SWE111: ENGINEERING MATHEMATICS I

CORRECTION OF THE FIRST SEMESTER EXAMINATION

ANSWER ALL QUESTIONS **DURATION: TWO HOURS**

1. i. Find, without using a calculator, the exact value of:

(a) $Log_2 6 - Log_2 15 + Log_2 20$ 2mks

$Log_2 6 - Log_2 15 + Log_2 20 = \log_2 \frac{6}{15} + \log_2 20 = \log_2 \frac{6 \times 20}{15} = \log_2 8 = \log_2 2^3 = 3$

(b) e^{-2ln5}

$e^{-2ln5} = e^{ln\frac{1}{5}} = \frac{1}{5} = e^{ln\frac{1}{5}} = \frac{1}{5}$ 1mk

(c) $\frac{Log_5 125}{Log_9 234}$

$\frac{Log_5 125}{Log_9 234} = \frac{Log_5 5^3}{Log_9 3^5} = \frac{3}{5 log_9 3} = \frac{3}{5} \frac{1}{\frac{log_3 3}{log_3 9}} = \frac{3}{5}\left(\frac{1}{1/2}\right) = \frac{6}{5}$ 2mks

ii. (a) Solve in $\mathbb{R}$ the equation $x^2 - 5x + 6 = 0$ 2mks

$(x-2)(x-3) = 0 \Longrightarrow x = 2, x = 3$

Hence, or otherwise solve each of the following in $\mathbb{R}$:

(b) $\ln^2 x - 5lnx + 6 = 0$

$Hence\ \ lnx = 2\ or\ lnx = 3 \Longrightarrow x = e^2\ or\ x = e^3$ 2mks

(c) $e^x + 6e^{-x} - 5 = 0$

$(e^x)^2 - 5(e^x) + 6 = 0 \Longrightarrow e^x = 2\ or\ e^3 \Longrightarrow x = ln2\ or\ ln3$ 3mks

---

2. Let $f(x,y) = x^3 + x^2y^3 - 2y^2$

i. Evaluate (a) $f(2,1)$

$f(2,1) = 2^3 + 2^2(1^3) - 2(1^2) = 10$ 2mks

ii. Find $\frac{\partial f}{\partial x}$ *and* $\frac{\partial f}{\partial y}$

$\frac{\partial f}{\partial x} = 3x^2 + 2xy^3\ and\ \frac{\partial f}{\partial y} = 3x^2y^2 - 4y$ 3mks

iii. Find the equation of the tangent plane to the surface $z = f(x,y)$ at the point $(2,1,3)$

$f_x(2,1) = 3(2)^2 + 2(2)(1^3) = 16$

$f_y(2,1) = 3(2)^2(1^2) - 4(2) = 8$

$z - 3 = f_x(2,1)(x-2) + f_y(2,1)(y-1)$

Tangent plane at $(2,1,3)$: $z - 3 = 16(x-2) + 8(y-1)$

$\Longrightarrow 16x + 8y - z = 37$ 3mk

iv. Find the gradient of f (i.e. $\nabla f(x,y)$)

$\nabla f(x,y) = f_x\mathbf{i} + f_y\mathbf{j} \Rightarrow \nabla f(x,y) = (3x^2 + 2xy^3)\boldsymbol{i} + (3x^2y^2 - 4y)\boldsymbol{j}$ 1mk

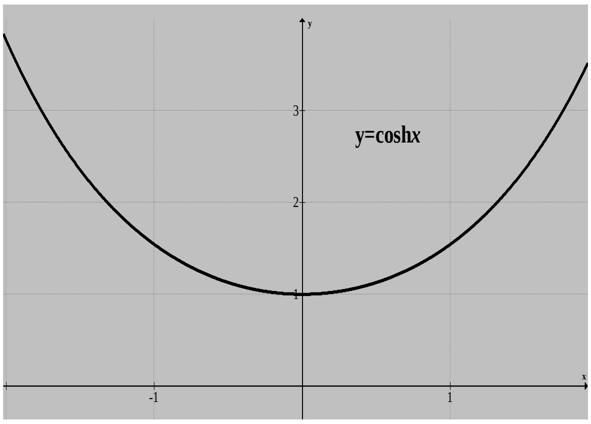
v. Find the unit vector in the direction $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j}$

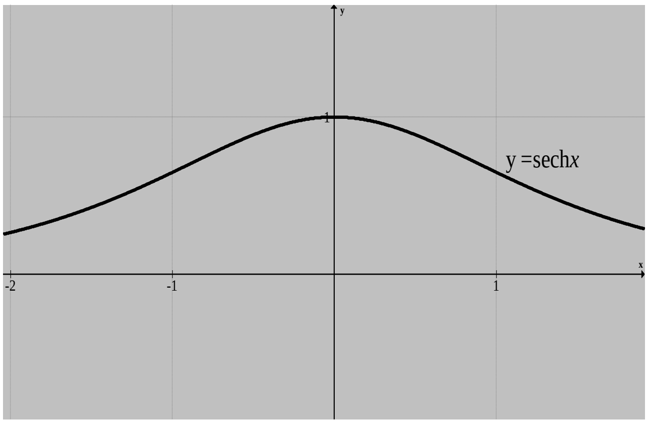
$\widehat{\boldsymbol{v}} = \frac{\boldsymbol{v}}{|v|} = \frac{3\boldsymbol{i}+4\boldsymbol{j}}{\sqrt{3^2+4^2}} = \boldsymbol{v} = \frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{j}$ 2mks

vi. Find the directional derivative of f in the direction

$D_{\hat{v}}f(x,y) = \nabla f(x,y) \cdot \widehat{\boldsymbol{v}} = \frac{3}{5}(3x^2 + 2xy^3) + \frac{4}{5}(3x^2y^2 - 4y)$ 2mks

3. i. Sketch the curves $y = coshx$ and $y = sechx$ on two separate diagrams. 2mks





ii. If $Acoshx - Bsinhx \equiv 4e^x - 3e^{-x}$, find the exact values of A and B.

$$Acoshx - Bsinhx \equiv 4e^x - 3e^{-x}$$

$$\frac{A}{2}(e^x + e^{-x}) - \frac{B}{2}(e^x - e^{-x}) \equiv 4e^x - 3e^{-x}$$

$$Ae^x + Ae^{-x} - Be^x + Be^{-x} \equiv 8e^x - 6e^{-x}$$

$$\begin{cases} A + B = 8 \\ A - B = -6 \end{cases} \Longrightarrow A = 1, B = -7$$ 4mks

iii. Show that $(coshx + sinhx)^n = \cosh(nx) + \sinh(nx)$

$$(coshx + sinhx)^n = \left(\frac{e^x+e^{-x}}{2} + \frac{e^x-e^{-x}}{2}\right)^{\mathrm{n}} = (e^x)^{\mathrm{n}} = \mathrm{e}^{\mathrm{nx}}$$

$$\cosh(nx) + \sinh(nx) = \frac{e^{nx}+e^{-nx}}{2} + \frac{e^{nx}-e^{-nx}}{2} = \mathrm{e}^{\mathrm{nx}}$$

Hence $(coshx + sinhx)^n \equiv \cosh(nx) + \sinh(nx)$ 4mks

iv. $coth(lnx) = \frac{x^2-1}{x^2+1}$

$$coth(lnx) = \frac{e^{lnx}+e^{-lnx}}{e^{lnx}-e^{-lnx}} = \frac{x+\frac{1}{x}}{x-\frac{1}{x}} = \frac{x^2+1}{x^2-1}$$ 3mks

4. i. Prove the identity $tan\theta sin\theta + cos\theta = sec\theta$

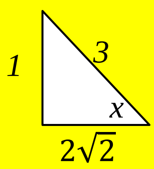
$$tan\theta sin\theta + cos\theta = \frac{sin\theta}{cos\theta} sin\theta + cos\theta$$

$$= \frac{\sin^2\theta + cos^2\theta}{\cos\theta}$$

$$= \frac{1}{cos\theta}$$

$$tan\theta sin\theta + cos\theta = sec\theta$$ 3mks

ii. If $sinx = \frac{1}{3}$ and $secy = \frac{5}{4}$, where x and y lie between are acute angles, evaluate $sin(x + y)$

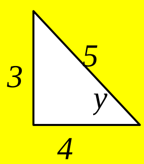


Pythagoras theorem

$$a = \sqrt{3^2 - 1^2}$$

$$a = \sqrt{8} = \sqrt{4 \times 2}$$

$$a = \sqrt{4}\sqrt{2} = 2\sqrt{2}$$



Pythagoras theorem

$$o = \sqrt{5^2 - 4^2}$$

$$o = \sqrt{9} = 3$$

$$sin(x + y) = sinxcosy + cosxsiny = \frac{1}{3} \times \frac{4}{5} + \frac{2\sqrt{2}}{3} \times \frac{3}{5} = \frac{4+6\sqrt{2}}{15}$$ 3mks

iii. If $y = \tanh(3x)$ show that $\frac{dy}{dx} = 3\,\mathrm{sech}^2(3x)$

$y = \tanh(3x) = \frac{\sinh(3x)}{\cosh(3x)}$

$\frac{dy}{dx} = \frac{\cosh(3x).\cosh(3x) - \sinh(3x).\sinh(3x)}{\cosh^2(3x)}$

$$\frac{dy}{dx} = \frac{\cosh^2(3x) - \sinh^2(3x)}{\cosh^2(3x)}$$

$\frac{dy}{dx} = \frac{1}{\cosh^2(3x)} = \operatorname{sech}^2(3x)$ 2mks

iv. Find $\int \cosh x \, dx$

$\int \cosh x \, dx = sinhx + k$ 1mks

v. Show that the function $f(x) = sinhx + xe^x$ is an odd function

$f(-x) = \sinh(-x) + (-x)e^{-x}$

$f(-x) = -(sinhx + xe^{-x})$

$f(-x) \neq -f(-x)$

$Hence\ f\ is\ not\ an\ odd\ function$ 2mks

---

5. Given that C_f is the curve of the function $f(x) = \frac{lnx}{x}$.

a) Determine the domain of f

f is defined when $x > 0\ and\ x \neq 0$

$D_f =]0, +\infty[$ 1mk

b) Evaluate $\lim_{x\to 0^+} f(x)$ and $\lim_{x\to+\infty} f(x)$.

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1}{x}\ln x = \left(\frac{1}{0^+}\right)(-\infty) = -\infty$$

$$\lim_{x\to+\infty} f(x) = 0$$

2mks

c) Deduce the vertical asymptote of $f(x)$.

$\lim_{x\to 0^+} f(x) = -\infty \Rightarrow x = 0$ is a vertical asymptote from above

$\lim_{x\to+\infty} f(x) = 0 \Rightarrow y = 0$ is a horizontal asymptote at $+\infty$ 1mk

d) Find the derivative, $f'(x)$ and the stationary points of C_f

$f'(x) = \frac{\frac{1}{x}.x - 1.lnx}{x^2} = \frac{1 - lnx}{x^2}$

At a stationary point $f'(x) = 0 \Longrightarrow 1 - lnx = 0 \Longrightarrow x = e,$

$y = f(e) = \frac{1}{e} \Longrightarrow Stationary\ point\ \left(e, \frac{1}{e}\right)$ 4mks

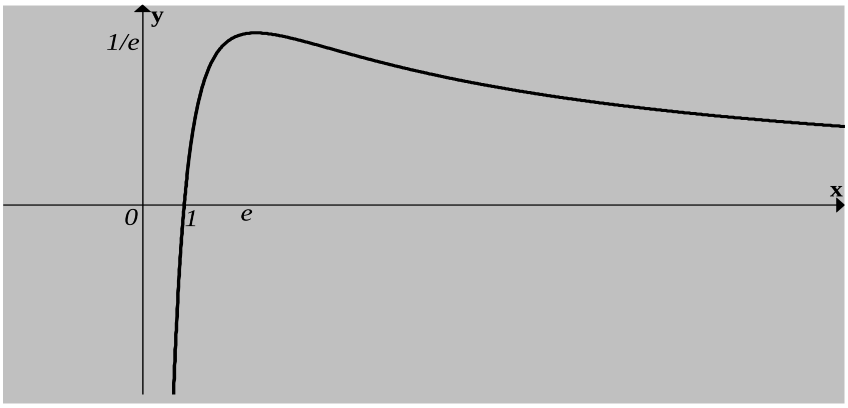
e) Construct the variation table of $f(x)$.

x	0		e		$+\infty$
$f'(x)$		$+$	0	$-$	
$f(x)$	$-\infty$	$\nearrow$	$1/e$	$\searrow$	0

Hence $\left(e, \frac{1}{e}\right)$ is a maximum turning point 1mk

f) Setch C_f showing clearing the turning point if any and the intercept.

Intercept: $y = 0 \Longrightarrow lnx = 0 \Longrightarrow x = 1 \Longrightarrow Point\ (1,0)$



2mk

6. The function $g(x) = x^2e^x$ is reperesented by the curve C_g.

a) Determine the domain of g

$D_g = \mathbb{R} =]-\infty, +\infty[$ 1mk

b) Evaluate $\lim_{x\to 0^+} g(x)$ and $\lim_{x\to +\infty} g(x)$.

$\lim_{x\to 0^+} g(x) = 0\times 1 = 0$

$\lim_{x\to +\infty} g(x) = (+\infty)(+\infty) = +\infty$ 2mks

c) Deduce the horizontal asymptote of $g(x)$.

$\lim_{x\to -\infty} g(x) = \lim_{x\to -\infty} \frac{x^2}{e^{-x}} = \lim_{x\to -\infty} \frac{2x}{-e^{-x}} = \lim_{x\to -\infty} \frac{2}{e^{-x}} = 0$

$y = 0$ is a horizontal asymptote at $-\infty$ 1mk

d) Find the derivative , $g'(x)$ and the stationary points of C_g

$g'(x) = 2xe^x + x^2e^x = x(2+x)e^x$

$g'(x) = 0 \Longrightarrow x(2+x) = 0 \Longrightarrow x = 0, x = -2$

$x = 0 \Longrightarrow y = 0 \Longrightarrow pt.(0,0)$

$x = -2 \Longrightarrow y = 4e^{-2} = \frac{4}{e^2} \Longrightarrow pt.\left(-2, \frac{4}{e^2}\right)$ 4mks

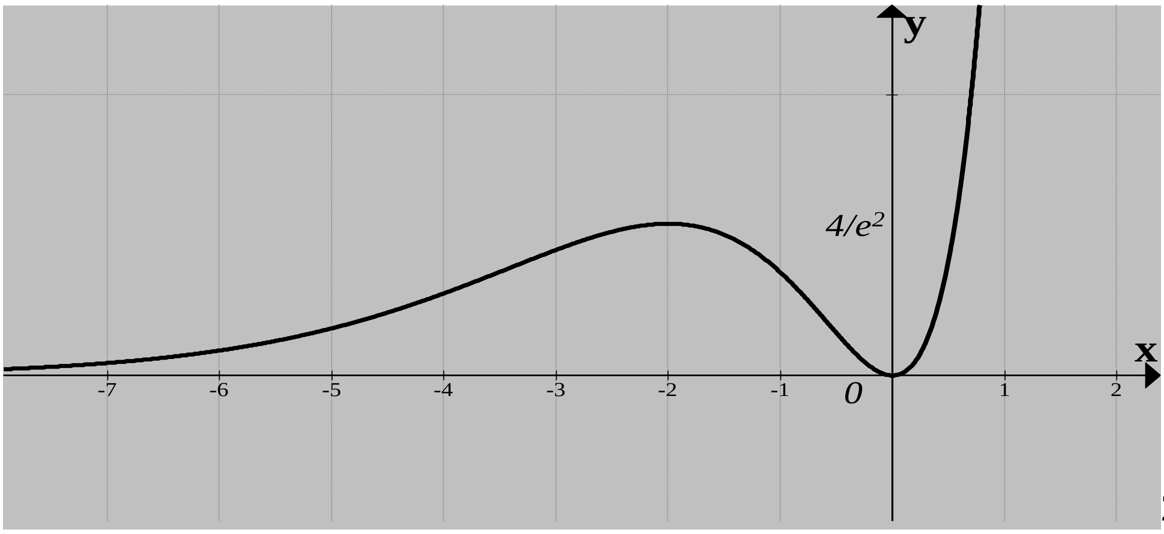
e) Construct the variation table of $g(x)$.

x	$-\infty$		-2		0		$+\infty$
$g'(x)$		$+$	0	$-$	0	$+$	
$g(x)$	0	$\nearrow$	$4/e^2$	$\searrow$	0	$\nearrow$	$+\infty$

Hence $\left(2, \frac{4}{e^2}\right)$ is a maximum turning point and $(0,0)$ is a minimum turning point

1mk

f) Setch C_g showing clearing the turning point(s) if any and the intercept.



2mk

S/M: JAFF LAWRENCE