

# 12.353

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**Question.** Which one of the following describes the relationship among the three vectors

$$\hat{i} + \hat{k} + \hat{k}, 2\hat{i} + 3\hat{j} + \hat{k}, 5\hat{i} + 6\hat{j} + 4\hat{k}$$

- 1) The vectors are mutually perpendicular
- 2) The vectors are linearly dependent
- 3) The vectors are linearly independent
- 4) The vectors are unit vectors

**Solution:**

Let us solve the given equation theoretically and then verify the solution computationally.

Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} \quad (1)$$

Let

$$\mathbf{M} = (\mathbf{A} \quad \mathbf{B} \quad \mathbf{C}) \quad (2)$$

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 5 \\ 1 & 3 & 6 \\ 1 & 1 & 4 \end{pmatrix} \quad (3)$$

Now applying row operations

$$R_2 \longrightarrow R_2 - R_1 \quad \text{and}$$

$$R_3 \longrightarrow R_3 - R_1$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 1 & 3 & 6 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} \quad (4)$$

Now doing row operation

$$R_3 \longrightarrow R_3 + R_2$$

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

Here the rank of the matrix is 2.

Since the rank is less than the number of vectors, the vectors are linearly dependent

3D Visualization of Linearly Independent Vectors

