4. Analysing data using k-means clustering, chi-squared test and choropleth maps

November 29, 2023

In this notebook I shall look at the question are there a higher proportion of under 35-year-olds in more urban areas of England compared with more rural areas. To do this I will classify areas as being more or less urban or rural based on their population density and the percentage of developed land.

I use k-means clustering to cluster Local Authorities in England into groups depending on how rural/urban they are and then use chi-squared test to check the null hypothesis that the degree of urbanisation and whether the proportion of the population aged 0-34 is above or below average are unrelated.

I then produce a number of choropleth maps to show visually how the areas with a high proportion of under 35s also tend to correlate with areas with a high population density and a high percentage of developed land.

```
import pandas as pd
import numpy as np
import scipy.stats
from scipy.stats import chi2_contingency
from sklearn import cluster
from sklearn.metrics import silhouette_samples
import json
import seaborn as sns
import matplotlib.pyplot as plt
import folium
```

```
[2]: combined_df = pd.read_csv(r"combined_df.csv", index_col=0)
combined_df.head()
```

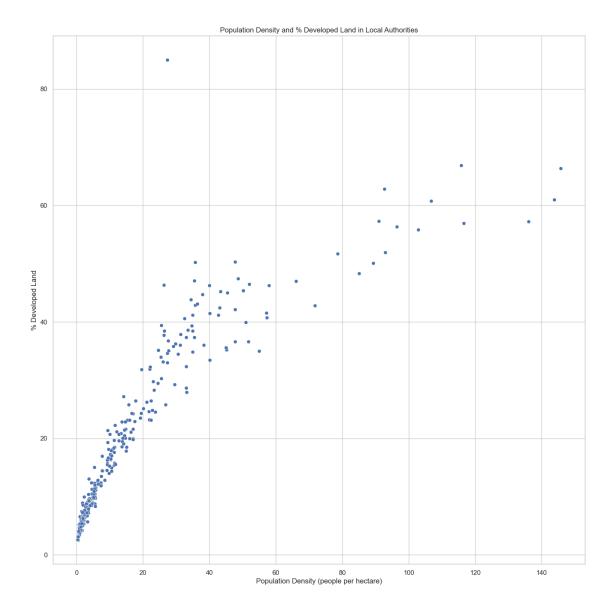
```
[2]:
                                        Total Land Area (hectares)
         ONS Code Local Authority Name
      E07000223
                                  Adur
                                                        4364.505691
     1 E07000026
                             Allerdale
                                                      132054.795869
     2 E07000032
                          Amber Valley
                                                       26543.663279
     3 E07000224
                                                       22450.444783
                                  Arun
     4 E07000170
                              Ashfield
                                                       10955.544524
        % Developed Land Proportion pop aged 0-34 All persons
     0
                   21.57
                                         36.279070
                                                           64500
```

1	3.67	35.379813	96100				
2	9.82	36.846276	126200				
3	12.43	34.344660	164800				
4	18.52	40.934283	126300				
	Population Density	(people per hectare)					
0	14.778306						
1	0.727728						
2	4.754430						
3	7.340612						
4		11.528409					

Now I would like to produce a scatterelot of my two variables for determining degree of urbanisation, population density and proportion of developed land in order to see whether any clusters of LAs can be seen by eye.

```
[3]: #Reset the chart plot size
sns.set(rc={"figure.figsize":(16, 16)},
style='whitegrid')

sns.scatterplot(data=combined_df, x="Population Density (people per hectare)",
y="% Developed Land").set(title='Population Density and % Developed Land in
Local Authorities');
```



The scatterplot shows an overall fairly strong positive correlation between population density and the proportion of developed land. There is one outlier with over 80% developed land but a lower population density than might be expected. I am going to find out what this LA is.

```
[4]: #Identify outlier in scatterplots of population density and proportion of developed land outlier = combined_df.loc[(combined_df['% Developed Land'] >= 80)] outlier
```

[4]: ONS Code Local Authority Name Total Land Area (hectares) \
56 E09000001 City of London 314.938269

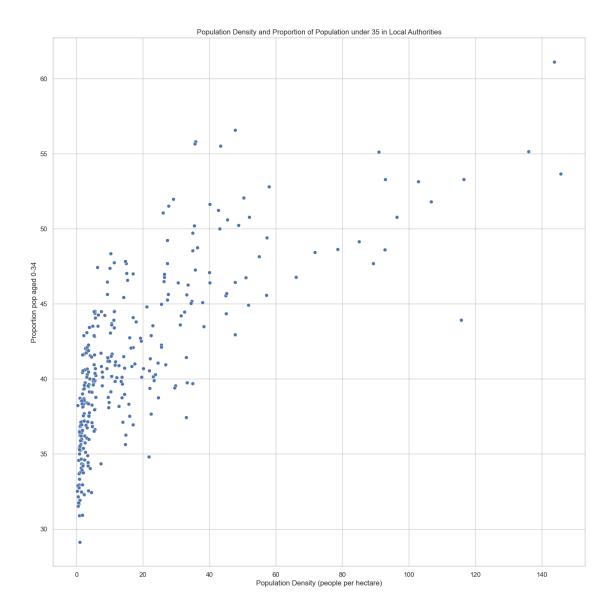
% Developed Land Proportion pop aged 0-34 All persons \

56 84.96 47.674419 8600

```
Population Density (people per hectare) 56 27.306939
```

The fact that the outlier is the City of London makes sense as this area has a very high proportion of developed land with office blocks etc but relatively few people live there compared to other areas of London.

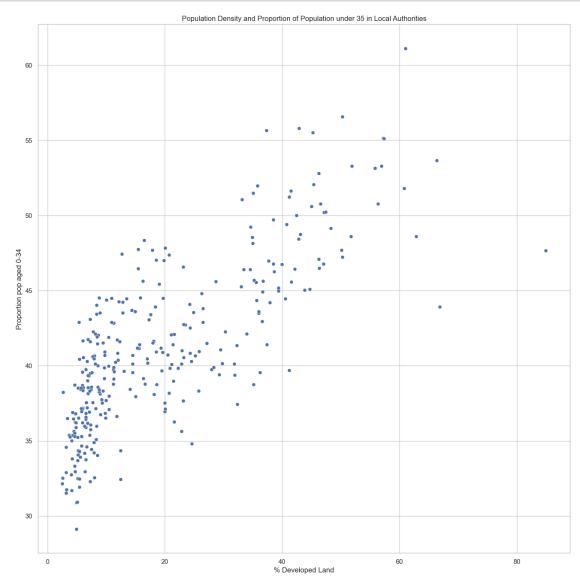
There do seem to be some clusters visible on the scatterplot. I would like to see whether the proportion of the population under 35 correlates with population density so I will produce a scatterplot to check this.



It appears to correlate but I will also check using Pearson's R correlation coefficient.

r: 0.7243610651560741, p: 1.6274380452504376e-51

There does appear to be fairly strong correlation at 0.724. Now I will check whether the proportion of the population under 35 correlates with the proportion of developed land so I will produce a scatterplot to check this.



Again it seems to correlate but I will check calculating the correlation coefficient.

```
print('r: {r}, p: {p}'.format(r=r, p=p))
```

r: 0.7824573582715375, p: 4.05672284260325e-65

This has a fairly high positive correlation of 0.78. Now I would like to use the k-means clustering algorithm to cluster the LAs into groups depending on their degree of urbanisation. I will try using various numbers for k.

Population Density (people per hectare)

18.52

Local Authority Name

Ashfield

 Adur
 14.778306

 Allerdale
 0.727728

 Amber Valley
 4.754430

 Arun
 7.340612

 Ashfield
 11.528409

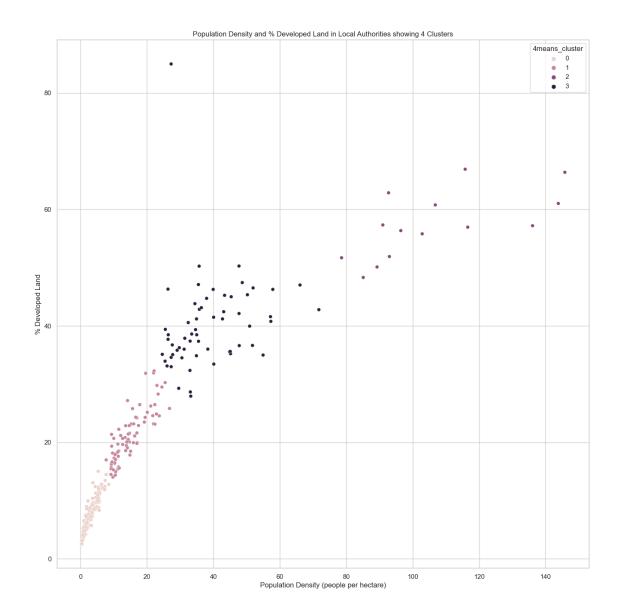
```
[10]: # Call the fit function on the clusterer to fit the data to the 4 means.
assignedClusters_clust = kmeans4.fit(clustering_df)

combined_df['4means_cluster'] = assignedClusters_clust.labels_
combined_df.head()
```

```
[10]:
         ONS Code Local Authority Name Total Land Area (hectares) \
                                   Adur
      0 E07000223
                                                        4364.505691
      1 E07000026
                              Allerdale
                                                      132054.795869
      2 E07000032
                           Amber Valley
                                                       26543.663279
      3 E07000224
                                   Arun
                                                       22450.444783
      4 F.07000170
                               Ashfield
                                                       10955.544524
```

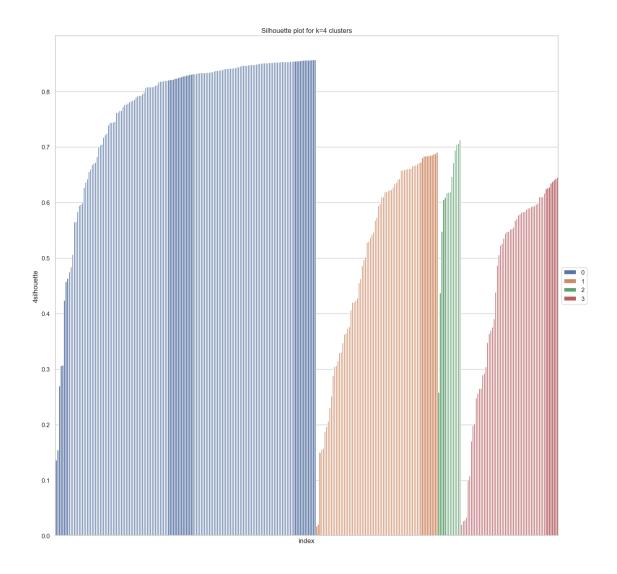
% Developed Land Proportion pop aged 0-34 All persons $\$

```
21.57
      0
                                            36.279070
                                                              64500
      1
                      3.67
                                            35.379813
                                                              96100
                      9.82
                                            36.846276
      2
                                                             126200
      3
                     12.43
                                            34.344660
                                                             164800
      4
                     18.52
                                            40.934283
                                                             126300
         Population Density (people per hectare)
                                                    4means_cluster
      0
                                         14.778306
      1
                                          0.727728
                                                                  0
      2
                                          4.754430
                                                                  0
      3
                                          7.340612
                                                                  0
      4
                                                                  1
                                         11.528409
[11]: #Reset the chart plot size
      sns.set(rc={"figure.figsize":(16, 16)},
              style='whitegrid')
      sns.scatterplot(data=combined_df, x="Population Density (people per hectare)", __
       _{\hookrightarrow}y= "% Developed Land", hue = "4means_cluster").set(title='Population Density_
       →and % Developed Land in Local Authorities showing 4 Clusters');
```



The diagram shows that a cluster of 4 might be appropriate for the data. I will now calculate the silhouette statistics to generate a silhouette plot to check its suitability.

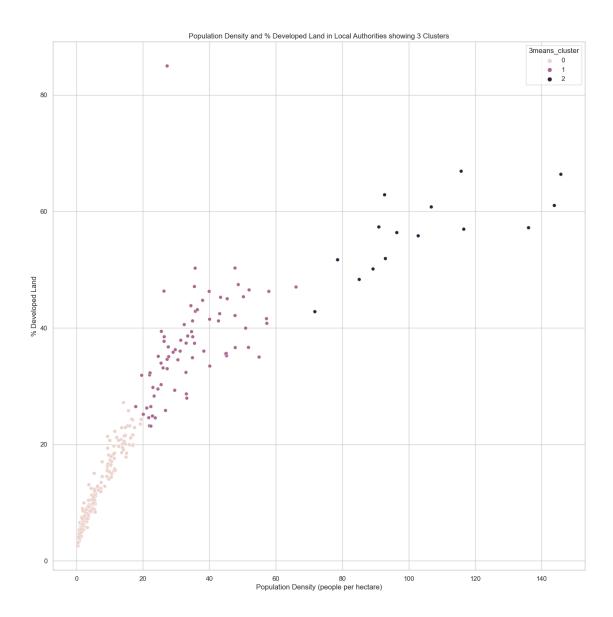
```
[12]:
          ONS Code Local Authority Name Total Land Area (hectares) \
      0 E07000223
                                                         4364.505691
                                   Adur
      1 E07000026
                              Allerdale
                                                       132054.795869
      2 E07000032
                           Amber Valley
                                                        26543.663279
      3 E07000224
                                                        22450.444783
                                   Arun
      4 E07000170
                               Ashfield
                                                        10955.544524
         % Developed Land Proportion pop aged 0-34 All persons \
      0
                    21.57
                                           36.279070
                                                            64500
                     3.67
                                                            96100
      1
                                           35.379813
      2
                     9.82
                                           36.846276
                                                           126200
      3
                    12.43
                                           34.344660
                                                           164800
      4
                    18.52
                                           40.934283
                                                           126300
         Population Density (people per hectare)
                                                   4means_cluster 4silhouette
      0
                                       14.778306
                                                                      0.684705
                                                                1
      1
                                         0.727728
                                                                0
                                                                      0.798607
      2
                                         4.754430
                                                                0
                                                                      0.743754
      3
                                        7.340612
                                                                0
                                                                      0.424257
      4
                                        11.528409
                                                                      0.546599
[13]: #Create new df and sort it to produce silhouette plot
      combined df sorted = combined df.sort_values(["4means_cluster", "4silhouette"])
      # Reset the index
      combined_df_sorted.reset_index(drop=True, inplace=True)
[14]: \#Produce\ silhouette\ plot\ for\ k=4\ clusters
      ax = sns.barplot(data=combined_df_sorted.reset_index(),
                       x="index",
                       y="4silhouette",
                       hue="4means cluster",
                       # The dodge=False parameter has a useful side effect of \Box
       → thickening the bars
                       dodge=False)
      #Suppress the x-axis tick labels
      ax.set(xticks=[],
             title="Silhouette plot for k=4 clusters")
      #Pop the legend out the the main plot frame
      ax.legend(loc="center left", bbox_to_anchor=(1, 0.5));
```



That looks suitable as all values are positive. Now I will look at k=3 and k=5.

```
[15]:
          ONS Code Local Authority Name Total Land Area (hectares)
      0 E07000223
                                    Adur
                                                          4364.505691
      1 E07000026
                               Allerdale
                                                        132054.795869
      2 E07000032
                            Amber Valley
                                                         26543.663279
      3 E07000224
                                    Arun
                                                         22450.444783
      4 E07000170
                                Ashfield
                                                         10955.544524
         % Developed Land
                           Proportion pop aged 0-34
                                                      All persons
      0
                    21.57
                                           36.279070
                                                             64500
      1
                     3.67
                                           35.379813
                                                             96100
      2
                     9.82
                                           36.846276
                                                            126200
      3
                    12.43
                                           34.344660
                                                            164800
      4
                    18.52
                                           40.934283
                                                            126300
         Population Density (people per hectare)
                                                   4means_cluster
                                                                    4silhouette
      0
                                        14.778306
                                                                       0.684705
                                                                 1
                                         0.727728
      1
                                                                 0
                                                                       0.798607
      2
                                         4.754430
                                                                 0
                                                                       0.743754
      3
                                         7.340612
                                                                 0
                                                                       0.424257
      4
                                        11.528409
                                                                 1
                                                                       0.546599
         3means_cluster
      0
                      0
      1
                      0
      2
                      0
      3
                      0
      4
                      0
[16]: #Reset the chart plot size
      sns.set(rc={"figure.figsize":(16, 16)},
              style='whitegrid')
      sns.scatterplot(data=combined_df, x="Population Density (people per hectare)", u
       y="% Developed Land", hue = "3means_cluster").set(title='Population Density⊔
       →and % Developed Land in Local Authorities showing 3 Clusters');
```

combined_df.head()



```
[17]: # Add the silhouette coefficients as a new column in the # combined DataFrame:

combined_df["3silhouette"] = silhouette_samples(

combined_df[["Population Density (people per hectare)", "% Developed_

→Land"]],

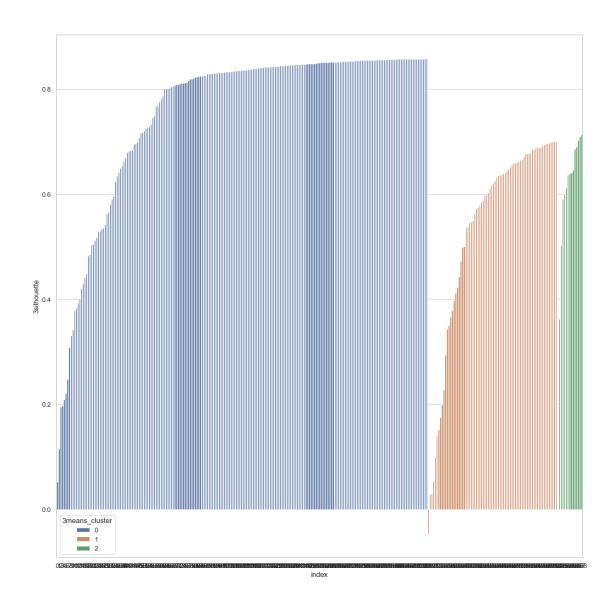
combined_df["3means_cluster"])

combined_df.head()
```

```
Amber Valley
      3 E07000224
                                                        22450.444783
                                    Arun
      4 E07000170
                               Ashfield
                                                        10955.544524
         % Developed Land
                           Proportion pop aged 0-34 All persons
                    21.57
      0
                                           36.279070
                                                            64500
                     3.67
                                           35.379813
                                                            96100
      1
      2
                     9.82
                                           36.846276
                                                           126200
      3
                    12.43
                                           34.344660
                                                           164800
      4
                    18.52
                                           40.934283
                                                           126300
         Population Density (people per hectare)
                                                   4means_cluster
                                                                   4silhouette \
      0
                                        14.778306
                                                                       0.684705
      1
                                         0.727728
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      2
                                         4.754430
                                                                0
                                                                       0.743754
      3
                                         7.340612
                                                                0
                                                                       0.424257
      4
                                                                       0.546599
                                        11.528409
                                                                1
         3means_cluster
                        3silhouette
      0
                            0.429623
                      0
                      0
                            0.819370
      1
      2
                      0
                            0.846533
      3
                      0
                            0.801439
                            0.624451
                      0
[18]: #Create new df and sort it to produce silhouette plot
      combined_df_sorted = combined_df.sort_values(["3means_cluster", "3silhouette"])
      # Reset the index
      combined_df_sorted.reset_index(drop=True, inplace=True)
[19]: #Produce silhouette plot for k=3 clusters
      x = sns.barplot(data=combined_df_sorted.reset_index(),
                       x="index",
                       y="3silhouette",
                       hue="3means_cluster",
                       # The dodge=False parameter has a useful side effect of
       ⇔thickening the bars
                       dodge=False)
      #Suppress the x-axis tick labels
      ax.set(xticks=[],
             title="Silhouette plot for k=3 clusters")
      #Pop the legend out the the main plot frame
      ax.legend(loc="center left", bbox_to_anchor=(1, 0.5));
```

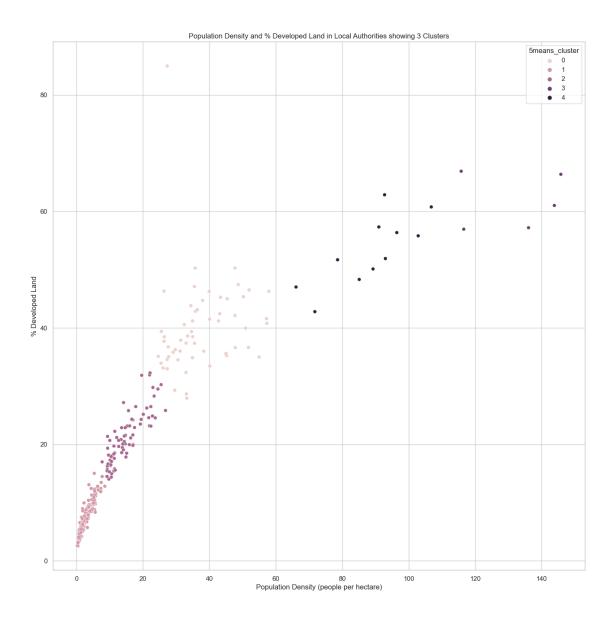
26543.663279

2 E07000032



```
combined_df.head()
          ONS Code Local Authority Name Total Land Area (hectares)
[20]:
      0 E07000223
                                   Adur
                                                         4364.505691
      1 E07000026
                              Allerdale
                                                       132054.795869
      2 E07000032
                           Amber Valley
                                                        26543.663279
      3 E07000224
                                   Arun
                                                        22450.444783
      4 E07000170
                               Ashfield
                                                        10955.544524
         % Developed Land Proportion pop aged 0-34 All persons
      0
                    21.57
                                           36.279070
                                                            64500
                     3.67
                                           35.379813
                                                            96100
      1
      2
                     9.82
                                           36.846276
                                                           126200
      3
                    12.43
                                           34.344660
                                                           164800
      4
                    18.52
                                           40.934283
                                                           126300
         Population Density (people per hectare)
                                                   4means_cluster
                                                                   4silhouette
      0
                                       14.778306
                                                                      0.684705
                                                                1
      1
                                        0.727728
                                                                0
                                                                      0.798607
      2
                                        4.754430
                                                                0
                                                                      0.743754
      3
                                        7.340612
                                                                0
                                                                      0.424257
      4
                                       11.528409
                                                                1
                                                                      0.546599
         3means_cluster
                        3silhouette
                                      5means_cluster
      0
                      0
                            0.429623
                            0.819370
                      0
                                                    1
      1
      2
                      0
                            0.846533
                                                    1
      3
                      0
                            0.801439
                                                    1
      4
                      0
                            0.624451
[21]: # Add the silhouette coefficients as a new column in the
      # combined DataFrame:
      combined_df["5silhouette"] = silhouette_samples(
          combined_df[["Population Density (people per hectare)", "% Developed ⊔
       combined_df["5means_cluster"])
      combined_df.head()
[21]:
          ONS Code Local Authority Name Total Land Area (hectares)
                                   Adur
      0 E07000223
                                                         4364.505691
      1 E07000026
                              Allerdale
                                                       132054.795869
      2 E07000032
                           Amber Valley
                                                        26543.663279
      3 E07000224
                                                        22450.444783
                                   Arun
      4 E07000170
                               Ashfield
                                                        10955.544524
         % Developed Land Proportion pop aged 0-34 All persons \
```

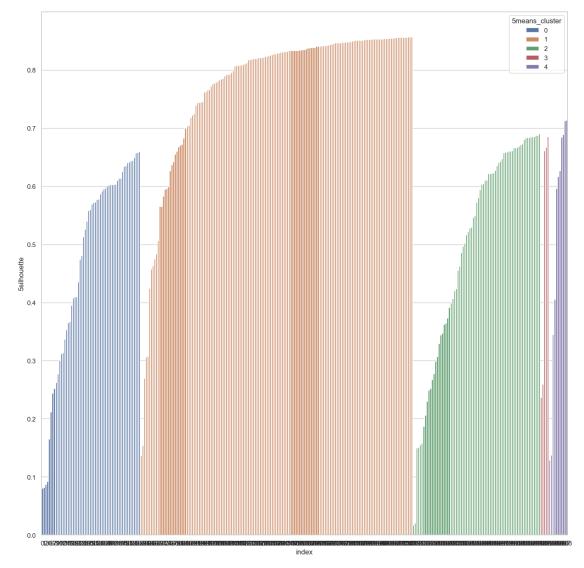
```
0
                    21.57
                                           36.279070
                                                             64500
      1
                     3.67
                                           35.379813
                                                             96100
      2
                     9.82
                                                            126200
                                           36.846276
      3
                    12.43
                                           34.344660
                                                            164800
      4
                    18.52
                                           40.934283
                                                            126300
         Population Density (people per hectare)
                                                   4means_cluster
                                                                    4silhouette \
      0
                                        14.778306
                                                                       0.684705
                                                                 1
      1
                                         0.727728
                                                                       0.798607
                                                                 0
      2
                                         4.754430
                                                                 0
                                                                       0.743754
      3
                                         7.340612
                                                                 0
                                                                       0.424257
      4
                                        11.528409
                                                                       0.546599
         3means_cluster
                         3silhouette
                                       5means_cluster 5silhouette
      0
                            0.429623
                                                    2
                                                           0.684705
                      0
      1
                      0
                            0.819370
                                                    1
                                                           0.798607
      2
                      0
                            0.846533
                                                    1
                                                           0.743754
      3
                      0
                            0.801439
                                                    1
                                                           0.424257
      4
                                                    2
                            0.624451
                                                           0.546599
[22]: #Reset the chart plot size
      sns.set(rc={"figure.figsize":(16, 16)},
              style='whitegrid')
      sns.scatterplot(data=combined_df, x="Population Density (people per hectare)", u
       y="% Developed Land", hue = "5means_cluster").set(title='Population Density⊔
       →and % Developed Land in Local Authorities showing 3 Clusters');
```



```
# The dodge=False parameter has a useful side effect of dodge=False)

#Suppress the x-axis tick labels
ax.set(xticks=[],
title="Silhouette plot for k=5 clusters")

#Pop the legend out the the main plot frame
ax.legend(loc="center left", bbox_to_anchor=(1, 0.5));
```



Now I will calculate the mean silhouette statistics to see which cluster works well.

```
[25]: #Calculate mean silhouette statistics for k=3
total3silo = combined_df['3silhouette'].sum()
mean_silo_3 = total3silo/len(combined_df.index)
print('Mean silhouette statistic for 3 clusters is',mean_silo_3)

#Calculate mean silhouette statistics for k=4
total4silo = combined_df['4silhouette'].sum()
mean_silo_4 = total4silo/len(combined_df.index)
print('Mean silhouette statistic for 4 clusters is',mean_silo_4)

#Calculate mean silhouette statistics for k=5
total5silo = combined_df['5silhouette'].sum()
mean_silo_5 = total5silo/len(combined_df.index)
print('Mean silhouette statistic for 5 clusters is',mean_silo_5)
```

```
Mean silhouette statistic for 3 clusters is 0.6939985652940626
Mean silhouette statistic for 4 clusters is 0.6377568217123043
Mean silhouette statistic for 5 clusters is 0.636503598970922
```

While 3 clusters has the highest mean silhouette statistic and the clustering looks suitable however in three clusters the vast majority of Local Authorities have been clustered into one group so it does not differentiate sufficiently between the LAs with the lowest proportion of developed land use and population density which were the vast majority of LAs. Therefore I am going to use four clusters which has the second highest mean silhouette statistic. Five clusters has a very similar mean silhouette statistic but it has two clusters with very few Local Authorities in them.

In order to check whether more urbanised areas do have a larger proportion of their population under 35 I am going to divide each LA into a category according to whether they have a proportion of 35 years old above or equal to the average or below the average. First I need to calculate the average.

```
[26]: #Calculate average proportion of 0-34 year olds
sum_prop_under_35 = combined_df['Proportion pop aged 0-34'].sum()
mean_prop_under_35 = sum_prop_under_35/len(combined_df.index)
print('Average proportion of under 35s is', mean_prop_under_35)
```

Average proportion of under 35s is 41.11009398543328

Now I will assign the category of above (and equal to) or below average proportion of population under 35.

```
[27]:
            ONS Code
                                       Local Authority Name
           E07000223
      0
                                                         Adur
      1
           E07000026
                                                    Allerdale
      2
           E07000032
                                                Amber Valley
      3
           E07000224
                                                         Arun
      4
           E07000170
                                                     Ashfield
      . .
      304
           E07000245
                                                West Suffolk
      305
           E06000060
                                             Buckinghamshire
      306
           E06000058
                       Bournemouth, Christchurch and Poole
      307
           E06000059
                                                       Dorset
      308
           E07000246
                                  Somerset West and Taunton
           Total Land Area (hectares)
                                          % Developed Land Proportion pop aged 0-34
      0
                            4364.505691
                                                      21.57
                                                                              36.279070
                                                       3.67
      1
                         132054.795869
                                                                              35.379813
      2
                           26543.663279
                                                       9.82
                                                                              36.846276
      3
                           22450.444783
                                                      12.43
                                                                              34.344660
      4
                           10955.544524
                                                                              40.934283
                                                      18.52
      . .
      304
                          103448.896657
                                                       7.29
                                                                              41.601780
                                                                              40.462846
      305
                          156488.645695
                                                       7.90
      306
                           17379.276560
                                                      29.77
                                                                              40.144891
      307
                          252036.384802
                                                       5.44
                                                                              32.481560
      308
                          121005.325243
                                                       4.84
                                                                              36.213469
                         Population Density (people per hectare)
           All persons
                                                                      4means_cluster
                  64500
      0
                                                          14.778306
                                                                                    1
                                                                                    0
      1
                  96100
                                                           0.727728
      2
                 126200
                                                           4.754430
                                                                                    0
      3
                 164800
                                                           7.340612
                                                                                    0
      4
                 126300
                                                          11.528409
                                                                                    1
      304
                 179800
                                                           1.738056
                                                                                    0
                                                           3.534442
                                                                                    0
      305
                 553100
      306
                 400300
                                                          23.033180
                                                                                    1
                                                           1.506132
      307
                 379600
                                                                                    0
      308
                 157400
                                                           1.300769
                                                                                    0
           4silhouette
                         3means_cluster
                                           3silhouette
                                                         5means_cluster
                                                                          5silhouette
      0
               0.684705
                                                                       2
                                        0
                                              0.429623
                                                                              0.684705
                                        0
                                                                       1
      1
               0.798607
                                                                              0.798607
                                              0.819370
      2
               0.743754
                                        0
                                              0.846533
                                                                       1
                                                                              0.743754
      3
                                        0
               0.424257
                                              0.801439
                                                                       1
                                                                              0.424257
                                        0
      4
               0.546599
                                              0.624451
                                                                              0.546599
      304
               0.848265
                                        0
                                              0.853586
                                                                       1
                                                                              0.848265
```

305	0.833326	0	0.857598	1	0.833326
306	0.376583	1	0.378123	2	0.344042
307	0.846451	0	0.844900	1	0.846451
308	0.834335	0	0.837885	1	0.834335

Above/below average proportion of pop aged 0-34 0 Below 1 Below 2 Below 3 Below 4 Below 304 Above 305 Below 306 Below 307 Below 308 Below

[309 rows x 14 columns]

Now I want to produce a cross-tabulation of my results in order to perform a chi-squared test to check the null hypothesis that the proportion of under 35 year olds in an English LA is independent of how urbanised that LA is.

```
[28]: Above/below average proportion of pop aged 0-34 Above Below 4means_cluster

0 27 133
1 38 37
2 14 0
3 54 6
```

Now to run the chi-squared test.

```
[29]: #Chi-squared test
chi2, p, dof, expected = chi2_contingency(contingency)
print (f'Chi-square Statistic : {chi2} ,p-value: {p}')
```

```
Chi-square Statistic: 118.95815218087833, p-value: 1.2936252717544955e-25
```

The p-value is very small which offers some evidence against the null hypothesis that the degree of urbanisation and whether the proportion of the population aged 0-34 is above or below average are unrelated. So it looks like there may be a link between the proportion of under 35 year olds in an Local Authority and its level of urbanisation.

Now I would like to produce three choropleth maps showing 1. Proportion of under 35 year olds

in an LA. 2. Population density of an LA 3. Proportion of developed land in an LA, to see if the areas of higher proportion of under 35 year olds correlate with areas with higher population densities and higher proportions of developed land.

```
[30]: # Opening JSON file
f = open('Data/LA_2021_UK.geojson')

# returns JSON object as a dictionary
data = json.load(f)

# Closing file
f.close()
```

[31]: <folium.folium.Map at 0x205574f0ac0>

```
fill_color='RdYlBu', fill_opacity=0.8, 
an_fill_color="transparent", 
legend_name='Population Density (average number_u)

of people per hectare)', 
threshold_scale=[0, 15, 30, 45, 60, 150])

choropleth.add_to(EnglandMap2)

EnglandMap
```

[32]: <folium.folium.Map at 0x205574f0ac0>

[33]: <folium.folium.Map at 0x2055bb04f40>

Looking at the three maps there does seem to be some correlation between the areas with the highest proportion of their population under 35 and the areas which have a high population density and have a large percentage of their total land as developed land. This leads some weight to my hypothesis that there are a higher proportion of under 35-year-olds in more urban areas of England compared with more rural areas.

```
[]:
```