SWE111: ENGINEERING MATHEMATICS I 08/02/20

FIRST SEMESTER

ASSIGNMENT TO BE CONSIDERED AS THE CONTINUOUS ASSESSMENT

ANSWER ALL QUESTIONS

A-SEVERAL REAL VARIABLE FUNCTIONS

1. Let $f(x,y) = ln(x+y-1)$
   - i. Evaluate (a) $f(1,1)$ (b) $f(e^3, 1)$ — 2mks
   - ii. Find and sketch the domain of f. — 2mks
   - iii. Find the range of f — 1mk
2. Given the function: $h(x,y) = \sqrt{16 - x^2 - y^2}$
   - a) Evaluate the value of h at the point $\left(\sqrt{3}, 2\right)$ — 1mk
   - b) Find and sketch the domain. — 2mks
   - c) Find the range — 1mk
3. Compute the given limits.
   a) $\lim\limits_{(x,y)\to(0,4)} \dfrac{\cosh(xy)}{x^2+y-1}$ b) $\lim\limits_{(x,y,z)\to(1,1,2)} \dfrac{e^{x+y-z}}{x-z}$ — 2mks
4. Find all the second partial derivatives of $f(x,y) = x^3y^5 + 2x^4y$ — 6mks
   Hence, verify that the conclusion of Schwarz theorem holds, that is $f_{xy} = f_{yx}$. — 1mk
5. Use implicit differentiation to show that if
   $x^2 + y^2 + z^2 = 3xyz$ then $\frac{\partial z}{\partial x} = \frac{2x-3yz}{3xy-2z}$ — 2mks
6. Determine whether $u = e^{-x}cosy - e^{-y}cosx$ is a solution ot Laplace's equation $u_{xx} + u_{yy} = 0$ — 2mks
7. Find the equation of the tangent plane to the surface
   $z = x^2 + xy + 3y^2$ at the point $(1,1,5)$ — 2mks
8. Find the linear approximation of the function $f(x,y) = \ln(x-3y)$ at $(7,2)$ and use it to approximate $f(6.9, 2.06)$ — 3mks
9. If $z = 5x^2 + y^2$ and (x,y) changes from $(1,2)$ to $(1.05, 2.1)$, compare the values of Δz and dz. — 3mks
10. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10cm high and 4cm in diameter if the metal in the top and bottom is 0.1cm thick and the metal in the sides is 0.05cm thick. — 3mks
11. Given the function $f(x,y) = x^2y^3 - y^4$, the point $P(2,1)$ and the vector $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j}$
    - a) Find the gradient of f. — 2mks
    - b) Evaluate the gradient of f at P. — 1mk
    - c) Find the directional derivative of f at or the rate of change of f at P in the direction of $\boldsymbol{v}$ — 2mks
12. Find and classify the critical points of $f(x,y) = x^3 - 12xy + 8y^3$ — 7mks