# 12.353

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# Question

Which one of the following describes the relationship among the three vectors

$$\hat{i} + \hat{k} + \hat{k}, 2\hat{i} + 3\hat{j} + \hat{k}, 5\hat{i} + 6\hat{j} + 4\hat{k}$$

1. The vectors are mutually perpendicular
2. The vectors are linearly dependent
3. The vectors are linearly independent
4. The vectors are unit vectors

# Equation I

Let

$$\mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \quad \textit{and} \quad \begin{pmatrix} 5 \\ 6 \\ 4 \end{pmatrix} \tag{1}$$

Let

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{C} \end{pmatrix} \tag{2}$$

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & 5 \\ 1 & 3 & 6 \\ 1 & 1 & 4 \end{pmatrix} \tag{3}$$

# Theoretical Solution

Now applying row operations
$R_2 \longrightarrow R_2 - R_1$ and
$R_3 \longrightarrow R_3 - R_1$

$$\begin{pmatrix} 1 & 2 & 5 \\ 1 & 3 & 6 \\ 1 & 1 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} \tag{4}$$

# Theoretical solution

Now doing row operation
$R_3 \longrightarrow R_3 + R_2$

$$\begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{5}$$

Here the rank of the matrix is 2.
Since the rank is less than the number of vectors , the vectors are linearly dependent

# C Code

```c
#include <math.h>

#define N 3
#define TOLERANCE 1e-9

int check_vectors(double v1[], double v2[], double v3[]) {
    double mat[N][N];

    // Create a matrix from the input vectors (as columns)
    for (int i = 0; i < N; i++) {
        mat[i][0] = v1[i];
        mat[i][1] = v2[i];
        mat[i][2] = v3[i];
    }
```

# C Code

```c
// --- Gaussian Elimination to find rank ---
for (int col = 0; col < N; col++) {
    int pivot_row = col;
    for (int i = col + 1; i < N; i++) {
        if (fabs(mat[i][col]) > fabs(mat[pivot_row][col])) {
            pivot_row = i;
        }
    }

    if (pivot_row != col) {
        for (int i = 0; i < N; i++) {
            double temp = mat[col][i];
            mat[col][i] = mat[pivot_row][i];
            mat[pivot_row][i] = temp;
        }
    }
```

# C Code

```c
    if (fabs(mat[col][col]) < TOLERANCE) continue;

        for (int i = 0; i < N; i++) {
            if (i != col) {
                double factor = mat[i][col] / mat[col][col];
                for (int j = col; j < N; j++) {
                    mat[i][j] -= factor * mat[col][j];
                }
            }
        }
    }

    int zero_rows = 0;
    for (int i = 0; i < N; i++) {
        int all_zeros = 1;
        for (int j = 0; j < N; j++) {
            if (fabs(mat[i][j]) > TOLERANCE) {
                all_zeros = 0;
```

# C Code

```c
        break;
 }
     }
     if (all_zeros) zero_rows++;
   }

   int rank = N - zero_rows;

   return (rank < 3) ? 2 : 3;
}
```

# Python Code

```
import ctypes
import os
import platform
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

# --- Function to get vector input from the user ---
def get_vector_from_user(vector_name: str) -> list[float]:
    Prompts the user for 3 vector components and returns them as a list.
    while True:
        try:
            input_str = input(fEnter the 3 components for vector {vector_name} (e.g., 1, 2, 3): )
            components = [float(item) for item in input_str.split(',')]
```

# Python Code

```
if len(components) == 3:
                return components
            else:
                print(Error: Please enter exactly 3 numbers separated by commas.)
        except ValueError:
            print(Error: Invalid input. Please enter numbers only.)

# --- Determine library name and path ---
lib_name = 'rank.so' if platform.system() != 'Windows' else 'rank.dll'
lib_path = os.path.join(os.getcwd(), lib_name)

# --- Load the C library ---
try:
    c_lib = ctypes.CDLL(lib_path)
```

# Python Code

```
    except OSError as e:
    print(fError loading shared library '{lib_name}': {e})
    print(Please ensure you have compiled 'vector_lib.c' in the same directory.)
    exit()

# --- Define the C function's signature for type safety ---
c_lib.check_vectors.argtypes = [
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double),
    ctypes.POINTER(ctypes.c_double)
]
c_lib.check_vectors.restype = ctypes.c_int

# --- Main execution block ---
if __name__ == __main__:
    print(This script checks if three 3D vectors are linearly dependent using a C function and then plots them.)
```

```
    # Get user input
  v1_list = get_vector_from_user(v1)
  v2_list = get_vector_from_user(v2)
  v3_list = get_vector_from_user(v3)

  # --- Part 1: C Function Call ---
  # Convert Python lists to C-compatible arrays
  V1_CArray = (ctypes.c_double * 3)(*v1_list)
  V2_CArray = (ctypes.c_double * 3)(*v2_list)
  V3_CArray = (ctypes.c_double * 3)(*v3_list)

  # Call the C function to get the result
  result_code = c_lib.check_vectors(V1_CArray, V2_CArray,
      V3_CArray)

  # --- Part 2: Display Result and Plot ---
  print(\n--- Analysis Result from C Function ---)
  is_dependent = (result_code == 2)
```

# Python Code

```python
    if is_dependent:
        print(The vectors are linearly dependent (coplanar).)
        print(Generating a 3D plot to visualize the vectors on
            their plane...)
else:
    print(The vectors are linearly independent.)
    print(Generating a 3D plot of the vectors...)

# Convert lists to NumPy arrays for plotting
v1 = np.array(v1_list)
v2 = np.array(v2_list)
v3 = np.array(v3_list)

fig = plt.figure(figsize=(10, 8))
ax = fig.add_subplot(111, projection='3d')
```

# Python Code

```
    origin = [0, 0, 0]
ax.quiver(*origin, *v1, color='r', label=f'v1: {v1}', length=
    np.linalg.norm(v1), normalize=True)
ax.quiver(*origin, *v2, color='g', label=f'v2: {v2}', length=
    np.linalg.norm(v2), normalize=True)
ax.quiver(*origin, *v3, color='b', label=f'v3: {v3}', length=
    np.linalg.norm(v3), normalize=True)

# If they are dependent, plot the plane they lie on
if is_dependent:
    normal_vector = np.cross(v1, v2)
    if np.linalg.norm(normal_vector) > 1e-6: # Ensure vectors
         are not collinear
        x_range = np.linspace(min(0, v1[0], v2[0], v3[0]), max
            (v1[0], v2[0], v3[0]), 5)
        y_range = np.linspace(min(0, v1[1], v2[1], v3[1]), max
            (v1[1], v2[1], v3[1]), 5)
        xx, yy = np.meshgrid(x_range, y_range)
```

# Python Code

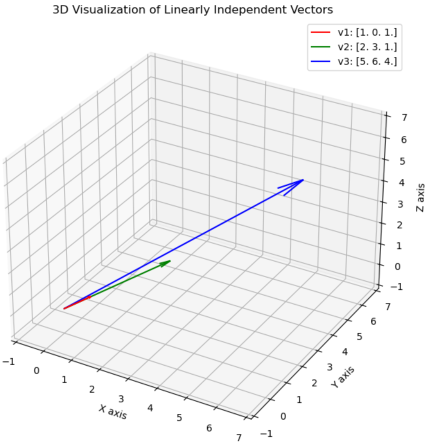
```
      xx, yy = np.meshgrid(x_range, y_range)
    if abs(normal_vector[2]) > 1e-6:
            zz = (-normal_vector[0] * xx - normal_vector[1] * yy) / normal_vector[2]
            ax.plot_surface(xx, yy, zz, alpha=0.2, color='gray', rstride=100, cstride=100)

# Formatting the plot
max_range = np.max(np.abs([v1, v2, v3])) * 1.1
ax.set_xlim([-max_range, max_range])
ax.set_ylim([-max_range, max_range])
ax.set_zlim([-max_range, max_range])
ax.set_xlabel('X axis')
ax.set_ylabel('Y axis')
ax.set_zlabel('Z axis')
ax.set_title('3D Visualization of Input Vectors')
```

# Python Code

```
    ax.legend()
ax.grid(True)
plt.savefig(/media/indhiresh-s/New Volume/Matrix/ee1030-2025/
    ee25btech11027/MATGEO/12.353/figs/figure1.png)
plt.show()
```

# Plot


3D Visualization of Linearly Independent Vectors
v1: [1. 0. 1.]
v2: [2. 3. 1.]
v3: [5. 6. 4.]
X axis
Y axis
Z axis