| & -2a < x < 2a \\ 0 & \text{otherwise} \end{cases}$

and find the LQ



Total area $= 2a \times 4a^2 = 8a^3 = 1 \Rightarrow \underline{\underline{a = 1/2}}$

$\because y = -x$

$$\int_{Q_1}^{0} (-x)\, dx = 1/4$$

$$\left[-\frac{x^2}{2} \right]_{Q_1}^{0} = 1/4 \Rightarrow \frac{Q_1^2}{2} = 1/4$$

$$\Rightarrow Q_1 = \pm\sqrt{1/2}$$

$$\Rightarrow Q_1 = -\sqrt{2}/2 \quad \because -2a < Q_1 < 0$$