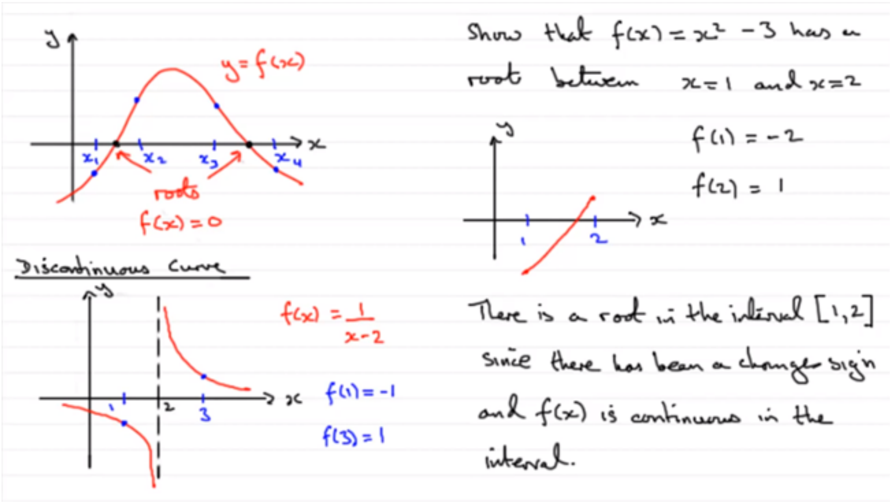
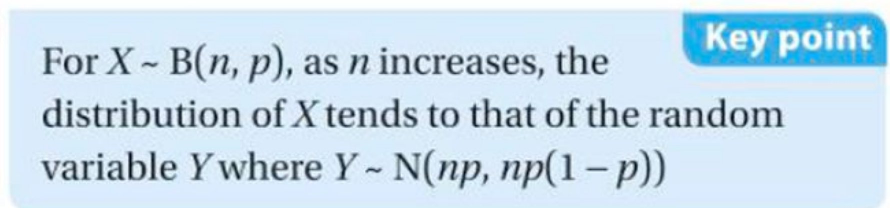
1.	Give an example of a dependent event	Picking sweets out of a bag **without replacement**. Delete
2.	Prove that sqrt(2) is irrational	Suppose $\sqrt{2}$ were rational $\rightarrow \sqrt{2} = \frac{n}{m}$, reduced $\rightarrow \left(\frac{n}{m}\right)^2 = 2 \quad \rightarrow n^2 = 2m^2$ $\rightarrow n^2$ is even $\quad \rightarrow n$ is even $\rightarrow n^2$ is divisible by 4 $\rightarrow m^2$ is even $\quad \rightarrow m$ is even $\rightarrow \frac{n}{m}$ is not reduced $\rightarrow \sqrt{2}$ is not rational Reduced is another word for saying n and m are coprime **AND** n, m =/= 0.
3.	Prove that there are infinitely many primes	Suppose there are only finitely many primes, let's say n of them. We denote them by $p_1, p_2, \cdots, p_n$. Now construct a new number $$p = p_1 \times p_2 \times p_3 \times \cdots \times p_n + 1.$$ Clearly, p is larger than any of the primes, so it doesn't equal one of them. Since $p_1, p_2, \cdots, p_n$ constitute *all* primes p can't be prime. Thus it must be divisible by at least one of our finitely many primes, say p_m (with $1 \leq m \leq n$). But when we divide p by p_m we get a remainder 1. That's a contradiction, so our original assumption that there are finitely many primes must be false. Thus there are infinitely many primes.
4.	What is the polynomial remainder theorem?	The factor theorem states that **Key point** $\mathrm{f}\left(\frac{b}{a}\right) = 0$ if and only if $(ax - b)$ is a factor of $\mathrm{f}(x)$ Since a polynomial f(x) can be written as… f(x) = (ax - b)g(x) + R

5.	How is change of sign used to find roots? And when does it not work / fail?	●Look for a change of sign between 2 points. ●Fails when… ■The interval was not narrow enough, captured an even number of roots (including repeated roots). ■The graph is discontinuous in the interval. 
6.	What does the Product Moment Correlation Coefficient (PMCC) describe? And how?	● It describes how correlated 2 variables are. ● It can take on any value between -1 and +1 inclusively where p = +1 means perfect positive correlation, p = -1 means perfect negative correlation, and p = 0 means no correlation.
7.	What is causal connection and spurious correlation?	●Casual connection - when a change in one variable affects the other. ●Spurious correlation - correlation without causal connection. *This relates to correlation doesn't imply causation.*
8.	How can the usage of the Normal Distribution as an approximate for the Binomial Distribution be refined?	1. Increasing number of events (n). 2. Having p closer 0.5 (more symmetrical).  Key point For $X \sim B(n, p)$, as n increases, the distribution of X tends to that of the random variable Y where $Y \sim N(np, np(1-p))$ The approximation can still be used if it's p = 0.2 but it won't be good.
9.	What is a parameter and a statistic? How are they used? (with example)	- A parameter is a number that describes the entire population. - A statistic is a number taken from a single sample. - You used a statistic to estimate a parameter - Example: the mean of a sample is an estimate of the population mean.

10. What does it mean when f''(x) < 0, = 0, > 0? And how do you find points of inflection?

f''(x) < 0 ⇒ concave ⇒ maximum point.
f''(x) = 0 ⇒ **MAYBE** a point of inflection **YET** point of inflection ⇒ f''(x) = 0. To make sure you have a point, check for a change in concavity either side of the point (if so, is a point of inflection) and consider points where the f''(x) is undefined.
f''(x) > 0 ⇒ convex ⇒ minimum point.

---

11. How are rectangles used to integrate?

$$\lim_{n\to\infty}\sum_{i=1}^{n} y_i\,\delta x = \int_a^b y\,dx$$

---

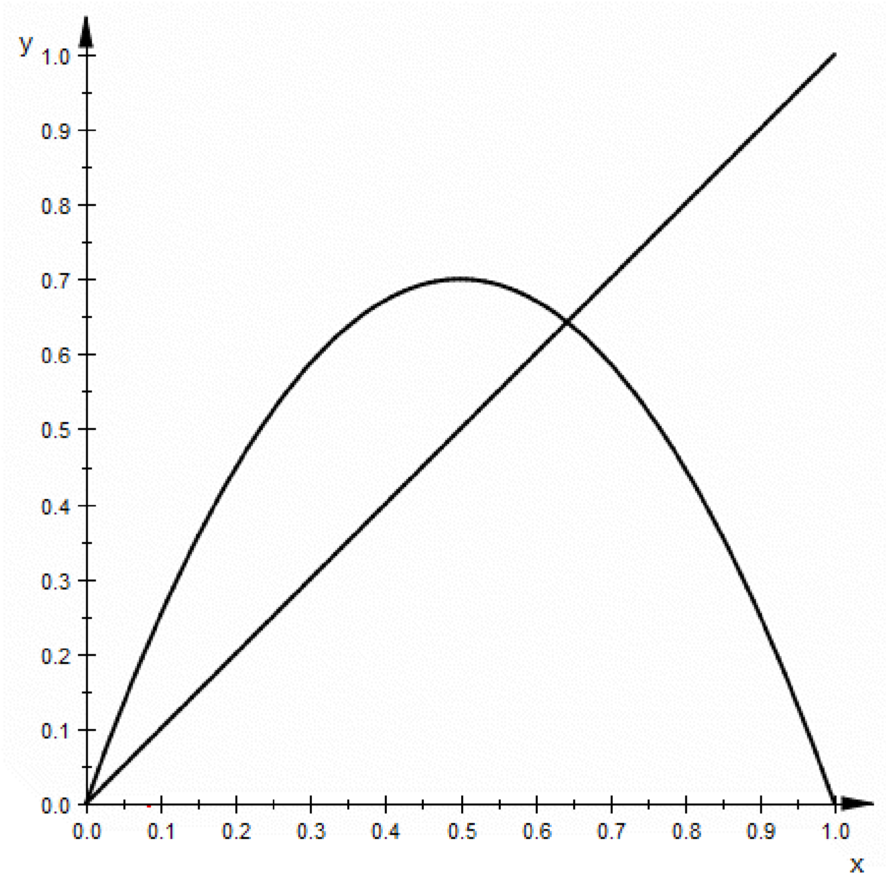
12. When does an iteration converge? How can this be drawn?

Let g(x) have a solution α

Key point

If $-1 < g'(x) < 1$ for all x in an interval which contains α and the starting value x_1, then $x_{n+1} = g(x_n)$ converges.

An animation of the cobweb is drawn below:



This works since finding the point of intersection is the same as finding roots.

13. How is a sample normally distributed? What is required for the population?	$$\text{If, } X \sim N(\mu, \sigma^2), \text{ then } \overline{X} \sim N(\mu, \frac{\sigma^2}{n})$$ This requires the population to also be normally distributed.
14. Describe quota sampling	Using opportunity sampling YET taking into account how many people of each group you want in your sample.
15. How can you find the limit of a sequence?	-As $n \to \infty$, $u_{n+1} = u_n = L$ -This substitution can be used to find the limit.
16. How does static friction relate to the force applied? What's its maximum? What happens when it's exceeded?	1. It is equal and opposite. 2. It is $F_{max} = uR$ (generally $F_s <= uR$ where for values smaller, it won't move). 3. Once moving a constant kinetic friction of $F_k = uR$ acts.
17. How should you describe correlation on a scatter graph? (in words)	- Strong, moderate, weak. - Positive, no correlation, negative.
18. What 4 conditions must be satisfied for a Binomial Distribution to be valid?	1. Two possible outcomes in each trial. 2. A fixed number of trials. 3. Independent trials. 4. Identical trails (same probability).
19. Describe simple random sampling	1. Allocate numbers to objects. 2. Use a random number generator. 3. Ignore repeats.
20. What product-based formula holds true if event A and event B are independent? Why?	$$P(A \cap B) = P(A) \cdot P(B)$$ This is because… $$P(A) = P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$ As they're independent. Finally, you can rearrange.
21. What are exhaustive events? (with example)	$$P(A \cup B) = 1$$ When rolling a six-sided die, the events 1, 2, 3, 4, 5, and 6 are collectively exhaustive, because they encompass the entire range of possible outcomes

22. What are complementary events? (with example)

Two outcomes of an event that are the **only two possible outcomes.** E.g., heads and tails of a coin.
This is a type of mutually exclusive AND exhaustive events.

---

23. What is a conjecture?

The claim you're testing.

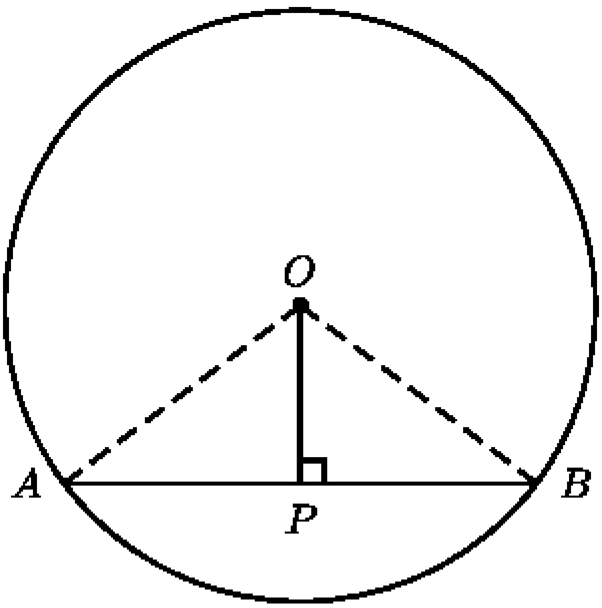
---

24. What are the derivatives of sin(kx) and cos(kx)?

f(x)	**f'(x)**
sin(kx)	kcos(kx)
cos(kx)	-ksin(kx)

---

25. How can we prove that a perpendicular line from the centre to a chord bisects the chord?



Draw OA and OB.

In $\triangle OPA$ and in $\triangle OPB$,

$$\begin{aligned} OA^2 &= OP^2 + AP^2 \quad \text{(Pythagoras)} \\ OB^2 &= OP^2 + BP^2 \quad \text{(Pythagoras)} \end{aligned}$$

and

$$\begin{aligned} OA &= OB \quad \text{(equal radii)} \\ \therefore AP^2 &= BP^2 \\ \therefore AP &= BP \end{aligned}$$

Therefore OP bisects AB.

26. How can you check if event A depends on event B?

<u>If dependent:</u>

$$P(A \mid B) \neq P(A)$$

As given B has occurred, then the probability of A changes.

<u>Otherwise, if independent then:</u>

$$P(A \mid B) = P(A)$$

As B occurring doesn't affect the probability of A occurring,

---

27. Derive the geometric sequence formula

$$S_n = a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1} \qquad [1]$$

$$\therefore S_n \times r = (a + ar + ar^2 + \ldots + ar^{n-2} + ar^{n-1}) \times r$$

$$S_n \times r = ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n \qquad [2]$$

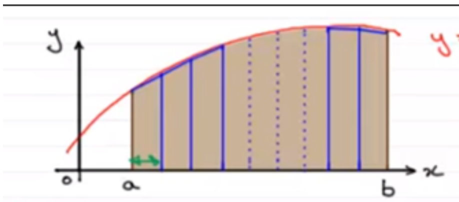
$[2]-[1]$:

$$(S_n \times r) - S_n = (ar + ar^2 + ar^3 + \ldots + ar^{n-1} + ar^n) - (a + ar + ar^2 + \ldots + ar^{n-1})$$

$$\therefore S_n(r-1) = ar^n - a \qquad \text{so } S_n = \frac{a(r^n - 1)}{r-1} \qquad (\text{provided } r \neq 1)$$

---

28. What is the trapezium rule?



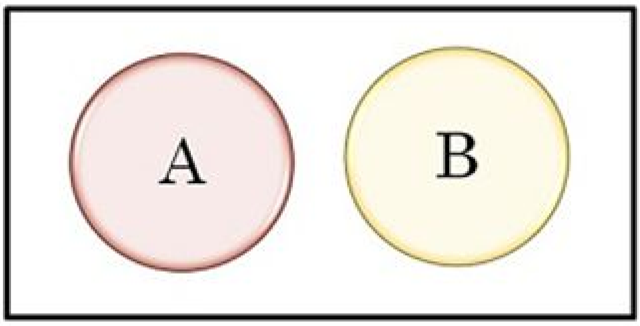
$$\int_a^b f(x)\,dx = \text{Area}$$

$$\boxed{\text{Area} = \frac{\text{width of strip}}{2}\left[1^{st}\text{ height} + 2(\text{sum of middle heights}) + \text{last height}\right]}$$

---

29. What are mutually exclusive events? (with example and formula)

Events that cannot occur at the same time. E.g.,



Or flipping a coin **<u>TWICE</u>**, you cannot have HH and TT at the same time meaning these events are also mutually exclusive. This can be given by:

$$P(A \cap B) = 0$$

30. What is the Bayes' Theorem for conditional probability?	$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$ *In other words, P (one event occurs | another occured) = P (both occurred) / P (given one occurred)* ***OR*** *the probability of one event occurring given another has already occured.*
31. Describe systematic sampling	Follow a system (e.g., selecting every k^{th} person along).
32. Describe stratified sampling	Having the groups in your sample proportional to the groups in the population.
33. Describe opportunity sampling	Asking people you have access to until you have a sample of desired size.
34. Describe cluster sampling	Splitting into clusters with similarities then sampling from each cluster.
35. How can you prove a number is prime by exhaustion?	Prove 97 is a prime. 97 / 2 = 48.5, 97 / 3 = 32.333, 97 / 5 = 19.4, 97 / 7 = 13.86. We don't need to go beyond 7 since 7 < sqrt(97) < 10. If there is a factor above 10, there must be one below 10.
36. What is consequence and equivalence?	-Consequence: $A \Rightarrow B$ means if A is true then B is also true. -Equivalence: $A \Leftrightarrow B$ means A implies B and B implies A. -Eg, $x = -1 \Rightarrow x^3 = x$ yet not $\Leftrightarrow$ since $x^3 = x$ can also have x = 0, 1.
37. What are the 3 types of sequences?	1.Increasing: $u_{n+1} > u_n$ 2. Decreasing: $u_{n+1} < u_n$ 3.Periodic: $u_{n+a} = u_n$ (usually has some trig function)
38. What are the all arithmetic sequences formulae?	see table below

$u_n = a + (n-1)d$	nth term of the sequence
$S_n = \frac{n}{2}(a+l)$	Sum of first n terms using first and last term
$S_n = \frac{n}{2}(2a+(n-1)d)$	Sum of first n terms using first term and common difference

Where a = first term, l = last term, d = difference.

39. What are the all geometric sequences formulae?

$u_n = ar^{n-1}$	nth term of the sequence
$S_n = \frac{a(1-r^n)}{1-r}\left(=\frac{a(r^n-1)}{r-1}\right)$	Sum of first n terms
$S_\infty = \frac{a}{1-r}, \|r\|<1$	Sum to infinity

Where a = first term, r = common ratio.

40. How can you express rsin(θ±α) or rcos(θ±α)?

Key point

$a\cos\theta \pm b\sin\theta$ can be written as $r\sin(\theta \pm \alpha)$ or $r\cos(\theta \pm \alpha)$, where r is positive and angle α is acute.

The algebra is easier with matching signs.

41. What is the arc length and area of segment formulae?

Key point

$$s = \frac{\theta}{2\pi} \times 2\pi r \Rightarrow s = r\theta$$

$$A = \frac{\theta}{2\pi} \times \pi r^2 \Rightarrow A = \frac{1}{2}r^2\theta$$

where θ is in radians.

42. How can you change the base of a log?

$$\log_a b = \frac{\log_k b}{\log_k a}$$

43. How are outliers **COMMONLY** found?

Outside the interval (Q_1 - 1.5 x IQR, Q_3 + 1.5 x IQR).
There may be other rules which you are told to apply.

44. What proportion lies within 1 and 3 σ's of the mean in a Normal Distribution?

± 1 σ is around 68%
± 3 σ is around 99.8%

45. What is the z-score? How is it calculated?

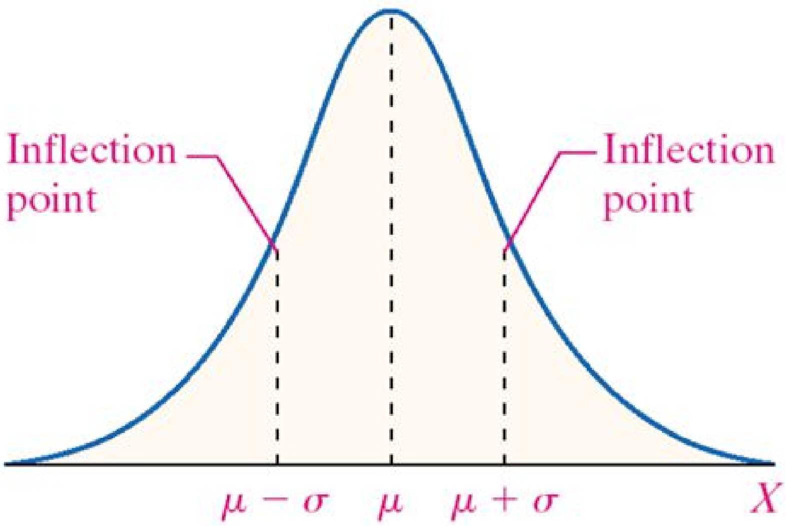
A measure of how many standard deviations a value is to the right of the mean which is calculated by:

$$z = \frac{x - \mu}{\sigma}$$

This is sometimes referred to as a test statistic in context.

---

46. Where are the points of inflection of a Normal Distribution?



---

47. What is the test statistic for a normally distributed sample that is used during a hypothesis test?

Key point

$z = \dfrac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$, $\bar{x}$ is the mean of the sample, μ_0 is the hypothesised mean of the distribution, σ^2 is the variance of the distribution and n is the sample size.

---

48. When do we divide by n or n - 1 for the variance?

- Either works.
- Divide by n - 1 when an unbiased estimator of the popular variance is required.

---

49. What is the probability of event A OR event B?

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For mutually exclusive, P(A n B) = 0.
