SWE121-1: ENGINEERING MATHEMATICS II

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COUSRE OUTLINE

I. ANALYSIS I: 3 CREDITS (45 HOURS); L, T, SPW

I.1. NUMERICAL FUNCTIONS OF A REAL VARIABLE:

I.1.1 Logarithmic and exponential functions

I.1.2 Reciprocal circular functions

I.1.3 Hyperbolic functions and their reciprocals.

I.2. SEVERAL REAL VARIABLE FUNCTIONS

I.2.1 First and second order partial derivative

I.2.2 Schwarz theorem

I.2.3 Differential applications

I.2.4 Composite functions

I.2.5 Differential forms

I.2. 6 Vector operators

I.3. TAYLOR SERIES AND LIMITS

I.4. INTEGRATION (SIMPLE AND MULTIPLE)

I.5. DIFFERENTIAL EQUATIONS

II. LINEAR ALGEBRA I: 2 CREDITS (30 HOURS); L, T, SPW

II.1 Vector space of finite dimensions N$\leq$ 4

II.2 Matrices

I. ANALYSIS I: 3 CREDITS (45 HOURS); L, T, SPW

I.1. NUMERICAL FUNCTIONS OF A REAL VARIABLE

I.1.1 Logarithmic and exponential functions

Objectives

By the end of this lesson you should be able to:

- ✓ Define exponential and logarithmic functions.
- ✓ Sketch the graphs of exp. and log. Functions.
- ✓ Investigate the properties of exp. And log. Functions.
- ✓ Simplify exp. and log. Expressions.
- ✓ Solve exp. and log. Equations and inequations.

Base, Index and Radicals

$8 = 2^3$ ← Index or exponent or power

Base

1. If $a > 0$ then $a^n > 0, \forall n \in \mathbb{R}$. A positive number raised to any power gives a positive value.

2. If $a < 0$ then $a^n > 0$ if n is even, i.e. a negative number raised to an even power gives a positive value.

3. If $a < 0$ then $a^n < 0$ if n is odd, i.e. a negative number raised to an odd power gives a negative value.

4. $\boxed{a^x = a^n \Rightarrow x = n}$

5. $x^n = a^n \Rightarrow x = a$ if n is odd

$x^n = a^n \Rightarrow x = \pm a$ if n is even.

Examples

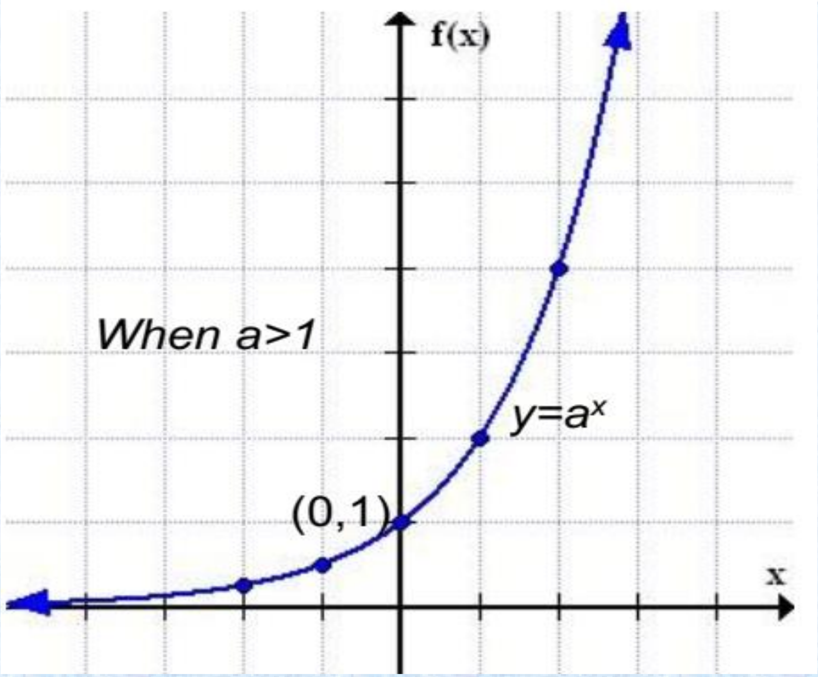
Solve

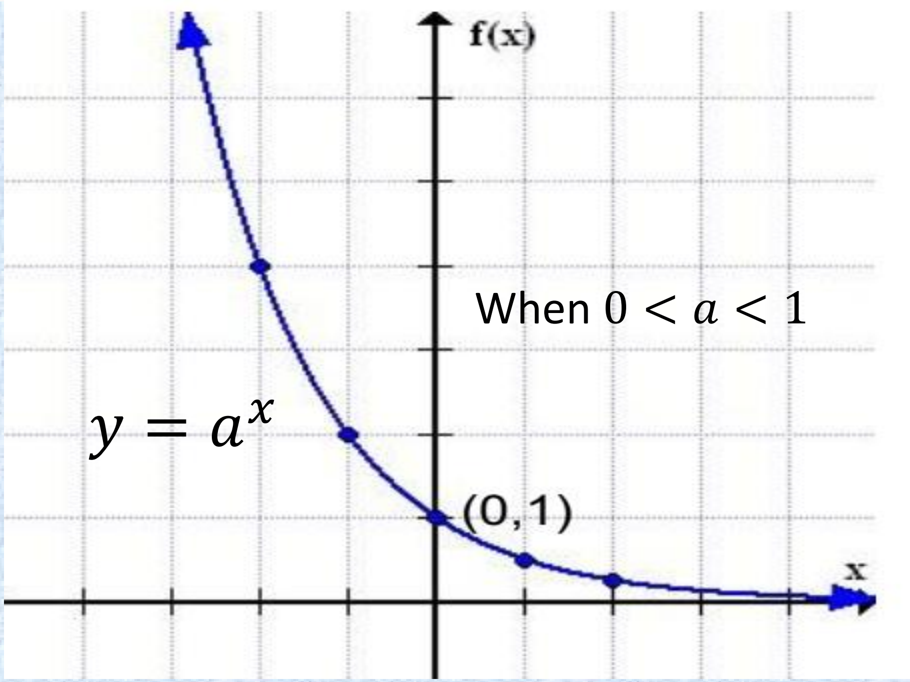
1. $4^{3x-2} = 2^{4x+2}$
2. $x^2 = 9$
3. $x^3 = 8$

Exponential functions

Definition: If a is a positive number and x is any number, we define the exponential function as

$$f(x) = a^x \text{ or } y = a^x$$





Properties of the Exponential Function $y = f(x) = a^x$

- Domain: $\mathbb{R}\ or\ (-\infty, +\infty)$
- Range: $\mathbb{R}^+\ or\ (0, +\infty)$
- y-intercept: $(0, 1)$
- Continuous in $\mathbb{R}\ or\ (-\infty, +\infty)$
- x-axis is a horizontal asymptote: $as\ x \to -\infty, a^x \to 0$
- Monotonicity:
  - Increasing if $a > 1$
  - Decreasing if $0 < a < 1$

Base and Index

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Laws of Exponents

1. $a^m \times a^n = a^{m+n}$

2. $\dfrac{a^m}{a^n} = a^{m-n}, a \neq 0$

3. $\left(a^m\right)^n = \left(a^n\right)^m = a^{mn}$

4. $a^0 = 1, \quad a \neq 0$

5. $a^{-n} = \dfrac{1}{a^n}, a \neq 0$

6. $(ab)^n = a^n b^n$

7. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}, b \neq 0$

8. $\left(\dfrac{a}{b}\right)^{-n} = \left(\dfrac{b}{a}\right)^n = \dfrac{b^n}{a^n}, a \neq 0, b \neq 0$

9. $\sqrt[n]{a} = a^{\frac{1}{n}}$ and $\sqrt{a} = a^{\frac{1}{2}}$

10. $\left(\sqrt[n]{a}\right)^m = \left(\sqrt[n]{a^m}\right) = a^{\frac{m}{n}}, n \neq 0$

Surds or Radicals

1. $\sqrt{ab} = \sqrt{a}\sqrt{b} : e.g. \sqrt{3} \times \sqrt{2} = \sqrt{6}$

2. $\sqrt{a} \times \sqrt{a} = a : e.g. \sqrt{2} \times \sqrt{2} = 2$

$\left(\sqrt{a}\right)^2 = a$ e.g. $\left(\sqrt{3}\right)^2 = 3$

3. $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}, b \neq 0 :$

e.g. $\sqrt{\frac{49}{169}} = \frac{\sqrt{49}}{\sqrt{169}} = \frac{7}{13}$

LOGARITHMS

Introduction

From indices: $16 = 2^4$

- The number *4* is called the power or exponent or index.
- If a positive number y is such that $y = a^x$, then x is called the logarithm of y to the base a

$$\text{If } y = a^x \text{ then } x = log_a y, \quad y > o$$

- Logarithmic and exponential functions are inverses. Thus if a and x are positive numbers, we defined the logarithmic function as

$$f(x) = log_a x \; or \; y = log_a x$$

- Definition

The logarithm of x to the base a is the power to which a is raised to give x.

Examples

1. $3^4 = 81 \Leftrightarrow Log_3 81 = 4$

2. $\log_{10} 10000 = 5$

3. $log_{10} o.oo1 = -3$

4. $\log_8 2 = \frac{1}{3}$

5. $\log_7 49 = 2$

Examples and Exercise

1) $\log_a a = 1$ because $a = a^1$

2) $\log_a 1 = 0$ because $1 = a^0$

3) $\log_a N = x \Rightarrow N = a^x$

4) From (3); $a^{\log_a N} = N$

5) The log of zero and negative numbers do not exist.

Exercise

Evaluate each of the following

1. $\log_3 9$ (2). $\log_8 2$ (3). $\log_{10} 10000$

(4) $a^{\log_a^5}$ (5) $\log_e x = 0 \Rightarrow \boxed{}$

Solution

1. $\log_3 9 = 2$

(2). $\log_8 2 = \log_8 8^{1/3} = 1/3$

(3). $\log_{10} 10000 = \log_{10} 10^4 = 4$

(4) $a^{\log_a^5} = 5$

(5) $\log_e x = 0 \Rightarrow x = 1$

Natural or Naperian Logarithm

Natural logarithms are logarithms to the base e where e is the irrational number 2.71828…
$\log_e N = LogN = \ln N$.

Using the calculator to find e

$\log_e e = 1 \Rightarrow \ln e = 1 \Rightarrow e = \boxed{1}\boxed{2^{nd}}\boxed{\ln}\quad 2.71828\ldots$

Laws of Logarithm

1) $\log ab = \log a + \log b$ OR

$\ln ab = \ln a + \ln b$

2) $\log\left(\frac{a}{b}\right) = \log a - \log b$ OR

$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$

3) $\log a^n = n \log a$ OR

$\ln a^n = n \ln a$, where $n \in \mathbb{Z}$

4) $\log_a b = \frac{\log_c b}{\log_c a}$ (Change of base)

$\Rightarrow \quad \log_{10} a = \frac{\ln a}{\ln 10}$

5) $\ln\left(\frac{1}{a}\right) = -\ln a$

6) $\ln\sqrt{a} = \frac{1}{2}\ln a$

7) $\ln a = \ln b \Rightarrow a = b$

8) $y = a^x \Rightarrow \ln y = x \ln a$

NB: $\ln^2 a = (\ln a)^2$

9) $a^x = e^{\ln a^x} = e^{x \ln a}$

Example A

1. Solve $\log(x-1)+\log(x+8)=2\log(x+2)$

2. Evaluate $$\frac{\log 25-\log 125+\frac{1}{2}\log 625}{3\log 5}$$

3. Solve the equation:
$\log\left(x^2-3\right)-\log x=\log 2$

Example **B**

- Use the properties of logarithms to simplify the following expressions:

  1. $\ln(3x^2 - 9x) + \ln(\frac{1}{3x})$

  2. $\ln \sec\theta + \ln \cos\theta$

  3. $3 \ln \sqrt[3]{x^2 - 1}$ -$\ln(x + 1)$

Example C

By using logarithms and exponentials properties as needed, solve the following for x:

1. $\log(x-2) - \log(2x-3) = \log 2$
2. $\log_8 x + \log_8(x+6) = \log_8(5x+12)$
3. $\ln(6x-5) = 3$
4. $27^x * 81^{x-2} = 9$

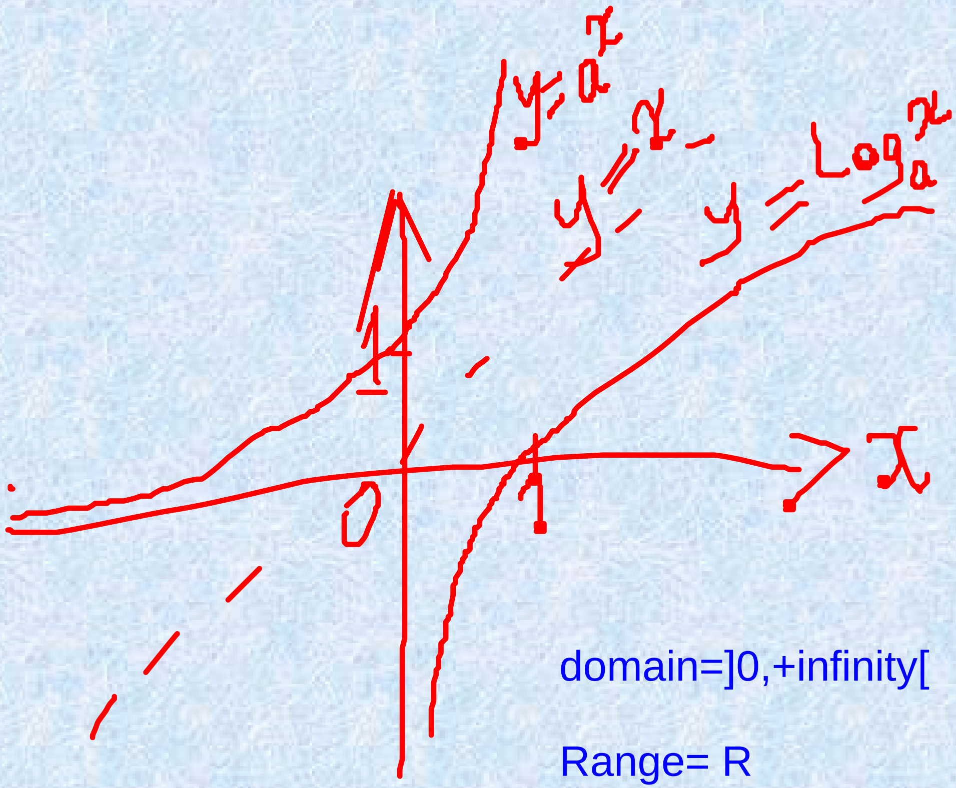
Exponential and Logarithmic Equations and Inequalities

An exponential or logarithmic equation can be solved by changing the equation into one of the following forms, where a and b are real numbers, $a > 0$, and $a \neq 1$.

1. $\boldsymbol{a^{f(x)} = b}$
Solve by taking the logarithm of each side.

2. $\boldsymbol{\log_a f(x) = \log_a g(x)}$
Solve $f(x) = g(x)$ analytically.

3. $\boldsymbol{\log_a f(x) = b}$
Solve by changing to exponential form $f(x) = a^b$.



domain=]0,+infinity[

Range= R
Function increasing