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Applications of quantum Fisher information and joint estimation in squeezed states

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We analyze the quantum Fisher information to describe the precision limit for the squeezed state. To realize the true simultaneous measurement of position and momentum, we construct two commutation operators on the system and joint ancillary system to measure position and momentum. Our measurement's classical Fisher information matrix is optimal and close to the quantum Fisher information in one direction. There is a minimum area of the ellipse of the matrix's quadratic form.

Quantum parameter estimation

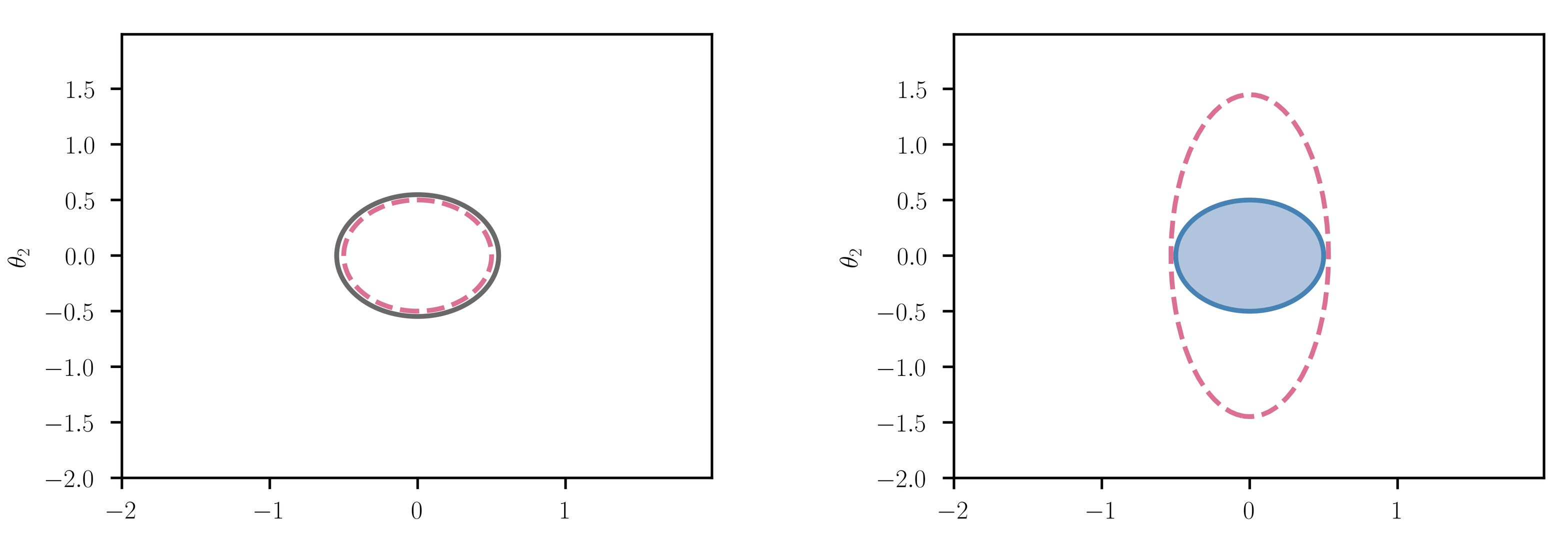
The state of a quantum system depends on an unknown vector parameter $\theta = (\theta_1, \theta_2, \dots \theta_n) \in \mathbb{R}^n$ and is described by a parametric family of density operators ρ_θ. Denote covariance by $\mathcal{E}_{jk}(M) \coloneqq \mathbb{E}_\theta[(\hat{\theta}_j - \theta_j)(\hat{\theta}_k - \theta_k)]$, classical Fisher information matrix (CFIM) by $F_{jk} \coloneqq \mathbb{E}_\theta\left[\frac{\partial \ln p_\theta(x)}{\partial \theta_j}\frac{\partial \ln p_\theta(x)}{\partial \theta_k}\right]$, and quantum Fisher information matrix (QFIM) by $\mathcal{F}_{jk} \coloneqq \mathrm{Tr}(L_j L_k \rho_\theta)$ under measurement M, where L_j satises $\partial\rho_\theta = (L_j\rho_\theta + \rho_\theta L_j)/2$. According to Cramér-Rao bound and quantum Cramér-Rao bound,

$$\mathcal{E} \geq \mathrm{F}^{-1} \geq \mathcal{F}^{-1}.$$

In statistics, the inverse of the covariance matrix is called precision matrix or concentration matrix Ω and its definition is $\Omega := \mathcal{E}^{-1}$. So the above inequality chain can be written as

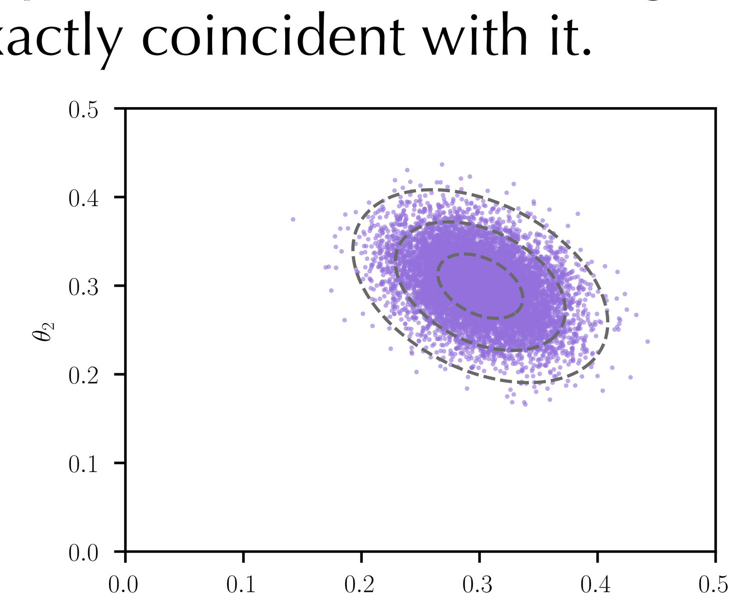
$$\Omega \leq F \leq \mathcal{F}.$$

Let quadratic form functions of these matrices be equal to 1 to get an ellipse equation to visualize the inequality as follows:



The gray ellipse is decided by Ω and the red ellipse is decided by F. The gray solid line ellipse is always outside the red dotted line ellipse because of the CRB. By choosing different measurement bases, the red ellipse may coincide with the gray ellipse exactly. The blue solid ellipse is decided by $\mathcal{F}$. The outer red ellipse can be as close as possible to the inner blue ellipse and at most, it is tangent to it in one direction, but not exactly coincident with it.

The ellipse decided by Ω is called error ellipse and it can be related to observed data falling within an ellipse due to the central limit theorem as left figure.



Squeezed state

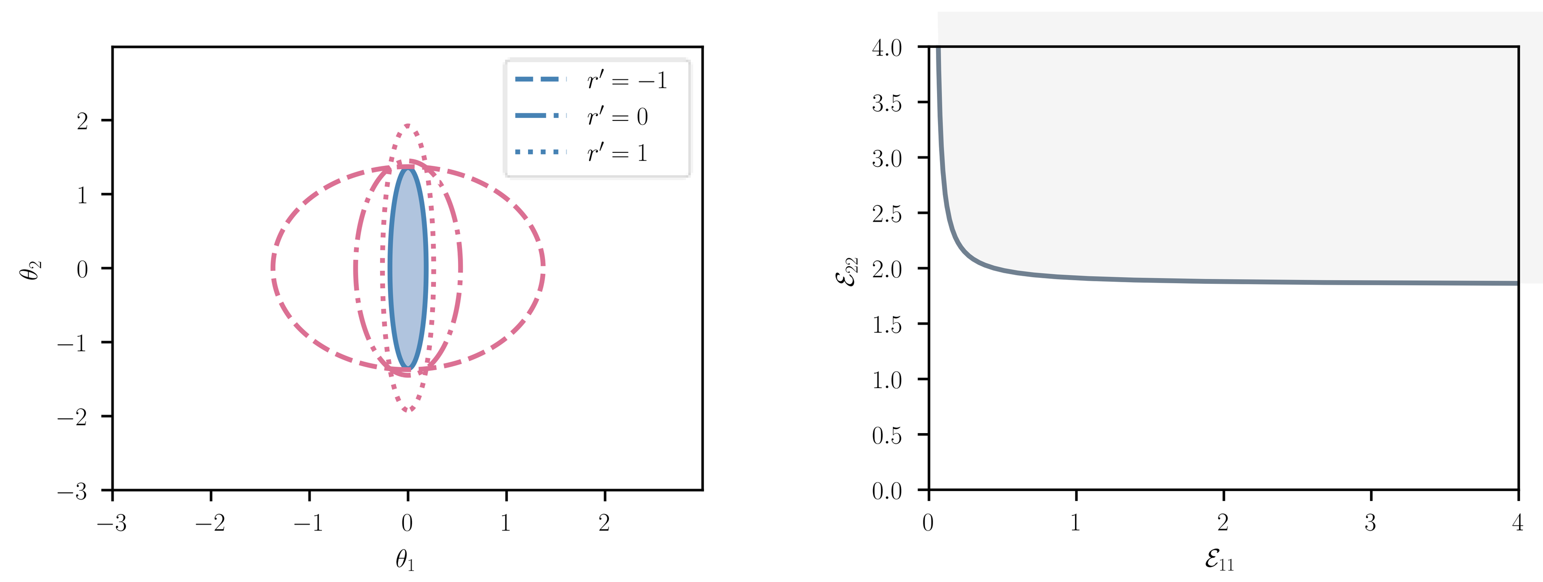
The squeezed state is defined as

$$|\alpha;\zeta\rangle = D(\alpha)S(\zeta)|0\rangle.$$

The parameters of interest here are the real and imaginary parts of α, i.e., $\theta_1 = \alpha_r, \theta_2 = \alpha_i$. Let $\mathcal{M}$ (denote the by letter) be the squeezed state considered to be measured by using an ancillary mode as A (denote the by letter with prime). Define the dimensionless coordinate and momentum operators for these two modes by

$$Q = \frac{a + a^\dagger}{2}, \qquad P = \frac{a - a^\dagger}{2i},$$
$$Q' = \frac{a' + a'^\dagger}{2}, \qquad P' = \frac{a' - a'^\dagger}{2i},$$

The two observable operators are $A = Q - Q', B = P + P'$. The normalized simultaneous eigenstate of A and B are $|\xi,\eta\rangle = \pi^{-1/2}\int e^{2i\eta q}|q\rangle_Q \otimes |q - \xi\rangle_{Q'}\mathrm{d}q$. Respectively, the ellipse decided by CFIM and QFIM is as follow (left):



By changing r', The outer red ellipse can be close to the inner blue ellipse and almost tangent to it. Under the above measurement scheme, the information regret constraint is shown as in above (right) figure. The region below the line is forbidden. The measurement is optimal because the point of this scheme falls at the gray line.

Conclusion

Under our joint estimation, the noncommutative operators P and Q are transposed to commutative operators and this measurement is optimal. The ellipse decided by the covariance matrix coincides with CFIM. The ellipse decided by CFIM is outer of QFIM and can be close to QFIM in one direction.

References

C. Helstrom, IEEE Transactions on Information Theory **14**, 234 (1968).
X.-M. Lu and X. Wang, Physical Review Letters **126**, 120503 (2021).

Our paper*: Applications of quantum Fisher information and joint estimation in squeezed states* (provisional) in production…