

12.561

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# Question

The following system of equations

$$2x - y - z = 0$$

$$-x + 2y - z = 0$$

$$-x - y + 2z = 0$$

- ① has no solution
- ② has a unique solution
- ③ has three solutions.
- ④ has an infinite number of solutions

# Equation I

The given equation can be given as:

$$\mathbf{Ax} = \mathbf{B} \quad (1)$$

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

# Theoretical Solution

Now forming the augmented matrix and performing row operations

$$\left( \begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array} \right) \xleftrightarrow{R_1 \leftrightarrow R_2} \left( \begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array} \right) \quad (3)$$

$$\left( \begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array} \right) \xleftrightarrow{R_3 \leftarrow R_3 - R_1} \left( \begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right) \quad (4)$$

$$\left( \begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right) \xleftrightarrow{R_3 \leftarrow R_3 + R_2} \left( \begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (5)$$

Here the rank of the matrix is 2 which is less than 3.

So the system of the equation has an infinite number of solutions

Intersection of Planes for the System of Equations

