

# Übungsblatt 2

Pascal Diller, Timo Rieke

1. Mai 2025

## Aufgabe 1

(i)

$$\begin{aligned}\lambda_1(x^2 + 5x + 1) + \lambda_2(9x + 1) + \lambda_3(3x^2 - 3x + 1) &= 2x^2 + x = 1 \\ (\lambda_1 + 3\lambda_3)x^2 + (5\lambda_1 + 9\lambda_2 - 3\lambda_3)x + (\lambda_1 + \lambda_2 + \lambda_3) &= 2x^2 + x = 1\end{aligned}$$

Gleichsetzen der Koeffizienten

$$\lambda_1 + 3\lambda_3 = 2 \tag{1}$$

$$\lambda_1 + 9\lambda_2 - 3\lambda_3 = 1 \tag{2}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = 1 \tag{3}$$

Aus (1):

$$\lambda_1 + 3\lambda_3 = 2 \Leftrightarrow \lambda_1 = 2 - 3\lambda_3$$

Einsetzen in (3)

$$\begin{aligned}(2 - 3\lambda_3) + \lambda_2 + \lambda_3 &= 1 \\ \Leftrightarrow \lambda_2 &= 1 - (2 - 3\lambda_3) - \lambda_3 = 2\lambda_3 - 1\end{aligned}$$

Einsetzen in (2)

$$\begin{aligned}5(2 - 3\lambda_3) + 9(2\lambda_3 - 1) &= 3\lambda_3 = 1 \\ 10 - 15\lambda_3 + 18\lambda_3 - 9 &= 3\lambda_3 = 1 \\ 1 &= 1 \rightarrow \text{unbestimmtes System}\end{aligned}$$

Sei  $\lambda_3 = t \in \mathbb{R}$   
dann:

$$\lambda_1 = 2 - 3t$$

$$\lambda_2 = 2t - 1$$

$$\lambda_3 = t$$

Sei  $t = 0$ , dann:

$$\lambda_1 = 2$$

$$\lambda_2 = 1$$

$$\lambda_3 = 0$$

$$2x^2 + x + 1 = 2(x^2 + 5x + 1) + 1(9x + 1)$$

**(ii)**

**(a)**

$$\begin{aligned}\lambda_1(x^3 + x^2 + x + 1) + \lambda_2(2x^2 - 4x + 6) + \lambda_3(-x^2 - 2x + 5) &= 2x^3 - x^2 - 2x + 13 \\ \lambda_1 x^3 + (\lambda_1 + 2\lambda_2 - \lambda_3)x^2 + (\lambda_1 - 4\lambda_2 - 2\lambda_3)x + (\lambda_1 + 6\lambda_2 + 5\lambda_3) &= 2x^3 - x^2 - 2x + 13\end{aligned}$$

Gleichstellen der Koeffizienten

$$\lambda_1 = 2 \tag{4}$$

$$\lambda_1 + 2\lambda_2 - \lambda_3 = -1 \tag{5}$$

$$\lambda_1 - 4\lambda_2 - 2\lambda_3 = -2 \tag{6}$$

$$\lambda_1 - 6\lambda_2 + 5\lambda_3 = 13 \tag{7}$$

Einsetzen in (2)

$$2 + 2\lambda_2 - \lambda_3 = -1 \Leftrightarrow \lambda_3 = 2\lambda_2 + 3$$

Einsetzen in (3)

$$2 - 4\lambda_2 - 2(2\lambda_2 + 3) = -2$$

$$\Leftrightarrow -8\lambda_2 - 4 = -2$$

$$\Leftrightarrow \lambda_2 = -\frac{1}{4}$$

Einsetzen in  $\lambda_3$

$$\lambda_3 = 2\left(-\frac{1}{4}\right) + 3 = 2.5 = \frac{5}{2}$$

Alle Koeffizienten wurden gefunden, also ist (a) ein Element von der Span.

**b**

**(iii)**

**a**

**b**