Voluntary Carbon Market Design*

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Abstract

We consider optimal certification design in voluntary carbon markets. A certifier commits to a menu of methodologies to certify the quality of negative emissions activities. Project developers engaging in such activities choose an activity and a certification methodology. After the chosen methodologies are applied to certify activities, a competitive market for certificates determines prices. Gains from trade are maximized by offering all possible methodologies at cost, and addition of certificates always increases gains from trade. This result fails to hold if the objective is not maximization of gains from trade, but emissions mitigation, in particular if emissions mitigation and gains from trade are not aligned. The certifier's problem is equivalent to a non-linear screening problem with non-monotonic valuations. For any such screening problem, menus that maximize such general objectives are *monotonic in objective* and any addition of a certificate that violates monotonicity reduces the certifier's objective. In particular, there are sufficient conditions on which certificates may be introduced into a market.

1 Introduction

Voluntary carbon markets are emerging as an essential tool in the global fight against climate change. These markets empower companies, individuals, and nations to go beyond regulatory compliance, reducing greenhouse gas emissions by purchasing and retiring carbon credits, thus becoming a driving force for climate mitigation. Participating in these markets offers not only an immediate solution for offsetting emissions but also stimulates innovative negative carbon emissions technologies to meaningfully contribute to mitigate global warming.

This article aims to provide a model of the design of voluntary carbon markets, delineating the role of different stakeholders, and discussing the objectives of certifiers in voluntary carbon markets.

At the center of voluntary carbon markets stand carbon credit certifiers such as *Verra* and *Gold Standard*, which issue certification standards for mitigation activities such as reforestation, and chemical carbon removal. For example, a certificate could provide a statement that a crystallization plant removes one metric ton of carbon dioxide from the atmosphere.

After certification, a downstream market allocates goods based on certification, with a price formation that is independent of, and not controllable by, the certifier. Therefore, independent of the objective that the certifier adopts, it will need to reason about how certification of certificate supply on market outcomes?

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What impact does the presence of certification have on outcomes in the downstream market? We answer this question of a monopolistic certifier and a competitive downstream market. In our model there is a mass of heterogeneous project developers, which we will call *producers*, each creating a single unit of a vertically-differentiated product. Producers can select the quality level of their products. The corresponding production cost is determined by the producer's type and is increasing in quality. A mass of unit-demand consumers purchases these projects, which we will also call *goods*. Consumers also have varying types which determine the strength of their preference for quality. Types are totally ordered and satisfy a single-crossing condition: higher-type producers are relatively more efficient at producing higher-quality goods, which higher-type consumers have relatively stronger preference for.

Quality cannot be verified directly; instead, a third-party certifier offers a menu of certification methodologies, each with its own requirements and price of certification. Producers can purchase certifications of their products' qualities from this menu. If their product surpasses the quality level of the certificate, it will be marketed as such. The certified goods, together with indistinguishable uncertified ones, are then sold in a competitive market.

In our model, we assume that expectations about quality are rational, following the Economics literature on certification Milgrom 1981; Grossman 1981; Lizzeri 1999; P. M. DeMarzo, Kremer, and Skrzypacz 2019. A big strand of empirical work considers the question of whether certifications are correctly interpreted in various markets (Wimmer and Chezum 2003 for race horses, Tadelis and Zettelmeyer 2015 on used-car auctions, Ramanarayanan and Snyder 2012 for dialysis screening centers, Luca 2016 for restaurant reviews, Elfenbein, Fisman, and McManus 2015 for seller ratings on Ebay, Conte and Kotchen 2010 for voluntary carbon credits). In our model, certificates are based on verifiable certification requirements, and at perfect Bayesian equilibrium all market participants hold rational beliefs about the distribution of quality implied by any given certificate. We note that such rational beliefs need not be aligned with official certificate descriptions. For example, in voluntary carbon markets, several empirical contributions point out that carbon certificate descriptions might not correspond to counterfactual mitigation outcomes, compare West, Börner, et al. 2020; West, Wunder, et al. 2023; Guizar-Coutiño et al. 2022; Clarke and Barratt 2021; Greenfield 2023; Greenfield 2021.

The presence of certificates clearly impacts the market equilibrium because it influences consumers' beliefs and hence willingness-to-pay. Our first result shows that, for any menu of certificates offered by the certifier, the equilibrium choice of certificates and the allocation outcome of the downstream market equilibrium is unique. Furthermore better producers (that is, those who can produce higher quality at lower cost) always purchase higher (more restrictive) levels of certification. This implies that the producer-consumer matching in equilibrium is assortative in a weak sense: There is a matching of trading producer-consumer pairs that is matched in quantiles. In such a matching, better producers sell to consumers with higher value for quality, and at higher quality levels.

We deduce from this analysis that a certifier maximizing gains from trade offers every certificate at cost. We also show that a certifying agent entering into the market can never lead to a loss of welfare, no matter what menu of certification options they make available. In addition, there are small menus yield that yield close to optimal welfare, and small menus that yield close to optimal revenue.

Our proof contains technical insights that may be of independent interest. Namely, we reduce the certifier's problem to one of a seller who wishes to sell a divisible good, facing buyers whose valuations are non-linear *and potentially non-monotone* in quantity. The seller corresponds to the certifier, and the buyers correspond to producer-consumer pairs. The quality of certificates corresponds to the quality of the good. This projection of producers and consumers into a single economic agent is enabled by the fact that the matching in equilibrium is unique and assortative. The buyer valuations in this reduced problem are concave but not necessarily monotone as a function of quantity, meaning that for different buyer types the valuation may reach its maximum at different quantities. The valuations do satisfy a single-crossing property, which implies that types are totally ordered and higher-type buyers have point-wise higher valuations and have weakly higher preferred quantities. Under these conditions, we show that the revenue-optimal menu may be non-linear but will exhibit prices that are monotone in quantity. Monotonicity of menus allows us to show that small menues achieve good revenue, and good welfare, generalizing results on non-linear pricing from Bergemann, Shen, Xu, and E. Yeh 2012; Bergemann, Shen, Xu, and E. M. Yeh 2012.

The menu of certifications offered by a revenue-maximizing third-party certifier can distort welfare and result in inefficient levels of trade by using its market power in the market for certification. But even with menus that achieve high gains from trade, the goal of voluntary carbon markets may not achieve high welfare. The reason for this is that there might be strong externalities: Carbon certificates that allow firms to claim net zero carbon emissions might be of high value despite having low actual emissions reduction. We find that climate objectives are incompatible with both revenue and gains-from-trade maximization, in that there are instances of the problem that achieve high value under one while achieving an arbitrarily little fraction under the other objective. Similarly, gains-from-trade and revenue objectives are incompatible.

1.1 Related Literature

This work contributes to the literature on certification. The early results Milgrom 1981; Grossman 1981 produce unraveling type results: In equilibrium, the quality of a good is fully revealed. The main intuition for these results in models of certification is that certifying non-informatively will be interpreted by the market as a sign of bad quality—adverse selection is extreme. In our model of a revenue-maximizing certifier, there might be other reasons for certification less informatively, as the price in the competitive market may depend on not only an individual seller's quality, but all the sellers in the market. Later contributions Lizzeri 1999; P. M. DeMarzo, Kremer, and Skrzypacz 2019; Acharya, P. DeMarzo, and Kremer 2011 allow for unsuccessful certifications, restricting the stark result obtained in Milgrom 1981; Grossman 1981.

More broadly, our work is related to mechanism and information design in the presence of an exogenously given game played after the design. We recommend Bergemann and Morris 2019 for a general overview of information design. Our work is especially related to the design of information provided to buyers and/or sellers of a good. Bergemann, Brooks, and Morris 2017 considers the design of information for a first-price auction, where a third party can reveal a signal correlated with a buyer's valuations, and fully characterizes the achievable revenue and payoffs. Alijani et al. 2022 extends the analysis to a scenario with multiple buyers. In a general mechanism design framework, Candogan and Strack 2021 develop an optimal disclosure policy for action recommendations in a game with private types and a hidden state. Dworczak 2020 designs mechanisms in a setting where players participate in a finite Bayesian game after participating in the mechanism, so that game outcomes are impacted by information revealed over the course of the mechanism. He finds a cutoff structure of optimal mechanisms in the first stage.

More generally, our work relates to the problem of how to sell payoff-relevant hard information to a potential buyer. Bergemann, Bonatti, and Smolin 2018 solve for the revenue-maximizing mechanism in binary environments, and Bergemann, Cai, et al. 2022 establish when full disclosure is approximately optimal under more general spaces of actions. In contrast, our certifying agent is selling a signal that is valuable in that it conveys information to other participants in a subsequent game.

In our mathematical analysis, we reduce the certifier's problem to a pricing problem with a non-linear valuation. The non-linear pricing literature following Mussa and Rosen 1978 (see also treatment in Dewatripont, Bolton, et al. 2005 and Börgers and Krahmer 2015, Chapter 2.3) studies a non-linear concave valuation with a quadratic cost. The functional form assumptions in these papers allow to characterize the optimal mechanism in closed form. Typically, the optimal menu of offered goods is a continuum, in contrast to the linear screening problem first studied in the influential Myerson 1981. Our analysis will show that also the class of models we consider may feature infinite menus.

Our results on the optimality of small menus relate to a literature on mechanism design with limited communication. The papers Bergemann, Shen, Xu, and E. Yeh 2012; Bergemann, Shen, Xu, and E. M. Yeh 2012 consider approximation of non-linear single- and multi-dimensional pricing environments with finite menus (in the papers called "finite information"). The papers make functional form assumptions similar to the ones in Mussa and Rosen 1978, and derive rates of approximation by finite menus in these finite menus.

Additionally, our reinterpretation of a certification design problem as screening problem relates to the paper Zapechelnyuk 2020 which reduces a certification problem in moral hazard to a delegation problem.

Finally, our main assumption guaranteeing uniqueness of our equilibrium is a single-crossing condition. Single-crossing conditions are important in several domains, among them the interdependent private values auctions (Milgrom and Weber 1982) and social choice and voting (Saporiti and Tohmé 2006). A recent line of work in algorithmic mechanism design has employed single-crossing conditions to enable approximately optimal designs in interdependent value settings Roughgarden and Talgam-Cohen 2013; Chawla, Fu, and Karlin 2014. Closest to the present paper, another implication of single-crossing is adverse selection in markets Mirrlees 1971; Spence 1974.

1.2 Outline

The rest of this article is structured as follows. We formalize our model in Section 2. In Section 3 we analyze the structure of equilibria given the certifier's offerings. Section 4 discusses our results for certification maximizing gains from trade. Section 5 contrasts this with the optimal design for welfare or for climate objectives. Finally, Section **??** studies revenue maximization of certifiers, Section **??** concludes. Section A contains additional results.

2 Model

Market We consider a continuum market between producers and consumers. Producers are unit-supply and parameterized by types $\psi \in \mathbb{R}_+$ with measure G. Consumers are unit-demand and parameterized by types $\phi \in \mathbb{R}_+$ with measure F. The type measures F and G are atomless and continuous with compact support.

Goods can be produced at different levels of quality, denoted $q \in [0, 1]$. Goods of higher quality are more valuable to consumers but more costly to produce. We write $g(q; \psi)$ for the cost incurred by a producer of type ψ when producing a good of quality q. We assume g is weakly convex and non-decreasing in q for every ψ and normalized so that $g(0;\psi) = 0$. We also write $f(q;\phi)$ for the value enjoyed by a consumer of type ϕ for a good of quality q, where f is weakly concave and non-decreasing in q for every ϕ and normalized so that $f(0;\phi) = 0$. We will scale valuations so that $f(q;\phi) \leq 1$ for all q and ϕ, which is without loss for bounded values.

We will assume that costs and valuations satisfy single-crossing with respect to the producer and consumer types, respectively. Roughly speaking, this means that producers (consumers) of higher types have lower marginal cost (higher marginal value) for producing higher-quality goods. More formally, for all $\phi_1 < \phi_2$ and $q_1 < q_2$, we have

$$f(q_2;\phi_2) - f(q_1;\phi_2) > f(q_2;\phi_1) - f(q_1;\phi_1).$$

Likewise, for all $\psi_1 < \psi_2$ and $q_1 < q_2$, we have

$$g(q_2;\psi_2) - g(q_1;\psi_2) < g(q_2;\psi_1) - g(q_1;\psi_1).$$

Transfers between producers and consumers are permitted. We assume that producers and consumers are risk-neutral and have quasi-linear preferences with respect to money. That is, if consumer ϕ purchases a product of quality q from producer ψ at a price of p, then the consumer enjoys utility

$$u_C((q,p);\phi) = f(q;\phi) - p$$

and the producer's utility is

$$u_P((q,p);\psi) = p - g(q;\phi).$$

Certification Crucially, producers and consumers cannot contract on quality. This means that a producer cannot credibly commit to the quality of the good they produce, and a consumer cannot verify quality at the point of trade. But there is a third-party certifier who is able to determine the quality of a producer's good. This verification is costly to the certifier, with cost $c \geq 0$.

After verifying the quality of a good, the certifier is able to assign to that good a signal (or certificate) $\sigma \in \Sigma$, where Σ is an arbitrary space of potential certificates. This certificate is visible to all producers and consumers. The certifier is permitted to collect payments from producers for this service, and these transfers can depend arbitrarily on the certificate produced.

The certifier can commit to a certification menu M, which is a collection of certificate-transfer pairs $(\sigma, t(\sigma))$ along with quality requirements for each certificate σ. Following the literature on information design and persuasion, we note that it is without loss of generality to associate each certificate signal σ with the set of quality levels that are eligible for that certificate. We will therefore assume without loss of generality that $\Sigma = 2^{[0,1]}$, the collection of all subsets of $[0,1]$, and each σ is a subset of $[0,1]$. The interpretation is that a producer can select an item from this menu, in which case she pays the certifier price (or transfer) $t(\sigma)$, the certifier verifies the good's quality q, and as long as $q \in \sigma$ the producer will receive certification σ. We will assume for technical convenience that each σ in the certifier's menu contains a minimum element, meaning that $\inf \sigma \in \sigma$.[1]

[1]This excludes certificates of the form "the quality q is strictly greater than 1/2," as opposed to "... at least 1/2." Certificates of the former type are inconvenient because there is no single least-cost choice of quality that satisfies the certification

We will write $\sigma_0 = [0, 1]$ for the trivial signal that conveys no additional information about quality. A good with certificate σ_0 is functionally equivalent to a good that has not been certified. It will be notationally convenient to assume that the certifier always offers signal σ_0 at cost 0. This allows us to think of every good as being certified, albeit possibly at the trivial level. If a producer attempts to purchase a certificate but does not satisfy that certificate's requirements, they will instead be assigned certificate σ_0; i.e., the certifier will not certify the good.

The Competitive Market All certification is assumed to occur simultaneously, and in advance of any trading between producers and consumers. Given the menu M of certificates and prices offered by the certifier, the producers' (production and certification) strategy is a mapping from producer type ψ to a choice of quality level q and certification σ. We will denote such a strategy $\Gamma : \psi \mapsto (q, \sigma)$, and restrict attention to measurable functions Γ. In a slight abuse of notation, we will also use Γ to denote the measure over pairs (q, σ) of products with corresponding quality and certification that result when producers apply strategy Γ.

Goods that are assigned the same certificate are indistinguishable by the consumers. After all certification is complete, each producer has a single unit of a good marked with a certification σ. For any given certification σ, write Γ_σ for the marginal distribution over quality q of Γ restricted to certificate σ. That is, fixing the choices of the producers, Γ_σ is the distribution of levels of quality for a product with certification σ. Then the value of a consumer of type ϕ for a good with certificate σ can be evaluated as

$$f(\sigma; \phi) = E_{q \sim \Gamma_\sigma}[f(q; \phi)].$$

That is, each consumer rationally evaluates the expected quality of each product given its certification level and the choices of the producers.

Since goods with the same certification are indistinguishable to consumers, we can view the competitive market between producers and consumers as a market for certificates σ. A Walrasian (or Competitive) equilibrium of the resulting market is an allocation $x(\phi)$ of a certificates to each consumer ϕ, along with a price p_σ for each certificate, such that:

- Demand satisfaction: every consumer purchases her most-preferred good. That is, for every consumer type ϕ, $f(x(\phi); \phi) - p_{x(\phi)} \geq f(\sigma; \phi) - p_\sigma$ for every $\sigma \in \Sigma$.
- Market Clearing: every good with a positive price is sold. That is, for all σ, the measure of consumers ϕ such that $x(\phi) = \sigma$ is at most the measure of producers ψ who select level of certification σ. If $p_\sigma > 0$ then these measures are equal.

Since buyers (consumers) are unit-demand and hence their preferences satisfy the gross substitutes condition, a Walrasian equilibrium is guaranteed to exist Gul and Stacchetti 2000. We will therefore assume that trade occurs between consumers and producers at competitive equilibrium prices given the choices made by the producers.

requirement, and hence no utility-maximizing choice of quality for the producer. We could handle such certificates in a straightforward but tedious way by relaxing our equilibrium notion and assuming that each producer selects an ϵ-approximately utility maximizing choice of quality for some arbitrarily small ϵ. For the remainder of the paper we will ignore such technical issues and simply assume that each σ includes a minimum element.

Timeline To summarize, the timing of the market with certification is as follows:

1. The certifier commits to a menu of certificates with corresponding prices.
2. Each producer ψ simultaneously and privately chooses whether to produce a good, and if so at what level of quality.
3. Each producer that chose to produce decides whether to certify their product, and at which certificate. These decisions are made simultaneously for all producers.
4. The certifier verifies the products of producers who choose to certify and assigns certificates. Any producer who does not successfully certify receives certificate σ_0.
5. Producers and consumers trade goods in a competitive market. I.e., trade occurs at market-clearing prices for the chosen levels of certification.

Since Walrasian equilibria are not unique in general, one might wonder if the outcome described in the final step is well-defined. We will show in the next section that the competitive market equilibrium described the final step exists and its resulting allocation is unique for any certificate menu chosen by the certifier and any choice of certification levels chosen by the producers.

3 The Certifier's Problem

In this section we describe the market outcome that will occur for any given menu of certificates offered by the certifier. We show that it is without loss of generality for the certifier to restrict to offering threshold certificates that guarantee that a product is at least a certain level of quality. We characterize the unique Walrasian market equilibrium allocation that results from any assignment of such certificates to producers. We then use that characterization to solve for each producer's utility-maximizing choice of certificate from the certifier's menu, which will also be unique. Finally, we use this characterization of producer behavior to interpret the certifier's menu-selection task as a non-linear screening problem.

3.1 Certifications as Minimum Quality Thresholds

A first simple observation is that since producer costs are increasing in quality level, and since goods at different quality levels but with the same certification are indistinguishable to consumers (and hence must sell at the same price), a producer who is assigned certificate σ will always choose to produce at the minimum quality level eligible for that certificate.

Observation 3.1. *Fix any certifier menu M and any production and certification strategy of the producers. Then for any producer ψ, selecting certificate σ and producing at quality $q > \min \sigma$ is dominated by selecting certificate σ and producting at quality $\min \sigma$.*

Given this observation, we know that for any menu M, any equlbrium strategy Γ for the producers, and any certificate σ, the marginal distribution over quality Γ_σ will be a point mass at $\min \sigma$. In particular, any two certificates with the same minimum will induce the same equilibrium beliefs over quality and hence have indistinguishable value to all consumers. It is therefore without loss of generality for the certifier to only offer certificates of the form $\sigma_q = [q, 1]$; i.e., certificates that are differentiated only with respect to their minimum values. If a producer selects certificate σ_q, then that producer's chosen quality level at equilibrium will necessarily be q. Any given certification menu M therefore reduces to a (possibly infinite) collection of quality levels in $[0, 1]$ to certify.

Motivated by this observation, we will assume for the remainder of the paper that all certificates are of the form $[q, 1]$, and associate each $\sigma = [q, 1]$ with its quality threshold q. We can then think of a certification menu M as a collection of pairs $\{(q_i, t_i)\}$, where q_i is a quality threshold and t_i is a corresponding price for certifying that quality is at least q_i.

3.2 Uniqueness and Assortativeness of Competitive Market Allocations

We now turn to an analysis of the competitive market outcome that will result given the strategies of the certifier and producers. Recall that we can restrict attention to certificates of the form $\sigma_q = [q, 1]$ and that any good with certificate σ_q will have quality q with probability 1, so for the remainder of the section we will think of a market outcome as an allocation x and prices p of quality levels. That is, $x(\phi) \in [0, 1]$ for all consumers ϕ, and for each $q \in [0, 1]$ in menu M there is an associated market price p_q. We emphasize that x is a mapping from consumers to the certified goods they buy at market, whereas Γ is a mapping from producers to the certificates that they choose from the certifier.

The following lemma shows that for any choice of certification menu M and production and certification strategy Γ of the producers, all competitive market equilibria in the resulting market have the same uniquely-determined allocation. This allocation will be assortative, with higher-type consumers purchasing the higher-quality certificates.

Lemma 3.2. *Fix any certifier menu M and any strategy Γ of the producers. Then in every competitive market outcome (x, p), the allocation x satisfies $x(\phi_1) \leq x(\phi_2)$ and $p_{x(\phi_1)} \leq p_{x(\phi_2)}$ for all $\phi_1 \leq \phi_2$.*

Proof. Since consumers are unit-demand and each producer has a single unit of good, a Walrasian equilibrium (x, p) of the market is guaranteed to exist. By the first welfare theorem, any such equilibrium must maximize the total welfare,

$$\int_{\phi} f(x(\phi); \phi) dF(\phi).$$

Suppose there exist types $\phi_1 < \phi_2$ with $x(\phi_1) > x(\phi_2)$. By the single-crossing condition, we have that

$$f(x(\phi_1); \phi_2) - f(x(\phi_2); \phi_2) > f(x(\phi_1); \phi_1) - f(x(\phi_2); \phi_1)$$

and hence

$$f(x(\phi_1); \phi_2) + f(x(\phi_2); \phi_1) > f(x(\phi_1); \phi_1) + f(x(\phi_2); \phi_2)$$

which contradicts the supposed welfare optimality of allocation x.

We now turn to prices. Fix any consumer types $\phi_1 < \phi_2$, so in particular we know $x(\phi_1) \leq x(\phi_2)$, and suppose for contradiction that $p_{x(\phi_1)} > p_{x(\phi_2)}$. By monotonicity of the value function f, we must have $f(x(\phi_1); \phi_1) \leq f(x(\phi_2); \phi_1)$. But this then means $f(x(\phi_2); \phi_1) - p_{x(\phi_2)} > f(x(\phi_1); \phi_1) - p_{x(\phi_1)}$, which violates the competitive market condition that consumer type ϕ_1 is choosing her most-preferred good. We therefore conclude that $p_{x(\phi_1)} \leq p_{x(\phi_2)}$, as claimed. $\square$

3.3 Uniqueness and Assortativeness of Certificate Selection

Given that market outcomes are well-defined, we next turn to the equilibrium choices of the producers when selecting quality levels and their corresponding certifications. We again show that for any menu M of certificates offered, the quality choices of producers are unique at equilibrium. Moreover, higher-type producers will always select (weakly) higher certificates. In the Lemma 3.2, we saw that higher-type consumers also purchase (weakly) higher certificates. As we discuss later, this means the matching in any equilibrium will be assortative and hence constrained-efficient given the available certificates.

Recall that a producer strategy Γ is a mapping from producer type ψ to a choice of certification and quality, which we know from will always coincide. We will therefore write $\Gamma(\psi) = q$ to mean that producer ψ produces at quality level q and purchases certificate σ_q. In particular, we must have $\Gamma(\psi) \in M$ for all ψ.

Lemma 3.3. *Fix any certification menu M offered by the certifier. Then there is a unique equilibrium strategy Γ for the producers, and $\Gamma(\psi)$ is weakly increasing in ψ.*

Proof. Fix strategy Γ, which implies the measure of certificates chosen by the collection of producers. Let (x, p) denote a Walrasian equilibrium in the resulting competitive market, and recall that x is uniquely determined.

We first show that Γ is weakly increasing is ψ. Assume for contradiction that there exist $\psi_1 < \psi_2$ with $q_1 = \Gamma(\psi_1)$ and $q_2 = \Gamma(\psi_2)$ with $q_2 < q_1$. Then by the single-crossing condition for producers, we have $g(q_1; \psi_1) - g(q_2; \psi_1) > g(q_1; \psi_2) - g(q_2; \psi_2)$. But then, if we let p_{q_1} and p_{q_2} denote the Walrasian equilibrium prices of q_1 and q_2 given Γ, we have

$$(p_{q_1} - g(q_1; \psi_1)) + (p_{q_2} - g(q_2; \psi_2)) < (p_{q_1} - g(q_1; \psi_2)) + (p_{q_2} - g(q_2; \psi_1))$$

which means that either

$$p_{q_1} - g(q_1; \psi_1) < p_{q_2} - g(q_2; \psi_1)$$

or

$$p_{q_2} - g(q_2; \psi_2) < p_{q_1} - g(q_1; \psi_2).$$

In other words, either producer ψ_1 or ψ_2 (or both) would strictly improve their utility by switching their choice of quality and certification. As such a swap has measure zero and does not influence the competitive equilibrium, this would be an improving deviation for the producer(s), violating the assumption that Γ is an equilibrium strategy for the producers.

We have shown that Γ is weakly increasing in ψ. On the other hand, we know from Lemma 3.2 that the market allocation of quality levels to consumers is weakly increasing in ϕ. This means that any equilibrium outcome of production and trade is equivalent to one in which consumers and producers are matched assortatively, with higher-type consumers trading with higher-type producers. In other words, for any producer type ψ, there is a consumer type $\phi = \phi(\psi)$ such that ψ always trades with $\phi(\psi)$. Specifically, $\phi(\psi)$ is such that $F(\phi(\psi)) = G(\psi)$ (treating F and G as cumulative distribution functions).

Given this, we claim that $\Gamma(\psi)$, the certification selected by producer ψ at equilibrium, will always be a certificate q_i from the certifier's menu $M = \{(q_i, t_i)\}$ that maximizes $f(q_i; \phi(\psi)) - g(q_i; \psi) - t_i$. To see why, suppose for contradiction that the producer instead chooses some other certificate q' at price t' such that $f(q'; \phi(\psi)) - g(q'; \psi) - t' < f(q_i; \phi(\psi)) - g(q_i; \psi) - t_i - \epsilon$ for some $\epsilon > 0$, and sells to consumer $\phi(\psi)$ at an assumed market-clearing price $p_{q'}$.[2] Then, the producer ψ could instead deviate to purchasing q_i at a price of t_i, and offering it on the competitive market at a price of $p_{q'} + g(q_i;\psi) - g(q';\psi) + (t_i - t') + \epsilon/2$. Note that if consumer $\phi(\psi)$ were to purchase from producer ψ at this price, then her utility would be

$$
\begin{aligned}
&f(q_i;\phi(\psi)) - [p_{q'} + g(q_i;\phi(\psi)) - g(q';\phi(\psi)) + (t_i - t') + \epsilon/2] \\
&= (f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i - \epsilon) + (g(q';\psi) + t') + \epsilon/2 - p_{q'} \\
&> f(q';\phi(\psi)) - (g(q';\psi) + t') + (g(q';\psi) + t') - p_{q'} + \epsilon/2 \\
&> f(q';\phi(\psi)) - p_{q'}.
\end{aligned}
$$

But since $(q', p_{q'})$ is the most-demanded offering to consumer $\phi(\psi)$ in the market equilibrium, this means that the offering of q_i at the proposed price would be the most-demanded offering to consumer $\phi(\psi)$ under this deviation. In particular this means that *some* consumer would want to purchase q_i at the suggested price, and therefore in the adjusted market equilibrium after this proposed deviation the price of q_i must be at least this high.

We conclude that the utility of producer ψ under this deviation is at least

$$
\begin{aligned}
[p_{q'} + (g(q_i;\psi) - g(q';\psi) + (t_i - t') + \epsilon/2] - g(q_i;\psi) - t_i &= p_{q'} - g(q';\psi) - t' + \epsilon/2 \\
&> p_{q'} - g(q';\psi) - t'
\end{aligned}
$$

and hence this deviation is strictly utility-improving for the producer, contradicting the assumption that Γ is an equilibrium.

We therefore conclude that at equilibrium, each producer ψ chooses whichever certificate q_i from the menu maximizes $f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i$. The choice of each producer is therefore unique, up to tie-breaking on sets of measure zero. □

An immediate corollary of Lemma 3.2 and Lemma 3.3 is that the equilibrium outcome for a given menu M is not only essentially unique (up to the choice of market-clearing prices), but also has a natural assortative interpretation. Each producer in the market has a corresponding consumer with whom they will always trade. The producer will select whichever certification level maximizes the gains from trade between themselves and their partner consumer, less the price of the certification.

Corollary 3.4. *For any menu $M = \{(q_i, t_i)\}$ of the certifier, the resulting equilibrium market outcome has producer ψ trade with consumer $\phi = \phi(\psi)$ where $G(\psi) = F(\phi)$. The level of quality at which ϕ and ψ trade maximizes $f(q_i;\phi) - g(q_i;\psi) - t_i$, and is weakly increasing in ψ.*

3.4 Reduction to Non-linear Pricing

To this point we have characterized the producer choices and downstream outcomes that result from the certifier's choice of menu. Given this understanding, we can now relate the certifier's design problem to a certain non-linear pricing problem between a single seller and a single buyer.

[2] Note that as we showed Γ is weakly increasing in ψ, the deviation can not change the ordering of firms in terms of the certificate they purchase and hence does not change the consumer to which they sell.

The non-linear pricing problem is as follows. The buyer seeks to buy a perfectly divisible good. The seller may commit to a menu of quantities and prices. If a buyer of type θ purchases quantity $q \in [0,1]$ of the good at a total price of t, then the buyer utility is

$$u((q,t);\theta) = v(q;\theta) - t,$$

where v is concave in quantity q but not necessarily non-decreasing, and $v(0;\theta) = 0$ for all θ. The valuations v satisfy a single-crossing condition, which is that for $\theta_1 < \theta_2$ and $q_1 < q_2$, we have

$$v(q_2;\theta_2) - v(q_1;\theta_2) > v(q_2;\theta_1) - v(q_1;\theta_1).$$

The principal faces a constant cost of production $c > 0$ for any non-zero quantity, and seeks to design a menu M over quantity-price pairs given a prior over buyer types. The principal receives a payoff that is determined by the menu item selected and the buyer's utility.

We claim that the problem faced by a certifier choosing a menu of certificates is equivalent to the problem faced by the seller in a corresponding instance of the non-linear pricing problem. Denote by $\mathcal{E}$ an instance of our market economy with a certifier, and denote by $\mathcal{E}'$ an instance of the non-linear pricing problem described above.

Proposition 3.5. *For any instance $\mathcal{E}$ of a market with certification there is a corresponding instance $\mathcal{E}'$ of the non-linear pricing problem described above such that*

- *there is a one-to-one mapping between buyer types in $\mathcal{E}'$ and pairs of producer/consumer types in $\mathcal{E}$;*
- *there is a one-to-one mapping between a menu M chosen by the certifier in $\mathcal{E}$ and a menu M' chosen by the seller in $\mathcal{E}'$;*
- *the outcomes chosen from menu M by a producer type correspond to the outcomes chosen from menu M' by the corresponding buyer type, and the resulting producer/consumer gains from trade in $\mathcal{E}$ correspond to buyer surplus in $\mathcal{E}'$.*

Proof. The certifier's problem is to design a certificate menu $M = \{(q_i, t_i)\}$.

By Corollary 3.4, given menu M, each producer ψ will purchase whichever certificate q_i maximizes $f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i$.

For $q \in [0,1]$, we can interpret ψ as a buyer type and define valuation function $v(q;\psi) = f(q_i;\phi(\psi)) - g(q_i;\psi)$. Then since f and g both satisfy single-crossing with respect to their corresponding types, valuation function v does as well. Moreover, v is concave and $v(0;\psi) = 0$. By definition, the producers' choices of certificates from menu M corresponds precisely to the buyer's choice of quantity when facing the same menu, interpreting each certificate quality threshold as a quantity. Thus the outcomes (menu item chosen and transfers made to the seller/certifier) in the two settings are equivalent. Finally, the gains from trade between the producer and consumer is precisely (by definition) the buyer utility under these equivalent outcomes. □

Note that an immediate implication of Proposition 3.5, given Corollary 3.4, is that for any menu M chosen by the seller, the choice of quantity purchased by the buyer is weakly increasing in buyer type.[3]

[3]Alternatively, this is a direct consequence of the single-crossing condition on valuation functions v.

Moreover, any menu-design problem faced by the certifier with an objective determined by revenue, quality level sold, and/or gains from trade is equivalent to a corresponding optimization problem faced by the seller in the non-linear screening scenario.

The next three sections characterize three different objectives and where they conflict.

4 Certification to Maximize Gains from Trade

A first objective, *gains from trade from certification* represents the effectiveness of the marketplace to serve the two sides that would like to trade. The gains from trade are the sum of utilities of the consumers and the producers. This means that certification cost does not improve gains from trade. For a certification menu M, we will write $\mathrm{GFT}(M)$ for the gains from trade that results in the unique market outcome resulting from menu M and will abbreviate gains from trade with GFT.

One structural property is that adding *any* certificate to the menu of certificates cannot reduce the sum of utilities of the consumers and producers, regardless of the prices selected. One interpretation of this is that welfare cannot be harmed by a revenue-maximizing certifier entering a market for certification in which some certification options are already available.

Proposition 4.1. *Consider two certification menus M and M' with $M \subseteq M'$. Then $\mathrm{GFT}(M) \leq \mathrm{GFT}(M')$.*

Proof. If producer ψ selects option (q,p) from certification menu M, then the gains from trade generated for the producer ψ and corresponding consumer ϕ, less the revenue raised by the certifier, is $f(q;\phi(\psi)) - g(q;\psi) - p$. By Corollary 3.4, producer ψ purchases precisely whichever menu item from M maximizes this quantity. Providing additional items can therefore only increase the gains from trade jointly enjoyed by producer type ψ and corresponding consumer $\phi(\psi)$. As this holds point-wise for every ψ, it holds in aggregate over all types as well. □

An additional consequence is aboout the optimal menu. From Corollary 3.4 the GFT-optimal choice of menu is to offer all possible certification levels, at the cost of verification c.

Theorem 4.2. *The GFT-optimal menu of certificates offers every possible certification level $q > 0$ at a cost of c, and level 0 at a cost of 0.*

Proof. If all quality levels were available, the GFT-maximizing outcome is would be for each producer ψ to trade with consumer $\phi(\psi)$ at whichever quality level q maximizes their gains from trade $f(q;\phi(\psi)) - g(q;\psi)$. However, since quality levels are hidden, producers and consumers can trade at a positive level of quality only if the cost of verification is paid. So if the maximum gains from trade is less than c, then it is preferable to trade at level 0. However, we observe that this is precisely the outcome implemented at equilibrium from the proposed certification menu, so it must be GFT-optimal over all possible menus. □

5 Certification to Maximize A Menu-Based Outcome

In many cases it might be the case that the certifier has an instrumental interest in the market's outcomes, but is interested in the quality and revenue gained from menus. In this section, we will consider the following class of objectives:

$$\mathbb{E}[o(q,t)]$$

where the expectation is taken over which certificates are traded in the market and o is a bounded function, $|o(q,t)| \leq M$ for all q, t. We will be particularly interested in the special case $o(q,t) = q + \lambda t$, which we call the *climate objective*. In this, q is a measure of real mitigation (hence can be interpreted as having unit tons of CO2), and λ is the amount of mitigation possible outside of the voluntary carbon market (hence having unit ton of CO2 per amount of currency).

5.1 Incompatibility with Gains From Trade Maximization

We first observe that these objectives may be at conflict with gains from trade maximization.

Proposition 5.1. *There are problem instances in which any menu that maximizes gains from trade is an arbitrarily poor approximation to the maximization of a menu-dependent objective, and vice versa.*

Proof. For the first direction of incompatibility, choose any arbitrarily small $\epsilon > 0$. Take $c = 0$ and suppose there is only a single consumer type ϕ with $f(q;\phi) = \min\{q, \epsilon\}$, and a single producer type ψ with $g(q;\psi) = \epsilon^2 q$. For any $\epsilon < 1$ the gains from trade between the producer and consumer is uniquely maximized at $q = \epsilon$. Recall from Theorem 4.2 that this outcome is implemented by a certificate menu that offers each $q \geq 0$ at price 0. But the alternative menu that offers only $q = 1$ at price 0 will induce trade at quality level $q = 1$. Thus the welfare-maximizing menu achieves only an ϵ fraction of the mitigation possible over all menu choices of the certifier.

For the reverse direction, take $c = 0$ and suppose $f(q;\phi) = \phi q$ and $g(q;\psi) = 0$ for all q and ψ. Our distribution over consumer types ϕ is an equal-revenue distribution supported on $[1, H]$; that is, $F(\phi) = 1 - \frac{1}{\phi}$ for all $\phi \in [1, H]$. Recalling Observation 5.2, it is revenue-optimal for the certifier to offer contract $q = 1$ at a price of H, for an expected gains from trade (and revenue) of 1. However, the optimal gains from trade, $\log(H)$, can be achieved by offering contract $q = 1$ at price 0. $\square$

5.2 Complexity of Menus

By Proposition 3.5, to solve the certifier's revenue maximization problem it suffices to optimize revenue in the non-linear pricing problem. As a first step, we note that in the special case where the valuations v are linear in quantity (which happens, for example, if the cost g and value f functions in our certification problem are both linear in q), this problem reduces to a standard pricing problem in mechanism design. A characterization due to Myerson Myerson 1981 then immediately establishes that it is revenue-optimal to choose a menu with only a single non-trivial item (q, p) with $q = 1$.

Observation 5.2. *If $v(q;\theta)$ is linear in q for all θ, then there is a revenue-optimal menu that offers only quantity $q = 1$ at some price p. This price p will be chosen to maximize $p \times \Pr_\theta[v(1;\theta) > p]$.*

However, in general, non-linearity substantially changes the problem relative to the linear case. In particular, it is not necessarily optimal to offer a single menu item.

Proposition 5.3. *There are problem instances in which posting any single menu item is an arbitrarily poor approximation to the optimal revenue. The approximation factor can be as large as $\Omega(\log(H))$, where $H = \max_{\theta_1,\theta_2} \frac{\max_q v(q;\theta_1)}{\max_q v(q;\theta_2)}$ is the ratio between the highest and lowest maximum values across buyer types.*

Proof. Choose $c = 0$ and consider the valuation function $v(q;\theta) = q$ if $q \le \theta$, and $v(q;\theta) = 2\theta - q$ if $q \ge \theta$. That is, v is piecewise linear for each θ, with maximum value θ occurring at $q = \theta$. Fix some $H \ge 1$ and suppose the type distribution is such that $\Pr[\theta > h] = 1/h$ for all $h \in [1, H]$. That is, the type distribution is equal-revenue on range $[1, H]$.

This valuation function is concave and satisfies $v(0;\theta) = 0$ for all θ. Moreover, it satisfies the single-crossing condition. Indeed, for any q and any $\theta_1 < \theta_2$, we note that $\frac{d}{dq}v(q;\theta_1) \le \frac{d}{dq}v(q;\theta_2)$, since for any θ this derivative is 1 for $q < \theta$ and -1 for $q > \theta$. Since v is also continuous in both q and θ, the single-crossing condition is implied.[4] Finally, we note that this valuation function v can indeed arise in our reduction from the certification problem with producers and consumers.[5]

We now show the desired gap in approximation. Consider any menu M with a single non-trivial menu item (q, p). Then the revenue achieved by the seller is at most the gains from trade generated by the optimal allocation of quality level q. Since $v(q;\theta) \le q$ for all θ, this is certainly at most q times the probability that $v(q;\theta) > 0$, which is $q \Pr[\theta > q/2] \le q(2/q) = 2$.

On the other hand, the seller could offer a menu that includes every quality level $q \in [1, H]$ at a price of $q/2$. A buyer of type θ would then choose to purchase quality level $q = \theta$ for a utility of $\theta/2$, generating revenue $\theta/2$.[6]

Posting a single menu item is therefore at best an $O(\log H)$ approximation to the optimal revenue, as claimed. $\square$

5.3 Optimality of Monotonic Menus

Definition 5.4. *We say a menu $M = \{(q_i, p_i)\}$ is* o-monotone *if for all (q_i, t_i), $(q_j, t_j) \in M$ such that $q_i < q_j$, we have $o(q_i, t_i) \le o(q_j, t_j)$.*

Lemma 5.5. *For any instance of the non-linear pricing problem there is a revenue-optimal menu that is o-monotone. Also, adding a menu item to the certificate that violates o-monotonicity will decrease the objective o.*

Corollary 5.6. *In particular, for any instance of the certifier's problem, there is a certification menu that is o-monotone.*

Proof of Lemma 5.5. Let M be a revenue-optimal menu, and write $M = \{(q_i, t_i)\}_{i \in \Lambda}$ where Λ is some (possibly uncountable) index set. It is without loss to assume Λ is a subset of $[0, 1]$ such that $q_i < q_j$ for all $i < j$ (e.g., by reindexing so that the index of q_i is equal to q_i). We can further assume without loss of generality that every item in M is purchased by some buyer type, as any element that is never purchased could be removed from M without impact.

[4]Technically our construction only satisfies weak single-crossing since the inequality in derivatives is not strict. We can make the example strict by perturbing the slope of the initial line segment by an infinitesimal amount so that it depends on the type θ. We omit these details for expositional clarity.

[5]In particular, take ϕ and ψ to be supported on $[1, H]$, define $f(q;\phi) = q$ for all ϕ and $g(q;\psi) = \max\{0, 2(q - \psi)\}$ for all ψ. Then f is concave (in fact, linear), g is convex, the single-crossing conditions are satisfied, and $v(q;\theta) = f(q;\phi(\theta)) - g(q;\theta)$ as required.

[6]Buying a higher quality level $q' > \theta$ is worse for the buyer because it generates less value at a higher price, whereas buying any quality level $q' < \theta$ generates utility $q' - q'/2 = q'/2 < \theta/2$.

By Corollary 3.4, quality levels purchased will be monotone non-decreasing in buyer type. This means that every item (q_i, t_i) is purchased by some contiguous interval of buyer types I_i (which may have measure zero).

Suppose that menu M is not o-monotone. This means that either there is an element $i < \sup \Lambda$ such that $o(q_i, t_i) > o(q_j, t_j)$ for all $j > i$, or else there exists a pair of menu items (q_i, t_i) and (q_j, t_j) with $j > i$ such that (a) there exists some $\ell \in \Lambda$ with $i < \ell < j$, and (b) $o(q_\ell, t_\ell) < \min\{o(q_i, t_i), o(q_j, t_j)\}$ for all $i < \ell < j$.

Consider the former case, where there is an element $i < \sup \Lambda$ such that $o(q_i, t_i) > o(q_j, t_j)$ for all $j > i$. In particular there must exist some $j \in \Lambda$ with $j > i$. Let M' be the menu $\{(q_\ell, p_\ell)\}_{\ell \in \Lambda, \ell \leq i}$. I.e., M' is M with all elements with quantities greater than q_i removed. Note that for all $j \leq i$, since the types I_j preferred element (q_j, t_j) to any other element in M, they prefer element (q_j, p_j) to any other element in M' as well. Moreover, since purchase decisions are monotone in buyer type, we conclude that all types $\theta \in I_\ell$ with $\ell > i$ will purchase element (q_i, t_i) from menu M'. But since $o(q_i, t_i) > o(q_\ell, t_\ell)$ for all $\ell > i$, this means that the revenue generated by menu M' is strictly greater than the revenue generated by menu M, contradicting the supposed optimality of menu M.

Next consider the other case, there exists a pair of menu items (q_i, t_i) and (q_j, t_j) such that $o(q_\ell, t_\ell) < \min\{o(q_i, t_i), o(q_j, t_j)\}$ for all $i < \ell < j$. Let M' be the menu $\{(q_\ell, t_\ell)\}_{\ell \in \Lambda, \ell \leq i} \cup \{(q_\ell, t_\ell)\}_{\ell \in \Lambda, \ell \geq j}$. That is, M' is menu M with all elements strictly between (q_i, t_i) and (q_j, t_j) removed. Then as in the previous case, for all $\ell \leq i$ and $\ell \geq j$, types I_ℓ all still prefer to purchase (q_ℓ, t_ℓ). In particular, types I_i purchase (q_i, t_i) and types I_j purchase (q_j, t_j). By monotonicity of purchasing decisions due to Corollary 3.4, all intermediate types $\theta \in I_\ell$ for $i < \ell < j$ must purchase either (q_i, t_i) or (q_j, t_j). As $o(q_i, t_i)$ and $o(q_j, t_j)$ are both larger than the prices of the elements those types were purchasing under menu M, the revenue of menu M' must be strictly larger, which is again a contradiction.

We conclude that the prices in menu M must be o-monotone in quality levels, as claimed. □

This result has several implications.

First, one for the menu complexity of the resulting menus.

Proposition 5.7. *For any o-monotone menu M, there exists a menu M' of size at most k such that $o(M') \geq o(M) - 2M/k$.*

Proof. Choose an o-monotone optimal menu $M = \{(q_i, p_i)\}_{i \in \Lambda}$. We can choose a sub-menu

$$M' = \{(q_1, t_1), (q_2, t_2), \ldots, (q_k, t_k)\}$$

of this menu such that such that for every $(q, t) \in M$, there is $i = 1, 2, \ldots, k$ such that

$$|o(q, t) - o(q_i, t_i)| \leq 2M/k.$$

(In fact, such a menu can be constructed greedily: Choose a first menu item, and add additional items that are not within $2M/k$ of any other item until all menu items are within $2M/k$ of one of the chosen items.) This menu is also o-monotone and we assume that it is ordered in objective, i.e. $o(q_i, t_i) \leq o(q_{i+1}, t_{i+1})$.

To bound the loss in objective from dropping menu items, denote the types that were buying menu items that have quantity between q_i and q_{i+1} by Θ_i. Denote by Θ_0 those that were buying a quality smaller than q_1 and by Θ_k that were buying the a quantity higher than q_k.

As in the proof of Lemma 5.5, we observe that all types in Θ_i will purchase either (q_i, t_i) or (q_{i+1}, t_{i+1}) and that all other types' purchase decisions are unaffected. Note also that because of o-monotonicity of the menu, the objective change of the change in purchase decision for each of these is bounded by $2M/k$. Hence, the total loss is bounded by

$$\sum_{i=0}^{k} F(\Theta_i) 2M/k \leq 2M/k.$$

□

Hence, small menus achieve high objectives, and they do so in an additive fashion, independent of the measure of quality.

Another implication is for policy: Where gains from trade were maximized by an arbitrarily large menu of certificates, here, there are clear cases for which certificates should not be added to any menu: Those that violate o monotonicity.

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A Results on Computation

We will make an additional assumption to provide a polynomial-time approximation scheme.

Assumption A.1. *There exists some $\lambda > 0$ such that $\frac{d}{dq}v(0;\theta) < \lambda$ for all θ.*

Theorem A.2. *A menu with revenue at least* $\mathrm{OPT} - \lambda\epsilon$ *can be found in time polynomial in λ and $1/\epsilon$. The menu can optionally be constrained to contain at most k quality levels, in which case* OPT *is the optimal revenue achievable with at most k quality levels.*

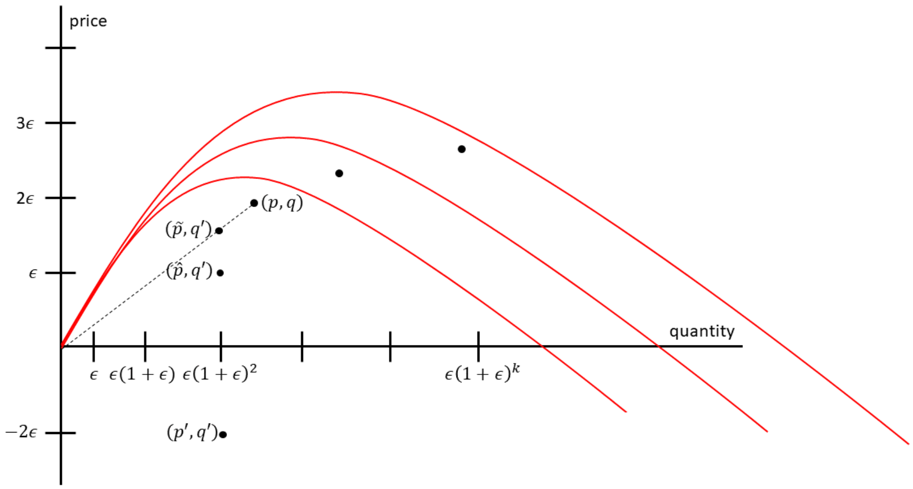

Figure 1: Illustration of the discretization procedure from Lemma A.3. Red curves denote buyer valuation functions. Menu item (p, q) is discretized in quantities to $(\tilde{p}, q')$ by shifting along a line to the origin, then discretized in price to $(\hat{p}, q')$. A final discount is applied to obtain the adjusted menu item (p', q').

We show that we can discretize the possible (quality, price) pairs that appear in our menu without losing too much revenue.

Lemma A.3. *For any monotone menu M of size k, there exists a menu M' such that*

- *M' has at most k elements,*
- *for each $(q, p) \in M'$, p is a multiple of ϵ and $q = \epsilon(1+\epsilon)^\ell$ for some integer $\ell \geq 0$, and*
- $\mathrm{Rev}(M') \geq \mathrm{Rev}(M) - O((k + \lambda)\epsilon)$.

Proof. Let us first give the high-level idea for the construction, which is illustrated in Figure 1. We will round each (quantity, price) pair on menu M in three steps. First, we will round each quantity down to an appropriate discretized grid, and then also lower the corresponding price *to keep constant the ratio of quantity to price.* The concavity of the value functions will then imply that buyer utilities cannot be reduced by too much, multiplicatively, as a result of this change. This first step discretized the quantities; in the second step we discretize prices by rounding each price down to an appropriate grid, which can only increase utilities. At this point we would be almost done, except for one complication: we must make sure that the (small) changes in buyer utility we induce do not result in buyers switching from more expensive items to significantly cheaper items from the menu. It is here where we use the price-monotonicity of the menu. Since the higher-quanitity items are the more expensive ones, in our third step we will provide discounts for the higher-quantity menu items, which will offset any utility perturbations due to discretization. These discounts are what lead to the loss term being proportional to $k\epsilon$, rather than ϵ, in our error bound.

We now move on to the formal construction. Fix menu M of size k, so that $M = \{(q_1, p_1), \ldots, (q_k, p_k)\}$ where $q_1 < q_2 < \ldots < q_k$. We can assume without loss of generality that $p_1 \leq p_2 \leq \ldots \leq p_k$, and that each element of M is purchased with positive probability.

We construct a new menu M' in the following sequence of steps. First, for each (q_i, p_i) with $q_i \geq \epsilon$, let q_i' be q_i rounded down to the nearest value of $\epsilon(1+\epsilon)^\ell$ where $\ell \geq 0$ is an integer. Then define $\tilde{p}_i = p_i \times (q_i'/q_i)$, noting that we chose $\tilde{p}_i$ so that $\tilde{p}_i/q_i' = p_i/q_i$. This element $(q_i', \tilde{p}_i)$ corresponds to rounding quantity but keeping the price to quantity ratio constant in our intuitive description above. For our second step, we take $\hat{p}_i$ to be $\tilde{p}_i$ rounded down to the nearest multiple of ϵ. Finally, in our third step, we introduce our discounts. For this we define $p_i' = \hat{p}_i - 3i\epsilon$, noting that the difference between p_i' and $\hat{p}_i$ is increasing in i. This completes our discretization procedure, so we add (q_i', p_i') to menu M'. We note that menu M' is not necessarily monotone, may contain elements that are preferred by no types, and may contain elements with negative prices.

We claim that for every type θ, if θ purchased item (q_i, p_i) in menu M with $q_i \geq \epsilon$, then θ will purchase some (q_j', p_j') in menu M' such that $j \geq i$. To see why, first consider what would happen if $(q_i', \tilde{p}_i)$ were on the menu. What is $u((q_i', \tilde{p}_i); \theta)$? Since valuation function v is concave and $v(0; \theta) = 0$, we must have $v(q_i'; \theta) \geq \frac{q_i'}{q_i} v(q_i; \theta)$. But since $q_i' \geq \frac{1}{1+\epsilon} q_i$, this implies

$$
\begin{aligned}
u((q_i', \tilde{p}_i); \theta) &= v(q_i'; \theta) - \tilde{p}_i \\
&\geq \frac{q_i'}{q_i} v(q_i; \theta) - \tilde{p}_i \\
&= \frac{q_i'}{q_i} \left(v(q_i; \theta) - p_i \right) \\
&\geq \frac{1}{1+\epsilon} u((q_i, p_i); \theta) \\
&\geq u((q_i, p_i); \theta) - \epsilon.
\end{aligned}
$$

Since we also know that $\tilde{p}_i - \epsilon \leq \hat{p}_i \leq \tilde{p}_i$ and $p_i' = \hat{p}_i - 3i\epsilon$, we have

$$
\begin{aligned}
u((q_i', p_i'); \theta) &= u((q_i', \hat{p}_i); \theta) + 3i\epsilon \\
&\geq u((q_i', \tilde{p}_i); \theta) + 3i\epsilon \\
&\geq u((q_i, p_i); \theta) + (3i - 1)\epsilon.
\end{aligned}
$$

On the other hand, for any $j < i$, the utility of purchasing $(q_j', \tilde{p}_j)$ is at most the utility of purchasing $(q_j, \tilde{p}_j)$, which is at most ϵ more than the utility of purchasing (q_j, p_j) (since the ratio between p_j and $\tilde{p}_j$ is no greater than $(1+\epsilon)$). We therefore have

$$
\begin{aligned}
u((q_j', p_j'); \theta) &= u((q_j', \hat{p}_j); \theta) + 3j\epsilon \\
&\leq u((q_j', \tilde{p}_j); \theta) + (3j + 1)\epsilon \\
&\leq u((q_i, p_i); \theta) + (3j + 2)\epsilon.
\end{aligned}
$$

Since we know that $u((q_i, p_i); \theta) \geq u((q_j, p_j); \theta)$ by assumption that θ purchases item (q_i, p_i) from menu M, we conclude that $u((q_i', p_i'); \theta) \geq u((q_j', p_j'); \theta)$ as well, since $(3j+2) \leq (3i-1)$ for $i > j$.

We conclude that each type θ that purchases (q_i, p_i) from M with $q_i \geq \epsilon$ will purchase a menu item (q_j', p_j') from M' such that $j \geq i$. Since prices are monotone in menu M, we conclude that the total loss in revenue can be at most the difference in price between p_i and p_i' for any i. This is at most $O(k\epsilon)$.

Finally, consider a type θ that purchases (q_i, p_i) from M with $q_i < \epsilon$. By Assumption 1, the maximum willingness to pay for any agent for quality level ϵ is $\lambda\epsilon$. These types therefore generate revenue at most $\lambda\epsilon$, thus regardless of their purchase behavior they account for a total loss in revenue of at most $O(\lambda\epsilon)$. $\square$

With Lemma A.3 in hand, we can complete the proof of Theorem A.2 by employing dynamic programming to determine the revenue-optimal mechanism with a given maximum-quality entry. One subtlety in the construction is that we must be careful to account for potential cannibalization by higher-quality elements in the menu. We handle this by insisting that the menu we construct contains only elements that are selected by a non-zero measure of buyer types, and we check this condition when recursively applying the dynamic program.

Proof of Theorem A.2. We show how to compute the optimal menu with qualities and prices chosen from a discrete indexed set of possible options, using dynamic programming. In general, given a quantity q and price p that lie in our discrete set of options, we will use Q and P to denote the integer indexing of q and p, respectively.

Given any choice of Q and P and some $k \geq 1$, write $M[Q, P, k]$ for the optimal revenue that can be obtained using a menu with at most k elements, of which the one with highest quality is the one indexed by Q and P. We will also write $L[Q, P, k]$ for the lowest type θ that purchases quality level Q in this optimal menu. We can compute $M[Q, P, k]$ and $L[Q, P, k]$ recursively as follows. If $k = 1$ then $M[Q, P, k]$ is precisely p times the probability that $v(q; \theta) \geq p$, and $L[Q, P, k]$ is precisely the infimum of types θ for which $v(q; \theta) \geq p$.

For $k > 1$, we will consider all possible options for the next-highest quality level on the menu given our discretization, say (q', p') with $Q' < Q$. For each choice of (Q', P'), we let $\theta(Q', P')$ denote the type that is indifferent between menu items (Q', P') and (Q, P), if any. Recall from the single-crossing condition that this choice of $\theta(Q', P')$ is unique if it exists. If there is no such $\theta(Q', P')$, then we disqualify menu item (Q', P') from consideration. Otherwise, we consider $L[Q', P', k-1]$, the lowest type that purchases element (Q', P') in the optimal menu with highest quality level Q' at price P'. If $L[Q', P', k-1] \geq \theta(Q', P')$, then again we disqualify menu item (Q', P') from consideration, as this means that the optimal menu containing menu item (Q, P) does not include any types that would purchase menu item (Q', P').

Otherwise, we have that $L[Q', P', k-1] < \theta(Q', P')$. We can therefore calculate the revenue from the optimal menu with highest and second-highest quality levels (Q, P) and (Q', P') as $R(Q', P') = M[Q', P', k-1] + (P - P')Pr[\theta > \theta(Q', P')]$. That is, the additional revenue gain or loss due to including menu item (Q, P) is $(P - P')Pr[\theta > \theta(Q', P')]$, the difference due to agents with type greater than $\theta(Q', P')$ switching from menu item (Q', P') to menu item (Q, P).

Finally, consider also the revenue that would be obtained by using only menu item (Q, P); call this R. If all potential choices of (Q', P') were eliminated or if $R > R(Q', P')$ for all potential choices of (Q', P'), then we set $M[Q, P, k] = R$ and set $L[Q, P, k]$ to be the infimum type θ such that $v(Q; \theta) \geq P$. This corresponds to the case that the optimal menu contains only the element (P, Q). Otherwise, let (Q', P') be the choice that maximizes $R(Q', P')$, which by assumption is larger than R. Then we take $L[Q, P, k] = \theta(Q', P')$ and $M[Q, P, k] = R(Q', P')$.

We conclude that we can fill tables M and L, with each entry taking time proportional to ϵ^{-2} (the time needed to consider every possible choice (Q', P')). As there are $k\epsilon^{-2}$ entries in total, the total time to fill the tables is at most $k\epsilon^{-4}$. The revenue-optimal mechanism with at most k menu items can then be obtained by taking the maximum of $M[Q,P,n]$ over all choices of Q and P.

Finally, by Lemma 5.7, we can take $k = 1/\sqrt{\epsilon}$ and our dynamic program will obtain a menu M such that $Rev(M)$ is at most $O(\lambda/\sqrt{\epsilon})$ less than the optimal revenue. An appropriate change of variables, taking $k = 1/\epsilon$ and discretizing to multiples of ϵ^2, then implies that our resulting menu is at most $O(\lambda\epsilon)$ less than that of the optimal menu. □

Theorem A.4. *Let M be the GFT-maximizing menu with at most k elements. A menu with M' with at most k elements and such that* $\mathrm{GFT}(M') \geq \mathrm{GFT}(M) - O(\lambda\epsilon)$ *can be found in time polynomial in* $1/\epsilon$.

Proof. The proof is very similar to the one for Theorem A.2, and strictly simpler, so we only briefly describe the differences here. First, it is without loss to restrict attention to menus that post price c for every non-trivial level of quality.

Given any such menu M, we can discretize potential levels of quality by rounding down to the nearest multiple of ϵ. By Assumption A.1 this reduces welfare by at most $\lambda\epsilon$, as the welfare from each menu item is reduced by at most this much and each producer selects her gains-from-trade-maximizing element from the menu.

Given such a discretization, one can express the welfare-optimal menu recursively via dynamic programming as in Theorem A.2, with the simplification that we need only index on quality rather than (quality, price) pairs (since all prices will be set to c). Rather than defining $M[Q,k]$ (and respectively $L[Q,k]$) to be the maximum revenue of a menu with k elements and maximum quality indexed by Q, it will be the maximum welfare of a menu with k elements and maximum quality indexed by Q. Our method of recursively computing $M[Q,k]$ (and $L[Q,k]$) then remains nearly unchanged relative to Theorem A.2. The only change to note is the actual welfare calculation, relative to the revenue calculation. In Theorem A.2 we used that the revenue obtained when agents of type $\theta > \theta'$ purchase certificate q at price p is p times $\Pr[\theta > \theta']$. In contrast, the welfare obtained when producers of type $\psi > \psi'$ all purchase certificate q at price c can be calculated in closed form as $\int_{\psi}^{\psi'} (f(q;\phi(\eta)) - g(q;\eta) - c)d\eta$. Substituting this welfare calculation for the revenue calculation completes the necessary changes. □