HW 1

1-1.

(1) $3n^2+10n \in O(n^2)$ (2) $\frac{n^2}{10}+2^n \in O(2^n)$ (3) $21+\frac{1}{n} \in O(1)$

(4) $\log n^3 = 3\log n \in O(\log n)$ (5) $10\log 3^n = 10\log 3 \cdot n = O(n)$.

1-2

如果 $f(n) \in O(1)$，则 $\exists c \in R_+, N \in N_+, \forall n \geq n_0 \quad f(n) \leq c \leq 2c \quad \therefore f(n) \in O(2)$.

所以 $O(1)$和$O(2)$所表示的函数差别应在于常数项 (比常数项更小的渐进项. eg. $\frac{1}{n}$)

1-3

$\because \lim\limits_{n\to\infty} \frac{2}{\log n} = 0$, $\lim\limits_{n\to\infty} \frac{\log n}{n^{2/3}} = \lim\limits_{n\to\infty} \frac{\frac{1}{n\ln 2}}{\frac{2}{3}n^{-\frac{1}{3}}} = \lim\limits_{n\to\infty} \frac{3n^{\frac{1}{3}}}{2\ln 2\cdot n} = 0$, $\lim\limits_{n\to\infty} \frac{n^{2/3}}{20n} = 0$, $\lim\limits_{n\to\infty} \frac{20n}{4n^2} = 0$

$\lim\limits_{n\to\infty} \frac{4n^2}{3^n} = 0$, $\lim\limits_{n\to\infty} \frac{3^n}{n!} = \lim\limits_{n\to\infty} \frac{3^n}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = \lim\limits_{n\to\infty} \frac{1}{\sqrt{2\pi n}}\left(\frac{3e}{n}\right)^n = 0$.

$\therefore 2 \in O(\log n)$, $\log n \in O(n^{\frac{2}{3}})$, $n^{\frac{2}{3}} \in O(20n)$, $20n \in O(4n^2)$, $4n^2 \in O(3^n)$, $3^n \in O(n!)$

$\therefore$ 渐近阶数从小到大为: $2, \log n, n^{\frac{2}{3}}, 20n, 4n^2, 3^n, n!$

1-4.

(1) $t_1 = 3\times 2^{n_0} = \frac{3\times 2^{n_1}}{64} \Rightarrow n_1 = n_0+6$ 可解决规模为 $n+6$ 的问题

(2) $t_2 = n_0^2 = \frac{n_1^2}{64} \Rightarrow n_1 = 8n_0$ 可解决规模为 $8n$ 的问题

(3) 可解决问题的规模不限.

1-7

由Striling公式，$\lim\limits_{n\to\infty} \frac{n!}{n^n} = \lim\limits_{n\to\infty} \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1+\Theta\left(\frac{1}{n}\right)\right)}{n^n} = \lim\limits_{n\to\infty} \frac{\sqrt{2\pi n}\left(1+\Theta\left(\frac{1}{n}\right)\right)}{e^n} = 0 \quad \therefore n! = o(n^n)$ 得证.

1-9.

由已知: $T_{avg}(n) = \Theta(f(n))$ 即 $\exists c_1, c_2 \in R_+, n_0 \in N_+, \forall n \geq n_0$ 有 $c_1 f(n) \leq T_{avg}(n) \leq c_2 f(n)$.

$\because T_{max}(n) \geq T_{avg}(n)$ $\therefore$ 则 $\exists c_3 = c_1 \in R_+, n_0 \in N_+, \forall n \geq n_0$, 有 $T_{max}(n) \geq T_{avg}(n) \geq c_3 f(n)$

$\therefore T_{max}(n) = \Omega(f(n))$