12.665

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Question.The product of eigenvalues of

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

1) -1
2) 1
3) 0
4) 2

Solution: Let

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \tag{1}$$

Let λ be the eigen value of the $\mathbf{A}$. Then,

$$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0 \tag{2}$$

$$\begin{vmatrix} -\lambda & 0 & 1 \\ 0 & 1-\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = 0 \tag{3}$$

$$-\lambda(1-\lambda)(-\lambda) + 1(-(1-\lambda)) = 0 \tag{4}$$

$$\lambda^2(1-\lambda) - (1-\lambda) = 0 \tag{5}$$

$$(\lambda^2 - 1)(1-\lambda) = 0 \tag{6}$$

$$\lambda = 1 \;\; and \;\; \lambda = -1 \tag{7}$$

Product of two eigen values is -1