SWE111: ENGINEERING MATHEMATICS I 08/02/20
FIRST SEMESTER
STUDY QUESTIONS

A-SEVERAL REAL VARIABLE FUNCTIONS

1. Let $f(x,y) = ln(x+y-1)$
   i. Evaluate (a) $f(1,1)$ (b) $f(e^3,1)$ 2mks
   ii. Find and sketch the domain of f. 2mks
   iii. Find the range of f 1mk
2. Given the function: $h(x,y) = \sqrt{16-x^2-y^2}$
   a) Evaluate the value of h at the point $(\sqrt{3},2)$ 1mk
   b) Find and sketch the domain. 2mks
   c) Find the range 1mk
3. Compute the given limits.
   a) $\lim\limits_{(x,y)\to(0,4)} \frac{\cosh(xy)}{x^2+y-1}$ b) $\lim\limits_{(x,y,z)\to(1,1,2)} \frac{e^{x+y-z}}{x-z}$ 2mks
4. Find all the second partial derivatives of $f(x,y) = x^3y^5 + 2x^4y$ 6mks
   Hence, verify that the conclusion of Schwarz theorem holds, that is $f_{xy} = f_{yx}$. 1mk
5. Use implicit differentiation to show that if
   $x^2+y^2+z^2 = 3xyz$ then $\frac{\partial z}{\partial x} = \frac{2x-3yz}{3xy-2z}$ 2mks
6. Determine whether $u = e^{-x}cosy - e^{-y}cosx$ is a solution ot Laplace's equation $u_{xx} + u_{yy} = 0$ 2mks
7. Find the equation of the tangent plane to the surface
   $z = x^2 + xy + 3y^2$ at the point $(1,1,5)$ 2mks
8. Find the linear approximation of the function $f(x,y) = \ln(x-3y)$ at $(7,2)$ and use it to approximate $f(6.9, 2.06)$ 3mks
9. If $z = 5x^2 + y^2$ and (x,y) changes from $(1,2)$ to $(1.05, 2.1)$, compare the values of Δz *and* dz. 3mks
10. Use differentials to estimate the amount of metal in a closed cylindrical can that is 10cm high and 4cm in diameter if the metal in the top and bottom is 0.1cm thick and the metal in the sides is 0.05cm thick. 3mks
11. Given the function $f(x,y) = x^2y^3 - y^4$, the point $P(2,1)$ and the vector $\boldsymbol{v} = \boldsymbol{i} + \boldsymbol{j}$
    a) Find the gradient of f. 2mks
    b) Evaluate the gradient of f at P. 1mk
    c) Find the directional derivative of f at or the rate of change of f at P in the direction of $\boldsymbol{v}$ 2mks
12. Find and classify the critical points of $f(x,y) = x^3 - 12xy + 8y^3$ 7mks

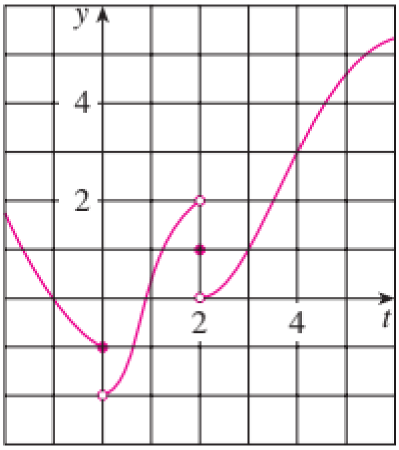
B-NUMERICAL FUNCTIONS OF A REAL VARIABLE

13. Find, without using a calculator, the exact value of:
    $(a)\ Log_2 6 - Log_2 15 + Log_2 20$ $(b)\ e^{-2ln5}$ $(c)\ \frac{Log_5 125}{Log_9 234}$ 2mks, 1mk, 2mks
14. Solve each of the following in $\mathbb{R}$ for x
    $(a)\ e^x = 5$ $(b)\ lnx = 2$ $(c)\ lnx + \ln(x-3) = \ln(6x) - \ln(x-2)$ 1mk,1mk, 2mks
15. Sketch the curve $y = lnx$. Hence, or otherwise solve $2 < lnx < 9$. 2mks
16. i. Sketch the curve $y = e^{2x}$. 2mks
    ii. Hence, or otherwise find the domain of the function $f(x) = \sqrt{3-e^{2x}}$. 1mk
17. i. Solve in $\mathbb{R}$ the equation $x^2 - 5x + 6 = 0$ 1mk
    ii. Hence, or otherwise solve each of the following in $\mathbb{R}$:
    $\ln^2 x - 5lnx + 6 = 0$ 1mk
    $e^x + 6e^{-x} - 5 = 0$ 1mk
    $3^{2x+2} - 5(3^{x+1}) + 6 = 0$ 1mk

18. Find, without using a calculator the exact value of:
$(a)\ \sec^{-1}(2)$ $(b)\ cosh(ln3)$ $(c)\ \sinh^{-1}(3)$ 1mk, 1mk, 1mk
19. (i) If $Acoshx - Bsinhx \equiv 4e^x - 3e^{-x}$, find the exact values of A and B. 2mks
(ii) Prove that $2\cosh^2 x - 1 = \cosh(2x)$ 2mks
(iii) Show that
$a)\ coshx + sinhx = e^x$ $(b)\ (coshx + sinhx)^n = \cosh(nx) + \sinh(nx)$ 6mks
$(c)\ \frac{1+tanhx}{1-tanhx} = e^{2x}$ $(d)\ tanh(lnx) = \frac{x^2-1}{x^2+1}$
(iv) Evaluate $\lim_{x\to\infty} \frac{sinhx}{e^x}$ 1mk
20. If $coshx = \frac{5}{3}$ and $x > 0$ find the exact values of
$(a)\ sinhx$ $(b)\ tanhx$ 3mks
21. Sketch $y = tanhx$ and $y = cothx$ on separate diagrams.
Hence, deduce the range of $f(x) = cothx$ 3mks
22. a) Find the derivative of $y = \sinh^{-1}\left(\frac{x}{a}\right)$ 2mks
b) Hence, show that $\int \frac{1}{\sqrt{x^2+a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right) + K$ 1mk
23. Express $\operatorname{sech}^{-1} x$ in logarithmic form. 2mks
24. i. Solve the equation $\sec^2\theta + 5tan\theta = 7\ in\ 0 < \theta < 360^o$ 3mks
ii. Find all values of x such that $\sin(2x) = sinx\ and\ 0 \le x \le 2\pi$ 3mks
iii. If $tanA = \frac{1}{3}$ and $tanB = \frac{1}{7}$, show that $2A + B = \frac{\pi}{4}$ 2mks
25. i. Prove the identities:
(a) $tan\theta sin\theta + cos\theta = sec\theta$ (b) $\frac{2tan\theta}{1+\tan^2\theta} = sin2\theta$
b) $\frac{\sin^2 x}{1+\cos(2x)} = \frac{1}{2}\tan^2 x$ 3mks
ii. If $sinx = \frac{1}{3}$ and $secy = \frac{5}{4}$, where $x\ and\ y$ lie between $0\ and\ \frac{\pi}{2}$,
evaluate $sin(x + y)$ 2mks
26. i. Solve in $\mathbb{R}^3$ the system $\begin{cases} 2x - y + z = 7 \\ x - 2y + z = 3 \\ x - y - 2z = -2 \end{cases}$ 2mks
ii. Hence, solve in $\mathbb{R}^3$ the system $\begin{cases} 2e^x - coshy + lnz = 7 \\ e^x - 2coshy + lnz = 3 \\ e^x - coshy - 2lnz = -2 \end{cases}$ 2mks
27. Given that C_f is the curve of the function $f(x) = \frac{lnx}{x}$.
Determine the domain of f and the limits at its boundaries. 2mks
Deduce the vertical asymptote of $f(x)$. 1mk
Find the derivative , $f'(x)$ and the stationary points of C_f 2mks
Construct the variation table of $f(x)$. 1mk
Setch C_f showing clearing the extrema if any and the intercept. 2mk
Given that $h(x) = \frac{1}{f(x)}$, deduce the curve C_h of $h(x)$ sketch it on a separate diagram. 1mk
28. The function $g(x) = x^2e^x$ is reperesented by the curve C_g.
i. Investigate the variation of $g(x)$ and construct its variation table. 6mks
ii. Sketch C_g showing clearly the intercepts and asymptotes if any. 1mk
iii. Deduce the intervals where g is increasing and the interval where it is decreasing. 2mks
29. i. If $sinx = \frac{1}{3}$ and $secy = \frac{5}{4}$, where $x\ and\ y$ lie between $0\ and\ \frac{\pi}{2}$, evaluate
$sin(x + y)$ 2mks
ii. Find all values of x such that $\sin(2x) = sinx\ and\ 0 \le x \le 2\pi$ 2mks
30. i. Solve the following equations:
a) $\log(x^2 + 8) - \log(2x) = log3$
b) $13e^{2x-1} = 7e^x$
c) $ln(x + 1)^2 = \ln(x + 1) - \ln(x + 2) + 2$ 6mks
31. Differentiate the following functions with respect to x

$a)\ y = e^{2x}\,\text{sech}(2x)$ $b)\ f(x) = 3\cosh^3(2x)$ $c)\ g(x) = 5\ln(sinhx)$ 6mks

32.



For the function g whose graph is given,

i. state the value of each quantity, if it exists. If it does not exist, explain why.

$(a)\ \lim_{t\to0^-} g(t)$ $(b)\ \lim_{t\to0^+} g(t)$ $(c)\ \lim_{t\to0} g(t)$ $(d)\ \lim_{t\to2^-} g(t)$ $(e)\ \lim_{t\to2^+} g(t)$ $(f)\ \lim_{t\to2} g(t)$

$(g)\ g(2)$ $(h)\ \lim_{t\to4} g(t)$ 8mks

ii. Identify the discontinuities of g 1mks

33. The electric scalar potential in a region of space is given by $\phi = x^2 + xy^2 + z^2$.
Determine, at the point $(-1, 2, -1)$

a) grad ϕ 2mks

b) the directional derivative of ϕ in the direction of the vector $\boldsymbol{a} = 2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$ **2mks**