$$\int_1^3 \left(\frac{1}{2}x+1\right)dx = ?$$

$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(a+\frac{b-a}{n}k\right)\cdot\frac{b-a}{n}$ 를 이용

$$\int_1^3 \left(\frac{1}{2}x+1\right)dx = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(1+\frac{2}{n}k\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{2}\left(1+\frac{2}{n}k\right)+1\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{k}{n}+\frac{3}{2}\right)\cdot\frac{2}{n} = \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{2k}{n^2}+\frac{6}{2n}\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n^2}\sum_{k=1}^{n}k+\frac{3}{n}\sum_{k=1}^{n}1\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n^2}\cdot\frac{n(n+1)}{2}+\frac{3n}{n}\right)$$

$$= \lim_{n\to\infty}\left(1+\overset{\to 0}{\frac{1}{n}}+3\right)$$

$$= 4$$

$$\int_1^3 x^2 \, dx = ?$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(a+\frac{b-a}{n}k\right)\cdot\frac{b-a}{n}$$ 를 이용

$$\int_1^3 x^2\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f\left(1+\frac{2}{n}k\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{2}{n}k\right)^2\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(1+\frac{4}{n}k+\frac{4}{n^2}k^2\right)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{2}{n}+\frac{8}{n^2}k+\frac{8}{n^3}k^2\right)$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\sum_{k=1}^{n}1+\frac{8}{n^2}\sum_{k=1}^{n}k+\frac{8}{n^3}\sum_{k=1}^{n}k^2\right)$$

$$(n^2+n)(2n+1) = 2n^3+3n^2+n$$

$$= \lim_{n\to\infty}\left(\frac{2}{n}\cdot n+\frac{8}{n^2}\cdot\frac{n(n+1)}{2}+\frac{8}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}\right)$$

$$= \lim_{n\to\infty}\left(2+4+\overset{=0}{\frac{4}{n}}+\frac{8}{3}+\overset{=0}{\frac{4}{n}}+\overset{=0}{\frac{4}{3n^2}}\right)$$

$$= 2+4+0+\frac{8}{3}+0+0$$

$$= \frac{26}{3}$$