

# Supplementary Material: Triangulated Relativistic Quantum Computation (TRQC) Proof-of-Concept

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## Abstract

This Supplementary Material provides the numerical Proof-of-Concept (PoC) accompanying the manuscript, *Triangulated Relativistic Quantum Computation: A Curvature-Modulated Unification of Quantum and Relativistic Computing*. The PoC confirms the consistency and operational semantics of the TRQC framework by simulating transport across representative graph families and verifying the core theoretical invariants, including causal order independence, remeshing robustness, and the expected  $\mathcal{O}(4^N)$  density-matrix scaling. We present detailed results for transport metrics and address the reviewer’s specific request for peak-over-steps arrival data to mitigate the effects of detector overshoot.

## 1 Code and Data Availability

### 1.1 Source Code and Replication

The complete Python code for the numerical Proof-of-Concept (PoC) is implemented as a self-contained **Jupyter Notebook** (`proof_of_concept.ipynb`). All simulations rely on the **PennyLane** quantum programming framework (using the `default.mixed` device for density-matrix evolution) and standard scientific libraries (`numpy`, `scipy`, `networkx`).

The code is publicly available and archived for persistence and reproducibility:

- **GitHub Repository:** <https://github.com/Ind50-UPM/2025-TRQC>
- **Archival DOI:** 10.5281/zenodo.17625496.

All figures, raw timeline data (.npz), and CSV summaries necessary for replication are generated upon execution of the notebook.

## 2 Design and Methodology of the Proof-of-Concept (PoC)

The PoC implements the curvature-guided and spacelike planned evolution described in Section 3 of the main manuscript.

### 2.1 Parameter Space

The simulations aggregate results over multiple folds (random graph realizations or random latent embeddings) across a fixed parameter space to test the framework across varying topologies and sizes (Tab. 1).

**Table 1** PoC Simulation Parameters and Metrics

Parameter/Metric	Values Used	Purpose
Graph Families	sphere, geometric2d, ER, scale-free	Test closed, boundary, homogeneous, and hub-dominated to...
Network Size ( $N$ )	{6, 8, 10, 12}	Verify $O(4^N)$ scaling and size-dependent transport.
Spacelike Rounds ( $R$ )	{2, 3}	Compare short vs. extended propagation schedules.
Folds per ( $N, R, \text{Family}$ )	5 for $N \leq 10$ ; 3 for $N = 12$	Stabilize statistical means (adjusted at $N = 12$ for memory)
Final-step Arrival	$\max_d P(\text{final step}, d)$	Conservative metric on fixed early-hop detectors.
Peak-over-steps Arrival	$\max_{t,d} P(\text{step } t, d)$	Captures true transport success (mitigates overshoot).
Participation Ratio (PR)	$1/\sum p_i^2$	Measures spatial delocalization (wavepacket spreading).

### 2.2 TRQC Implementation Details

- **Curvature Estimation:** Intrinsic Gaussian curvature ( $K_v$ ) is derived from **vertex angle deficits** in latent triangulation, consistent with the discrete Gauss-Bonnet identity (Theorem 3.2, Proposition 3.1). This field is slice-wise normalized and used to define the edge curvature average  $\kappa_e$  (Eq. 4).
- **Dynamics:** Local evolution is governed by the GKSL generator modulated by curvature (Definition 3.4), ensuring that  $\kappa_e$  modulates the rates of phase damping ( $\gamma_\phi$ ) and amplitude damping ( $\gamma_{\text{amp}}$ ) while guaranteeing **CPTP** well-posedness (Proposition 3.4).
- **Causal Scheduling:** Transport across the slice is implemented via a sequence of maximal matchings (spacelike SWAP layers), ensuring order independence and causal factorization within the step (Theorem 3.7).

### 3 Confirmation of Theoretical Invariants

The numerical results confirm the mathematical guaranties of the TRQC framework.

#### 3.1 Algebraic Invariance Checks

The PoC includes specific diagnostics to test structural stability:

- **Order Independence (Theorem 3.7):** The simulation checks for a non-trivial step where two pairs of commuting SWAPs/channels are applied. Applying the step in reverse order yields a final state difference of  $\|\Delta\rho\|_F = \mathbf{0.000e+00}$ , confirming that the global channel is independent of the ordering within the slice when the operators act on disjoint tensor factors.
- **Remeshing Robustness (Proposition 3.1):** Curvature fields computed from a rotated and slightly jittered embedding are compared. The resulting final states  $\rho$  differ by a Frobenius norm of  $\|\rho_0 - \rho_1\|_F = \mathbf{7.514e-05}$ , confirming the robustness of the dynamics to small, physically realistic perturbations in the latent coordinates.
- **Runtime Scaling:** The wall-clock runtime ( $t_{\text{seconds}}$ ) as a function of  $N$  confirms the expected growth  $\propto 4^N$  typical of density-matrix simulations on  $N$  qubits, without unexpected complexity anomalies.

#### 3.2 Curvature-Driven Transport

The statistical summary (Tab. 2) illustrates the basic hypothesis that curvature guides transport and delocalization.

**Table 2** Summary of Best-Performing Simulation Runs by Arrival Mean ( $\pm$  Std)

N	Rounds	Family	arrival <sub>mean</sub>	PR <sub>mean</sub>	t <sub>mean</sub> (s)
6	3	<b>sphere</b>	<b>0.5719 <math>\pm</math> 0.3209</b>	1.9864	0.20
8	3	<b>scalefree</b>	0.1596 $\pm$ 0.3305	1.5457	1.19
10	3	<b>er</b>	0.1657 $\pm$ 0.3704	1.5218	18.69
12	2	<b>geometric2d</b>	0.2592 $\pm$ 0.4489	1.6575	390.04

- **The highest success in Small  $N$ :** **\*\*sphere\*\*** (closed manifold, highly symmetric) with rounds  $R = 3$  achieves the highest overall final-step arrival, consistent with coherent advancement when the path length is short.
- **Detector Overshoot:** At  $N = 12$  with  $R = 3$ , the final-step arrival for **all families collapses to**  $\approx 0.0$ , despite many runs having  $\text{PR}_{\text{mean}} \approx 2$  (widespread wavepacket). This confirms that the excitation **overshoots** the conservative early-hop detector set, a phenomenon visually confirmed by the **Peak-over-steps Arrival** data logged in the accompanying figures.

## 4 Addressing the Key Suggestion: Peak-over-Steps Arrival

The PoC collects the complete history of the probability of detection (logged as `arrival_timeline.npy`) to provide the most informative metric of the capability of transport.

### 4.1 New Metrics and Conclusion

- **The final step metric is Conservative:** The conservative metric (arrival) is necessary for the initial metric in the paper, but its limitation is exposed at high  $N/R$ .
- **Peak-over-steps Metric:** The tracked timeline explicitly captures the **\*\*maximum arrival probability over all intermediate spacelike steps\*\*** and serves as the key metric to report true transport success. Supplementary figures (e.g., `arrival_vs_steps...`.pdf) visually confirm that the peak often occurs far from the final step, validating the need for this alternative metric to truly assess propagation capability in TRQC.