

## SF - Exponential Distribution

<b>What is exponential distribution used to do?</b>	Describe the waiting times between Poisson events.
<b>What is the cdf of the exponential distribution?</b>	$F(x; \lambda) = \begin{cases} 1 - e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$ <p>Where <math>F(x)</math> is the probability of an event occurring in the time <math>x</math>.</p> <p><i>Where <math>\lambda</math> is the mean no. of occurrence in a unit time.</i></p>
<b>What is the pdf of the exponential distribution?</b>	$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0, \\ 0 & x < 0. \end{cases}$ <p><i>This is by differentiating <math>1 - e^{-\lambda t}</math>.</i></p>

How can the mean and variance for exponential distribution be derived?

$$\begin{aligned}
 E(X) &= \int_0^{\infty} x f(x) dx = \lambda \int_0^{\infty} x e^{-\lambda x} dx \\
 &= \lambda \left\{ \left[ x \frac{e^{-\lambda x}}{-\lambda} \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\lambda x}}{-\lambda} dx \right\} \\
 &= \lambda \left\{ 0 - \left[ \frac{e^{-\lambda x}}{\lambda^2} \right]_0^{\infty} \right\} = \lambda \left( \frac{1}{\lambda^2} \right) \\
 &= \frac{1}{\lambda}
 \end{aligned}$$

$$\begin{aligned}
 E(X^2) &= \int_0^{\infty} x^2 f(x) dx = \lambda \int_0^{\infty} x^2 e^{-\lambda x} dx \\
 &= \lambda \left\{ \left[ x^2 \frac{e^{-\lambda x}}{-\lambda} \right]_0^{\infty} + \int_0^{\infty} \frac{e^{-\lambda x}}{\lambda} 2x dx \right\} \\
 &= \lambda \left\{ 0 - 2 \left( \left[ x \frac{e^{-\lambda x}}{\lambda^2} \right]_0^{\infty} - \int_0^{\infty} \frac{e^{-\lambda x}}{\lambda^2} dx \right) \right\} \\
 &= \lambda \left\{ 0 - 2 \left( 0 + \left[ \frac{e^{-\lambda x}}{\lambda^3} \right]_0^{\infty} \right) \right\} = \lambda \frac{2}{\lambda^3} = \frac{2}{\lambda^2} \\
 \text{Var}(X) &= E(X^2) - \{E(X)\}^2 = \frac{2}{\lambda^2} - \left( \frac{1}{\lambda} \right)^2 = \frac{1}{\lambda^2}
 \end{aligned}$$

*The derivation for  $E(X)$  requires integration by parts once. For  $E(X^2)$ , it's required twice.*

What is the memoryless function of the exponential distribution?

Say an event hasn't occurred after 30 seconds after  $t = 0$ . The probability of an event occurring in at least 10 seconds equals observing the event 10 seconds after  $t = 0$ .