## DD - Linear Programming

How should you structure a linear programming problem?  What is a basic variable under simplex?	<ol> <li>State the the variables and what they represent.         <ul> <li>Eg, "let x = the number of frogs, y = the number of rocks".</li> </ul> </li> <li>Find constraints (including x, y ≥ 0).</li> <li>Find the objective function.</li> <li>Write the conclusion.         <ul> <li>"Maximise/minimise [objective function] subject to"</li> </ul> </li> <li>A variable that has a 1 in their row and zeros in the rest of their column.</li> </ol>							
	An example	is snow	n belo	w:				
	variable	D v v c t						Row
		1	-2	0	0	4	2000	$R4 = R1 + 8 \times R6$
	S	0	$2\frac{1}{2}$	0	1	$-1\frac{1}{2}$	750	$R5 = R2 - 3 \times R6$
	у	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	250	$R6 = \frac{R3}{2}$
How is the Simplex Algorithm performed? (with example)	<ol> <li>Write the constraints and objective functions as equations in standard form using slack variables.</li> <li>Transfer (1) to a simplex tableau where the slack variables form the basis.</li> <li>Choose column with most negative coefficient in object row. This is the PIVOT COLUMN.         <ul> <li>This generally tells you the corresponding variables have to be increased to the optimal solution.</li> </ul> </li> <li>Choose the row giving the smallest θ-value. This is the PIVOT.         <ul> <li>This is found by dividing the each value by the positive numbers in the pivot column.</li> </ul> </li> <li>Divide the pivot row by the pivot</li> <li>Combine suitable multiples of the new row with other rows to make all other values zeros in the pivot column.</li> <li>This replaces the basic variable, eg, it may go from s</li> </ol>							

to x.

7. If there are **NO NEGATIVE** coefficients in the object row, the solution is optimal. Otherwise go to step 3.

## A DETAILED EXAMPLE IS SHOWN BELOW:

Maximise 
$$P = 6x + 8y$$

subject to 
$$4x+3y \le 1500$$

$$x+2y \le 500$$

$$x \ge 0, y \ge 0$$

Becomes...

Maximise 
$$P = 6x + 8y$$

subject to 
$$4x + 3y + s = 1500$$

$$x + 2y + t = 500$$

$$x \ge 0, y \ge 0, s \ge 0, t \ge 0$$

Hence...

$$P - 6x - 8y - 0s - 0t = 0$$

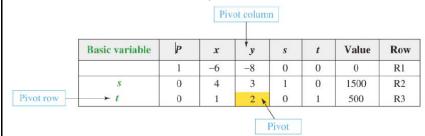
$$4x + 3y + s + 0t = 1500$$

$$x + 2y + 0s + t = 500$$

and the simplex tableau is

Basic variable	P	x	y	s	t	Value	Row
	1	-6	-8	0	0	0	R1
s	0	4	3	1	0	1500	R2
t	0	1	2	0	1	500	R3

And thus the pivot is chosen...



Since the  $\vartheta$ -value for s = 1500/3 = 500 and t = 500/2 so the final row is chosen.

Dividing by the pivot row by the pivot and getting rid of the remaining numbers in the pivot column yields...

Basic variable	P	x	y	s	t	Value	Row
	1	-2	0	0	4	2000	$R4 = R1 + 8 \times R6$
s	0	$2\frac{1}{2}$	0	1	$-1\frac{1}{2}$	750	$R5 = R2 - 3 \times R6$
у	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	250	$R6 = \frac{R3}{2}$

Repeating the change of basis for the x-column. We get...

Basic variable	P	x	у	s	t	Value	Row
	1	0	0	$\frac{4}{5}$	$2\frac{4}{5}$	2600	$R7 = R4 + 2 \times R8$
x	0	1	0	$\frac{2}{5}$	$-\frac{3}{5}$	300	$R8 = \frac{R5}{2\frac{1}{2}}$
у	0	0	1	$-\frac{1}{5}$	$\frac{4}{5}$	100	$R9 = R6 - \frac{1}{2} \times R8$

You now have x = 300, y = 100, s = 0, t = 0, giving P = 2600

## How are solutions for the Simplex Algorithm given?

With the basis variables equalling the value of the row and the rest equalling 0 as shown:

Step 6

B.V.	P	x	у	z	5	t	и	Value	Row
	1	0	0	1 5	3 5	1 5	0	44	$R9 = R5 + \frac{R11}{3}$
У	0	0	1	4 5	2 5	- <u>1</u>	0	16	$R10 = R6 - \frac{R11}{3}$
×	0	1	0	- <u>2</u> -5	- <u>1</u>	3   5	0	12	$R11 = \frac{R7}{1\frac{2}{3}}$
и	0	0	0	3 <del>2</del> 5	1 5	<u>3</u> - <u>5</u>	1	18	R12=R8-R11

**Step 7** There are no negative numbers in the top row so the solution is optimal.

The optimal solution is y = 16, x = 12, u = 18, z = 0, s = 0, t = 0, giving P = 44

## When can the standard Simplex Algorithm be used?

- 1. You need to maximise the object function.
- 2. Every non-trivial constraint is an inequality using ≤.
- 3. All variables are  $\geq 0$  (including the slack).
- 4. The origin is a vertex of the feasible region.

How can you apply the standard Simplex Algorithm when 'the objective function is being minimised' or 'an inequality involves ≥ instead'?

Minimising the objective function C is equivalent to maximising the objective function P = -C

maximising the objective function I

1.

2. You rewrite the inequality using  $\leq$  by multiply through by -1.

Key point

Both of these are applied below:

