

## Hands On 12

Hw 12

### a. aggregate method

In the aggregate method, first we have to calculate the total cost of performing a sequence of operations & then divide by the number of operations to get the amortized cost per operation.

#### 1. Cost Analysis

- We denote  $n$  as the no. of elements inserted &  $m$  as the no. of resizing operations.
- Let  $c_i$  be the cost of the  $i$ th insertion operation.
- When inserting the  $i$ th element, if a resize operation is not needed then the cost is  $O(1)$ . If a resize happens cost is  $O(i)$  as it involves copying the existing elements to the new table of size  $2^k$  ( $k$  is the number of resizes performed.)

#### 2. total cost

$$\begin{aligned}
 \sum_{i=1}^n c_i &= O(n) + O(2) + O(4) + \dots + O(2^m) \\
 &= O(n + 2 + 4 + \dots + 2^m) \\
 &= O(n + 2^{(m+1)} - 1) \\
 &= O(n + 2^{(m+1)} - 1)
 \end{aligned}$$

#### 3. Amortized cost per operation:

Since the number of resizes is at most  $\log_2(n)$ , the amortized cost per insertion is  $O(1)$ .

### b. Accounting method:-

Pseudo code -

for  $i = 1$  to  $n$

if table is full

new table = create new table with size  $2 \times$  current size

then copy elements from old table to new table

table = new table

Insert element  $i$  into table

initial charge = 0



for  $i = 1$  to  $n$ :

  charges  $+= 2$

  if table doubled in size from  $m$  to  $2m$

    credits  $+= m$

$$\text{total charge} = 2 \times n = O(n)$$

$$\text{total credits} = m + 2m + \dots + n/2 \times m = O(n)$$

$$\text{Amortized cost per insertion} = \text{total} / n$$

$$= O(n/n)$$

$$= O(1)$$

$$\text{runtime per insertion} = O(1)$$

$$\text{total time} = O(n)$$