Übung 5

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Aufgabe 1

(i)

I.A.

$$\left|\sum_{k=1}^{1} a_k\right| \leq \sum_{k=1}^{1} |a_k| = 1 \leq 1$$

I.V.

$$\left|\sum_{k=1}^{n} a_k\right| \leq \sum_{k=1}^{n} |a_k|$$

I.S

$$\begin{aligned}
&|\sum_{k=1}^{n+1} a_k| \leq \sum_{k=1}^{n+1} |a_k| \\
&= \left|\left(\sum_{k=1}^{n} a_k\right) + a_{n+1}\right| \leq \left(\sum_{k=1}^{n} |a_k|\right) + |a_{n+1}| \\
&\overset{I.V.}{=} \left|\sum_{k=1}^{n+1} a_k\right| \leq \sum_{k=1}^{n} |a_k| + |a_{n+1}| \\
&= \left|\sum_{k=1}^{n+1} a_k\right| \leq \sum_{k=1}^{n+1} |a_k|
\end{aligned}$$

Somit ist gezeigt, dass $A(n) \implies A(n+1)$.

(ii)

Zu zeigen:

$$\sum_{k=0}^{n}(-1)^k \binom{n}{k} = 0$$

Binomischer Lehrsatz:

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Seien $x = -1$ und $y = 1$:

$$\begin{aligned}(-1+1)^n &= \sum_{k=0}^{n} \binom{n}{k} (-1)^k 1^{n-k} \\ = 0^n &= \sum_{k=0}^{n} \binom{n}{k} (-1)^k \\ = 0 &= \sum_{k=0}^{n}(-1)^k \binom{n}{k}\end{aligned}$$