Mid Exam Simulation 2

1. Show that $(p{\rightarrow}q)\wedge(q{\rightarrow}r)\rightarrow(p{\rightarrow}r)$ is a tautology.
2. Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

   a) $A\cap(B{-}C)$
   b) $(A\cap B)\cup(A\cap C)$

3. Determine whether each of these functions is a bijection from R to R.
   a) $f(x) = -3x+4$
   b) $f(x) = -3x^2+7$

4. Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her
   a) mobile phone number.
   b) student identification number.
   c) final grade in the class.
   d) home town.

5. Prove that $1^2-2^2+3^2-\cdots+(-1)^{n-1}n^2 = (-1)^{n-1} n(n + 1)/2$ whenever n is a positive integer.

6. Give a recursive definition of
   a) the set of odd positive integers.
   b) the set of positive integer powers of 3.

7. How many strings of three decimal digits
   a) begin with an odd digit?
   b) have exactly two digits that are 4s?

8. How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

9. Assume that the probability a child is a boy is 0.51 and that the sexes of children born into a family are independent. What is the probability that a family of five children has
   a) exactly three boys?
   b) at least one boy?
   c) at least one girl?

10. Use Adjoint to compute inverse of the following matrix:

$$A = \begin{bmatrix} 1 & -3 & 0 \\ 1 & 2 & 2 \\ 2 & 1 & -1 \end{bmatrix}$$