SWE121: ENGINEERING MATHEMATICS I 03/08/20

SECOND SEMESTER

ASSIGNMENT TO BE CONSIDERED AS THE CONTINUOUS ASSESSMENT

ANSWER ALL QUESTIONS and SUBMIT BEFORE FRIDAY 7TH AUG. 2020

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1. Given the vectors $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ and $\vec{b} = 2\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$. Find:

   a) $\vec{a} \cdot \vec{b}$ (b) $\vec{a} \times \vec{b}$ (c) $|\vec{a}|$ (d) A unit vector in the direction $\vec{a}$

2. i. Solve the equation $\log(3x+8) - \log(x-2) = \log(2x+2)$

   ii. Given the matrices $A = \begin{pmatrix} 5 & 1 \\ 7 & 3 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. Find

a) Det (A) b) The transpose of A. c) The inverse of A d) The trace of A e) $A - \lambda I$

3. i. Given that α and β are acute angles such that $sin\alpha = \frac{3}{5}$ and $sin\beta = \frac{5}{13}$. Find $\sin(\alpha - \beta)$.

   ii. Show that (a) $sin2\theta = \frac{2tan\theta}{1+\tan^2\theta}$ (b) $\frac{cos\theta+1}{sin\theta} = cot\left(\frac{\theta}{2}\right)$

4. i. Given that $u = e^{xyz}$. Find $\frac{\partial u}{\partial z}$ and $\frac{\partial^2 u}{\partial x \partial z}$

   ii. Given that $z = e^y coshx$ $where$ $x = lnt$ and $y = t^2$. Find $\frac{dz}{dx}$.

5. i. Given $x^3 + 3x^2y + 6xy^2 + y^3 = 1$. Find $\frac{dy}{dx}$

   ii. Solve the system of equations below

$$\begin{cases} x - 2y - 3z = 1 \\ 2x - y - 2z = 2 \\ 3x - y - 3z = 3 \end{cases}$$

6. Given the surface $x^2 + 2y^2 + 3z^2 = 12$ and the $point$ $P(1,2,-1)$

   a) Find the tangent plane to this surface at P.

   b) Find the normal line to this surface at P

7. Locate and determine the nature of the stationary points of $f(x,y) = x^3 + y^3 - 6xy$

8. Given the surface $\emptyset(x,y,z) = xy^3z^2 - 4$ and the point $P(-1,-1,2)$

   a) Find the gradient of $\emptyset$ at P.

   b) Find the directional derivative of $\emptyset$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$.

   c) State the direction in which the rate of change of $\emptyset$ is greatest at P

   d) Find the greatest rate of change of $\emptyset$ at P.

9. Find the divergence and curl of $v = (xyz)\hat{\imath} + (3x^2y)\hat{\jmath} + (xz^2 - y^2z)\hat{k}$ at $(2,-1,1)$.

10. i. A force field is solenoidal if its divergence is zero. Find the value of a if

    $\vec{F} = (5x + 7z^2)\hat{\imath} + (4x^2 + ay)\hat{\jmath} + (7z - 2xy)\hat{k}$ is solenoidal.

    ii. A vector function is irrotational if its curl is zero. Find the values of a, b and c if

    $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is an irrotational vector function.

END