AIL 722: Assignment 1

Anamitra Singha
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Contents

1 Sample Trajectories and Observations

1.1 Sensor Detection Probability Visualized.

The sensor probabilities are visualised as heatmaps with yellow values indicating higher probabilities and green colour indicating lower probabilities. The probabilities here are the probability of sensing by a sensor

1.2 Sampling of trajectories of length 30.

The table in 1 show the sampled trajectories. Every element of the list of list is the coordinates in gridworld

1.3 Plots showing the sampled trajectories.

The plots in figure 5 and 6 show the sampled trajectories plotted on the grid

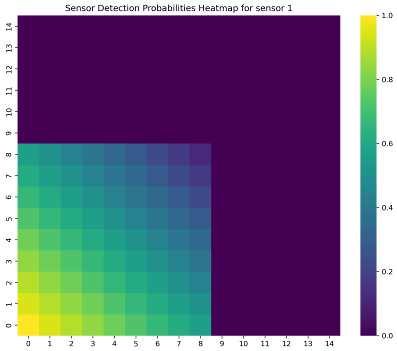

Figure 1: Sensor 1 Detection Heatmap

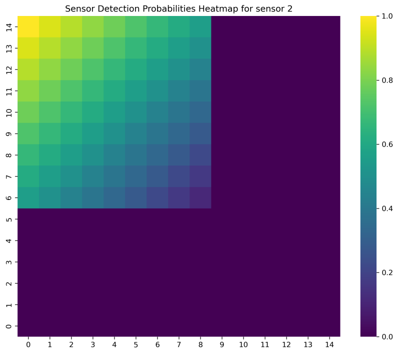

Figure 2: Sensor 2 Detection Heatmap

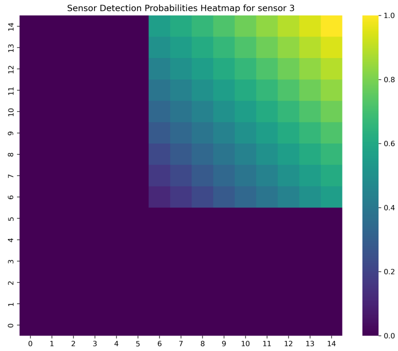

Figure 3: Sensor 3 Detection Heatmap

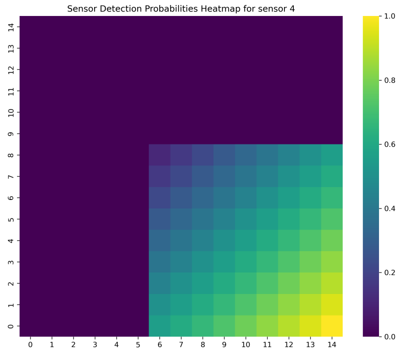

Figure 4: Sensor 4 Detection Heatmap

Trajectory 1

```
[
    [1, 1], [1, 1], [2, 1], [3,
   1], [4, 1], [5, 1], [6, 1],
   [5, 1], [6, 1], [7, 1], [6,
   1], [7, 1], [7, 1], [8, 1],
    [8, 1], [8, 2], [8, 3], [9,
   3], [9, 4], [9, 4], [8, 4],
   [8, 4], [9, 4], [9, 4], [10,
   4], [9, 4], [10, 4], [11, 4],
    [12, 4], [13, 4]
]
```

Trajectory 2

```
[
    [1, 1], [1, 2], [2, 2], [1,
   2], [1, 3], [1, 4], [1, 4],
   [2, 4], [2, 4], [3, 4], [3,
   4], [4, 4], [4, 5], [5, 5],
    [5, 6], [4, 6], [4, 6], [3,
   6], [4, 6], [5, 6], [6, 6],
   [6, 7], [7, 7], [8, 7], [7,
   7], [7, 8], [8, 8], [9, 8],
    [10, 8], [10, 7]
]
```

Table 1: Sample Trajectories

1.4 Sampling of sensor observations.

The table in 2 show the sampled observations. Every element of the list of list is the sensor outputs at a time step

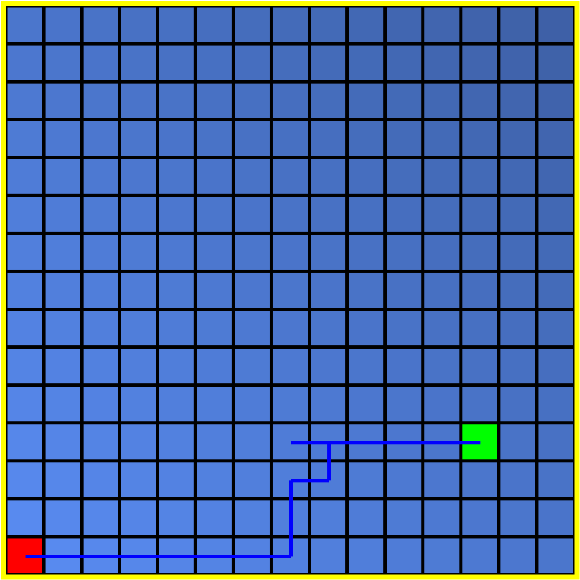

Figure 5: Trajectory 1

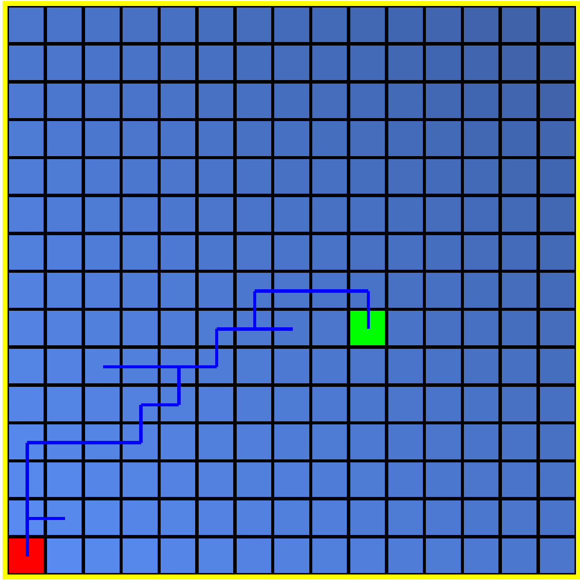

Figure 6: Trajectory 2

Sensor Observations for trajectory 1

```
[
    [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
    [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 1, 0, 0],
   [1, 0, 0, 0], [1, 1, 0, 0],
    [1, 1, 0, 0], [1, 1, 0, 0],
   [1, 1, 0, 0], [1, 1, 0, 0],
   [1, 1, 0, 0], [1, 1, 0, 0],
    [1, 1, 0, 0], [1, 1, 0, 0],
   [1, 1, 0, 0], [1, 1, 0, 0],
   [1, 1, 0, 0], [1, 1, 0, 0],
    [0, 1, 0, 0], [1, 1, 0, 0],
   [0, 1, 0, 0], [0, 1, 0, 0],
   [0, 1, 0, 0], [0, 1, 0, 0]
]
```

Sensor Observations for trajectory 2

```
[
    [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
    [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
    [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 0],
    [1, 0, 0, 0], [1, 0, 0, 0],
   [1, 0, 0, 0], [1, 0, 0, 1],
   [1, 1, 1, 1], [1, 1, 1, 1],
    [1, 1, 1, 1], [1, 1, 1, 1],
   [1, 1, 1, 1], [1, 1, 1, 1],
   [0, 1, 1, 0], [0, 1, 1, 0]
]
```

Table 2: Sampled Observations for sampled trajectories

2 Likelihood Estimation

2.1 Calculation of Likelihood Using the Forward Algorithm

Let N be the number of hidden states and T be the length of the observation sequence $O = (O_1, O_2, \ldots, O_T)$.

1. Initialization:

$$\alpha_1(j) = \pi_j \cdot B_j(O_1) \quad \text{for } j = 1, 2, \ldots, N$$

2. Recursion:

$$\alpha_t(j) = \left(\sum_{i=1}^{N} \alpha_{t-1}(i) \cdot T_{ij} \right) \cdot B_j(O_t) \quad \text{for } t = 2, 3, \ldots, T \quad \text{and } j = 1, 2, \ldots, N$$

3. Termination:

$$P(O; \lambda) = \sum_{j=1}^{N} \alpha_T(j)$$

where $\lambda = (T, B, \pi)$ represents the HMM.
The likelihood values for all the trajectories are provided in the table 3

Trajectory Number	Likelihood Value
1	$1.524631143942145 \times 10^{-11}$
2	$1.6261572273875116 \times 10^{-17}$
3	$9.122975152435896 \times 10^{-15}$
4	$4.861371504908632 \times 10^{-26}$
5	$1.9218169100102714 \times 10^{-12}$
6	$3.9335941230409524 \times 10^{-10}$
7	$3.951581101030434 \times 10^{-9}$
8	$3.4130221174873404 \times 10^{-12}$
9	$4.5440909305471274 \times 10^{-9}$
10	$2.0444022372018324 \times 10^{-16}$
11	$3.5882975291608605 \times 10^{-25}$
12	$3.492616240923471 \times 10^{-17}$
13	$3.163020611888358 \times 10^{-17}$
14	$9.550545030942883 \times 10^{-37}$
15	$2.138038039063107 \times 10^{-22}$
16	$2.7384552644979547 \times 10^{-9}$
17	$4.749269286682186 \times 10^{-9}$
18	$6.938432439422844 \times 10^{-22}$
19	$2.036198428016649 \times 10^{-20}$
20	$2.697322227895803 \times 10^{-24}$

Table 3: Likelihood vs Trajectory Number

2.2 Inference from observed likelihood values

The likelihood values are quite small, this might be due to the reason that the likelihood of an observation sequence is calculated by multiplying the probabilities of the hidden state transitions and the emmision transitions which in itself might be small values. Longer observation sequences lead to more multiplications of small probabilities, further reducing the likelihood value.

3 Decoding

3.1 Decoding Using the Viterbi Algorithm

The Viterbi algorithm is used to find the most likely sequence of hidden states **S** given a sequence of observations **O** in a Hidden Markov Model (HMM). It can be described as follows:

1. Initialization:

For each state i at time $t = 1$:

$$v_1(i) = \pi_i \cdot b_i(o_1)$$

$$bt_1(i) = 0$$

where π_i is the initial probability of state i, and $b_i(o_1)$ is the emission probability of observing o_1 given state i.

2. Recursion:

For each time $t = 2, 3, \ldots, T$ and each state j:

$$v_t(j) = \max_i [v_{t-1}(i) \cdot a_{ij}] \cdot b_j(o_t)$$

$$bt_t(j) =_i [bt_{t-1}(i) \cdot a_{ij}] \cdot b_j(o_t)$$

where a_{ij} is the transition probability from state i to state j, and $b_j(o_t)$ is the emission probability of observing o_t given state j.

3. Termination:

Find the maximum probability over all states at the final time step T:

$$P^* = \max_i v_T(i)$$

$$q_T^* = \arg\max_i v_T(i)$$

where P^* is the best score and q_T^* is the start of the backtrace

3.2 Mean Manhattan Distance

The mean Manhatten Distances across all the 20 trajectories between the decoded trajectory and the original trajectories with respect to the 30 time steps can be found in table 4

3.3 Plotting the True vs. Decoded State Sequences

The decoded trajectories for the trajectories we had sampled and plotted in 5 and 6 can be found in 9

Time Step	Mean Manhatten Distance
1	0.0
2	0.75
3	1.6
4	1.95
5	2.65
6	2.9
7	3.25
8	3.45
9	3.35
10	3.85
11	4.15
12	4.35
13	4.4
14	4.5
15	4.8
16	5.0
17	5.3
18	5.25
19	5.15
20	5.2
21	5.6
22	5.5
23	6.0
24	6.1
25	6.4
26	6.75
27	7.2
28	7.9
29	8.5
30	3.85

Table 4: The mean Manhatten Distance across all sequences between the predicted and the original trajectories with respect to time steps

4 Learning Parameters

For all the parts, the T and the B matrices are filled with values from a uniform distribution of probablities i.e in case of T, the values are filled in the way that at any state there is a $\frac{1}{5}$ chance to move to left, right, up, down or remain at the same position. In case of the B matrix, for any state the probabilities assigned is $\frac{1}{16}$ to all the 16 possible observation states

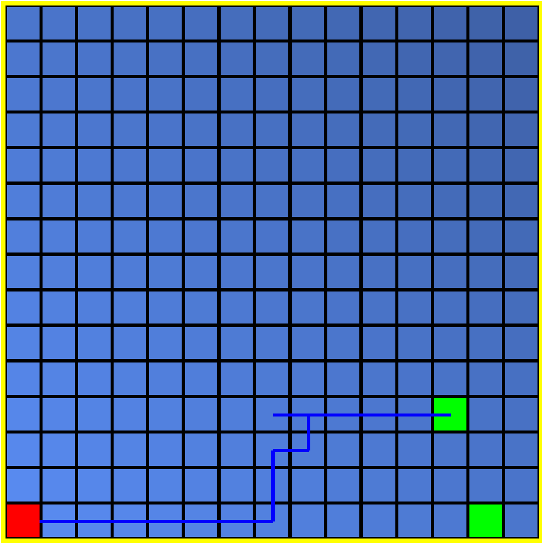

Figure 7: Trajectory 1

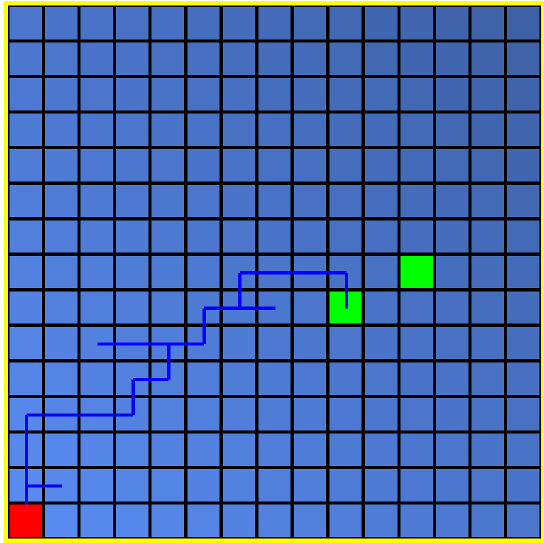

Figure 8: Trajectory 2

Figure 9: True vs Decoded state sequences in gridworld

4.1 Baum-Welch Algorithm for Estimating State Transition Matrix

The average KL Divergence between the T matrix and the B matrix is plotted across all the 20 iterations in 10

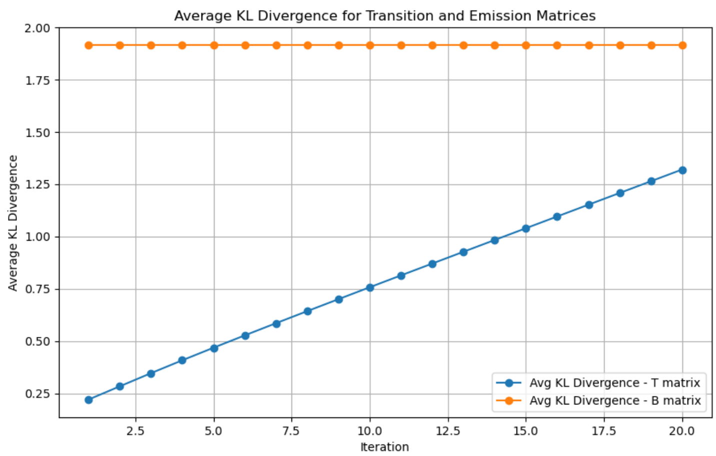

Figure 10: Average KL divergence with keeping B fixed

4.2 Joint Estimation of Transition and Emission Matrices

The average KL Divergence between the T matrix and the B matrix is plotted across all the 20 iterations in 11

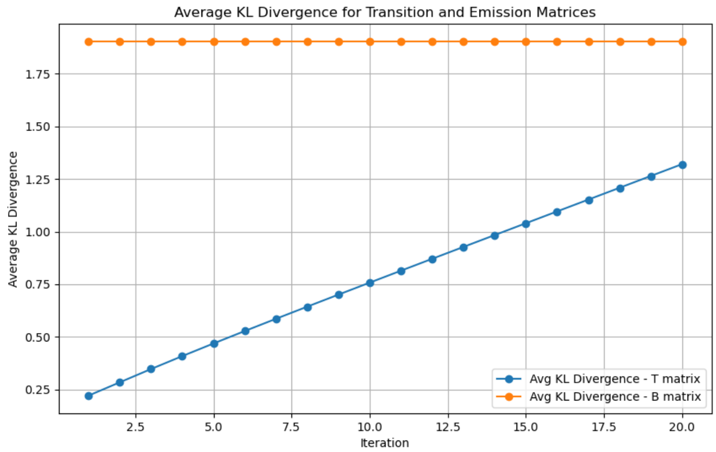

Figure 11: Average KL divergence

References

- Speech & Language Processing by Jurafsky and Martin
- https://adeveloperdiary.com/data-science/machine-learning/derivation-and-implementation-of-baum-welch-algorithm-for-hidden-markov-model/
- https://bmcbioinformatics.biomedcentral.com/articles/10.1186/s12859-021-04080-0