# Introductory Linear Algebra for AI

## - Useful Matrix Properties

Find the rank of $A$ and also the rank of $A^{\top}$:
($q$ is unknown)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 1 & q \end{bmatrix}$$

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Can you find the matrix $A$ that satisfies
"The only solution of $Ax = [1,2,3]^\top$ is $x = [0,1]^\top$"?

The complete solution to $Ax = [1,3]^\top$ is $x = [1,0]^\top + c[0,1]^\top$. Find $A$

Show that $\{v_1, v_2, v_3\}$ are independent but $\{v_1, v_2, v_3, v_4\}$ are not.

$$v_1 = [1,0,0]^\top, v_2 = [1,1,0]^\top,$$
$$v_3 = [1,1,1]^\top, v_4 = [2,3,4]^\top$$

Prove that $if\ a = 0\ or\ d = 0\ or\ f = 0$, the colums of U are dependent:

$$U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

**Is it possible to construct 3x3 matrix that satisfies:**

Column space contains: $[1,2,-3]^{\top},[2,-3,5]^{\top}$

**Null-space contains:** $[1,1,1]^{\top}$

If $A^\top Ax = 0$ then $Ax = 0$. Why?

Find $\hat{x}$ that makes $\|Ax - b\|^2$ as small as possible

Suppose $A^\top + A = 0$. Then, $x^\top A x = ?$

# Linear Systems and Stability

$$V(\mathbf{x}) = \mathbf{x}^T \mathbf{P} \mathbf{x}, \qquad \mathbf{P} = \mathbf{P}^T > 0$$

$$\dot{V}(\mathbf{x}) = \mathbf{x}^T \left( \mathbf{A}^T \mathbf{P} + \mathbf{P} \mathbf{A} \right) \mathbf{x}$$