

① a)  $5n^3 + 2n^2 + 3n = O(n^3)$

$$5n^3 + 2n^2 + 3n \leq 5n^3 + 2n^3 + 3n^3$$

$$5n^3 + 2n^2 + 3n \leq 10n^3$$

$$c = 10$$

$$5 + 2 + 3 \leq 10$$

$$\boxed{10 \leq 10} \quad n_0 = 1$$

$$n \leq n_0$$

b)  $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

$$c_1 \cdot n \leq \sqrt{7n^2 + 2n - 8} \leq c_2 \cdot n$$

$$\text{let } n_0 = 1$$

$$c_1 \leq \sqrt{1} \leq c_2$$

$$\boxed{c_1 \leq 1 \leq c_2}$$

$$n_0 = 1$$

$$c_1 = 0$$

$$c_2 = 2$$

c)  $d(n) = O(f(n))$

$$\downarrow$$

$$\text{for } n \geq n_0 \quad d(n) \leq c \cdot f(n)$$

$e(n) = O(g(n))$

$$\downarrow$$

$$\text{for } n \geq m_0 \quad e(n) \leq k \cdot g(n)$$

$$e(n) \cdot d(n) \leq k \cdot c \cdot f(n) \cdot g(n)$$

$$\hookrightarrow d(n)e(n) = O(f(n)g(n))$$

②

def example 1(1st) has run time  $\Theta(n^2)$

def example 2(1st) has run time  $\Theta(n)$

def example 3(1st) has run time  $\Theta(n^2)$

def example 4(n) has run time  $\Theta(n \log n)$