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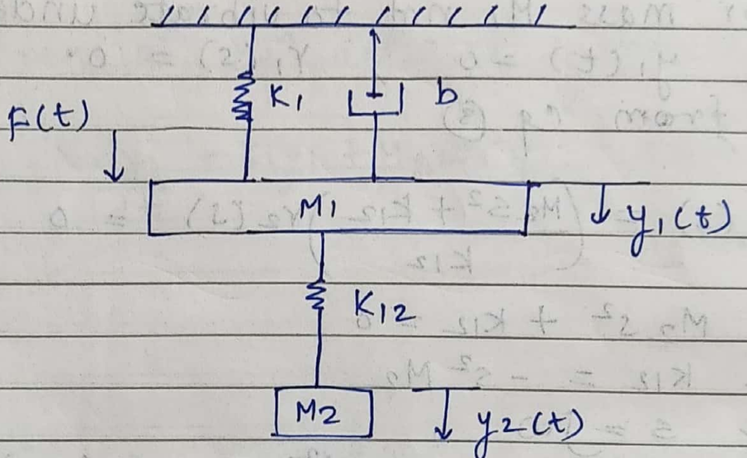
Q 1)

A dynamic vibration absorber is shown in fig. This system is representative of many situation involving the vibration of machine containing unbalanced component. The parameter M2 & K12 maybe choosen so that the main mass M1 does not vibrate in steady state when F(t) = 2 * sin (10 *t)

Obtain differential eqn for system.
Simulate system for 10 sec. $M_1 = 100$ $K_1 = 50$ $b = 50$
Find optimal value of M2 & K12 So that system does not vibrate



Ans: Applying Newton's second law of motion on M_1

$$M_1 \frac{d^2 y_1}{dt^2} = F - k_1 y_1 - b\frac{dy_1}{dt} - k_{12}(y_1 - y_2)$$

$$M_1 \frac{d^2 y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + k_{12}(y_1 - y_2) = F \quad \text{—— (1)}$$

Applying Newton's Second Law of Motion on M_2 -

$$M_2 \frac{d^2 y_2}{dt^2} + k_{12}(y_2 - y_1) = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + k_{12} y_2 = k_{12} y_1 \quad \text{—— (2)}$$

Take Laplas transform

$$M_2 s^2 Y_2(s) + k_{12} Y_2(s) = k_{12} Y_1(s)$$

$$Y_2(s) = \frac{k_{12}}{M_2 s^2 + k_{12}} Y_1(s) \quad \text{--- (3)}$$

The force = $F = f(t) = a \sin \omega_0 t$.

Take Laplas transform of force function

$$F(s) = \frac{a\omega_0}{s^2 + \omega_0^2}$$

for mass M_1 not to vibrate under steady state,

$y_1(t) = 0$, $Y_1(s) = 0$

$\therefore$ from eq (3)

$$Y_1(s) = \left(\frac{M_2 s^2 + k_{12}}{k_{12}}\right) Y_2(s) = 0$$

$\therefore M_2 s^2 + k_{12} = 0$

$\therefore k_{12} = -s^2 M_2$

& $s = j\omega$

$\therefore k_{12} = -(j\omega_0)^2 M_2 = -(-\omega_0)^2 M_2$

$= \omega_0^2 M_2$

recall eq (2) & substitute $k_{12} = \omega_0^2 M_2$, $F = a \sin \omega_0 t$

$\therefore$

$$M_1 \frac{d^2 y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + \omega_0^2 (y_1 - y_2) = a \sin \omega_0 t$$

$$M_1 \frac{d^2 y_1}{dt^2} + b\frac{dy_1}{dt} + k_1 y_1 + M_2 \omega_0^2 y_1 = a \sin \omega_0 t + M_2 \omega_0^2 y_2$$

Recall eq ②

$$M_2 \frac{d^2 y_1}{dt^2} + K_{12} y_2 = K_{12} y_1$$

Substitute $K_{12} = \omega_0^2 M_2$

$$M_2 \frac{d^2 y_2}{dt^2} + \omega_0^2 M_2 y_2 = \omega_0^2 M_2 y_1$$

$$\frac{d^2 y_2}{dt^2} + \omega_0^2 y_2 = \omega_0^2 y_1$$

∴ differential eqⁿ defining system are,

$$M_1 \frac{d^2 y_1}{dt^2} + b \frac{dy_1}{dt} + K_1 y_1 + M_2 \omega_0^2 y_1 = a \sin \omega_0 t + M_2 \omega_0^2 y_2 \quad \text{——— (A)}$$

$$\frac{d^2 y_2}{dt^2} + \omega_0^2 y_2 = \omega_0^2 y_1 \quad \text{——— (B).}$$