

12.561

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Question. The following system of equations

$$\begin{aligned} 2x - y - z &= 0 \\ -x + 2y - z &= 0 \\ -x - y + 2z &= 0 \end{aligned}$$

- 1) has no solution
- 2) has a unique solution
- 3) has three solutions.
- 4) has an infinite number of solutions

Solution:

Let us solve the given equation theoretically and then verify the solution computationally. The given equation can be given as:

$$\mathbf{Ax} = \mathbf{B} \quad (1)$$

$$\begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (2)$$

Now forming the augmented matrix and performing row operations

$$\left(\begin{array}{ccc|c} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array} \right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array} \right) \quad (3)$$

$$\left(\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ -1 & -1 & 2 & 0 \end{array} \right) \xrightarrow{R_3 \leftarrow R_3 - R_1} \left(\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 2 & -1 & -1 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right) \quad (4)$$

$$\left(\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & -3 & 3 & 0 \end{array} \right) \xrightarrow{R_3 \leftarrow R_3 + R_2} \left(\begin{array}{ccc|c} -1 & 2 & -1 & 0 \\ 0 & 3 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \quad (5)$$

Here the rank of the matrix is 2 which is less than 3.

So the system of the equation has an infinite number of solutions

From the figure it is clearly verified that the theoretical solution matches with the computational solution.

Intersection of Planes for the System of Equations

