$9 - 2 = 7$. Thus the equilibrium price is 9 and the equilibrium quantity is 7.

Example 18.

A monopolist faces the demand curve given by $p = 20 - q$ and his cost function is given as $C = q^2 + 8q + 2$. Determine the profit maximising output and the corresponding price.

Solution :

Let π represent total profit of the monopolist and R represent total revenue. Then

$$\pi = R - C = pq - C = (20 - q)\,q - q^2 - 8q - 2$$

$$\text{or, } \pi = 20q - q^2 - q^2 - 8q - 2 = -2q^2 + 12q - 2$$

This shows that π is a function of q only. The objective of the monopolist is to maximise profit. To maximise π the first order condition requires $\frac{d\pi}{dq} = 0$. or, $-4q + 12 = 0$ or, $q = 3$.

The second order condition for maximisation requires that $\frac{d^2\pi}{dq^2} < 0$. Here $\frac{d^2\pi}{dq^2} = -4 < 0$. Hence the profit maximising level of output of the monopolist is 3 units. The corresponding price $p = 20 - q = 20 - 3 = 17$.

Example 19.

A profit maximising monopolist has the following demand and total cost functions : demand function : $p = \frac{3}{q}$ and cost function : $C = 2q + 3q^2$. Is it possible for this monopolist to produce positive amounts for which his profit would be maximum? Give reasons for your answer.

Solution :

Let π represent total profit and R represent total revenue of the monopolist. Then

$$\pi = R - C = pq - C = \frac{3}{q} \cdot q - 2q - 3q^2 = 3 - 2q - 3q^2.$$

This shows that π is a function of q only. To maximise π the first order condition requires that

$$\frac{d\pi}{dq} = 0. \text{ or, } -2 - 6q = 0 \quad \text{or, } q = -\frac{1}{3}.$$

Thus it is seen that $\frac{d\pi}{dq}$ is zero for a negative value of q. $\frac{d\pi}{dq}$ is not zero for any positive value of q. Hence it is clear that it is not possible for the monopolist to produce positive amount of the commodity if he wants to maximise his profit.

An alternative explanation can be given as follows : Here total revenue $R = pq = \frac{3}{q} \cdot q = 3$. Hence marginal revenue $\frac{dR}{dq} = 0$. To maximise profit marginal revenue must be equal to marginal cost. But $MC = \frac{dC}{dq} = 2 + 6q$. From the MC function it is seen that MC cannot be zero for any positive value of q. Thus for any positive level of output the equality of MR and MC cannot be achieved.

Example 20.

Given the following demand and cost functions $p = 250 - 3q$ and $C = 3q + 5q^2$ respectively, find the profit maximising price and output. How would the firm adjust its price and output, if a tax of Rs. 4 per unit of output be imposed on the firm?

Solution :

Let π represent total profit and R represent total revenue of the monopolist. Then

$$\pi = R - C = pq - C \qquad \text{or, } \pi = (250 - 3q)q - 3q - 5q^2$$

$$\text{or, } \pi = 250q - 3q^2 - 3q - 5q^2 \qquad \text{or, } \pi = -8q^2 + 247q$$

This shows that π is a function of q only. To maximise π the first order condition requires $\frac{d\pi}{dq} = 0$, i.e,

$$-16q + 247 = 0 \quad \text{or, } 16q = 247 \quad \text{or, } q = \frac{247}{16} = 15 \cdot 4$$

The second order condition is also fulfilled since $\frac{d^2\pi}{dq^2} = -16 < 0$. Hence the profit maximising level of output = 15·4 units. The corresponding price, $p = 250 - 3 \times 15{\cdot}4 =$ Rs. 203·8.

When the unit tax of Rs. 4 per unit is imposed the cost function becomes $C = 3q + 5q^2 + 4q = 7q + 5q^2$. The new profit function becomes

$$\pi' = (250 - 3q)q - 7q - 5q^2 = -8q^2 + 243q.$$

The first order condition for maximising π' requires $\frac{d\pi'}{dq} = 0$, i.e,

$$-16q + 243 = 0. \quad \text{or, } 16q = 243, \quad \text{or, } q = \frac{243}{16} = 15 \cdot 2 \text{ units.}$$

Therefore, when the tax is imposed the output level becomes 15·2 units. The corresponding price $p' = 250 - 3 \times 15 \cdot 2 = \text{Rs.}204 \cdot 4$. Thus as a result of the imposition of the tax the output level decreases from 15·4 to 15·2 units and the price increases from Rs. 203·8 to Rs. 204·4.

P 100·4 100·4

Thus Lerner's index of monopoly power = 0·10.

Example 22.

A monopolist faces two demand functions $p_1 = 12 - q_1$ and $p_2 = 20 - 3q_2$ in two markets. Suppose his total cost function is $C = 3 + 2 \cdot (q_1 + q_2)$. Determine the prices the monopolist will charge in the two markets if his objective is to maximise profit.

Solution :

Let π represent total profit and R_1 and R_2 represent total revenues in the two markets. Then

$$\pi = R_1 + R_2 - C = p_1 q_1 + p_2 q_2 - C$$

or, $$\pi = (12 - q_1)q_1 + (20 - 3q_2)q_2 - 3 - 2(q_1 + q_2).$$

or, $$\pi = 12q_1 - q_1^2 + 20q_2 - 3q_2^2 - 3 - 2q_1 - 2q_2$$

or, $$\pi = 10q_1 + 18q_2 - q_1^2 - 3q_2^2 - 3$$

This shows that π is a function of q_1 and q_2. To maximise π the first order conditions require

$$\frac{\delta \pi}{\delta q_1} = 10 - 2q_1 = 0 \text{ or, } q_1 = 5, \qquad \frac{\delta \pi}{\delta q_2} = 18 - 6q_2 = 0 \text{ or, } q_2 = 3.$$

Since $\dfrac{\delta^2 \pi}{\delta q_1^2} = -2 < 0$, $\dfrac{\delta^2 \pi}{\delta q_2^2} = -6 < 0$, second order conditions are also fulfilled. Thus the profit maximising output levels in the two markets are 5 and 3 units respectively. Now, $p_1 = 12 - q_1 = 12 - 5 = 7$ and $p_2 = 20 - 3q_2 = 20 - 3(3) = 11$. Thus the prices in the two markets are Rs. 7 and Rs. 11 respectively.

le 23