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Question. A plane contains the following three points: $\mathbf{P}(2,1,5), \mathbf{Q}(-1,3,4)$ and $\mathbf{R}(3,0,6)$. The vector perpendicular to the above plane can be represented as
Solution:
Let us solve the given equation theoretically and then verify the solution computationally.
Given points are:

$$\mathbf{P} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \mathbf{Q} = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix} \quad and \quad \mathbf{R} = \begin{pmatrix} 3 \\ 0 \\ 6 \end{pmatrix} \tag{1}$$

The equation of plane through the points $\mathbf{P}$,$\mathbf{Q}$ and $\mathbf{R}$ can be given as

$$\begin{pmatrix} \mathbf{P} & \mathbf{Q} & \mathbf{R} \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{2}$$

Where $\mathbf{n}$ is the normal to the plane

$$\begin{pmatrix} 2 & -1 & 3 \\ 1 & 3 & 0 \\ 5 & 4 & 6 \end{pmatrix}^T \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{3}$$

$$\begin{pmatrix} 2 & 1 & 5 \\ -1 & 3 & 4 \\ 3 & 0 & 6 \end{pmatrix} \mathbf{n} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{4}$$

Now forming the augmented matrix and performing row operations

$$\left(\begin{array}{ccc|c} 2 & 1 & 5 & 1 \\ -1 & 3 & 4 & 1 \\ 3 & 0 & 6 & 1 \end{array}\right) \xleftrightarrow{R_2 \leftarrow -R_2} \left(\begin{array}{ccc|c} 2 & 1 & 5 & 1 \\ 1 & -3 & -4 & -1 \\ 3 & 0 & 6 & 1 \end{array}\right) \tag{5}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 5 & 1 \\ 1 & -3 & -4 & -1 \\ 3 & 0 & 6 & 1 \end{array}\right) \xleftrightarrow[R_1 \leftarrow R_1 - 2R_2]{R_3 \leftarrow R_3 - 3R_2} \left(\begin{array}{ccc|c} 0 & 7 & 13 & 3 \\ 1 & -3 & -4 & -1 \\ 0 & 9 & 18 & 4 \end{array}\right) \tag{6}$$

$$\left(\begin{array}{ccc|c} 0 & 7 & 13 & 3 \\ 1 & -3 & -4 & -1 \\ 0 & 9 & 18 & 4 \end{array}\right) \xleftrightarrow{R_3 \leftarrow 7R_3 - 9R_1} \left(\begin{array}{ccc|c} 0 & 7 & 13 & 3 \\ 1 & -3 & -4 & -1 \\ 0 & 0 & 9 & 1 \end{array}\right) \tag{7}$$

$$\left(\begin{array}{ccc|c} 0 & 7 & 13 & 3 \\ 1 & -3 & -4 & -1 \\ 0 & 0 & 9 & 1 \end{array}\right) \xleftrightarrow[R_1 \leftarrow \frac{1}{7}R_1]{R_3 \leftarrow \frac{1}{9}R_3} \left(\begin{array}{ccc|c} 0 & 1 & \frac{13}{7} & \frac{3}{7} \\ 1 & -3 & -4 & -1 \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \tag{8}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & \frac{13}{7} & \frac{3}{7} \\ 1 & -3 & -4 & -1 \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \xleftrightarrow[R_2 \leftarrow R_2 + 4R_3]{R_1 \leftarrow R_1 - \frac{13}{7}R_3} \left(\begin{array}{ccc|c} 0 & 1 & 0 & \frac{2}{9} \\ 1 & -3 & 0 & -\frac{5}{9} \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \tag{9}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 0 & \frac{2}{9} \\ 1 & -3 & 0 & -\frac{5}{9} \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \xleftrightarrow{R_2 \leftarrow R_2 + 3R_1} \left(\begin{array}{ccc|c} 0 & 1 & 0 & \frac{2}{9} \\ 1 & 0 & 0 & \frac{1}{9} \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \tag{10}$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 0 & \frac{2}{9} \\ 1 & 0 & 0 & \frac{1}{9} \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \xleftrightarrow{R_2 \longleftrightarrow R_1} \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{1}{9} \\ 0 & 1 & 0 & \frac{2}{9} \\ 0 & 0 & 1 & \frac{1}{9} \end{array}\right) \tag{11}$$

The equation of plane can be given as:

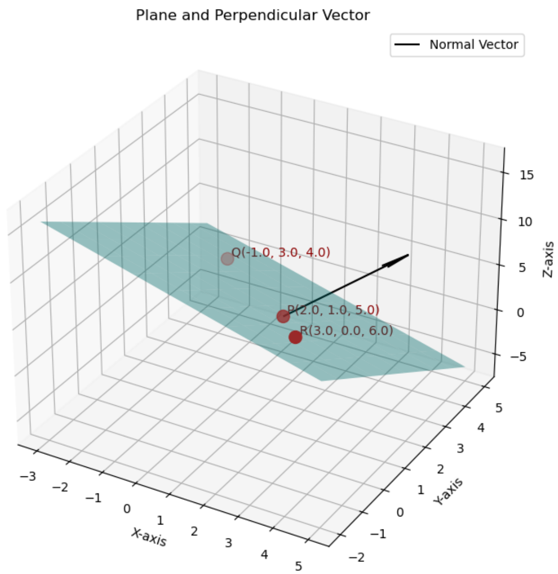
$$\begin{pmatrix} 1 & 2 & 1 \end{pmatrix} \mathbf{x} = 9 \tag{12}$$

Normal of the plane is:

$$\mathbf{n} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \tag{13}$$

This normal vector is perpendicular to the plane.

From the figure it is clearly verified that the theoretical solution matches with the computational solution.


Plane and Perpendicular Vector
Normal Vector
Q(-1.0, 3.0, 4.0)
P(2.0, 1.0, 5.0)
R(3.0, 0.0, 6.0)
X-axis
Y-axis
Z-axis