SD - Continuous random variables

What two conditions must hold true for all pdfs (probability density functions)?	$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad f(x) \geq 0, \forall\, x$
How can you find the mode of a CRV?	1. Differentiate the **<u>PROBABILITY DENSITY FUNCTION</u>** 2. Find where it equals 0. *While the probability of getting this exact value is negligible, a lot of values will occur around here.*
What is the Var(g(x)) and E(g(x)) of a CRV?	**Key point** If X is a random variable with probability density function $f(x)$, then the mean value and variance of $g(X)$ are given by $\mathrm{E}(g(X)) = \int_{-\infty}^{\infty} g(x)f(x)\,dx$ $\mathrm{Var}(g(X)) = \mathrm{E}(\{g(X)\}^2) - \{\mathrm{E}(g(X))\}^2$ $= \int_{-\infty}^{\infty} \{g(x)^2\} f(x)\,dx - \{\mathrm{E}(g(X))\}^2$ *Essentially, it's each value multiplied by its probability of occurring.* **<u>Example:</u>** A continuous random variable X has probability density function $f(x) = \frac{x+1}{6}$ for x between 1 and 3 Find the expected value of **a** $2X^2$ **b** $X + \frac{15}{X^3}$ **a** $\mathrm{E}(2X^2) = \int_1^3 2x^2\left(\frac{x+1}{6}\right)dx$ $= \frac{1}{3}\int_1^3 (x^3 + x^2)\,dx$ $= \frac{1}{3}\left[\frac{x^4}{4} + \frac{x^3}{3}\right]_1^3$ $= 9.56$ (2 dp) **b** $\mathrm{E}\left(X + \frac{15}{X^3}\right) = \int_1^3 \left(x + \frac{15}{x^3}\right)\left(\frac{x+1}{6}\right)dx$ $= \frac{1}{6}\int_1^3 (x^2 + x + 15x^{-2} + 15x^{-3})\,dx$ $= \frac{1}{6}\left[\frac{x^3}{3} + \frac{x^2}{2} - 15x^{-1} - \frac{15}{2}x^{-2}\right]_1^3$ $= \frac{44}{9}$ Or use $\mathrm{E}\left(X + \frac{15}{X^3}\right) = \mathrm{E}(X) + \mathrm{E}\left(\frac{15}{X^3}\right)$

What is a cdf (cumulative distribution function)?	$$F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x)\, dx$$ *This is useful for finding medians and quartiles (e.g., F(x) = 0.5).*
What is a continuous uniform distribution / rectangular distribution?	The area is (b - a)h = 1 so h = 1/(b-a). $$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & otherwise \end{cases}$$
What is the mean of a discrete uniform distribution and its derivation?	$$E(X) = \int_{a}^{b} x f(x)\ dx = \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b = \frac{1}{b-a} \cdot \frac{b^2 - a^2}{2} = \frac{a+b}{2}$$
What is the variance of a discrete uniform distribution and its derivation?	$$E(X^2) = \int_{a}^{b} x^2 f(x)\ dx = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b = \frac{1}{b-a} \cdot \frac{b^3 - a^3}{3} = \frac{1}{b-a} \cdot \frac{(b-a)(b^2+ab+a^2)}{3} = \frac{b^2+ab+a^2}{3}$$

	$$\begin{aligned} Var(X) &= E(X^2) - [E(X)]^2 \\ &= \frac{b^2 + ab + a^2}{3} - \frac{(b+a)^2}{4} \\ &= \frac{(a-b)^2}{12} \end{aligned}$$