# A SOLUTION TO SOME REPRESENTATIONAL AND ALGORITHMIC QUESTIONS IN INFLUENCE DIAGRAMS AND MECHANISM DESIGN

ANDREAS HAUPT

Abstract. We present five questions in algorithmic mechanism design whose answer is related to the representation, information design and algorithmics of mechanisms with sparse communication. The appendix contains definitions of multi-agent influence diagrams without (known) resp. with commitment (new). All results are denoted conjectures, many of these we are confident to be able to prove in the moment of writing.

## 1. Representation

1.1. **Which commitment is needed?** Given a multi-agent influence diagram with commitment, and given that this commitment might be computationally expensive (see answer to Question 4):

**Question 1.** *What is the complexity of finding the set of vertices where commitment cannot be dropped without altering equilibrium?*

More concretely:

**Definition.** *Let $G$ be an influence diagram with commitment and $v$ a commitment node. Let $G'$ be the influence diagram that is the same as $v$ except that $v$ is a non-commitment node. We call $v$* superfluous *if the sets of equilibria of $G$ and $G'$ coincide for any functional specification.*

In the following, we assume that the structural influence diagram (an unlabeled graph) is input to the algorithm.

**Conjecture.** *We can compute the set of superfluous nodes in $O(n^2)$.*

1.2. **Attainable Ex-Post Mechanisms.** Quantifying over choices of commitment node strategies and setting chance node values deterministically, one arrives for each specified influence diagram at families of mappings chance node values $\mapsto$ equilibrium expected utilities. If there is a choice of commitment node strategies and of equilibrium such that a certain mapping

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is realised, we call it *attainable mappings*. Given that canonical communication schemes are often sought:

**Question 2.** *Which choice nodes can be added without changing the set of attainable mappings? Which choice nodes can be deleted so that the set of attainable mappings is only expanded? Is there a unique, non-trivial inclusion-maximal set of attainable mappings? Can it be found efficiently?*

For an affirmative answer, the graph(s) that correspond to the inclusion-maximal set of attainable mappings are informative in that they represent the most general/canonical communication.

**Conjecture.**

- *Non-commitment action nodes that have only non-commitment action nodes of the same player as parents may be deleted and their parents pairwise connected to their children.*
- *For any choice node, all incoming edges can be replaced by an edge that is subdivided by an action node from the same player. This operation can be reversed.*
- *There is a non-trivial, unique, inclusion-maximal set of attainable mappings. It is the set of attainable mappings of the influence diagram that arises from the aforementioned operations such that no choice node has a child that is a choice node of the same player and all non-commitment choice nodes have in-degree one.*

## 2. Communication

### 2.1. Natural Bidding Languages.

> "The choice of a [bidding] language is somewhat arbitrary and there are various reasonable ways to strike the expressiveness vs. simplicity tradeoff, depending on the application." [NR01, p. 10].

Both simplicity and expressiveness in this formulation are not defined unambiguously. By defining a *natural* bidding language for a game that in a natural way is the most expressive for a set of utilities we lower the mentioned arbitrariness. To formulate our definition, we need to restrict to a class of mechanisms. For ease of formulation, we assume a static mechanism and symmetric, selfish agents. There are $n$ agents and a principle. Agents have types $t_i$ from identical type spaces, send a message, and the principle commits beforehand to a mapping of these messages to allocations. We consider cominant incentive compatible mechanisms. All agents have the same utility function that only depends on the agents' own outcome.

In such a setting, we define a $k$-natural bidding language to be a concise representation of a partition of the type space induced by the pre-images of (symmetric) equilibrium strategies of agents *when the message space is constrained to be of cardinality* $k$. This bidding language will crucially depend on utilities. Natural bidding languages represent concisely what agents most importantly would like to disclose about their preferences.

**Question 3.** *What are natural bidding languages for popular computational mechanism design models? How hard is the problem to decide whether a language is a natural bidding language?*

**Example** (voting, single-peaked preferences on finite ordered set)**.** *Set of connected partitions with respect to the order underlying the single-crossingness condition.*

**Conjecture** (multi-item auction; single-minded bidders)**.** *Some upward closure of a single minded bid (unclear!).*

By reduction from SAT:

**Conjecture.** *Even for a single agent in a combinatorial auction, deciding for a utility function and a partition whether the partition is a natural bidding language is NP-hard.*

## 3. Computation

3.1. **Mechanism Design is Harder than Nash.** Mechanism design is characterised as *reverse* game theory. Given multi-agent influence diagrams with and without commitment, we can study hardness of finding equilibria in games in comparison to finding mechanisms.

**Question 4.** *Are there classes of multi-agent influence diagrams such that the computation of equilibria with one node of commitment is much harder than without?*

Again by reduction from SAT:

**Conjecture.** *There is a class of multi-agent influence diagrams whose equilibrium can be found in linear time but where the calculation for one node of commitment is NP-complete.*

3.2. **Approximating mechanism outcomes in a non-standard multidimensional mechanism design problem.** Consider the multi-agent influence diagram with commitment given in Figure 1 (with commitment, the choice nodes $t_i$ can be spared).

**Question 5.** *Is there a constant-factor approximation algorithm for the commit strategy of the lighter agent?*[1]

[1]We give an equivalent formulation of the same problem: Let $B$ and $N$ be not necessarily independent random variables. Find random variables $X_1, \dots, X_n$ and numbers $t_1, t_2, \dots, t_n$ such that $X_i$ is a deterministic function of $N$, that for $i \in \arg\min_i \mathcal{W}^1(X_i, N) - t_i$ (Wasserstein-1-distance=total variation distance) we have $\mathbb{E}[t_i]$ maximal. This is a tough problem, although it seems that at least for the mapping $X_i \mapsto t_i$, mutual information seems to be promising heuristic.

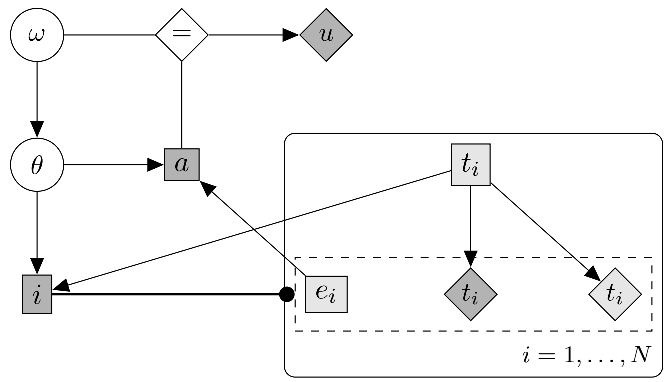


Figure 1. A two-agent influence diagram. As in [KM03], we draw nature nodes as circles, choice nodes as rectangles and utility nodes as diamonds. We use shades to denote choice and utility nodes for different agents. For plates, we adopt the standard notation as in [KFB09], for gates the one in [MW09]. The domains are $\omega, \theta, a \in \Omega$, $i \in [N]$, $t_i, u \in \{0, 1\}$.

## Appendix A. Background on Influence Diagrams

A.1. **Influence Diagrams.** A *structural multi-agent influence diagram* for agents $i = 1, 2, \ldots, n$ is a directed, acyclic graph $G$ on node set $\mathcal{N} \cup \bigcup_{i=1}^{n} \mathcal{A}_i \cup \bigcup_{i=1}^{n} \mathcal{U}_i$ with the property that all nodes in $\bigcup_{i=1}^{n} \mathcal{U}_i$ are leaves and all nodes are finite sets. Denote $\delta^{-}(x)$ the incoming nodes of a node $x \in V$ and $\mathcal{M}(A)$ the discrete probability distributions on $A$.

For a relational multi-agent influence diagram, a *functional specification* is a node labelling $f$

$$
\begin{aligned}
f(n) &\in \{p | p\colon \bigtimes_{x \in \delta^{-}(u_i)} x \to \mathcal{M}(n)\mu\}, n \in N \\
f(a_i) &\in \{A || A| < \infty\} \\
f(u_i) &\in \{u | u\colon \bigtimes_{x \in \delta^{-}(u_i)} x \to \mathbb{R}\}, u_i \in U_i
\end{aligned}
$$

We call a relational multi-agent influence diagram with a functional specification a specified multi-agent influence diagram. Nodes $n$ are called *nature* or *chance* nodes, $a_i$ are called *choice* or *action*Âănodes of player $i$ and $u_i$ are called utility nodes of player $i$.

An *equilibrium* of a specified multi-agent influence diagram is an assignment $\sigma_{a_i}$ from choice nodes to probability kernels given parent values such that the following maximisation problem is solved by

$$
\sum_{u \in \mathcal{U}_i} \prod XXX \tag{1}
$$

In addition, some choice nodes can be commitment nodes. These are considered fixed and to be optimised in the equilibrium computation.

## References

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University of Frankfurt, Frankfurt 60486, Germany
*Email address*: `s3339284@uni-frankfurt.de`