

# 10.4.3

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**Question.** The point at which the normal to the curve  $y = x + \frac{1}{x}$ ,  $x > 0$  is perpendicular to the line  $3x - 4y - 7 = 0$

**Solution:**

Let us solve the given equation theoretically and then verify the solution computationally. The given curve be rearranged as:

$$x^2 - xy + 1 = 0. \quad (1)$$

This can be expressed in the form:

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2)$$

Where:

$$\mathbf{v} = \begin{pmatrix} 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \text{ and } f = 1 \quad (3)$$

The required direction of normal which is perpendicular to the line  $3x - 4y - 7 = 0$

$$\mathbf{m} = \begin{pmatrix} 1 \\ -\frac{4}{3} \end{pmatrix} \quad (4)$$

$$\mathbf{m} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (5)$$

Now the equation of normal to the conic at the point of contact  $\mathbf{q}$  is given by:

$$(\mathbf{V}\mathbf{q} + \mathbf{u})^T \mathbf{R}(\mathbf{x} - \mathbf{q}) = 0 \quad (6)$$

In the normal equation  $\mathbf{V}\mathbf{q} + \mathbf{u}$  is proportional to the direction vector of the normal. So,

$$\mathbf{V}\mathbf{q} + \mathbf{u} = k\mathbf{m} \quad (7)$$

$$\mathbf{q} = \mathbf{V}^{-1}(k\mathbf{m} - \mathbf{u}) \quad (8)$$

$\mathbf{q}$  lies on the curve. So substituting Eq.8 in Eq.2:

$$(\mathbf{V}^{-1}(k\mathbf{m} - \mathbf{u}))^T \mathbf{V} \mathbf{V}^{-1}(k\mathbf{m} - \mathbf{u}) + 2\mathbf{u}^T \mathbf{V}^{-1}(k\mathbf{m} - \mathbf{u}) + f = 0 \quad (9)$$

$$(\mathbf{V}^{-1}(k\mathbf{m} - \mathbf{u}))^T (k\mathbf{m} - \mathbf{u}) + f = 0 \quad (10)$$

$$\left( \begin{pmatrix} 0 & -2 \\ -2 & -4 \end{pmatrix} \left( k \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \right)^T \left( k \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) + 1 = 0 \quad (11)$$

$$k^2 \begin{pmatrix} 3 & -4 \end{pmatrix} \begin{pmatrix} 0 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} + 1 = 0 \quad (12)$$

$$k^2 = \frac{1}{16} \quad (13)$$

$$k = \frac{1}{4} \text{ and } k = -\frac{1}{4} \quad (14)$$

Now substitute the corresponding values in the Eq.8 to get the point

$$\mathbf{q} = \begin{pmatrix} 0 & -2 \\ -2 & -4 \end{pmatrix} \left( k \begin{pmatrix} 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right) \quad (15)$$

$$\mathbf{q} = k \begin{pmatrix} 0 & -2 \\ -2 & -4 \end{pmatrix} \begin{pmatrix} 3 \\ -4 \end{pmatrix} \quad (16)$$

$$\mathbf{q} = k \begin{pmatrix} 8 \\ 10 \end{pmatrix} \quad (17)$$

When  $k = \frac{1}{4}$

$$\mathbf{q} = \begin{pmatrix} 2 \\ \frac{5}{2} \end{pmatrix} \quad (18)$$

When  $k = -\frac{1}{4}$

$$\mathbf{q} = \begin{pmatrix} -2 \\ -\frac{5}{2} \end{pmatrix} \quad (19)$$

Given that  $x > 0$  . So the point of contact is

$$\mathbf{q} = \begin{pmatrix} 2 \\ \frac{5}{2} \end{pmatrix} \quad (20)$$

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

