SWE121 II PROBABILITY AND STATISICS 03/08/20

SECOND SEMESTER

ASSIGNMENT TO BE CONSIDERED AS THE CONTINUOUS ASSESSMENT

ANSWER ALL QUESTIONS and SUBMIT BEFORE FRIDAY 7TH AUG. 2020

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1. i. In a class of 12 students 5 are boys and the rest are girls. Find the probability that a student selected at random is a girl.

   ii. If A denotes the event of getting an odd number and B denote the event of getting an even number when a die is tossed. List the elements of the sets $A, B, and\ A \cap B$. Hence, find the probability of getting an odd number or a number less than 4 when a die is tossed.
2. Three machines A, B and C manufacture 40%, 50% and 10% of the total production. The percentages of defective items produced by A, B and C are 2, 4 and 1 percent respectively. An item is chosen at random:
   a) Draw a tree diagram to show all the possible outcomes.
   b) Hence or otherwise find
      i. The probability that the item is defective
      ii. The probability that the item is produced by machine A given that it is defective.
      iii. The probability that the item is defective or produced by machine A
3. A biased coin is weighted such that the probability of it showing a head is $\frac{1}{3}$
   i. What is the probability of it showing a tail?
   ii. If this coin is tossed 6 times:
      a) What is the probability that ONLY the first two will yield heads?
      b) What will be the expected number of heads?
   iii. If the coin is tossed 30000 times, what will be the expected number of heads?
   iv. Comment on the possibility of the results in ii (b) and (iii).
4. Given that $P(A) = 0.1,\ P(B) = 0.2\ and\ P(A \cap B) = 0.05$.
   i. Find a) $P(A \cup B)$ (b) $P(A^C \cap B^C)$ (c) $P(A \cap B^C)$ (d) $P(A/B)$
   ii. Show that A and B are neither independent nor mutually exclusive.
5. i. Find the number of arrangements of the letters of the word TOMORROW.
   iii. There are 15 boys and 13 girls in a class.
   a) Find the number of ways of selecting 5 students from the class.
   b) Find the probability that three of the students selected are girls and the rest are boys.
6. A random variable has the following probability distribution

x	*0*	*1*	*2*	*3*
$P(X = x)$	*k*	*2k*	*3k*	*k*

   i. Find (a) the value of k b) $P(1 \leq X < 3)$ c) $P(X > 2)$
   ii. Find the mean, variance and standard deviation of X.
   iii. Find the distribution function of X and represent it graphically.

7. The probability density function of a continuous random variable X is $f(x) = \begin{cases} ax(3-x), & 0 \le x \le 3 \\ 0, & elsewhere \end{cases}$

i. Find the value of a

ii. Find $P(1 < X < 2)$ and $P(X = 2)$

iii. Find the mean and variance of X.

iv. Sketch the probability density function and state the mode.

v. Find the distribution function of X and $P(1 \le X \le 2.5)$

8. i. Given that $Y = a + bX$ and $that$ $E(X) = 2, E(Y) = 7, Var(X) = \frac{2}{3}$ and $Var(Y) = 8/3$

Find the values of the constants a and b

ii. Given the join probability distribution below

X \ Y	2	5
0	0.3	0.1
1	0.5	0.1

Find:

a) The marginal probability distributions of X and Y

b) $E(X),\ Var(X),$ and $standard$ $deviation$ of X

c) $E(Y),\ Var(Y),$ and $standard$ $deviation$ of Y

d) $E(XY)$ and $Cov(X,Y)$

e) The correlation coefficient r

f) The regression line y on x

9. The joint probability density function of two random variables X and Y is given by

$$f(x,y) = k(xy + x^2), 0 \le x \le 2,\ 0 \le y \le 1$$

Find:

a) The value of k

b) P$(X < 1)$

c) $P\left(X < 1, Y < \frac{1}{2}\right)$

10. The probability generating function of a discrete random variable, x is given by

$G(t) = k(3 + 4t + 2t^2)$

a) Show that $k = \frac{1}{9}$

b) Find $P(X = 2)$

c) Find $E(X)$ and Var(X)