Untitled

September 3, 2022

Resource state:
Here we will take any arbitary state $a|0\rangle + b|1\rangle$ in the first qubit and $|+\rangle$ in the second qubit. Then apply control-z gate. This will create the resource state

$$a|0\rangle_1 |+\rangle_2 + b|1\rangle_1 |-\rangle_2 \tag{1}$$

Now, after some calculation we will be able to represent this state as

$$\frac{1}{\sqrt{2}}|+\rangle_1 (a|+\rangle_2 + b|-\rangle_2) + \frac{1}{\sqrt{2}}|-\rangle_1 (a|+\rangle_2 - b|-\rangle_2) \tag{2}$$

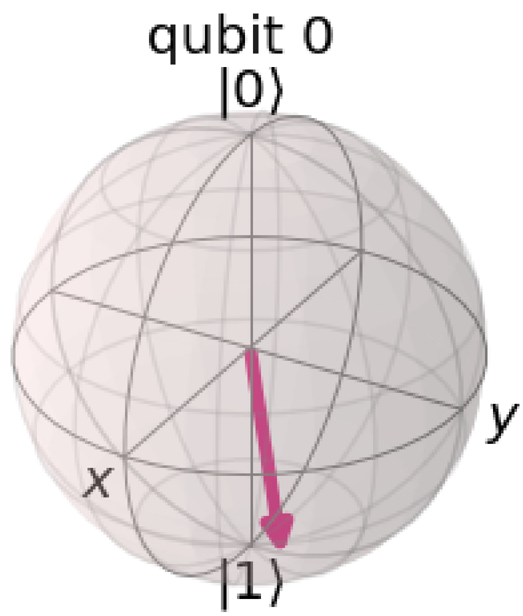
We have to make measurement on qubit 1 in the X basis.
1. If qubit 1 comes to $|+\rangle$ the qubit 2 will collapse to $a|+\rangle_2 + b|-\rangle_2$
2. If qubit 1 comes to $|-\rangle$ the qubit 2 will collapse to $a|+\rangle_2 - b|-\rangle_2$
So we have successfully teleported unknown state from qubit 1 to qubit 2 with some operation.

[1]:
```
from qiskit import QuantumCircuit, QuantumRegister, ClassicalRegister, Aer
from qiskit.quantum_info import random_statevector
from qiskit.visualization import plot_bloch_multivector
from qiskit.extensions import Initialize
instate = random_statevector(2)
qr = QuantumRegister(2)
cr = ClassicalRegister(1)
qc = QuantumCircuit(qr, cr)
qc.append(Initialize(instate), [0])
qc.h(1)
qc.cz(0, 1)
qc.h(0)
qc.measure(0, 0)
qc.h(0)
qc.h(1)
qc.z(1).c_if(cr, 1)
```

[1]:
```
<qiskit.circuit.instructionset.InstructionSet at 0x16fc31713c0>
```

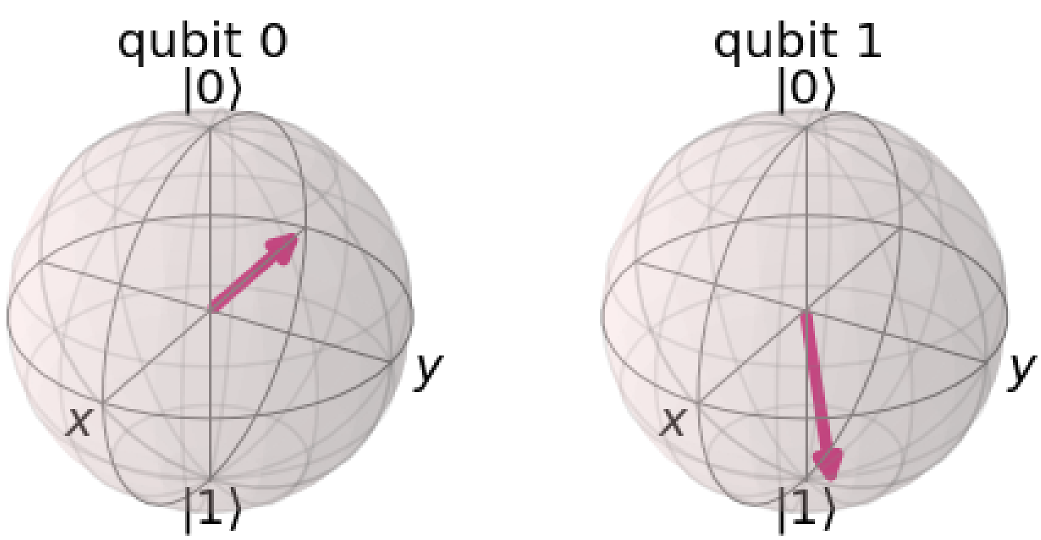
[2]:
```
plot_bloch_multivector(instate)
```

[2]:



[3]:
```
sim = Aer.get_backend('aer_simulator')
qc.save_statevector()
out_vector = sim.run(qc).result().get_statevector()
plot_bloch_multivector(out_vector)
```

[3]:



[ ]:
```
```

1 Quantum Error correction in measurement-based quantum computing

1.1 Resource state

Initial state they have taken is the traditional box-cluster state:

$$|\psi_{box}\rangle = \frac{1}{2}[|0\rangle_1 |+\rangle_2 |+\rangle_3 |0\rangle_4 + |0\rangle_1 |-\rangle_2 |-\rangle_3 |1\rangle_4 + |1\rangle_1 |-\rangle_2 |-\rangle_3 |0\rangle_4 + |1\rangle_1 |+\rangle_2 |+\rangle_3 |1\rangle_4] \quad (3)$$

How to produce this state?
1. Initialize all the qubit in $|+\rangle$ state
2. Then apply Control-Z gate between qubit- 1 and 2, 2 and 4, 3 and 4, 1 and 3 3. We know the operation of Control-Z gate as: $|x, y\rangle = (-1)^{xy} |x, y\rangle$

1.2 Error correction protocol

In the quantum error correction code we follow 3 steps:
1. encoding 2. syndrome measurement 3. decoding

1.2.1 Encoding

The encoding has been accomplished by measuring the qubit 1 in the basis $\alpha^* |0\rangle + \beta^* |1\rangle, \beta |0\rangle - \alpha |1\rangle$. We can think of it in slightly different way in the light of simple quantum mechanics. If $|\psi_1\rangle = \alpha^* |0\rangle + \beta^* |1\rangle$ and $|\psi_2\rangle = \beta |0\rangle - \alpha |1\rangle$ then state after encoding ψ_1 and ψ_2 in the qubit 1 the resultant state will be $\langle\psi_1| |\psi_{box}\rangle$ and $\langle\psi_2| |\psi_{box}\rangle$ respectively.(simple inner product) Suppose, we want $|\psi_1\rangle$ to encode then the remaining three-qubit state will be

$$\psi_3 = \frac{\alpha}{\sqrt{2}}(|{+}{+}\rangle_{23} |0\rangle_4 + |{-}{-}\rangle_{23} |1\rangle_4) + \frac{\beta}{\sqrt{2}}(|{-}{-}\rangle_{23} |0\rangle_4 + |{+}{+}\rangle_{23} |1\rangle_4) \quad (4)$$

1.2.2 Syndrome measurement and decoding

In the error correction code, they have checked out two kinds of error which is going to be corrected. The first one is $e^{-i\frac{\pi}{2}z}$ which is basically a phase error.
Now the operation of this kind of error basically changes the state $|+\rangle$ to $|-\rangle$. Thus, when this kind of error happens in qubit 2 then the state $|psi_3\rangle$ becomes

$$\psi_3 = \frac{\alpha}{\sqrt{2}}(|{-}{+}\rangle_{23} |0\rangle_4 + |{+}{-}\rangle_{23} |1\rangle_4) + \frac{\beta}{\sqrt{2}}(|{+}{-}\rangle_{23} |0\rangle_4 + |{-}{+}\rangle_{23} |1\rangle_4) \quad (5)$$

Now, the qubit 2 and 3 are measured, If they are found to be at state $|{-}{+}\rangle$ then the state after measurement will be $\alpha |0\rangle + \beta |1\rangle$. So basically, no correction is needed. If they are found to be at state $|{+}{-}\rangle$ the state after measurement will be $\alpha |1\rangle + \beta |0\rangle$. So the error is corrected by just applying a X gate.