Competitive Auctions and Digital Goods

ACM-SODA '01

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Discussed by Andreas Haupt

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Stanford Computer Science meets Game Theory

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- Features of robustly approximately optimal auctions: non-uniform prices and randomization

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Robustness in Mechanism Design:

- Correlation robustness: Carroll EMA '17

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Other approximation results: Approximate IC/IR, optimization on average across repeated auctions (no-regret learning of auctions)

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- Call an auction ϕ competitive for a set $\tilde{\Theta}$ if

$$\min_{\theta \in \tilde{\Theta}} \frac{R_\phi(\theta)}{\mathcal{F}(\theta)} \in \Omega(1),$$

$\frac{R_\phi(\theta)}{\mathcal{F}(\theta)}$ the competitive ratio.

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- Using a fixed reserve price r is not competitive:
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- Need reserve prices that are estimated from other bids

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- Experiments

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- Reject bids in B or do a symmetric estimation
- Observe: Auction is randomized

The Random Sampling Auction

Theorem

The random sampling auction is competitive for $\tilde{\Theta} = \{\theta \in \Theta^n | \alpha h \leq \mathcal{F}(\theta)\}$ *with high as* $\alpha \to \infty$ *.*

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- This does not use any distributional assumption on θ except for boundedness

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Theorem

For any truthful deterministic auction ϕ, there is $\theta \in \{\theta | \alpha h \leq \mathcal{F}(\theta)\}$ such that $\frac{R_\phi(\theta)}{\mathcal{F}(\theta)} \in O(\frac{1}{h})$.

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- Prior-independent approximation does not have this property

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Good further reading:

J. Hartline, *Mechanism Design and Approximation*, http://jasonhartline.com/MDnA/