# Pricing Network Effects: Competition AEJ: Microeconomics

Fainmesser, Galeotti, AEJ: Micro '20 Discussed by Andreas Haupt

July 26, 2021 Harvard Theory Reading Group

### Prologue

https://www.youtube.com/watch?v=NokEE3I4z0Y

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- Main Question: How do horizontal price competition and competition with network effects interact?

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- Expectations
- ▶ Model here is parsimonious, reduced form
- Interior solution
- Does not come at a cost: Taste heterogeneity is strong compared to network externality

Hotelling Style Setup with Network effects

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  - Firms maximize profit

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- Every parent of a user that consumes 1 (resp. 0) gives  $\gamma$  utility for consuming 1 (resp. 0)

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- Homogeneity of the network, in particular no homophily
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  - bounded strategic complementarity of consumers' decisions, which gives existence and uniqueness of equilibrium
- No endogenous choice of users whether to disclose influence

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Main intuition: A new Inefficiency

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- ► Information acquisition by firms is strategically complementary
- ▶ Hence firms invest much into learning about influence

# Demands (Prop. 1)

If the horizontal differentiation is strong enough compared to the network externality,  $\gamma \mathbb{E}[I] < \tau$ , and a technical condition

$$D_i(\mathbf{p}^0, \mathbf{p}^1) = \frac{1}{2} \left( 1 - \frac{1}{\tau} \Delta \rho_i - \frac{\gamma \mathbb{E}[I]}{\tau (\tau - \gamma \mathbb{E}[I])} \overline{\Delta \rho} \right)$$

# Prices (Prop. 2)

If network effects are weak,  $\gamma I_{\rm max} < \frac{1}{2}$ , then, there is a unique equilibrium in the pricing stage. This equilibrium is symmetric.

$$p = \tau - \gamma \mathbb{E}[I]$$

is the non-targeted price.

$$p(l) = p + \frac{\gamma(\mathbb{E}[l] - l)}{2 - w}$$

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- ▶ Varying w

The welfare loss from misallocation is

$$M(w) = \frac{1}{2} \frac{w(1-w)}{\tau} \operatorname{Var}(p(I)) = \frac{\gamma^2}{(2-w)^2} \operatorname{Var}(I)$$

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- ► Supermodular, symmetric game ∴ symmetric equilibrium
- Simplify to  $w^0 = w^1 = w$ , and solve FOC to get internal solution  $w^*$

The set of stable equilibria depend on  $K = \frac{\gamma^2 \operatorname{Var}(l)}{2\tau\alpha}$  (Prop. 7):

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Only producer surplus changes compared to exogenous w.

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- Fainmesser, Galeotti AEJ: Micro forthcoming: Modelling concerns of influencers that they might loose their followers

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- Robust Comparative Statics
- Do the comparative statics break with arbitrarily small levels of homophily?
- Where outside of influencer marketing can this model be applied?
- And many other questions which would re-introduce expectations.

## **Epilogue**

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