Hydraulics:

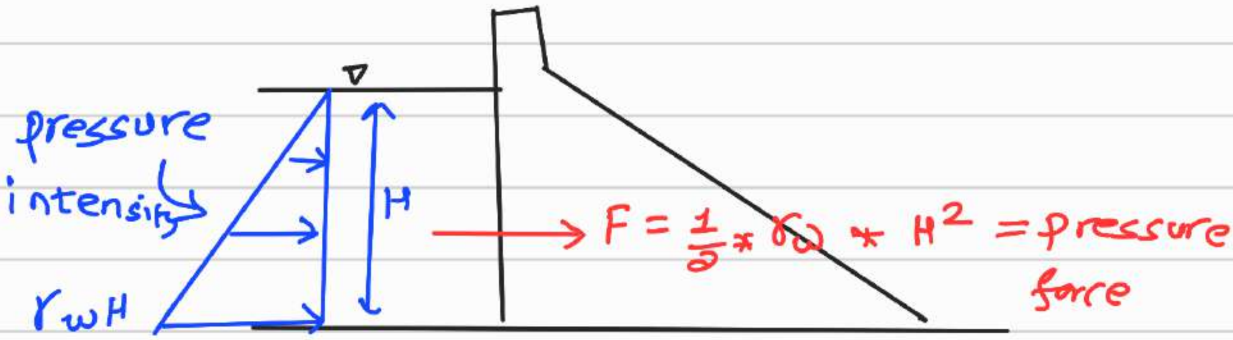
Fluid Pressure:

① Pressure intensity: $p = \dfrac{\text{Pressure force}}{\text{Area}}$

* Unit = N/m2

② Pressure force / Hydrostatic pressure, F
= force exerted on any surface due to pressure.

$$\boxed{F = \text{pressure force} = \int p * dA}$$



Measurement of Pressure:

* Pressure measurement is carried out by:
  a. Manometer
  b. Mechanical Gauges.

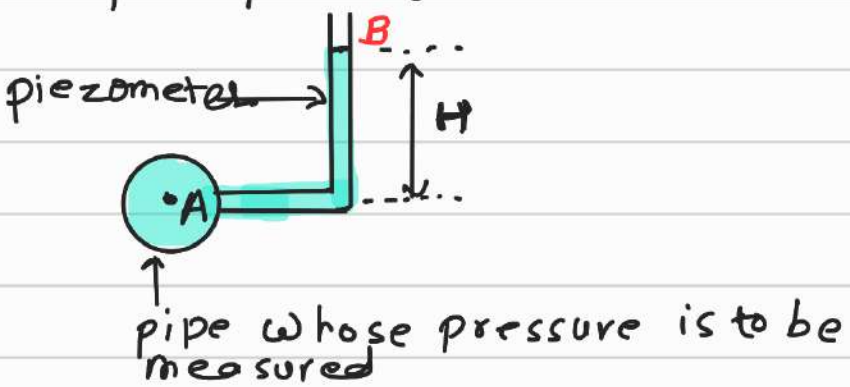
The brief description of these equipments is given by.

a. Manometer:

* They work on the principle of liquid column balance.

i. Piezometer:

* It is a simple tube which is connected to the



pipe, whose pressure is to be measured.

* $P_A = P_B + \gamma * H$

But; $P_B = P_{atm}$

$\therefore P_A = P_{atm} + \gamma * H$ ← Absolute pressure

or $P_A = \gamma * H$ ← Gauge pressure.

H = height of liquid column above centreline of pipe.

γ = unit wt. of liquid.

<u>Uses:</u> → For measurement of low to moderate pressure.

Limitations:

1. Can not measure gas pressure.
2. Can not measure negative pressure
3. Not suitable for high pressure.

-1 = suction

-100 = suction

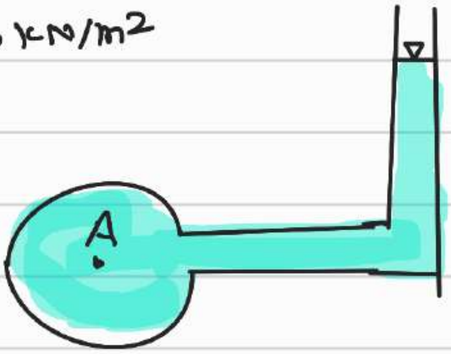
e.g. pressure = 500 kN/m²

$P = \gamma * H$

let liquid = water.

$h = \frac{P}{\gamma} = \frac{500}{9.81}$

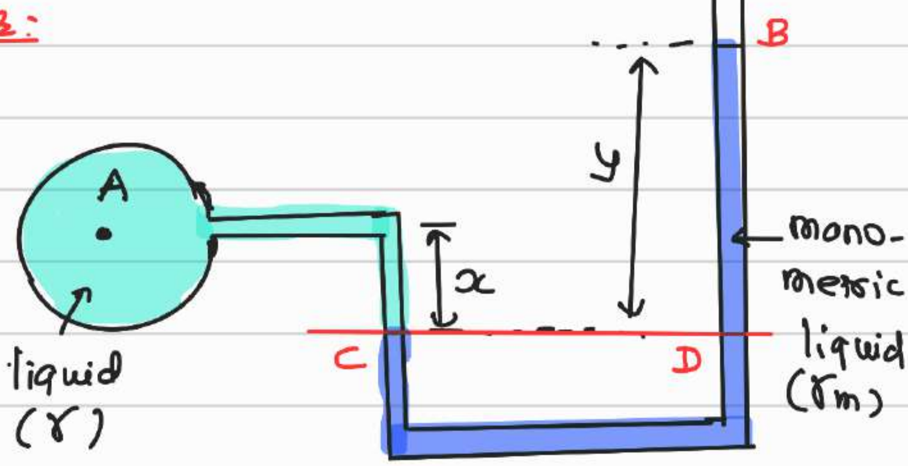
$\simeq \frac{500}{10} = 50m$



b. U-tube Manometer:

* In this manometer, manometric liquid is filled inside U-tube.

$P_A = P_B + \gamma_m * y - \gamma * x$



$\Rightarrow \quad P_A = P_{atm} + \gamma_m * y - \gamma * x \leftarrow$ Absolute

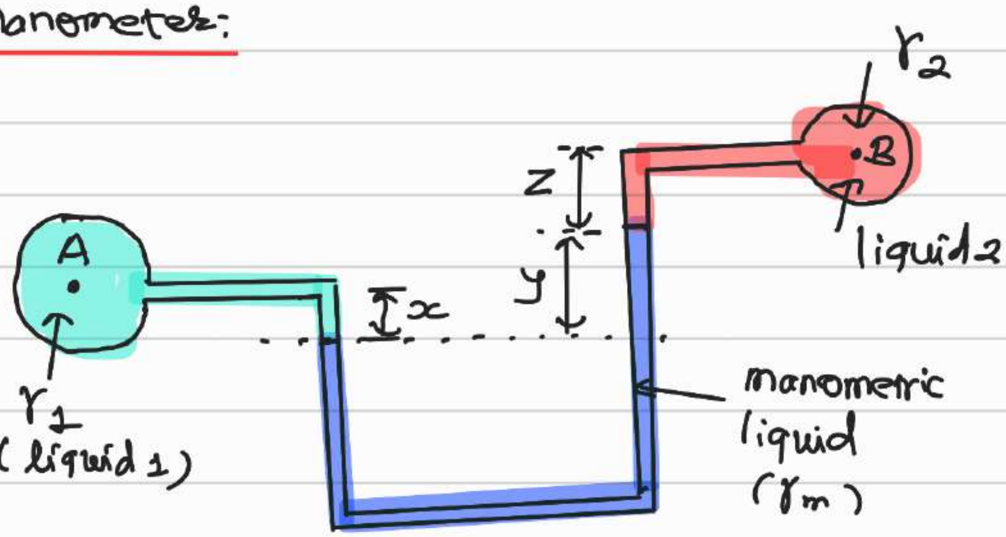
$\Rightarrow \quad P_A = \gamma_m * y - \gamma * x \leftarrow$ Gauge.

$\gamma_m =$ unit weight of monometric liquid

$$\boxed{\begin{aligned} P &= \rho g h \\ &= \gamma * h \\ \gamma &= \rho g \end{aligned}}$$

c. Differential manometer:

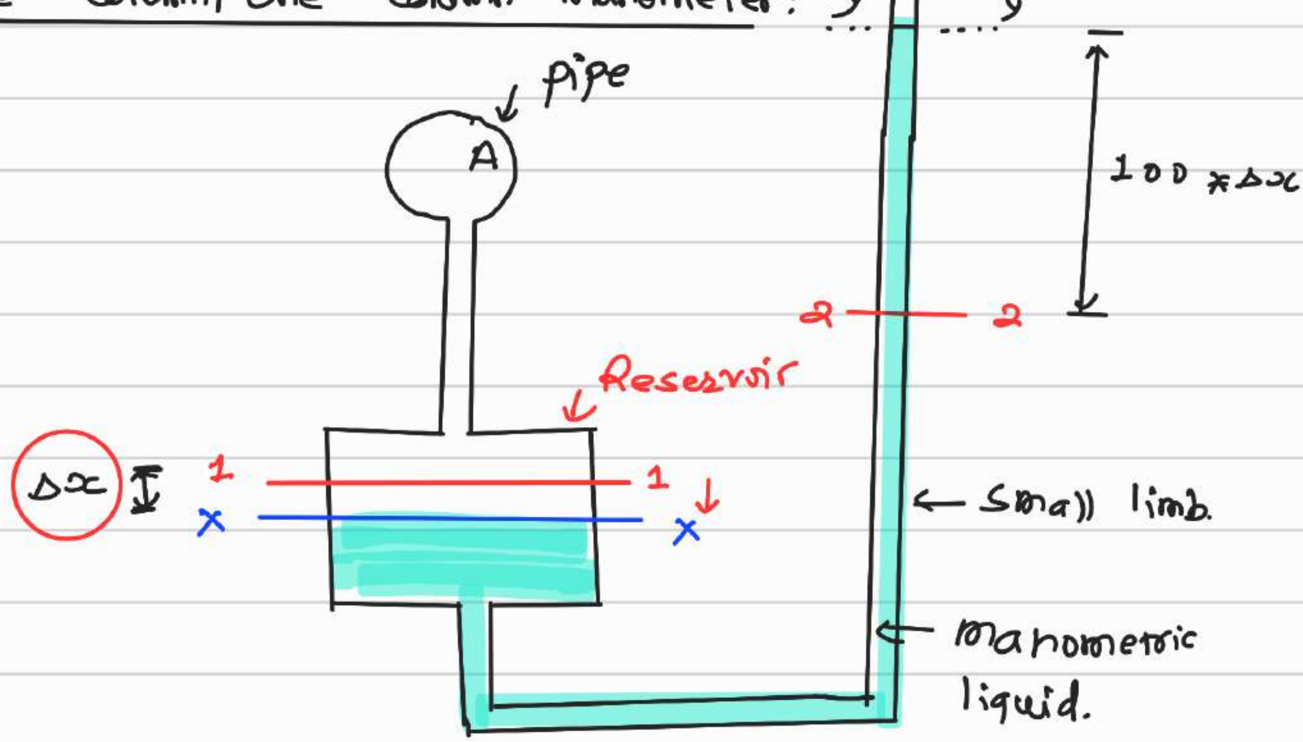
* A differential manometer is used to measure pressure difference between two different pipes or two different points of same pipe.



* $\boxed{P_A + \gamma_1 * x - \gamma_m * y - \gamma_2 * z = P_B}$

$\Rightarrow \quad P_A - P_B = \gamma_m * y + \gamma_2 * z - \gamma_1 * x$

d. Single Column / One Column manometer:



* In this manometer, a large reservoir of 100 times area approx. than that of tube is connected to one limb.
* As pressure in the pipe changes, the major change in liquid column occurs is small tube (right limb) whereas change of liquid level in reservoir is very less.

right limb

* So, reading of only one column is enough to calculate the pressure so, it is also called single column manometer.

e. Micromanometer:

* The manometer, which can measure small difference in pressure with large accuracy is called micro-manometer.

2. Mechanical Gauges:

* The mechanical gauges are used for measurement of very high pressure but their accuracy is low.
* Mechanical gauges use the principle of spring / metal displacement or dead weight balance.
* Mostly used mechanical gauges are:
  a. bourdon tube pressure gauge.
  b. Bellows pressure gauge
  c. Diaphragm pressure gauge.
  d. Dead weight pressure gauge.

Pressure Force/ Hydrostatic Pressure:

a. Pressure force on Plane Surface:

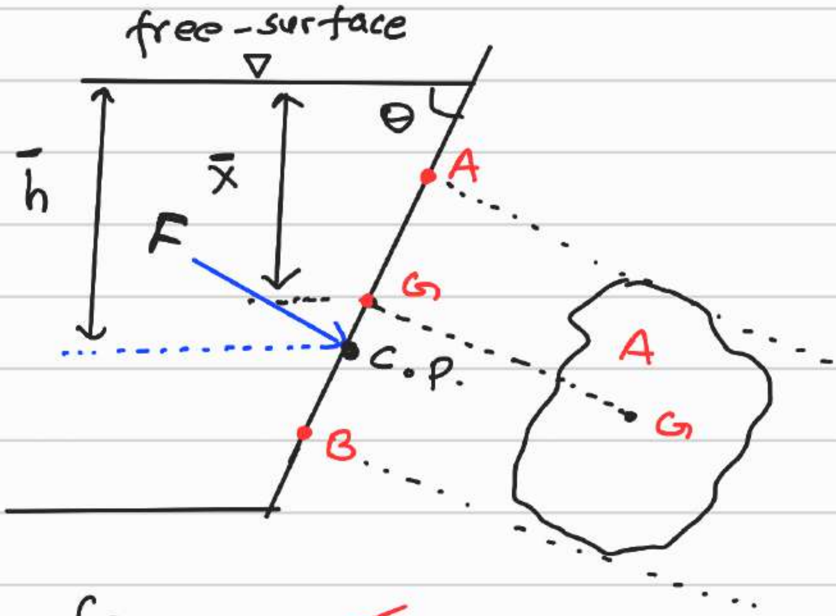
Pressure force on submerged plane surface AB

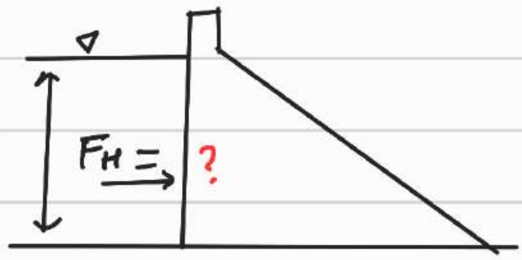
$F = \gamma * A * \bar{x}$

where, γ = unit wt. of liquid

A = Area of surface

$\bar{x}$ = vertical depth of C.G. of area from free surface.

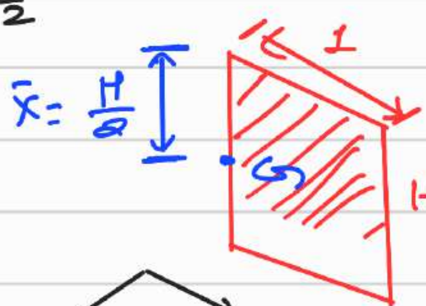




$F_H = \gamma * A * \bar{x}$

$= \gamma * 1 * H * \frac{H}{2}$

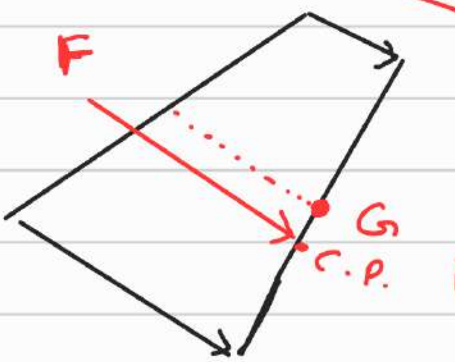
$= \frac{1}{2} \gamma * H^2$

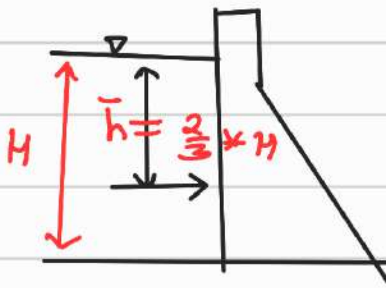


Centre of Pressure:

* The point through which resultant pressure force acts.
* The height of centre of pressure from free surface:

$$\boxed{\bar{h} = \bar{x} + \frac{I_{CG} * \sin^2\theta}{A\bar{x}}}$$

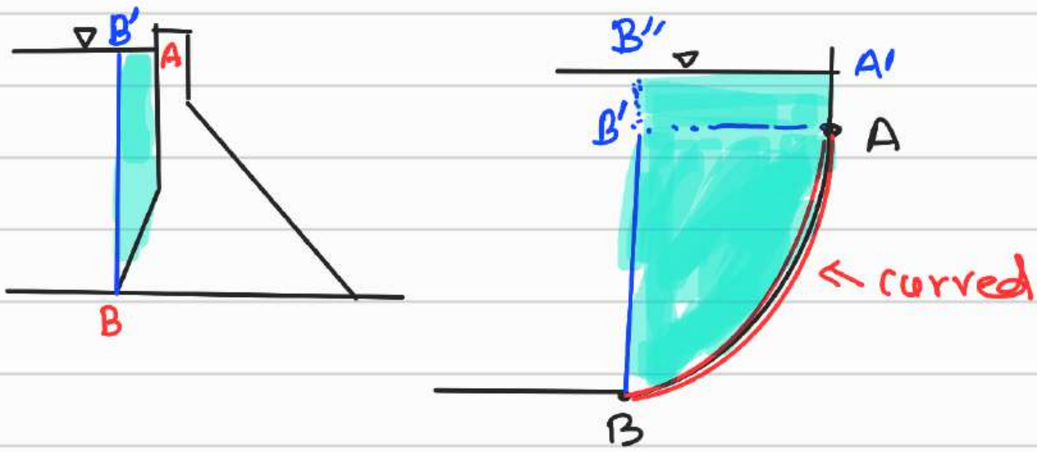




I_{CG} = Moment of inertia of the area about C.G.

θ = inclination of surface with horizontal.

Pressure force for Curved Surfaces:



* In Curved surface, the direction of pressure intensity changes from point to point. So, the pressure force is divided into two components.

a. Horizontal Pressure force (F_H) of curved surface
$= \gamma A \bar{x}$; where A = projected area in vertical plane
= area of BB'

$\bar{x}$ = vertical height of C.G. of projected area from free surface.

b. Vertical Pressure force (F_V)

F_V = Weight of liquid supported by the curved face.
= Weight of liquid in A'AB'BB'' (in fig a)
= weight of liquid in ABB'

Bernoulli's Equation and it's Application

* Bernoulli's equation is based on principle of conservation of energy.
* It is derried from Euler's equation.

Assumptions:

1. Fluid is non-viscous.
2. Fluid is incompressible.
3. flow is along streamlines or one-dimensional.
4. Flow is steady.
5. Flow is irrotational.

Statement:

* The sum of static head/potential head, pressure head and kinematic head of a non-viscous, incompressible, steady and one-dimensional flow is always constant.

i.e., $z + \frac{p}{\gamma} + \frac{v^2}{2g} = \text{constant}$ .... is

z = Static head

$\frac{p}{\gamma}$ = pressure head

$\frac{v^2}{2g}$ = velocity head.

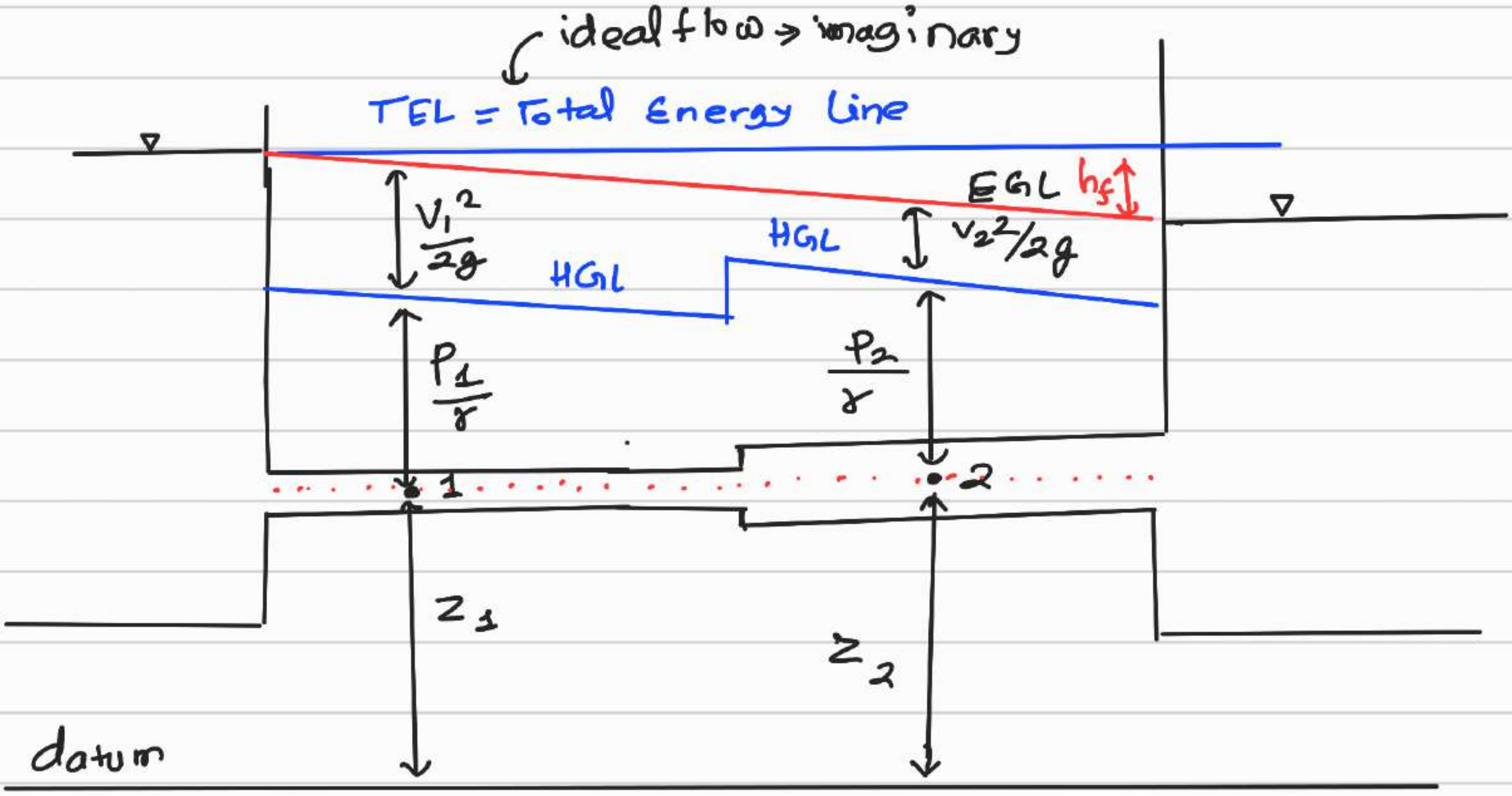
Equation is is Bernoulli's equation for ideal fluid. But, for real fluid, the Bernoulli's eqn may be written as:

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_f$$

[flow from 1 to 2]

where; h_f = head loss between section 1 and 2.

Total energy line (TEL), Energy Grade Line (EGL) and Hydraulic Gradient line (HGL)



Total Energy line: (TEL) * The line representing sum of static head, pressure head and velocity head for ideal fluid.

* It is horizontal.

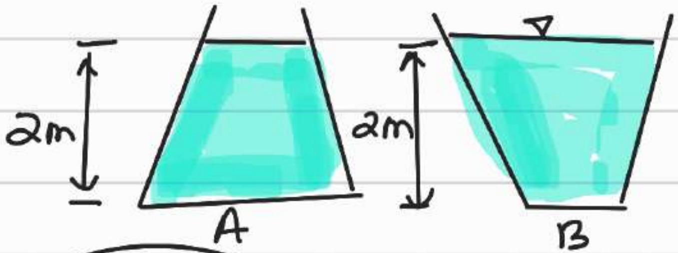
Energy Grade line: (EGL)

* $EGL = z + P/\gamma + V^2/2g$
* The line representing sum of all three heads for real fluid is energy grade line.
* It is sloping downwards.

Hydraulic Gradient line (HGL) / Piezometric Head Line

* The line representing sum of static head and pressure head is called hydraulic gradient line.

* HGL = $z + P/\gamma$
* In open channel flow, HGL coincides with water surface.
  * In siphon, HGL lies below centre line.
  * In general pipe flow HGL lies above centre line.



$$\cancel{z_1} + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \cancel{z_2} + \frac{P_2}{\gamma} + \frac{V_2^2}{2g}$$

$$\frac{P_1}{\gamma}\downarrow + \frac{V_1^2}{2g}\uparrow = \frac{P_2}{\gamma}\uparrow + \frac{V_2^2}{2g}\downarrow$$

$P_A = P_B$

↑

Hydrostatics.

↓

depth.

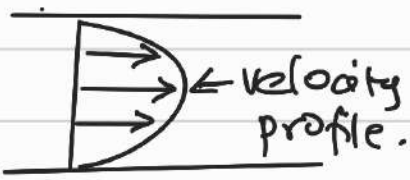
$V_1 > V_2$

$$\frac{V_1^2}{2g} > \frac{V_2^2}{2g} \Rightarrow \frac{P_1}{\gamma} < \frac{P_2}{\gamma}$$

Limitations of Bernoulli's Equation:

① In real fluids, there is always head loss due to viscosity and turbulence. But, Bernoulli's equation assumes constant head.

② The actual kinetic head is



Averag velocity:
$V = \frac{Q}{A}$

different than the kinetic head suggested by Bernoulli since the velocity is not uniform across the section. So, K.E. correction factor is needed.

③ The effect of rotation is not considered by Bernoulli.

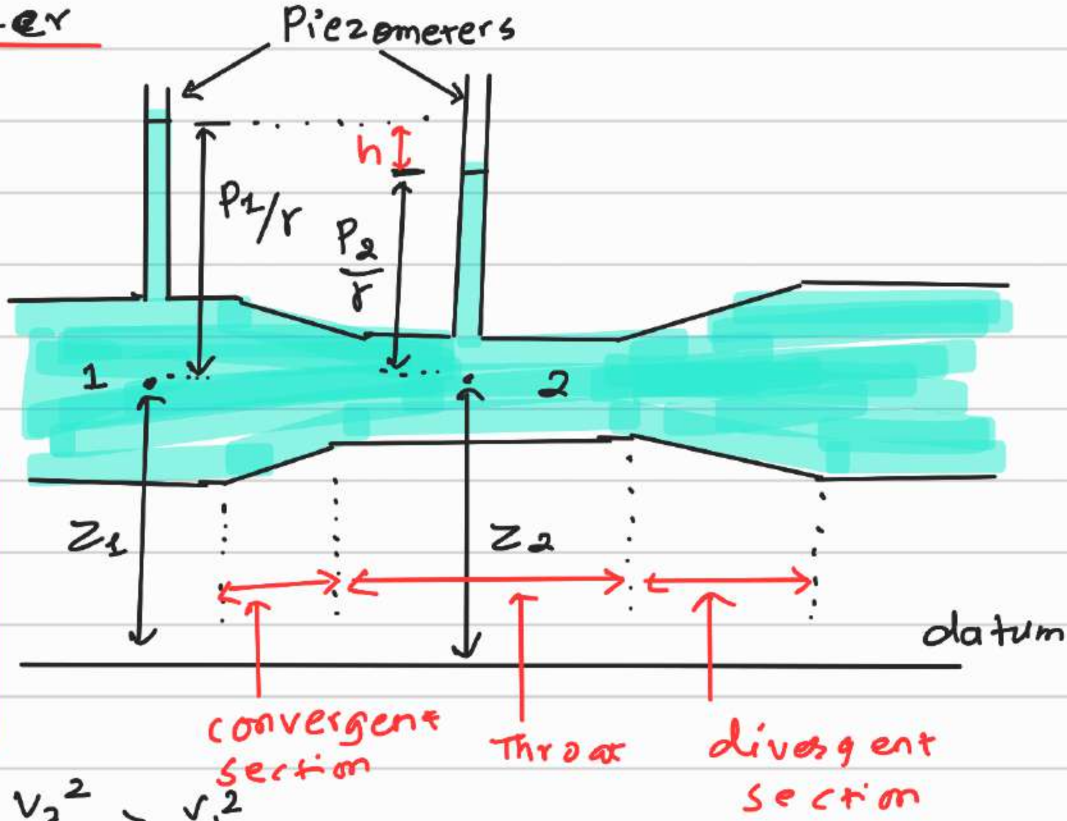
Application of Bernoulli's Equation

* Bernoulli's equation is widely applicable in the field of engineering that includes fluid related design. There are some equipments which work on Bernoulli's equation

i) Venturimenter

$$\cancel{z_1} + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \cancel{z_2} + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + h_f \rightarrow 0$$

$$\boxed{\frac{p_1}{\gamma}\uparrow + \frac{v_1^2}{2g}\downarrow = \downarrow\frac{p_2}{\gamma} + \frac{v_2^2}{2g}\uparrow}$$



$v_2 > v_1 \Rightarrow$ so; $\frac{v_2^2}{2g} > \frac{v_1^2}{2g}$

* It is an instrument used for measurement of discharge in pipes.
* It consists of convergent section, throat and divergent section and two piezometers or differential manometers.
* It works on Bernoulli's eqn, continuity equation and manometric equation.

from Bernoulli's eqn:

$$z_1 + \frac{p_1}{\gamma} + \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + \cancel{h_f}$$

$$\boxed{\left(z_1 + \frac{p_1}{\gamma}\right) - \left(z_2 + \frac{p_2}{\gamma}\right) = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}}$$

or $h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$ .....i), where $h = (z_1 + \frac{p_1}{\gamma}) - (z_2 + \frac{p_2}{\gamma})$ = piezometric head difference

Continuity eqn: $\boxed{Q_1 = Q_2 = Q}$

or $Q = A_1 V_1 = A_2 V_2$

$\Rightarrow V_2 = \frac{Q}{A_2}$; $V_1 = \frac{Q}{A_1}$

from eqn i) $h = \frac{1}{2g} * \left\{ \frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} \right\}$

$$\Rightarrow Q = \frac{A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}} \quad \text{.....ii)}$$

← Theoritical discharge.

But; actual discharge;

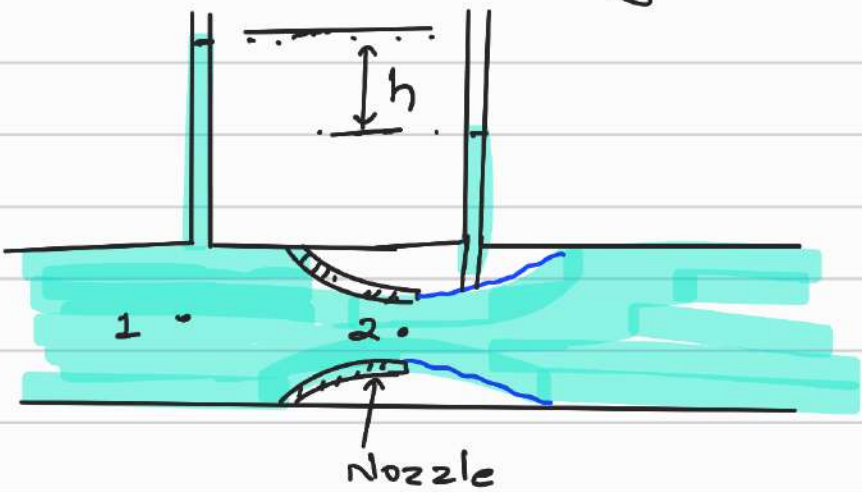
$$\boxed{Q = \frac{C_d * A_1 A_2 * \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}}$$

← Actual discharge.

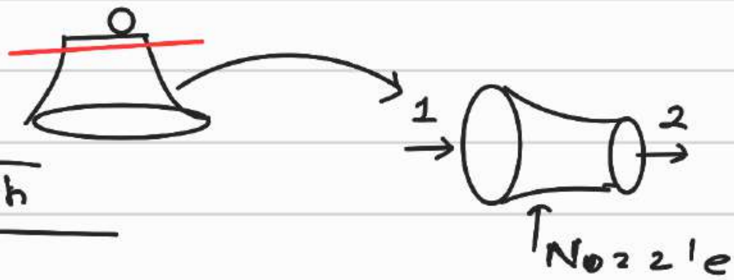
where C_d = coefficient of discharge

= 0.95 to 0.99 (Gerally 0.98)

2. Nozzle meter



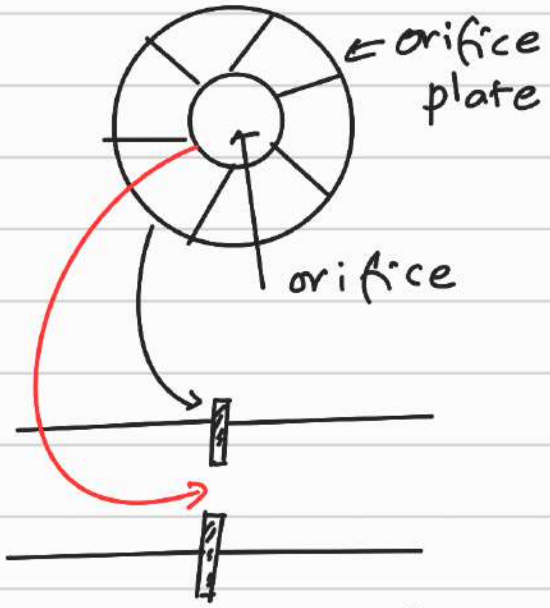
* It is also used for measurement of discharge.
* Working principle same as venturimeter.
* A nozzle is fitted inside pipe, which reduces flow area.

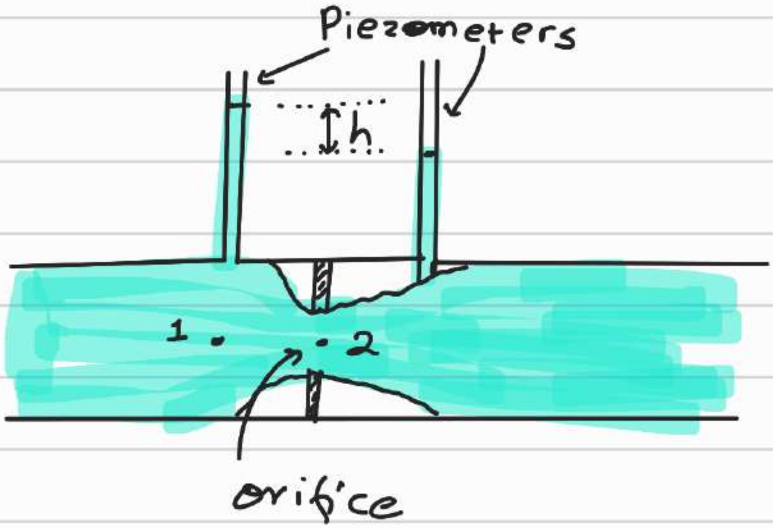


* $Q = \frac{C_d * A_1 A_2 \sqrt{2gh}}{\sqrt{A_1^2 - A_2^2}}$

C_d = coefficient of discharge = 0.7 to 0.8

3. Orifice meter:





* Same function and working principle as venturimeter.

* An orifice plate, which contains an orifice of diameter 0.4 to 0.8 times the pipe diameter is fitted inside the pipe.

* $Q = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} * \sqrt{2gh}$

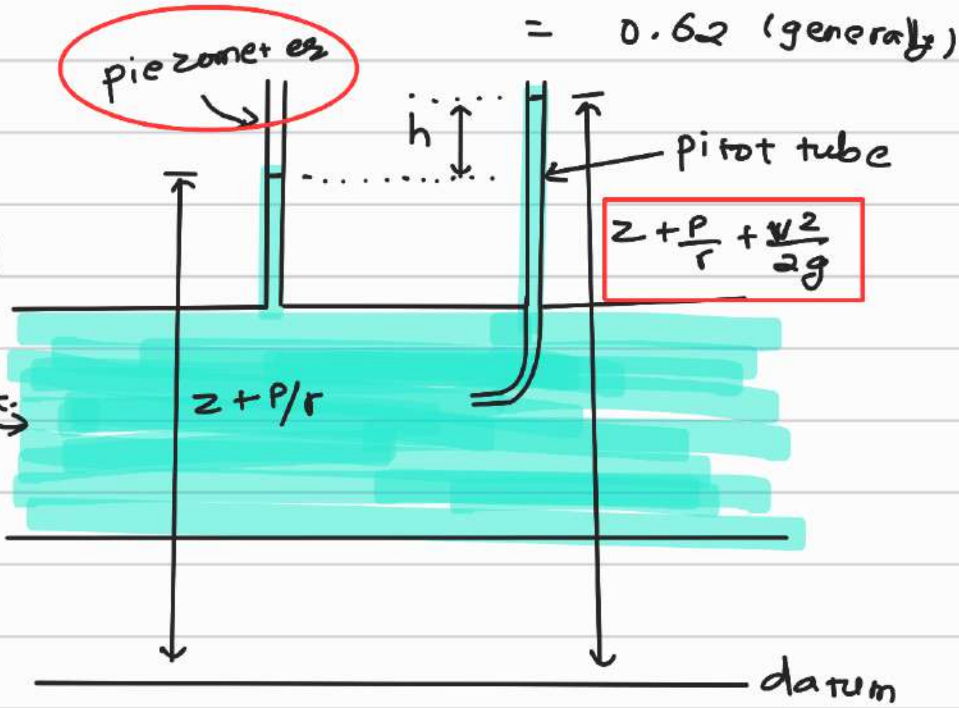
C_d = Coefficient of discharge = 0.61 to 0.65

= 0.62 (generally)

4. Pitot tube:

* It is a tube, which is bent at lower end that is used for velocity measurement.

* Water enters the pitot tube at full velocity and as it enters the tube, water becomes stagnant. The velocity head of water is converted into pressure head.

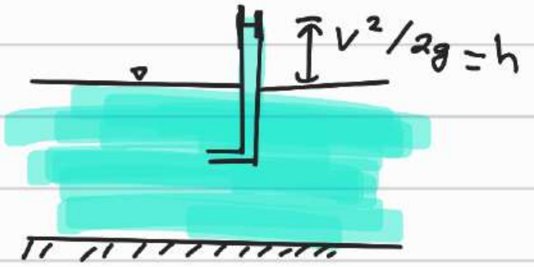


from above fig:

$$h = \frac{v^2}{2g}$$

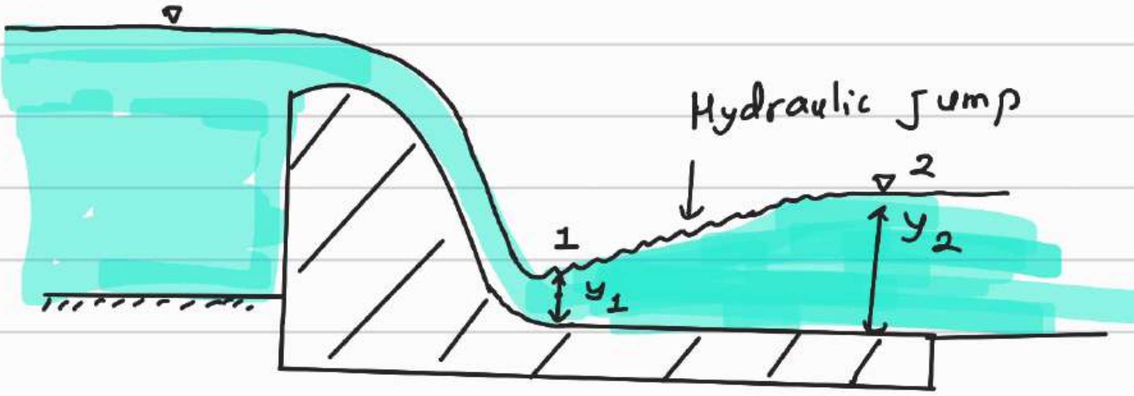
$$\Rightarrow v = \sqrt{2gh}$$

where v = velocity



open channel

Hydraulic Jump:



* Hydraulic jump is rapidly varied non uniform flow in which flow changes from super-critical to sub critical.
* Hydraulic jump is one of the method of energy dissipation, in which energy is dissipated due to turbulent action of water and formation of rollers.

* Hydraulic jump forms on horizontal floor or mild slope.

Equations of Hydraulic Jump:

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g} \quad \cdots\cdots \text{i)}$$

Where; y_1 = pre-jump depth

y_2 = post jump depth

q = discharge per unit width.

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8F_{r1}^2}\right] \quad \cdots\cdots \text{ii)}$$

$$y_1 = \frac{y_2}{2}\left[-1+\sqrt{1+8Fr_2^2}\right] \quad \ldots\ldots iii)$$

$$Fr_1 = \frac{v_1}{\sqrt{g*y_1}} = \text{froude no. at 1}$$

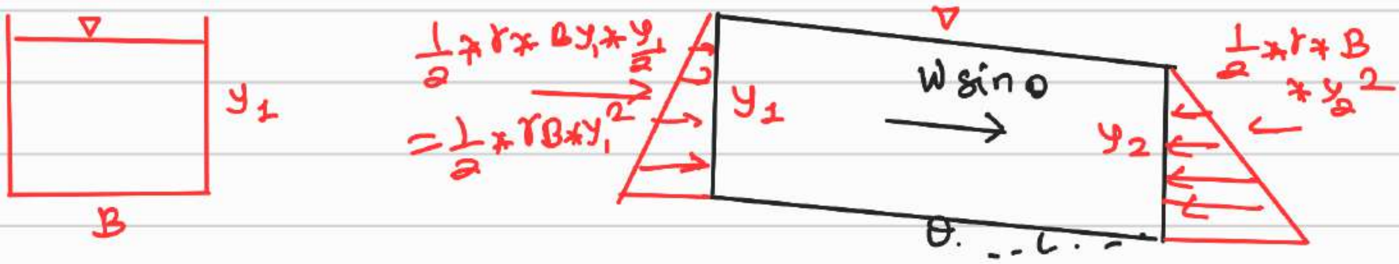
$$Fr_2 = \frac{v_2}{\sqrt{g*y_2}} = \text{froude no. at 2}$$

$$\text{head loss} = h_f = \frac{(y_2-y_1)^3}{4y_1y_2} \quad ; h_f = \text{head loss} \quad \ldots\ldots iv)$$

⇒ To get equation is or to Calculate flow depths y_1 and y_2 ⇒ We use momentum equation. We can't use Bernoulli's eqn since head loss is initially unknown.

⇒ The two depths having same specific force are called Conjugate or Sequent depths. For example, the depths y_1 & y_2 in hydraulic jump are sequent/conjugate depths.

⇒ The two depths having same specific energy are called alternate depths.



rate of change of momentum = net forces.

$$P_1 - P_2 + W\sin\theta - F = \rho Q(V_2 - V_1)$$

$$P_1 + \rho Q V_1 = P_2 + \rho Q V_2$$

$$\gamma A_1\bar{z}_1 + \rho Q * \frac{Q}{A_1} = \gamma A_2\bar{z}_2 + \rho Q * \frac{Q}{A_2}$$

⇒

$$A_1\bar{z}_1 + \frac{Q^2}{gA_1} = A_2\bar{z}_2 + \frac{Q^2}{gA_2} \quad \text{or}$$

$$\text{or} \quad F_1 = F_2$$

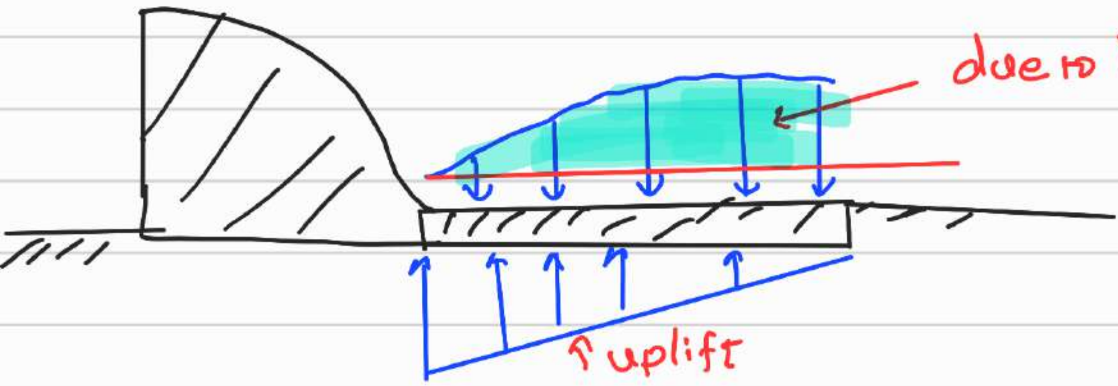
Where; $F_1 = A_1\bar{z}_1 + \dfrac{Q^2}{gA_1}$ = specific force at 1

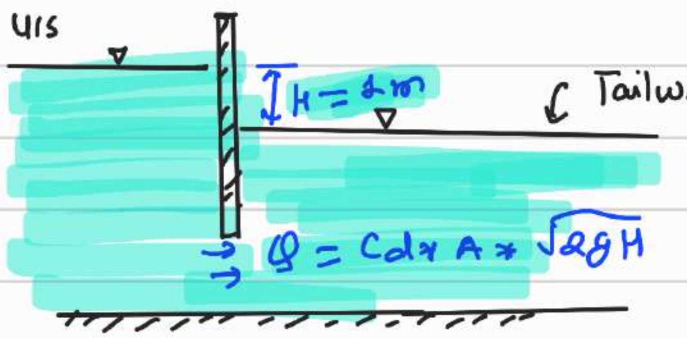
$F_2 = A_2\bar{z}_2 + \dfrac{Q^2}{gA_2}$ = " " at 2.

$$\text{specific force} = \frac{\text{force}}{\text{Unit wt.}}$$

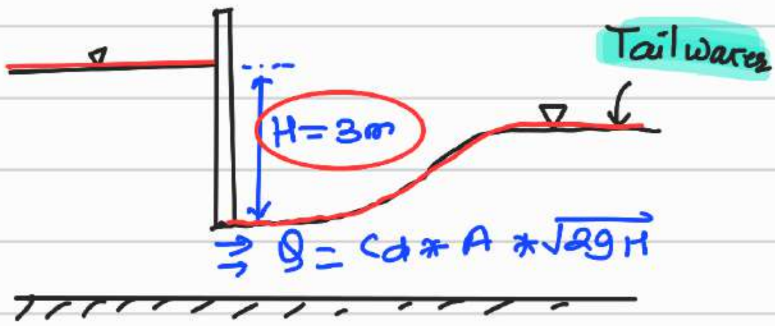
<u>Uses of Hydraulic Jump:</u>

1. It is used for dissipation of energy of water that flows over weir, barrage, spillway, fall, etc.
2. Used for mixing of chemicals in industrial applications.
3. Used to raise water depths in canals so that water can be easily fed to the distributaries.
4. It counterbalances uplift force on floors.
5. It increases discharge of sluice gates.





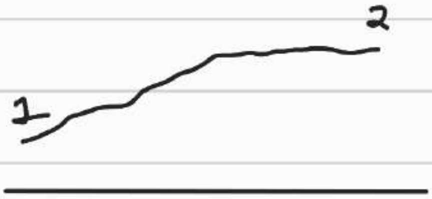
without hydraulic jump
less H, less Q.
$Q = C_d * A * \sqrt{2gH}$



with Hydraulic jump
more H, more Q

Types of Hydraulic Jump

1. Based Froude Number → Fr_1



a. Imposible Jump

* $Fr_1 < 1$
* Jump is not formed.

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}}$$ → more V_1, less y_1 and more Fr_1

b. Undulas Jump

* $Fr_1 = 1$ to 1.7
* Energy loss $< 5\%$

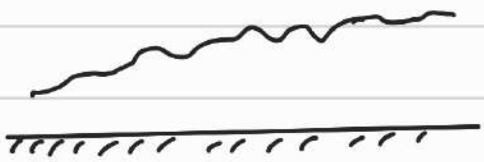
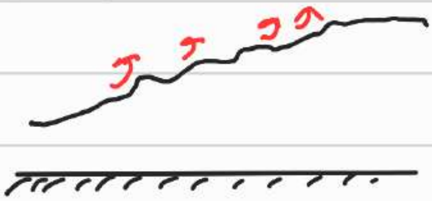


fig: undulas Jump.

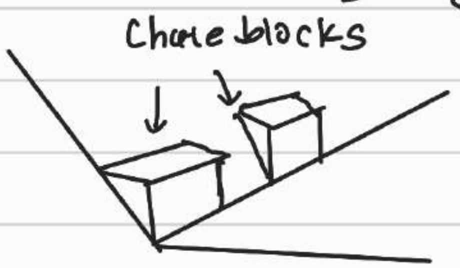
c. Weak Jump:

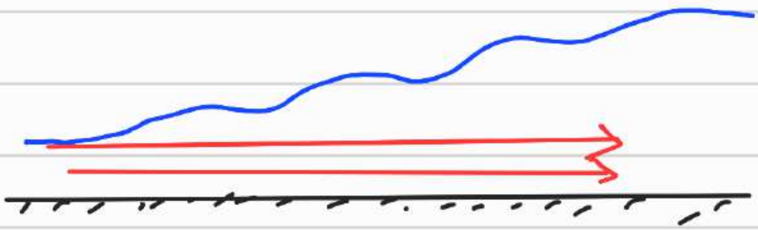


* $Fr_1 = 1.7$ to 2.5
* Energy loss $= 5$ to 15%
* The surface rises smoothly with small rollers.

d. Oscillating Jump:

* $Fr_1 = 2.5$ to 4.5
* Energy loss $= 15$ to 45%
* Pulsations are caused by jets entering the bottom of stilling basins generate large waves that travel for miles and damage earthen banks.
* To control it's effect, chute blocks are provided in stilling basin.





e. Steady Jump

* $Fr_1 = 4.5$ to 9
* energy loss $= 45$ to 70%

* Jump is stable, well balanced and intensive to downstream conditions.
* Recommend range for stilling basins.

f. Strong Jump:

* $Fr_1 > 9$
* energy loss = 70 to 85%.
* Very effective in energy dissipation but uneconomical for stilling basin due to large water heights.

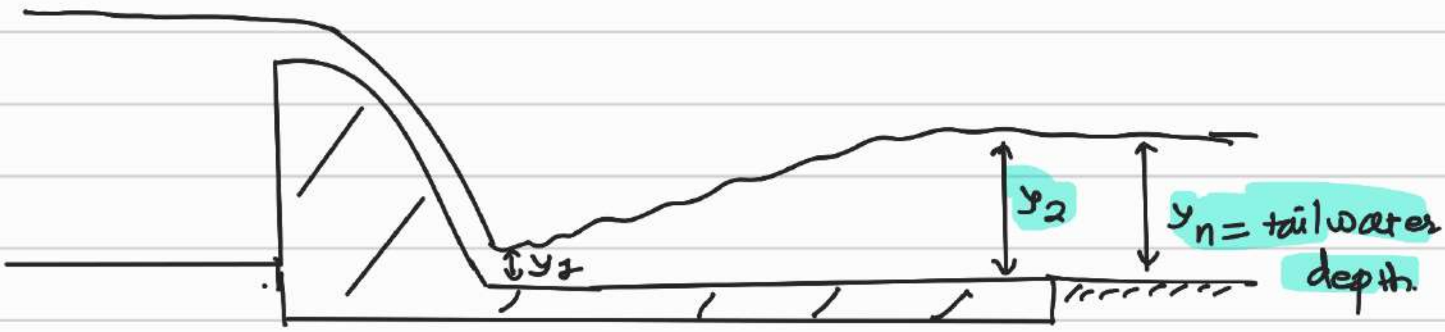
$y_1 = 0.5m$

$Fr_1 = 5 \Rightarrow y_2 = ?$

$Fr_1 = 15;\ y_2 = ?$

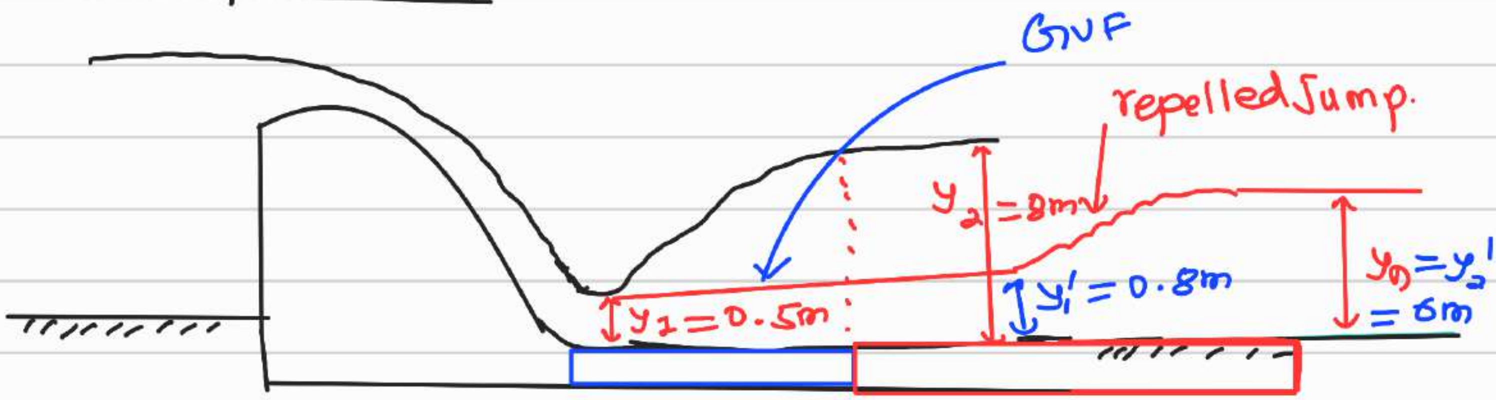
2. Based on Tailwater Depth:

a. Perfect Jump



* When Jump depth (y_2) = Tailwater depth (y_n), it is called perfect jump.

b. Repelled Jump:



* When $y_2 > y_n$: the jump is formed certain distance away from toe. This is called repelled jump.
* It increases length of floor.
* To avoid this, depressed floor is provided.

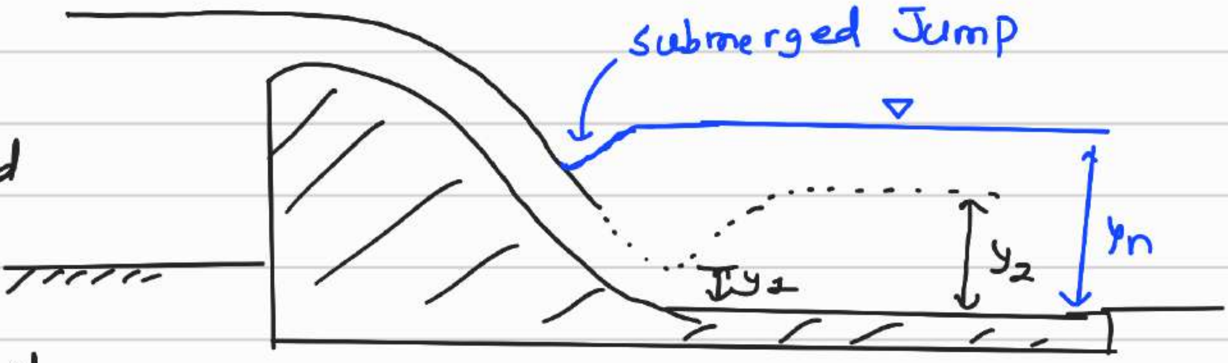
$\boxed{y_1 = 0.5m \;;\; y_2 = 8m.}$

$\boxed{y_n = 6m \;;\; y_1' = 0.8m}$
$= y_2'$

0.5 m to 0.8 m ⇒ GVF, which travels for long distance.

3. Submerged Jump:

* When $y_n > y_2$
⇒ the jump is submerged in tail water.



* To avoid this a sloping floor is provided above d/s bed or roller bucket type energy dissipator is used.

Q. Hydraulic jump can dissipate energy. Justify this statement with appropriate illustrations, figures, examples, equations where relevant.

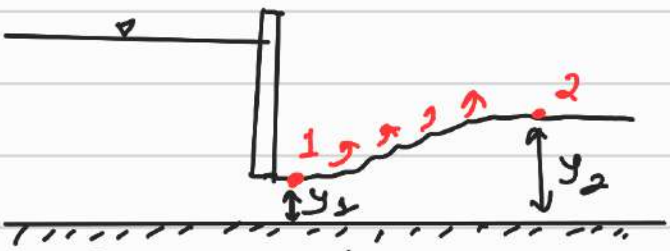


Fig: Hydraulic jump at d/s of sluice gate

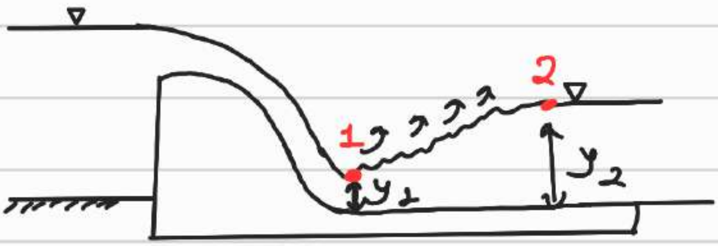


Fig: Hydraulic Jump at toe of spillway.

$\therefore$ 2 = sub-critical flow

1 = super critical flow

<u>Hydraulic Jump:</u> * It is the turbulent passage of flow from super-critical to sub-critical stage. It is rapidly varied non-uniform flow as the flow depth changes by large amount in a short length.

* Hydraulic Jump dissipates energy due to formation of turbulent rollers as a fast moving super-critical flow strikes a tranquil / sub-critical or streaming flow.

* Hydraulic Jump can dissipate upto 85% energy. The dissipation of energy by formation of hydraulic jump is achieved at toe of spillway, downstream of sluice gates, etc.

* The examples of energy dissipation by hydraulic jump include the stilling basins formed at toe of spillway, d/s floors at the toe of a fall structures or head regulators.

* The relevant equations of hydraulic jump are:

$$y_1 y_2 (y_1 + y_2) = \frac{2q^2}{g}$$

$$y_2 = \frac{y_1}{2}\left[-1 + \sqrt{1 + 8Fr_1^2}\right]$$

$$y_1 = \frac{y_2}{2}\left[-1 + \sqrt{1 + 8Fr_2^2}\right]$$

$$h_f = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

where, y_1, y_2 = sequent / conjugate

q = specific discharge

h_f = head loss.

However, to increase efficiency of energy dissipation, certain structures such as chute blocks, friction blocks and dentated sills are provided in stilling basins

Open Channel Flow:

Pipe flow	Open Channel flow
1. Top surface of flow is not exposed to air.	1. Top surface of flow is exposed to atmosphere.
2. The pressure is generally more than atmospheric pressure.	2. Pressure of flow surface is always atmospheric.
3. Flow occurs due to pressure difference	3. Flow occurs due to gravity.
4. HGL line may lie above or below centre-line of pipe	4. HGL coincides with flow surface.
5. Full flow in pipes, Siphon aqueducts, etc.	5. e.g: flow in canals, drains, rivers, partial flow in pipes, flow in sewers

$$\text{Total Energy} = z + \frac{p}{\gamma} + \frac{v^2}{2g}$$

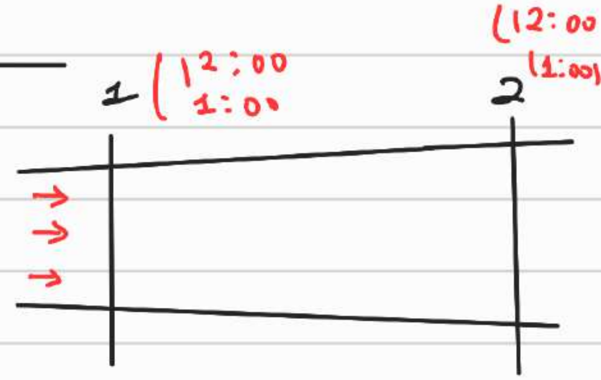
$z \downarrow$ low

$\frac{p}{\gamma} \downarrow$ high.

min. pressure

Types of open channel flow:

1. Steady and Unsteady flow:



Steady flow: flow parameters like depth, velocity, area, discharge, etc. at a point in channel do not change with time.

$$\frac{\partial y}{\partial t}=0;\quad \frac{\partial V}{\partial t}=0;\quad \frac{\partial A}{\partial t}=0,\quad \frac{\partial Q}{\partial t}=0$$

Unsteady flow: flow parameters like depth, velocity, area, discharge, etc. at a point in channel change with time.

$$\frac{\partial y}{\partial t}\neq 0;\quad \frac{\partial V}{\partial t}\neq 0;\quad \frac{\partial A}{\partial t}\neq 0,\quad \frac{\partial Q}{\partial t}\neq 0$$

2 Uniform and Non-Uniform flow:

a. Uniform flow:

* flow parameters donot change in space or along the length of channel or from point to point in the channel.
* $$\frac{\partial y}{\partial s}=0,\quad \frac{\partial V}{\partial s}=0,\quad \frac{\partial A}{\partial s}=0,\quad \frac{\partial Q}{\partial s}=0$$
* For flow to be uniform,

A ← i. Channel must be prismatic.

s ← ii. Slope of channel must be constant.

n ← iii. material of channel must be uniform throughout.

$$V=\frac{1}{n}R^{2/3}S^{1/2}$$

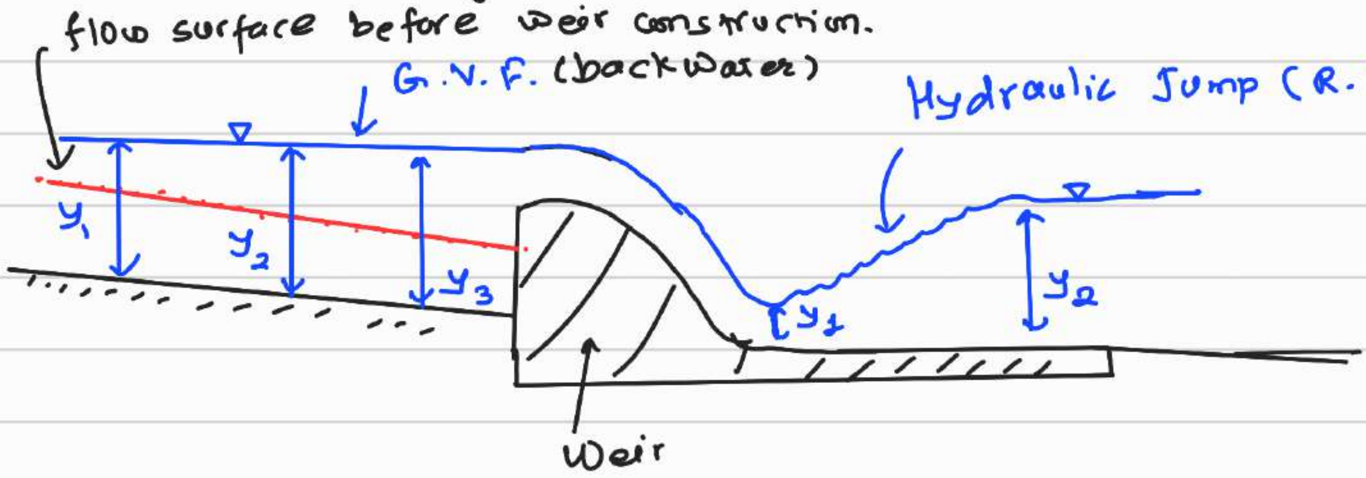
b. Non-uniform flow:

* flow parameters donot remain constant in space at a time

or along the length of Channel.

* $\frac{\partial y}{\partial s} \neq 0$, $\frac{\partial V}{\partial s} \neq 0$, $\frac{\partial Q}{\partial s} \neq 0$; $\frac{\partial A}{\partial s} \neq 0$.

i) Gradually Varied flow (G.V.F.)



* The depth of flow changes by small magnitude in a large length of channel.
* e.g; back water behind a weir/barrage

ii) Rapidly varied flow: (RVF)

* flow depth changes by large magnitude over a small length of channel.
* e.g; hydraulic jump, drop of water from channel into a pool, etc.

3. Based on Froude Number:

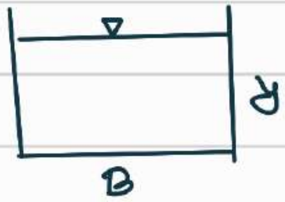
Froude Number = $Fr = \sqrt{\frac{\text{Inertia force}}{\text{viscous force}}}$

$$= \frac{V}{\sqrt{g * D}}$$

Where; D = hydraulic depth = $\frac{\text{Area}}{\text{top width}} = \frac{A}{T}$

But, for rectangular Channel,

$$D = \frac{A}{T} = \frac{B*y}{B} = y$$



$\therefore$ For rectangular channel, $\boxed{Fr = \frac{V}{\sqrt{g*y}}}$ ← For rectangular channel only.

① Critical flow:

* $Fr = 1$
* Occurs on critical slope.
* Normal depth is equal to critical depth.

- A = Adverse
- H = Horizontal
- M = mild
- C = critical
- S = steep ← Height
  - ↑ depth
  - कम fight उठाता
  - ↓ velocity

② Sub-critical flow

* $Fr < 1$ $\left[\frac{V}{\sqrt{gy}} < 1 \right]$
* Occurs on mild slope.
* Also called streaming or tranquil flow.
* Normal depth is more than critical depth of flow. $(y_n > y_c)$

③ Super-critical flow:

* $Fr > 1$ $\left[\frac{V}{\sqrt{gy}} > 1 \right]$
* Occurs on steep slope.
* Also called shooting depth.
* Normal depth is less than critical depth. $(y_n < y_c)$

4. Based on Reynold's no:

* Reynold's no. $= \frac{\text{inertia force}}{\text{viscous force}}$
* $Re = \frac{\rho V R}{\mu}$; ρ = density, V = velocity
  R = hydraulic mean depth. $= A/P$

a. **laminar flow:**

* $Re < 500$
* flow occurs in layers.

b. **Turbulent flow:**

* $Re > 2000$
* flow occurs in zig-zag way.

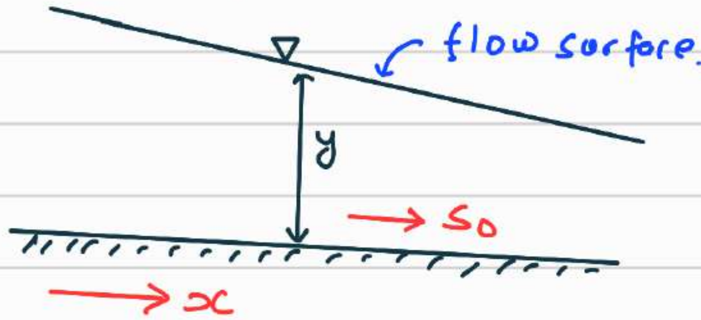
c. **Transitional flow**

* $Re = 500$ to 2000

Flow Profiles: → Related to G.V.F.

* The plot of flow surface of liquid in gradually varied flow (GVF) are called flow profiles.
* The governing equation for flow profile computation in GVF is:

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2}$$



where; y = flow depth

x = distance along channel.

S_0 = channel bed slope

S_f = energy or friction slope $= \frac{h_f}{L}$

$$= \frac{(z_1 + P_1/\gamma + v_1^2/2g) - (z_2 + P_2/\gamma + v_2^2/2g)}{L}$$

$$S_f = \frac{V^2 n^2}{R^{4/3}}$$

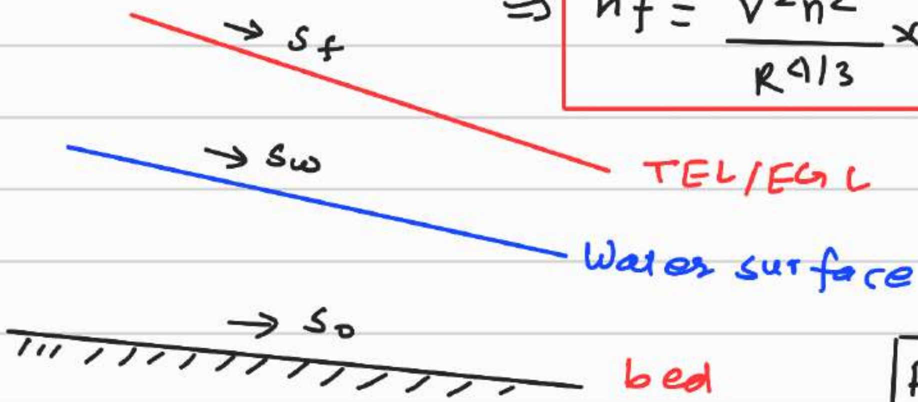
$$\left[V = \frac{1}{n} R^{2/3} S_f^{1/2} \right.$$

$$\Rightarrow S_f^{1/2} = \frac{V \times n}{R^{2/3}}$$

$$\text{or } S_f = \frac{V^2 n^2}{R^{4/3}}$$

But; $S_f = \frac{h_f}{L} \Rightarrow h_f = S_f * L$

$$\Rightarrow \boxed{h_f = \frac{V^2 n^2}{R^{4/3}} \times L}$$



Profile 1: rising
Profile 2: falling
Profile 3: rising

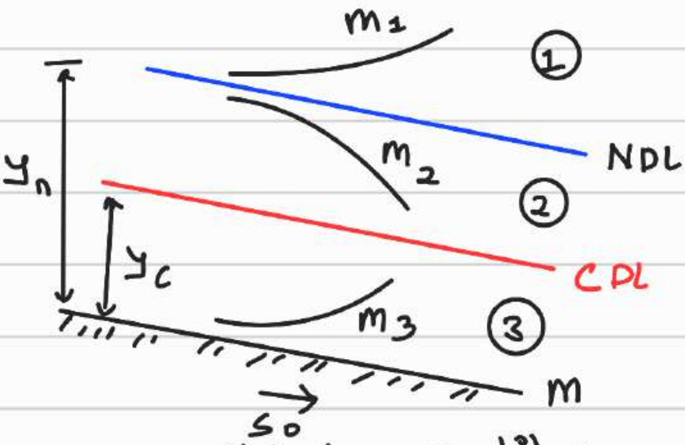


Fig: mild slope profiles

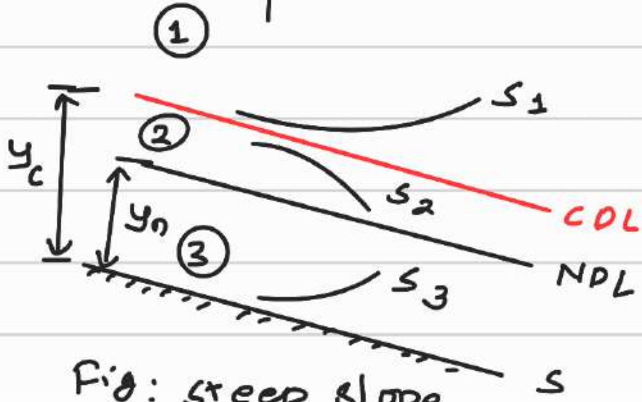


Fig: steep slope profiles

NDL = normal depth line
CDL = critical " "

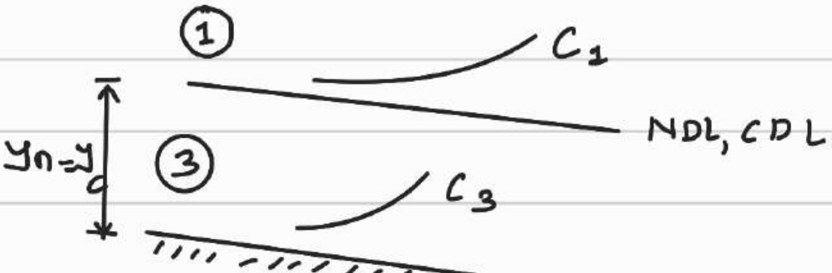


fig: critical slope profiles

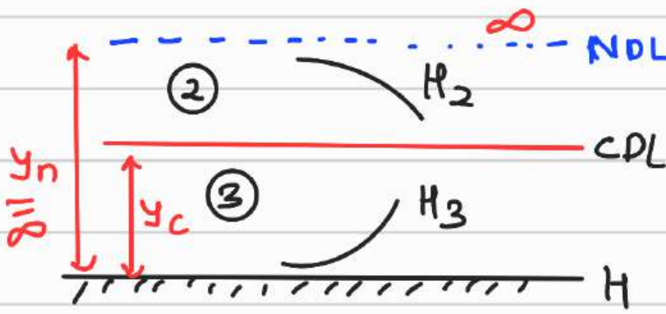


fig: horizontal slope profile

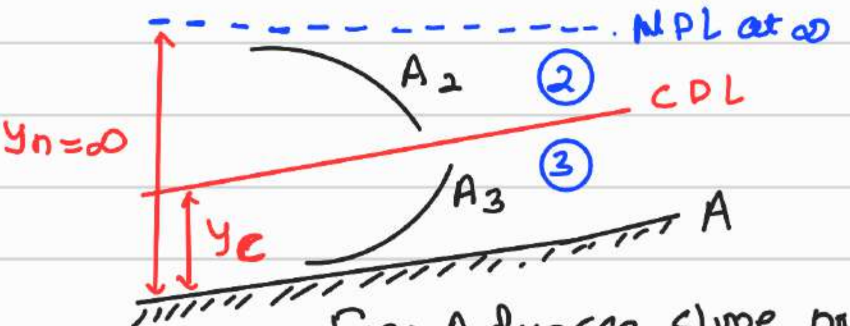
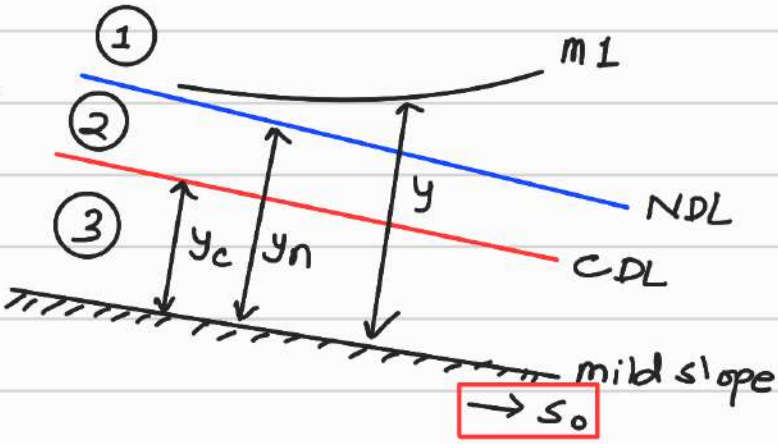


Fig: Adverse slope profiles

Mild slope Profiles:

1. M-1 Profile:



a. lies in region 1 (above NDL).

b. flow depth, $y > y_c$
and $y > y_n$.

c. Flow is sub-critical
since $y > y_c$.

d. $Fr < 1$

e. $\boxed{S_f < S_o}$ since; $\boxed{y > y_n}$

y_n corresponds to S_o or bed slope.

y corresponds to S_f or energy slope.

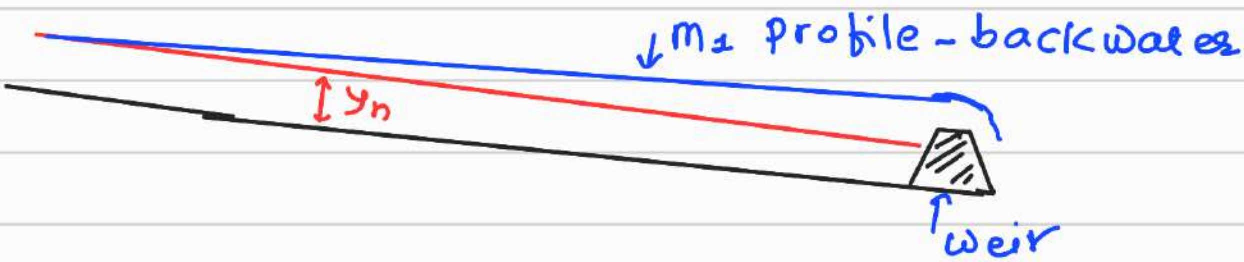
As depth increases, S_f decreases

$\because$ $\boxed{S_f = \dfrac{V^2 n^2}{R^{4/3}}}$

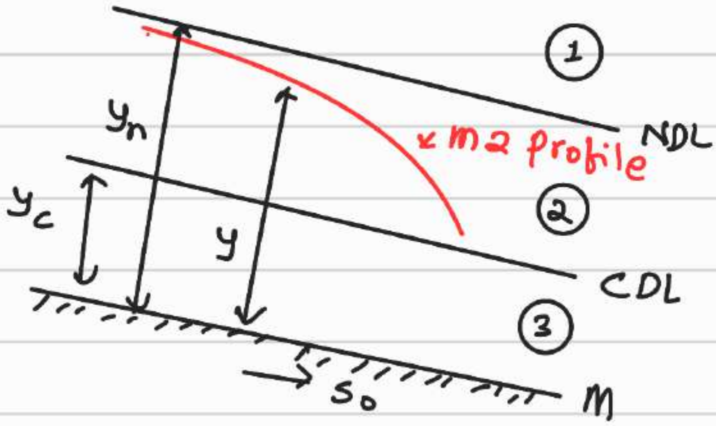
f. $$\frac{dy}{dx} = \frac{S_o - S_f}{1 - Fr^2} = \frac{+ve}{+ve} = +ve$$

$\Rightarrow$ flow depth increases along flow direction.

g. This profile is developed when an obstruction such as hump, weir, barrage is placed across a channel or when a even milder slope follows the given slope.



2. m2 Profile:



a. This profile lies in region 2 (betn NDL and CDL).

b. flow depth, $y > y_c$ and $y < y_n$.

c. Flow is subcritical since $y > y_c$

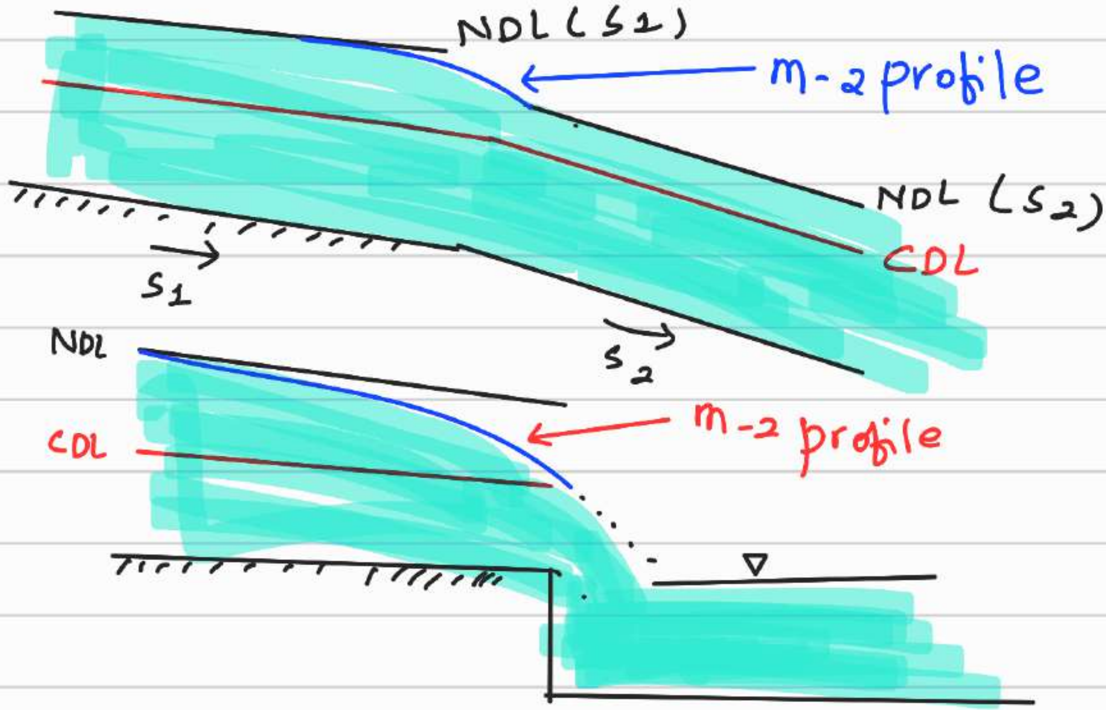
d. $Fr < 1$

e. $S_f > S_0$ because $y < y_n$

f. $$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - Fr^2} = \frac{-ve}{+ve} = -ve$$

⇒ flow depth decreases in flow direction.

g. This profile is developed when a channel drops into a pool or when a mild slope follows steeper slope.



3. m3 Profile:

a. lies in region 3 (below CDL)

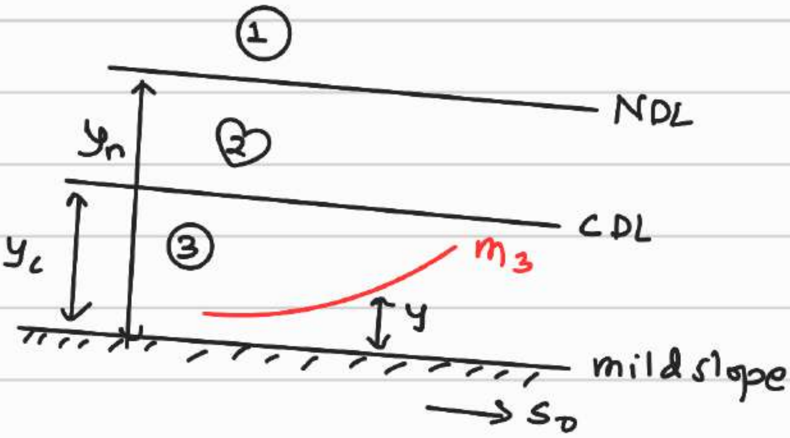
b. flow depth; $y < y_n$ and $y < y_c$.

c. flow is supercritical since; $y < y_c$.

d. $F_r > 1$

e. $S_f > S_0$ since $y < y_n$.

f. $\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2} = \frac{-ve}{-ve} = +ve$



$\Rightarrow$

$Q = \frac{A}{n} R^{2/3} S_0^{1/2} \rightarrow$ for uniform flow only.

$S_0 \rightarrow$ uniform flow हुँदाखेरीको energy slope

$S_f \rightarrow$ Actual flow हुँदाखेरीको energy slope

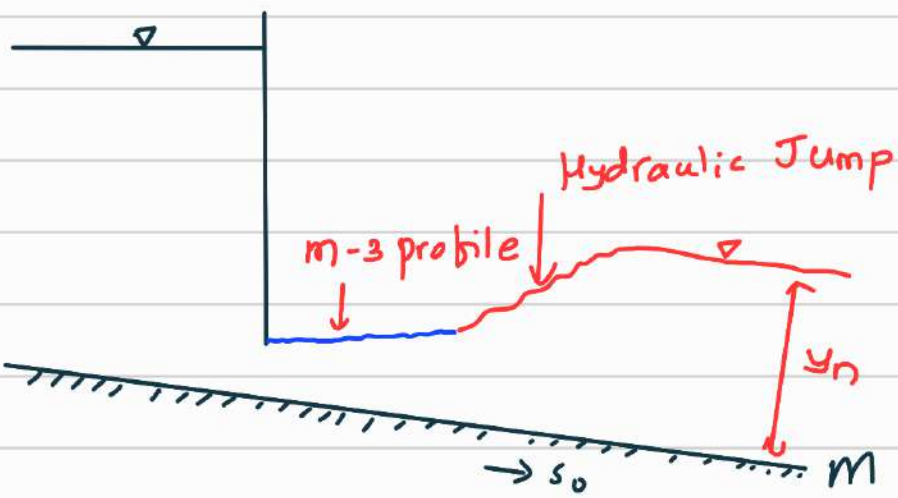
In uniform flow, $S_0 = S_f = S_w$

↑ water surface slope.

$Q = \frac{A}{n} R^{2/3} S_f^{1/2} \rightarrow$ for uniform as well as non-uniform flow.

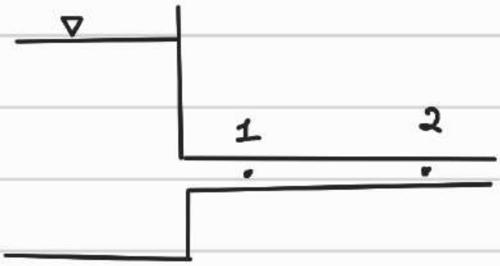
$\Rightarrow$ flow depth increases in flow direction.

g. This profile is developed when the water is released in a mild slope at super-critical velocity from sluice gate or spill way.



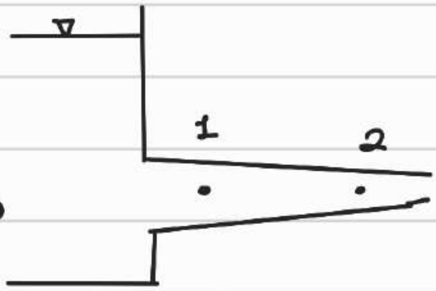
Classification of flow:

1. steady and unsteady flow.



a. Steady flow:

* flow parameters such as velocity, discharge, pressure, density, etc. remain constant at a point with respect to time.



* $\frac{\partial v}{\partial t}=0;\ \frac{\partial Q}{\partial t}=0;\ \frac{\partial P}{\partial t}=0;\ \frac{\partial \rho}{\partial t}=0$

b. Unstedy flow

* flow parameters at a point change with respect to time.

* $\frac{\partial v}{\partial t}\neq 0;\ \frac{\partial Q}{\partial t}\neq 0;\ \frac{\partial P}{\partial t}\neq 0;\ \frac{\partial \rho}{\partial t}\neq 0.$

Note: for steady flow, discharge must be constant.

2. Uniform and Non uniform flow:

a. Uniform flow:

* Flow parameters at a time remain constant in space or along length of flow.
* Diameter must be constant.

b. Non-uniform flow:

* flow parameters at any instant of time donot remain constant in space or along length of flow.

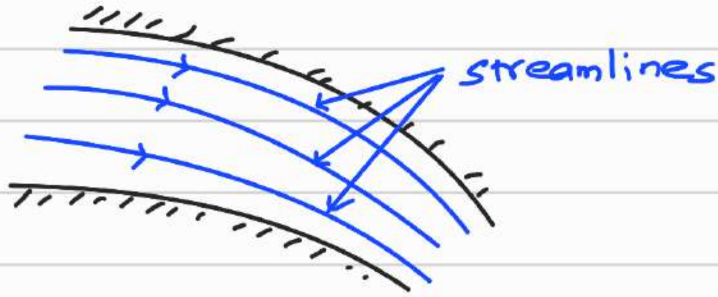
3. Based on Reynold's no:

* $Re = \frac{\rho v d}{\mu}$ in pipe flow $= \frac{vd}{\nu}$ in pipe flow.

v = velocity, ρ = density; d = diameter, μ = dynamic viscosity, ν = coeff. of kinematic viscosity

a. laminar flow:

* flow occurs in layers.
* one layer does not cross another layer.
* Streamlines exist.
* Re < 2000 for pipe flow
* Re < 500 for open channel flow.



b. Turbulent flow:

* flow occurs in zig-zag way.
* streamlines donot exist
* one layer crosses another.
* Re > 4000 for pipe flow.
* Re > 2000 for open channel flow.

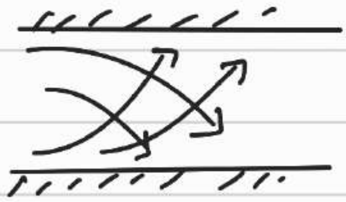


fig: turbulent flow.

c. Transitional flow:

* Re = 2000 to 4000 for pipe flow
* Re = 500 to 200 for open channel flow.

4. Compressible and Incompressible Flow:

incompressibile = no change in volume

$$\rho = \frac{mass}{volume}$$

a. Compressible flow: Density changes from point to point.

b. Incompressible flow: Density remains constant at each point.

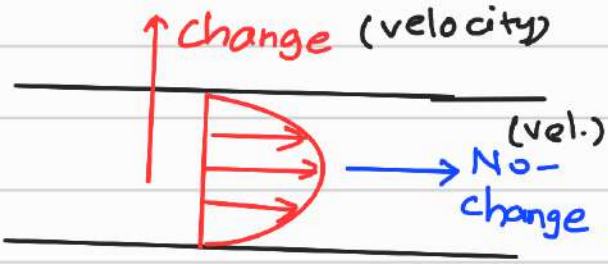
5. Rotational and Irrotational flow:

a. Rotational or Vortex flow: Net rotation about centre of particle is not zero.

b. Irrotational flow:- Net rotation about centre of particle is zero.

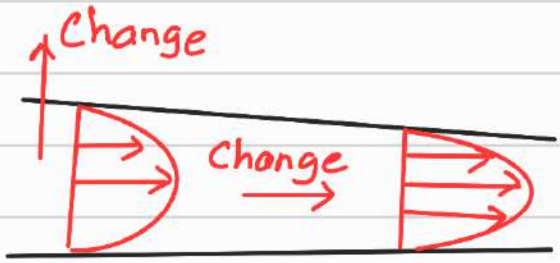
6. One - Two and Three Dimensional Flow:

a. One dimensional flow:



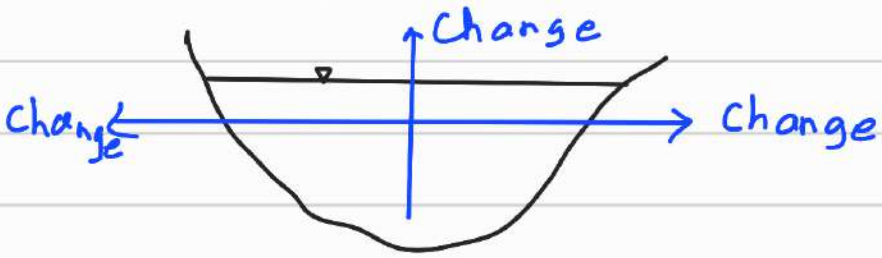
* flow parameters depend on one dimension only.
  $v = f(x, t)$. [t = time]
* e.g; flow in pipe of uniform diameter, flow in canal

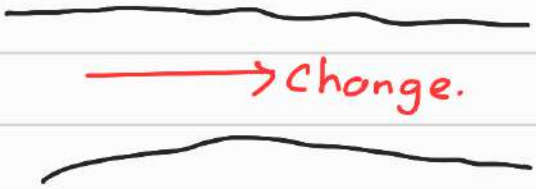
b. Two - Dimensional flow:



* flow parameters depend on two space variables.
* $v = f(x, y, t)$
* e.g; flow in convergent / divergent pipe.

c. Three - Dimensional flow:



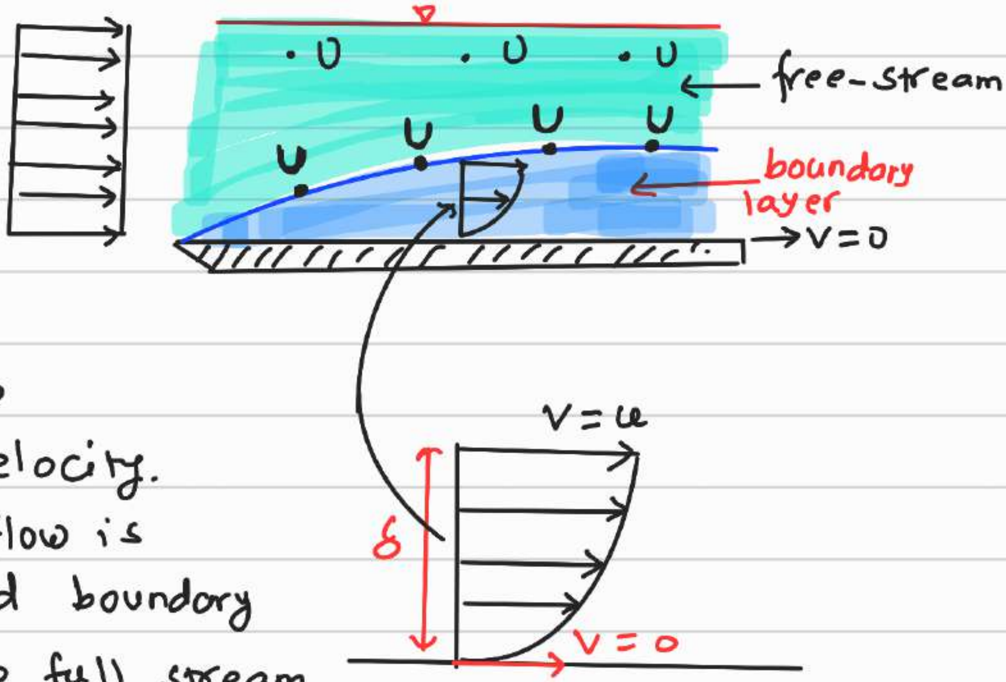


* flow parameters depend on three space variables
* $v = f(x, y, z, t)$.
* e.g; flow in rivers during flood.

Boundary layer Theory:

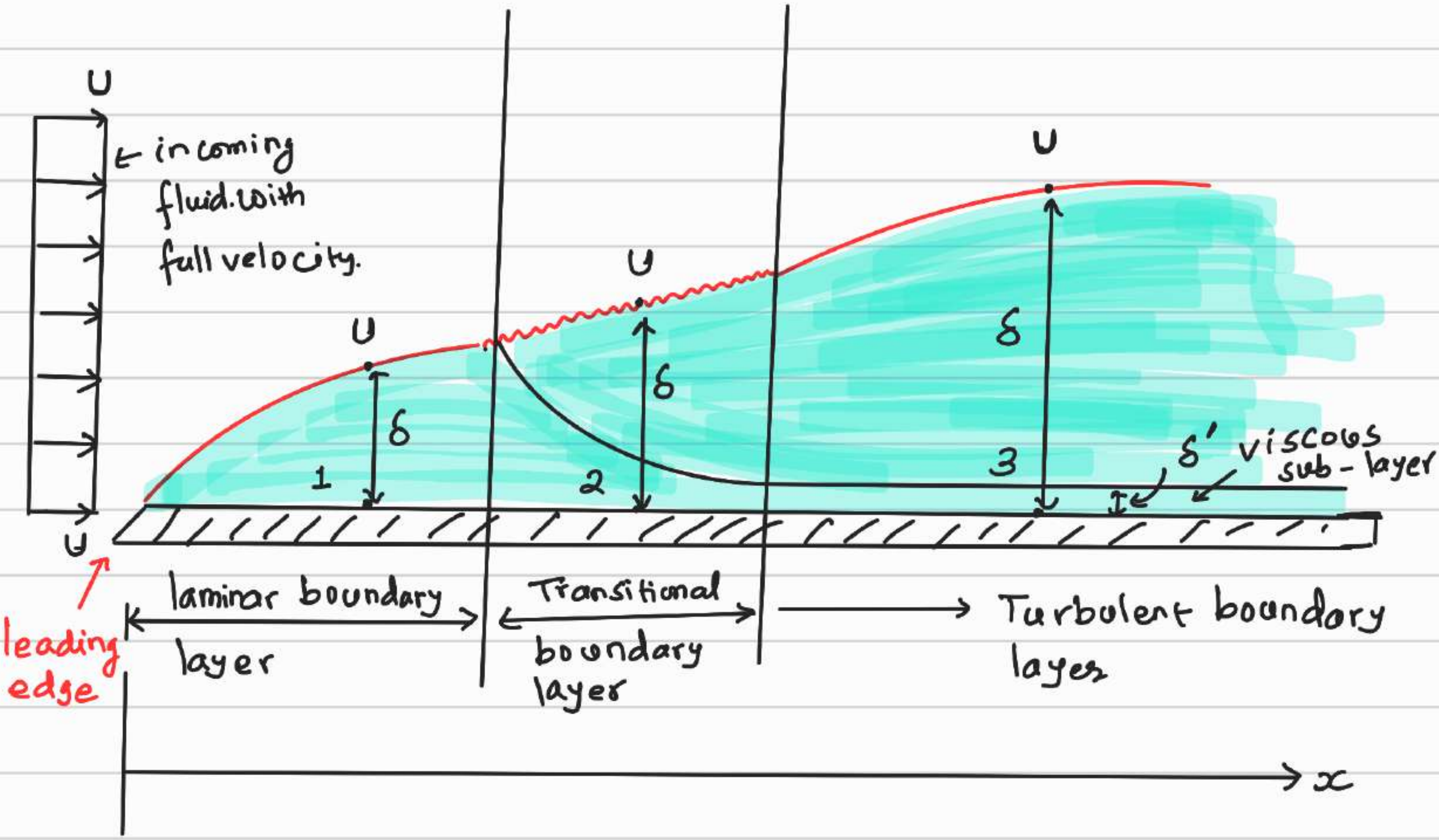
Boundary Layer:

* It is a narrow layer of fluid around a solid boundary, in which velocity changes from zero to full stream velocity.



* The velocity of flow is zero at the solid boundary and increases to full stream velocity in the boundary layer.

Boundary Layer Development Along a thin Plate



* When flow enters a solid plate, boundary layer is developed around it in which velocity changes from zero to full stream velocity.
* As the distance from leading edge increases, boundary layer grows in thickness.

Laminar Boundary Layer:

* The boundary layer will be laminar upto $Re = 5 \times 10^5$

where; $Re =$ Reynold's no $= \dfrac{\rho U x}{\mu} = \dfrac{U x}{\nu}$

where, x = distance from leading edge.

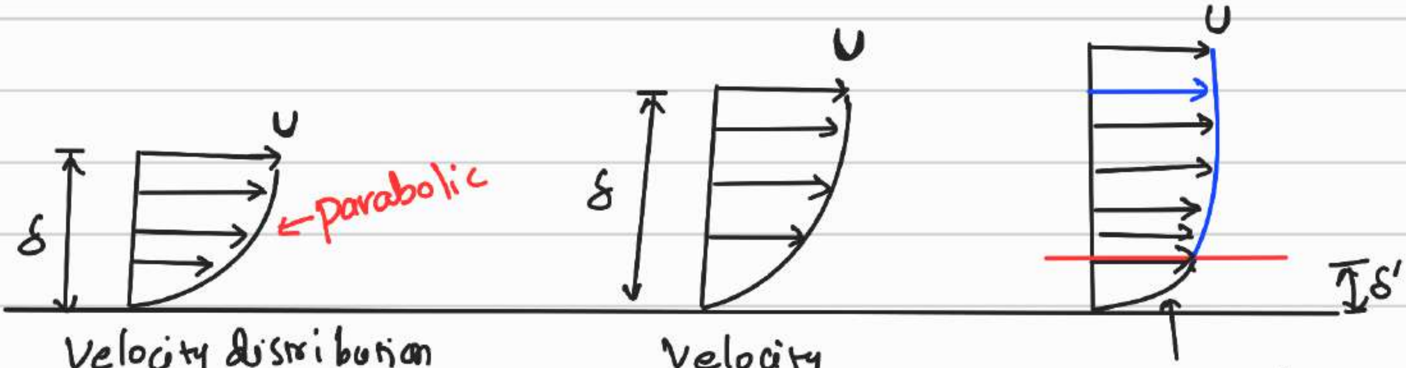
* In this zone, velocity distribution is parabolic.

Transitional Boundary layer:

* As the thickness of boundary layer continues to grow, it tends to change from laminar to turbulent.
* A small portion of boundary layer is separated which is laminar.

Turbulent Boundary layer:

* After $Re > 10^6$, boundary layer becomes turbulent and velocity distribution is logarithmic.
* A thin layer called laminar/viscous sub-layer still remains in which flow is laminar.



Velocity distribution at 1

Velocity distribution at 2

Velocity distribution at 3

Numerical Parameters:

* Thickness of laminar sub-layer,

$$\delta' = \frac{11.6 * \nu}{u*} \; ; \quad \nu = \text{coefficient of kinematic viscosity.}$$

$u* =$ shear friction velocity

$= \sqrt{\tau/\rho}$

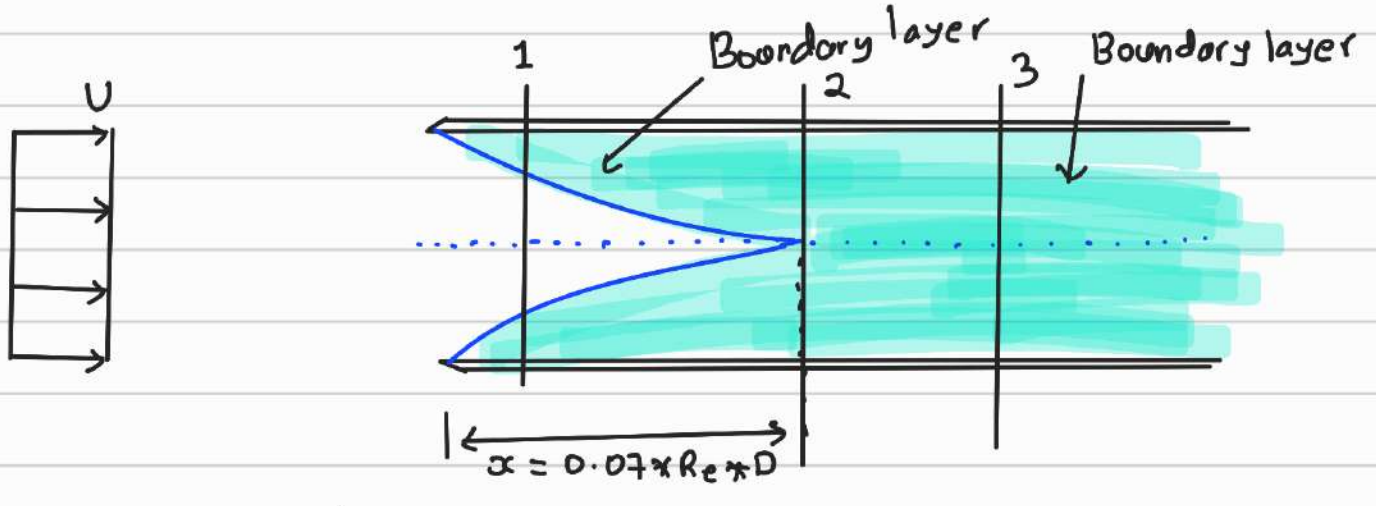
* Thickness of laminar boundary layer:

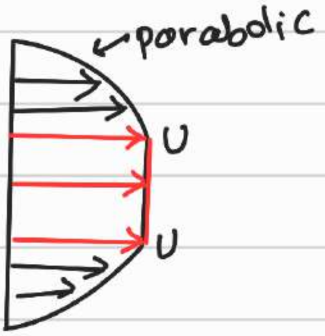
$$\frac{\delta}{x} = \frac{5}{\sqrt{Re}} \; ; \quad \delta = \text{thickness of boundary layer}$$

* Thickness of turbulent Boundary layer:

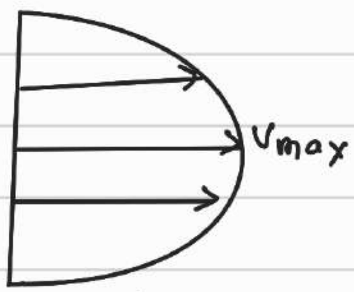
$$\frac{\delta}{x} = \frac{0.376}{Re^{1/5}} \; ; \quad \delta = \text{thickness of Boundary layer.}$$

Boundary Layer Along A Pipe:

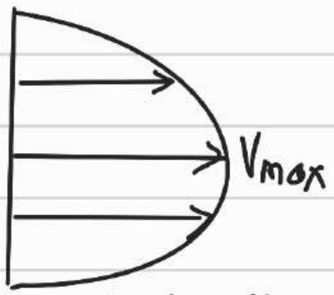




Velocity distribution at 1

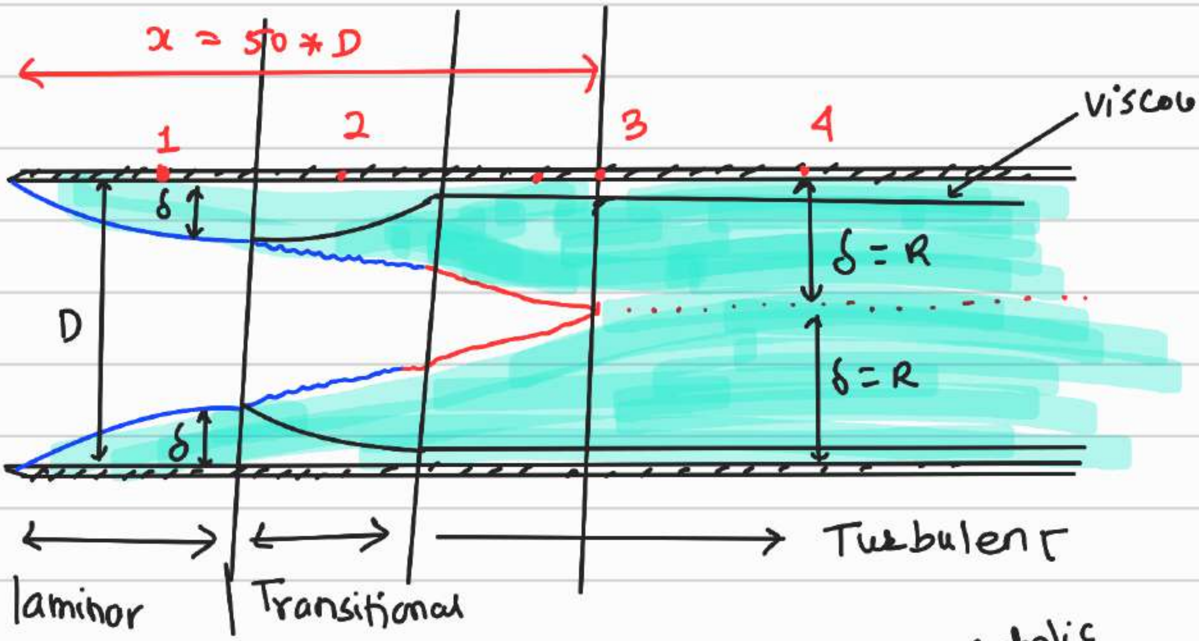


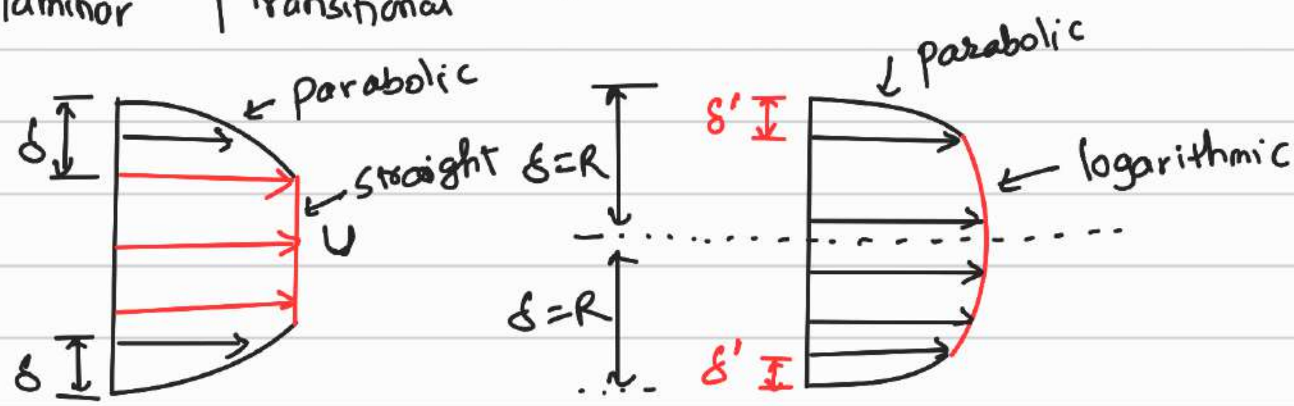
Velocity distribution at 2



Velocity distribution at 3

Boundary layer along a pipe for Turbulent flow:





At 1

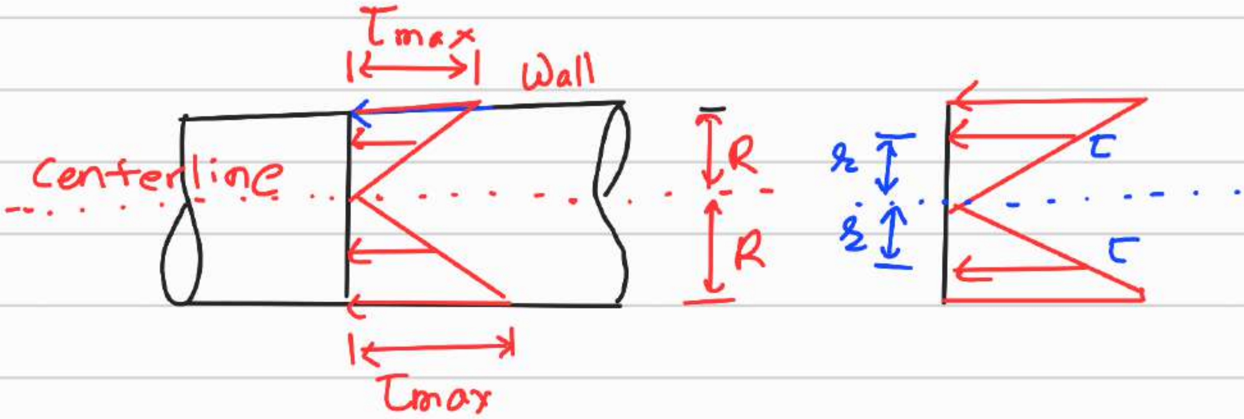
At 4

Laminar and Turbulent flow in Pipes:

Characteristics of laminar flow:

* flow in pipe will be laminar if $Re < 2000$.

a. Shear Stress Distribution:



* shear stress distribution in laminar flow in pipe is linear.
* maximum shear stress occurs at walls.
* At centre, shear stress is zero
* Shear stress at any distance r from centre of pipe is given by:

$$\tau = -\gamma * \frac{dh}{dx} * \frac{r}{2}$$

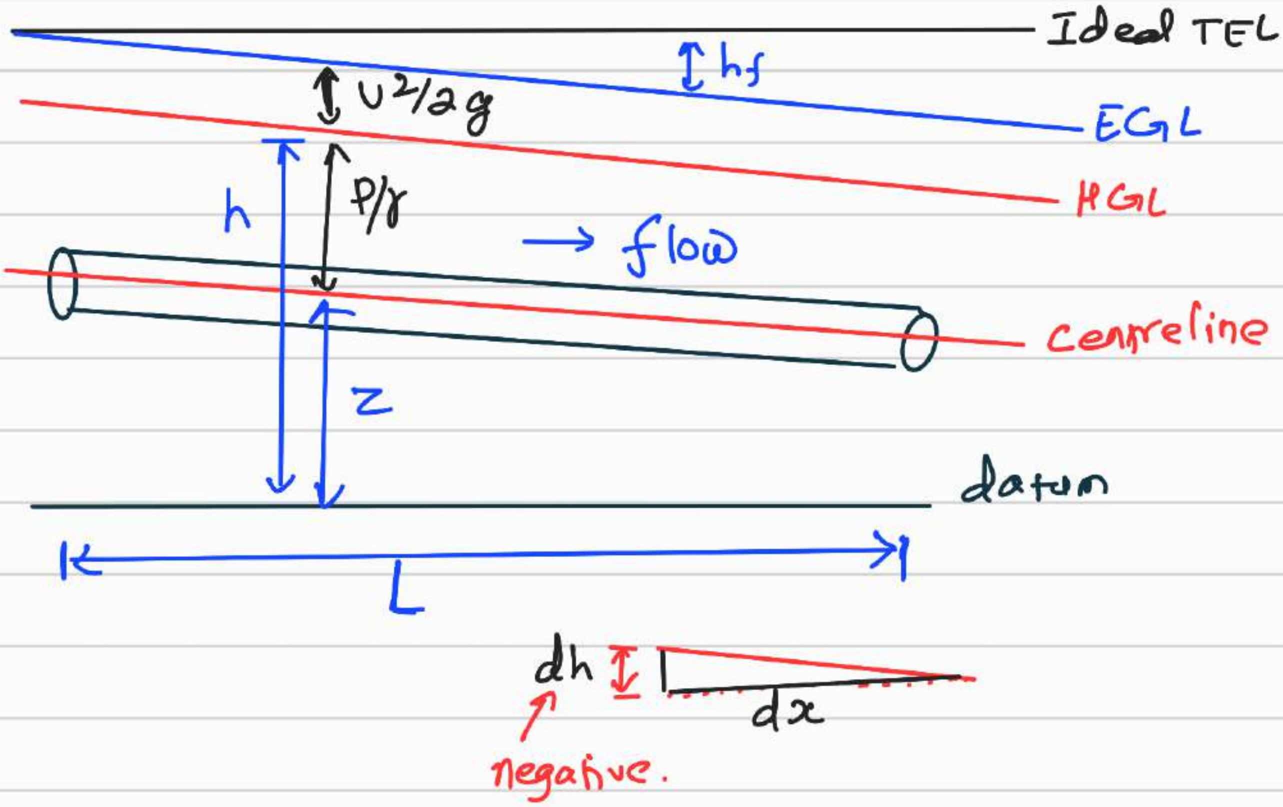
γ = unit weight of fluid

$$\frac{dh}{dx} = \frac{d}{dx}\left[z + \frac{p}{\gamma}\right]$$ = rate of change of piezometric head along direction of flow.

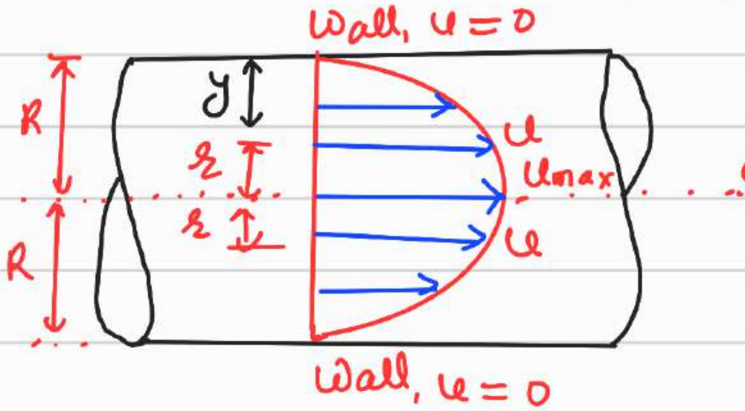
-ve sign shows that the flow takes place at expense of piezometric head. Or piezometric head is decreasing along flow direction.

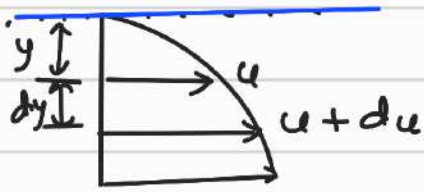
At centre; $r = 0$; $\tau = -\gamma * \frac{dh}{dx} * \frac{0}{2} = 0$

At walls; $r = R$; $\tau = -\gamma * \frac{dh}{dx} * \frac{R}{2}$



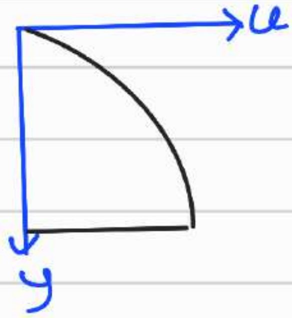
b. Velocity distribution





* Velocity distribution for laminar flow in pipe is given by:

$$u = \frac{-\gamma}{4\mu} * \frac{dh}{dx} * (R^2 - r^2)$$



shear stress, $\tau = \mu * \frac{du}{dy}$

we know; $\tau = -\gamma * \frac{dh}{dx} * \frac{r}{2}$

from fig;

$$y = R - r$$

$$\therefore \quad -\gamma * \frac{dh}{dx} * \frac{r}{2} = \mu * \frac{du}{dy}$$

$$or \quad du = -\frac{\gamma}{\mu} * \frac{dh}{dx} * \frac{r}{2} * dy$$

We know; $y = R - r$

$$dy = d(R - r) = -dr$$

$$\therefore du = -\frac{\gamma}{\mu} * \frac{dh}{dx} * \frac{r}{2} * -dr$$

$$du = \frac{\gamma}{\mu} * \frac{dh}{dx} * \frac{r}{2} * dr$$

$$u = \frac{\gamma}{\mu} * \frac{dh}{dx} * \frac{r^2}{4} + C \quad \ldots\ldots i)$$

At walls, $u = 0$, $r = R$

so, $0 = \frac{\gamma}{\mu} * \frac{dh}{dx} * \frac{R^2}{4} + C$

$$\Rightarrow \quad C = -\frac{\gamma}{4\mu} * \frac{dh}{dx} * R^2$$

from i) $u = \frac{\gamma}{4\mu} * \frac{dh}{dx} * r^2 - \frac{\gamma}{4\mu} * \frac{dh}{dx} * R^2$

$$\boxed{\boxed{u} = -\frac{\gamma}{4\mu} * \frac{dh}{dx} * \left[R^2 - \boxed{r^2} \right]}$$

* At centre, $r = 0$

$\therefore u = -\frac{\gamma}{4\mu} * \frac{dh}{dx} * R^2 \quad \ldots\ldots ii)$ maximum velocity

At walls, $u = 0$ ...... minimum velocity.

* Velocity distribution is parabolic
* Maximum velocity = 2 * Average velocity.
* $V_{max} = 2 * V_{avg}$

Average velocity = $\frac{\text{Discharge}}{\text{Area}}$

$$* \; V_{avg} = \frac{1}{2} * V_{max} = \frac{1}{2} * \frac{-\gamma}{4\mu} * \frac{dh}{dx} * R^2$$

$$= -\frac{\gamma}{8\mu} * \frac{dh}{dx} * R^2$$

* Average velocity occurs at $\frac{R}{\sqrt{2}}$ distance from center.

C. Head loss in laminar flow in Pipe:

we know;

$$u_{avg} = -\frac{\gamma}{8\mu} * \frac{dh}{dx} * R^2$$

Take $u_{avg} = V$

$$\therefore \; V = -\frac{\gamma}{8\mu} * \frac{dh}{dx} * R^2$$

$$\Rightarrow \frac{dh}{dx} = \frac{-8\mu}{\gamma * R^2}$$

$$\Rightarrow dh = \frac{-8\mu}{\gamma R^2} * dx * V$$

$$\text{But; } \int_1^2 dh = \int_1^2 \frac{-8\mu}{\gamma * R^2} * dx$$

$$\Rightarrow [h_2 - h_1] = -\frac{8\mu}{\gamma R^2} * V [x_2 - x_1]$$

But $x_2 - x_1 = L$ = length of pipe

$$h_2 - h_1 = -[h_1 - h_2] = -h_f$$

$$\therefore \; -h_f = \frac{-8\mu * V}{\gamma * R^2} * L$$

$$\Rightarrow h_f = \frac{8\mu * V}{\gamma * R^2} * L$$

$$\boxed{h_f = \frac{32 * \mu * V * L}{\gamma D^2}}$$

← Hazen poiseuille eqn

$\boxed{* \text{ head loss, } h_f \propto V}$

* Also in laminar flow, Darcy weisbach eqn also works for head loss, which gives:

* $h_f = \dfrac{fLV^2}{2gD}$; where; $\boxed{f = \dfrac{64}{Re}}$ ← compare h_f from Darcy weisbach & Hazen.

Turbulent flow in Pipes.

Basic characteristic of Turbulent flow in pipes:

a. Irregularity and randomness in time and space.

b. Diffusivity and rapid mixing

c. Reynold's no:, $Re > 4000$

d. Three dimensional velocity fluctuation.

e. Dissipation of energy by viscous shear stress.

f. Head loss is proportional to v^n where, $n = 1.85$ to 2

e.g., in Darcy Weisbach eqn:

$$\boxed{h_f = \frac{fLV^2}{2gD} \; ; \; h_f \propto V^2}$$

g. Turbulence is property of flow but not the property of fluid.

h. The friction factor, "f" in turbulent flow depends on Reynold's no, Re and relative roughness of pipe (K/D) ratio. where, K = average height of roughness
D = diameter of pipe.

* For hydrodynamically smooth pipes, "f" depends on Reynold's number only and for rough pipes. "f" depends on K/D ratio.

i. The friction is developed due to normal viscosity as well as eddy viscosity

$$t = \mu * \frac{du}{dy} + \eta * \frac{du}{dy}$$

dynamic viscosity   eddy viscosity.

Head loss

a. Major loss

* It occurs due to friction.

1. Darcy weisbach eqn.
   * Applicable for both laminar and turbulent flow.
   * $h_f = \frac{f l v^2}{2gd}$

$$= \frac{8 f l Q^2}{\pi^2 * g * d^5} = \frac{f l Q^2}{12.1 * d^5}$$

2. Hazen william formula. (open channel flow also)

$$V = 0.849 * C * R^{0.63} * S^{0.54}$$

C = roughness coeff.

R = Hydraulic mean depth $= \frac{D}{4}$

S = Slope of energy grade line.

$$= \frac{h_f}{L}$$

$$V = 0.849 * C * \left(\frac{D}{4}\right)^{0.63} * \left(\frac{h_f}{L}\right)^{0.54}$$

$$\therefore h_f = \frac{10.68 * L}{D^{4.87}} * \left(\frac{Q}{C}\right)^{1.852}$$

3. Manning's formula (for open channel flow)

$$* \quad V = \frac{1}{n} R^{2/3} S^{1/2}$$

But, S = slope of energy grade line $= \frac{h_f}{L}$

$$\therefore V = \frac{1}{n} * R^{2/3} * S^{1/2}$$

$$\Rightarrow S = \frac{V^2 n^2}{R^{4/3}}$$

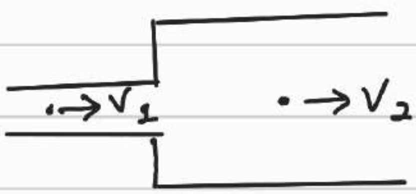
$$\Rightarrow \frac{h_f}{L} = \frac{V^2 n^2}{R^{4/3}} \Rightarrow \boxed{h_f = \frac{V^2 n^2}{R^{4/3}} * L}$$

L = length of channel
n = manning's roughness coeff.
R = Hydraulic mean depth.

2. minor head loss:

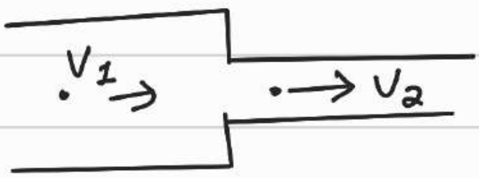
a. Head loss due to sudden expansion:

$$h_f = \frac{(V_1 - V_2)^2}{2g}$$



b. Head loss due to sudden contraction

$$* \; h_f = \left[\frac{1}{C_c} - 1\right]^2 * \frac{V_2^2}{2g}$$

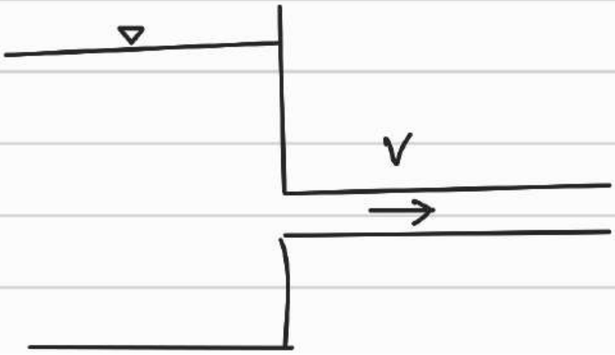


C_c = Coefficient of contraction
= 0.62 if not given

c. Entry loss:

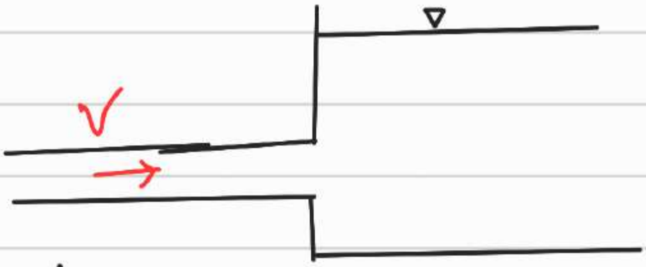
* extreme contraction

* entry loss $= 0.5 * \frac{V^2}{2g}$



d. exit loss:

* exit loss is the loss of velocity head when pipe discharges into reservoir or the velocity head of water at exit end.

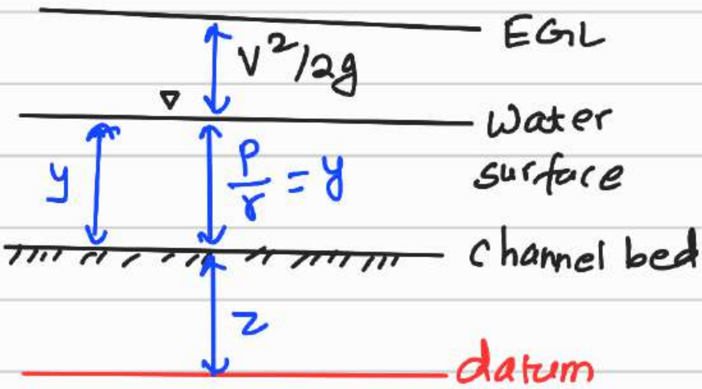


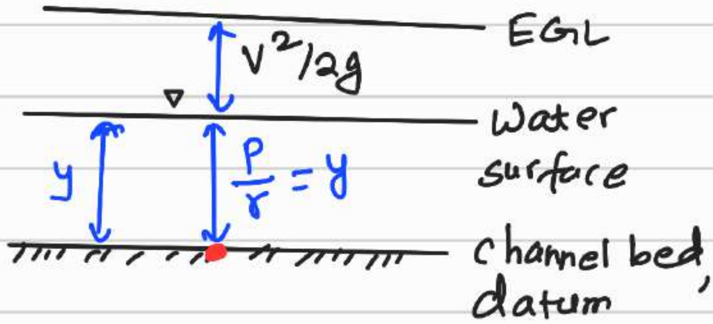
* exit loss $= \frac{V^2}{2g}$

e. Bend loss, valve loss, etc.

$h_f = K * \frac{V^2}{2g}$; where $K =$ loss coefficient

Specific Energy:



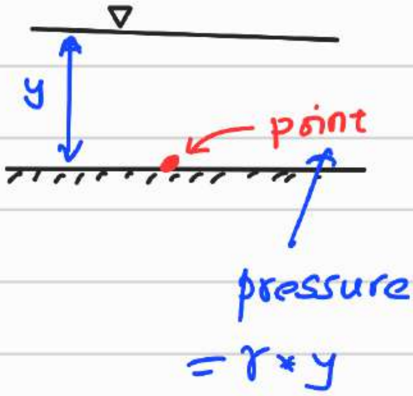


* Specific energy is the energy per unit weight of a liquid (Head) by taking channel bed as it's datum.

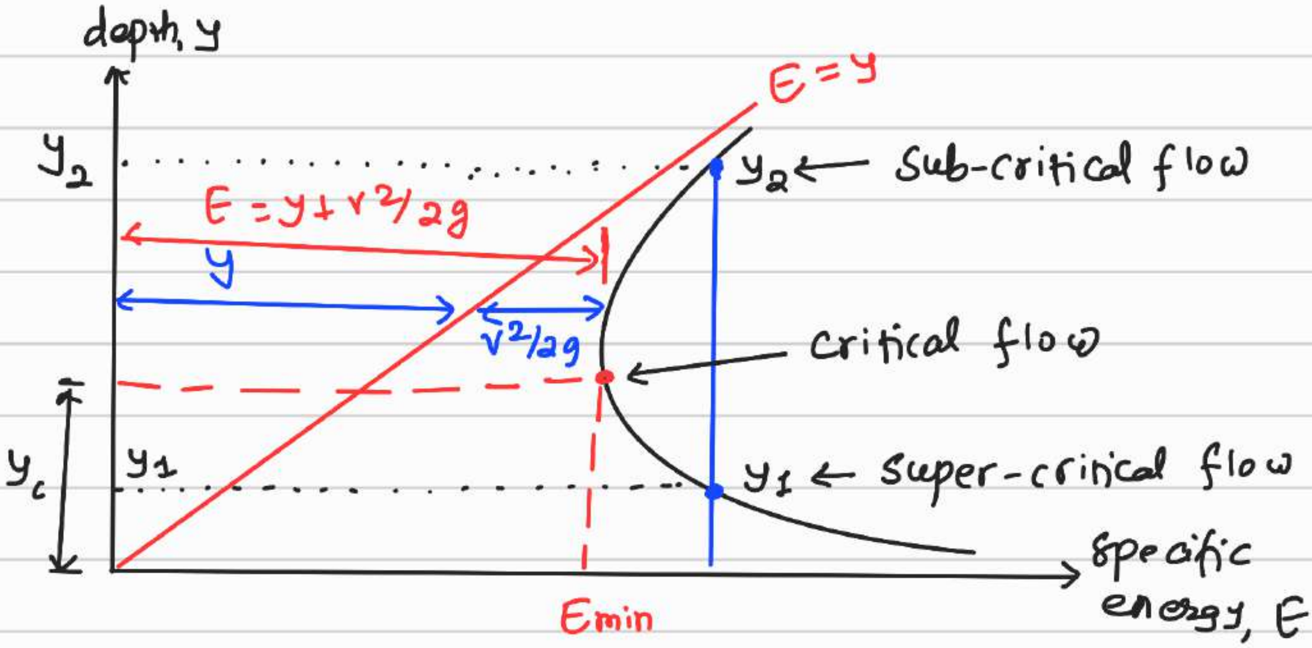
* Specific energy, $E = P/\gamma + V^2/2g$

But; in open channel flow; $\frac{P}{\gamma} = \frac{\gamma * y}{\gamma} = y$

$$\boxed{\therefore\ E = y + \frac{v^2}{2g}}$$



Variation of specific Energy



* The minimum specific energy of flow occurs for critical flow or $Fr = 1$.
* For same specific energy water can have

two depths, called alternate depths. One of those depths is in supercritical flow, y_1 and another is in sub-critical flow y_2.

Specific energy & Critical depth:

$$E_{min} = y_c + \frac{v_c^2}{2g}$$ ← Critical velocity.

specific energy min. (→ E_{min})

critical depth (→ y_c)

In rectangular channel,

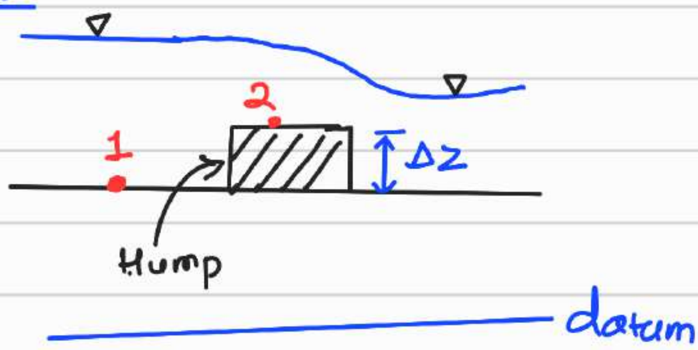
$E_{min} = 1.5 * y_c$

$v_c = \sqrt{g * y_c}$

$y_c = \left(\frac{q^2}{g}\right)^{1/3}$

Uses of Specific Energy:

1. To Calculate Height of Hump



We know, specific energy, $E = y + \frac{v^2}{2g}$

$= \text{Total energy} - z$

* The specific energy at 1 & 2 are linked by:

$E_1 = E_2 + \Delta z$

* Δz will be maximum when E_2 is minimum.

* Minimum value of $E_2 = E_c$

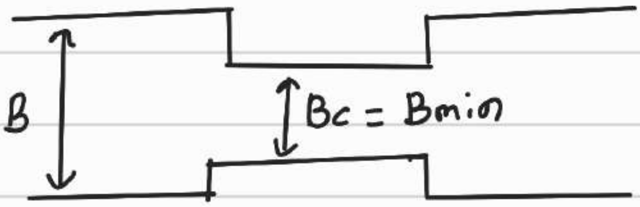
$$\boxed{\therefore \Delta z_{max} = E_1 - E_c}$$

2. To Calculate the floor level in regulators, etc.

floor level $=$ FSL of Canal $- E_f$

where; E_f = specific energy

3. To calculate minimum width of fluming or constriction in any channel without affecting upstream water level.



* For this, the flow in constricted part is critical.

Open channel flow velocity formula:

1. manning's : $\boxed{C = R^{1/6}/n}$ $\quad V = C\sqrt{RS}\ ;\ C = \frac{R^{1/6}}{n}$
2. Chezy
3. Kutter's formula: $V = \boxed{\dfrac{\frac{1}{n} + 23 + \frac{0.00155}{S}}{1 + \left(23 + \frac{0.00155}{S}\right) * \frac{n}{\sqrt{R}}}} * \sqrt{R*S}$ $\leftarrow C$

4. Bazin's formula:

$$V = C\sqrt{RS}$$

where; $C = \dfrac{157.6}{1.81 + \frac{m}{\sqrt{R}}}$

where; m = Bazin's constant
R = Hydraulic mean depth.

5. Hazen william formula:

$$V = 0.849 * C * R^{0.63} * S^{0.54}$$

C = roughness coeff. of pipe/ channel.
R = Hydraulic mean depth.
S = bedslope of channel/ slope of EGL.