A Theory of Voluntary Carbon Market Design*

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Abstract

A principal would like to use private funding to reduce the cost of socially beneficial activities, e.g., negative-emissions activities, by establishing property rights over those activities. We first show that in a large set of settings in which buyers are vertically differentiated by how much they benefit from owning a certificate of a particular quality, a competitive market for certificates, akin to voluntary carbon markets, is sufficient to implement the principal's objectives. We then show that the problem of certification design is equivalent to selling surplus from trade of an activity of a particular quality to producer-buyer pairs. This interpretation means that there are benefits of not making all qualities, for example those for low-quality actions, certifiable. Finally, we give a reasoning for greenwashing in this model: If the social benefit is small in monetary terms, the qualities that producers choose is depressed compared to what they would certify if all qualities were certifiable. Conversely, if the benefit of positive activities is high, the quality that a producer chooses to certify is higher if all qualities were certifiable. We interpret regulations that restrict the use of certification of low quality in light of our results.

1 Introduction

One way to reduce the social mitigation cost of reducing carbon emissions is by harnessing private investments and establishing property rights over emission reduction activities which are used to "offset" positive-emissions activities. Negative emissions technologies are allocated as *certificates* (Blaufelder et al. 2021) of differing quality (Buma et al. 2024), and the observed quality of activities is low (Greenfield 2021; Greenfield 2023; Calel et al. 2025). This paper provides a model of certification that aims to explain these three features.

At the core of our model is a two sided market. Producers can engage in negative-emissions activities of differing quality. The marginal cost of quality is increasing in the type of the producer. For funding, there is an organization that can assign (against payment) property rights on negative emissions technologies. Given by these property rights, buyers "offset" positive-emissions activities they engage in. Their valuation for quality of negative-emissions activities is increasing in type, but marginal valuations are decreasing in type.

Importantly, the principal has a valuation for the quality of the activities that differ from that of the market participants. That could mean, for example, that buyers in the market do not share an interest high quality of activities that the principal has.

Our analysis starts with a general model of mechanisms that the principal may employ. We show first that a particular structure of mechanism, which we observe in reality, is sufficient to implement all implementable mechanisms, independent of the goal of the principal (Theorem 3.1): a market for certificates. In this mechanism, the certifier chooses minimum quality standards, which are certified for a fee, and then traded in a competitive market. Such a mechanism resembles what we see in the world of voluntary carbon markets.

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If the designer maximizes the gains from trade of the market, they want to offer as many certificates as possible, to allow producers to distinguish themselves (Theorem 4.1). For other objectives, this may not hold.

Finally, we show that the certification problem can be viewed in a different way: In our movel of vertical differentiation, which seller sells from which buyer does not change with the certification regime. Always, the most quality-conscious buyers, and the most efficient producers will trade, as we show (Corollary 5.4).

Finally, we use this to make a prediction about the market. Certification may be further depressed from a market in which producers can distinguish themselves for free if the principal is sufficiently uninterested in quality of the actions taken. If they are interested, they

1.1 Related Literature

Certification. This work contributes to the literature on certification. The early results Milgrom 1981; Grossman 1981 produce unraveling type results: In equilibrium, the quality of a good is fully revealed. The main intuition for these results in models of certification is that certifying non-informatively will be interpreted by the market as a sign of bad quality—adverse selection is extreme. In our model of a revenue-maximizing certifier, there might be other reasons for certification less informatively, as the price in the competitive market may depend on not only an individual seller's quality, but all the sellers in the market. Later contributions Lizzeri 1999; DeMarzo, Kremer, and Skrzypacz 2019; Acharya, DeMarzo, and Kremer 2011 allow for unsuccessful certifications, restricting the stark result obtained in Milgrom 1981; Grossman 1981.

Aftermarkets. More broadly, our work is related to mechanism and information design in the presence of fixed play of a game *after* the mechanism. We recommend Bergemann and Morris 2019 for a general overview of information design. Our work is especially related to the design of information provided to buyers and/or sellers of a good. Bergemann, Brooks, and Morris 2017 considers the design of information for a first-price auction, where a third party can reveal a signal correlated with a buyer's valuations, and fully characterizes the achievable revenue and payoffs. Alijani et al. 2022 extends the analysis to a scenario with multiple buyers. In a general mechanism design framework, Candogan and Strack 2021 develop an optimal disclosure policy for action recommendations in a game with private types and a hidden state. Dworczak 2020 designs mechanisms in a setting where players participate in a finite Bayesian game after participating in the mechanism, so that game outcomes are impacted by information revealed over the course of the mechanism. He finds a cutoff structure of optimal mechanisms in the first stage. Several papers also consider the interaction of information design and two-sided matching Gomes and Pavan 2016; Banerjee et al. 2017; Valenzuela-Stookey 2020.

Selling Hard Information. For-profit certification relates to the sale of hard information, which has been studied in the context of competitive markets. Ali et al. 2022 consider a seller holding a good of uncertain quality. The seller can purchase a quality-correlated signal from a revenue-maximizing intermediary before bringing the good to market. In general the resulting equilibria are not unique and can vary substantially, but by employing noisy signals the intermediary can robustly guarantee high revenue. Our model differs in that any certification options are made available to the entire market and product quality is endogenous, so the certifying agent can impact welfare. Bergemann, Bonatti, and Smolin 2018 solve for the revenue-maximizing mechanism for information sale in binary environments, and Bergemann, Cai, et al. 2022 establish when full disclosure is approximately optimal under more general spaces of actions. In contrast, our certifying agent is selling a signal that is valuable in that it conveys information to other participants in a subsequent game. Similar to our paper, Condorelli and Szentes (2023) finds an assortative matching as a result of an optimal information design problem.

Non-Linear Pricing. In our mathematical analysis, we reduce the certifier's problem to a pricing problem with a non-linear valuation. The non-linear pricing literature following Mussa and Rosen 1978 (see also treatment in Dewatripont, Bolton, et al. 2005 and Börgers and Krahmer 2015, Chapter 2.3) studies a non-linear concave valuation with a quadratic cost. The functional form assumptions in these papers allow one to characterize the optimal mechanism in closed form. Typically, the optimal menu of offered goods is a continuum, in contrast to the linear screening problem first studied in the influential Myerson 1981. Our analysis will show that also the class of models we consider may feature infinite menus. Additionally, our reinterpretation of a certification design problem as screening problem relates to the paper Zapechelnyuk 2020 which reduces a certification problem in moral hazard to a delegation problem.

Mechanism Design with Limited Information. Our results on the optimality of small menus relate to a literature on mechanism design with limited communication. The papers Bergemann, Shen, Xu, and E. Yeh 2012; Bergemann, Shen, Xu, and E. M. Yeh 2012 consider approximation of non-linear single- and multi-dimensional pricing environments with finite menus (in the papers called "finite information"). The papers make functional form assumptions similar to the ones in Mussa and Rosen 1978, and derive rates of approximation by finite menus.

Empirics. Several branches of literature consider the effects of certification. (Wimmer and Chezum 2003 for race horses, Tadelis and Zettelmeyer 2015 on used-car auctions, Ramanarayanan and Snyder 2012 for dialysis screening centers, Luca 2016 for restaurant reviews, Elfenbein, Fisman, and McManus 2015 for seller ratings on Ebay, Vatter 2022 for healthcare plans, and Conte and Kotchen 2010; Rodemeier 2023 for voluntary carbon credits). For a survey of the literature environmental market design, see Cantillon and Slechten 2023. Buma et al. (2024) shows differences in quality of nature-based negative emissions activities.

Single-Crossing and Assortativity. Finally, our main assumption guaranteeing uniqueness of our equilibrium is a single-crossing condition (also known as increasing difference, compare Bolton and Dewatripont 2004, Section 2.1.3 for the uses of the word). Single-crossing conditions are important in several domains, among them interdependent private values auctions Milgrom and Weber 1982 and social choice and voting Saporiti and Tohmé 2006. A recent line of work in algorithmic mechanism design has employed single-crossing conditions to enable approximately optimal designs in interdependent value settings Roughgarden and Talgam-Cohen 2013; Chawla, Fu, and Karlin 2014. Compare also Quah and Strulovici 2012 on the aggregation of single-crossing functions. Closest to the present paper, another implication of single-crossing is adverse selection in markets Mirrlees 1971; Spence 1974 and in Becker (1973)'s theory of marriage.

1.2 Outline

The rest of this article is structured as follows. We formalize the general model of the principal and our certification model in Section 2. We show that certification is sufficient for implementation in Section 3. Section 4 shows when design of certificates is *not* necessary. If the principal wishes to maximize surplus from trade between producerrs and buyers, it is optimal to offer all certificates, and allow the producers to distinguish the quality of their activities to an arbitrary quality. In Section 5 we analyze the structure of equilibria given the certifier's offerings, showing how to interpret the certifier's design problem as a sale of surplus to pairs of market participants. Section 6 uses the reinterpretation of the problem to show a qualitative property of the design compared to fully informative certification. In particular, we show when the optimal menu for a certifier maximizing a weighted average of revenue and the sum of the quality of actions (read: carbon mitigated) decreases or increases the quality choices by producers compared to the full-information benchmark. We discuss implications of our results in Section 7. Appendix A contains omitted proofs.

2 Model

Participants We consider a continuum market between producers and consumers. Producers are unit-supply and parameterized by types $\psi \in \mathbb{R}_+$ with measure G. Consumers are unit-demand and parameterized by types $\phi \in \mathbb{R}_+$ with measure F. The type measures F and G are atomless and continuous with compact support.

Goods can be produced at different levels of quality, denoted $q \in [0,1]$. Goods of higher quality are more valuable to consumers but more costly to produce. We write $c(q;\psi)$ for the cost incurred by a producer of type ψ when producing a good of quality q. We assume c is weakly convex and non-decreasing in q for every ψ and normalized so that $g(0;\psi) = 0$. We also write $v(q;\phi)$ for the value enjoyed by a consumer of type ϕ for a good of quality q, where v is weakly concave and non-decreasing in q for every ϕ and normalized so that $f(0;\phi) = 0$. We scale valuations so that $f(q;\phi) \leq 1$ for all q and ϕ, which is without loss for bounded values.

We will assume that costs and valuations satisfy single-crossing with respect to the producer and consumer types, respectively. Roughly speaking, this means that producers (consumers) of higher types have lower marginal cost (higher marginal value) for producing higher-quality goods. More formally, for all $\phi_1 < \phi_2$ and $q_1 < q_2$, we have

$$f(q_2;\phi_2) - f(q_1;\phi_2) > f(q_2;\phi_1) - f(q_1;\phi_1).$$

Likewise, for all $\psi_1 < \psi_2$ and $q_1 < q_2$, we have

$$g(q_2;\psi_2) - g(q_1;\psi_2) < g(q_2;\psi_1) - g(q_1;\psi_1).$$

Transfers between producers and consumers are permitted. If consumer ϕ purchases a product of quality q from producer ψ at a price of p, then the consumer enjoys utility

$$u_C((q,p);\phi) = v(q;\phi) - p$$

and the producer's utility is

$$u_P((q,p);\psi) = p - c(q;\psi).$$

We study two problems in this paper: First, a full mechanism designer's problem, then a certification problem.

Mechanism Design The integrated market designer's problem is a slight variant of our primary model, with the same primitives. The certifier (i.e., the designer), which in this case can also propose transfers, commits to four functions

$$\begin{aligned} q_C &: \mathbb{R}_+ \to [0,1] \\ p_C &: \mathbb{R}_+ \to \mathbb{R}_+ \\ q_P &: \mathbb{R}_+ \to [0,1] \\ p_P &: \mathbb{R}_+ \to \mathbb{R}_+. \end{aligned}$$

which are, respectively, allocations and transfers for consumers and producers. That is, $q_C(\phi)$ and $p_C(\phi)$ are the allocation to and payment from consumer type ϕ, and $q_P(\psi)$ and $p_P(\psi)$ are the production from and transfer to producers of type ψ. Allocations and transfers must be such that product markets clear, i.e. for all $A \subseteq [0,1]$,

$$F(\{\phi : q_C(\phi) \in A\}) = G(\{\psi : q_P(\psi) \in A\}), \tag{1}$$

and the mechanism yields non-negative revenue,

$$\mathbb{E}[p_C(\phi)] \geq \mathbb{E}[p_P(\psi)]. \tag{2}$$

Sellers and buyers report their types ϕ, ψ to the mechanism to maximize their utility $u_C((q,p);\phi) = f(q_C(\phi);\phi) - p_C(\phi)$ and $u_P((q,p);\psi) = p_P(\psi) - g(q_P(\psi);\psi)$. Participation in the mechanism is optional. This implies, as standard in mechanism design, incentive compatibility (IC) and individual rationality (IR) constraints, compare Börgers and Krahmer 2015. The IC constraints are

$$\begin{aligned} f(q_C(\phi);\phi) - p_C(\phi) &\geq f(q_C(\phi);\phi) - p_C(\phi) \\ p_P(\psi) - g(q_P(\psi)) &\geq p_P(\psi') - g(q_P(\psi');\psi), \end{aligned}$$

and the IR constraints are $f(q_C(\phi);\phi) - p_C(\phi) \geq 0$, $p_P(\psi) - g(q_P(\psi)) \geq 0$. We call this the *Mechanism Designer's Problem.* We also consider a more restricted model.

Certification In the second model we consider, there is a third-party certifier who is able to commit to tests of quality of the producer's good. In particular, the certifier is able to choose a menu M of Blackwell experiment-transfer pairs (σ, t), where

$$\sigma\colon [0,1] \to \Delta(S), q \mapsto s.$$

and

$$t\colon [0,1] \to \mathbb{R}$$

for a signal space S, which we will assume without loss to be $S = [0,1]$. This certificate is visible to all producers and consumers. Hence, the certifier is permitted to collect payments from producers for certification depending on their produced quality, and these transfers can depend arbitrarily on the certificate produced.

The producers have an outside option of no certification (σ_0, t_0), where $\sigma_0(q) = 0$ for all q and $t_0(q) = 0$ for all q.

The Competitive Market All certification is assumed to occur simultaneously, and in advance of any trading between producers and consumers. Given the menu M of certificates and prices offered by the certifier, the producers' (production and certification) strategy is a mapping from producer type ψ to a choice of quality level q and certification σ. We will denote such a strategy $\Gamma : \psi \mapsto (q, \sigma)$, and restrict attention to measurable functions Γ. In a slight abuse of notation, we will also use Γ to denote the measure over pairs (q, σ) of products with corresponding quality and certification that result when producers apply strategy Γ.

Goods that are assigned the same certificate are indistinguishable by the consumers. After all certification is complete, each producer has a single unit of a good marked with a certification σ. For any given certification σ, write Γ_σ for the marginal distribution over quality q of Γ restricted to certificate σ. That is, fixing the choices of the producers, Γ_σ is the distribution of levels of quality for a product with certification σ. Then the value of a consumer of type ϕ for a good with certificate σ can be evaluated as

$$f(\sigma;\phi) = E_{q\sim\Gamma_\sigma}[f(q;\phi)].$$

That is, each consumer rationally evaluates the expected quality of each product given its certification level and the choices of the producers.

Since goods with the same certification are indistinguishable to consumers, we can view the competitive market for goods as a market for certificates σ. A Walrasian (or Competitive) equilibrium of the resulting market is an allocation $x(\phi)$ of a certificates to each consumer ϕ, along with a price p_σ for each certificate, such that:

- Demand satisfaction: every consumer purchases her most-preferred good. That is, for every consumer type ϕ, $f(x(\phi);\phi) - p_{x(\phi)} \geq f(\sigma;\phi) - p_\sigma$ for every $\sigma \in \Sigma$.
- Market Clearing: every good with a positive price is sold. That is, for all σ, the measure of consumers ϕ such that $x(\phi) = \sigma$ is at most the measure of producers ψ who select level of certification σ. If $p_\sigma > 0$ then these measures are equal.

Since buyers (consumers) are unit-demand and hence their preferences satisfy the gross substitutes condition, a Walrasian equilibrium is guaranteed to exist (Gul and Stacchetti 2000). We will therefore assume that trade occurs between consumers and producers at competitive equilibrium prices given the choices made by the producers.

Timeline To summarize, the timing of the market with certification is as follows:

1. The certifier commits to a menu of certificates with corresponding prices.
2. Each producer ψ simultaneously and privately chooses whether to produce a good, and if so at what level of quality.
3. Each producer that chose to produce decides whether to certify their product, and at which certificate. These decisions are made simultaneously for all producers.
4. The certifier verifies the products of producers who choose to certify and assigns certificates. Any producer who does not successfully certify receives certificate σ_0.
5. Producers and consumers trade goods in a competitive market. I.e., trade occurs at market-clearing prices for the chosen levels of certification.

Since Walrasian equilibria are not unique in general, one might wonder if the outcome described in the final step is well-defined. We will show in the next section that the competitive market equilibrium described the final step exists and its resulting allocation is unique for any certificate menu chosen by the certifier and any choice of certification levels chosen by the producers.

3 Sufficiency of Certification

We first show that it is sufficient for the principal to certify quality for a fee (or with a subsidy). This means, that mechanisms beyond certification cannot benefit the principal.

Theorem 3.1. *For the mechanism designer's problem, there is always a solution ofis outcome-equivalent to the Mechanism Designer's Problem. That is, for any mechanism (q_C, p_C, q_P, p_P), there is an allocation menu M with subsidies such that all producer and consumer types have the same allocations and transfers, and vice versa.*

This result shows that there that policy interventions that allow subsidies for particular negative-emissions activities are as effective as moving to a full-scale centralized integrated market.

Proof of Proposition 3.1. First for the easier direction, let M be a certification menu for the Certification Problem with Subsidies, and let $q_C(\phi), q_P(\psi)$ be the allocations for types ϕ, ψ. By market clearing, it must be that (1) holds. Also define the transfers $p_C(\phi)$ and $p_P(\psi)$ as the sum of certification transfers and market prices. Note that market prices are zero-sum. The agents maximize their utility by reporting their correct types as they chose optimally in the certification problem.

Next, let (q_C, t_C, q_P, t_P) be a mechanism for the Mechanism Designer's problem. We first observe that it must be that the allocation for both sides of the market is monotonic in their type, using a standard argument, Börgers and Krahmer 2015, Lemma 2.1. That is, the mappings $\phi \mapsto q_C(\phi)$ and $\psi \mapsto q_P(\phi)$ must be non-decreasing. As a direct consequence of monotonicity, it must be the case that quantile pairs (ϕ, ψ) such that $F(\phi) = G(\psi)$ trade the same quantity. We define the menu as: $\{((q_C(\phi), p_P(\phi) - p_C(\psi)) : \phi, \psi \text{ s.th. } F(\phi) = G(\psi)\}$. It must be the case it remains to show that a producer-consumer pair (ϕ, ψ) would indeed choose the menu entry $(q_C(\phi), t_P(\phi) - t_C(\psi))$. By incentive compatibility for both sides, we have that

$$\begin{aligned} f(q_C(\phi); \phi) - p_C(\phi) &\geq f(q_C(\phi'); \phi) - p_C(\phi') \\ p_P(\psi) - g(q_P(\psi)) &\geq p_P(\psi') - g(q_P(\psi'); \psi) \end{aligned}$$

Summing these constraints, we get, recalling $p(\phi) = p_P(\phi) - p_C(\psi)$,

$$f(q_C(\phi); \phi) - g(q_P(\psi); \psi) - p(\phi) \geq f(q_C(\phi'); \phi) - g(q_P(\psi'); \psi) - p(\phi') \tag{3}$$

which is the condition that means optimality of the choice of certificate. A similar argument summing individual rationality constraints shows that the incentives for the choice of the zero certificate are maintained. $\square$

It is insufficient to only charge a fee for certificates. Sometimes, certificates may need to be subsidized to reach the optimality.

Example 3.2 (Necessity of Subsidies)**.** *Let $\varepsilon > 0$. The designer's goal is to maximize total quality q. There is a single producer type ψ with $g(q;\psi) = 2\varepsilon q$, and two consumer types, ϕ_H and ϕ_L, with mass ε and $1-\varepsilon$, respectively. Values are $f(q;\phi_H) = q$ and ϕ_L, with $f(q;\phi_L) = \varepsilon q$.*

Any certification mechanism is equivalent to a mechanism that serves consumer-producer pairs, each of which has a gains-from-trade valuation v. An ε fraction of the pairs include a producer and a high-type consumer, yielding valuation $v(q) = (1-\varepsilon)q$. The remaining $(1-\varepsilon)$ fraction of the pairs involves a low-type consumer, yielding valuation $v(q) = (\varepsilon - 2\varepsilon)q = -\varepsilon q < 0$. Any certification menu must, in equilibrium, allocate quality $q = 0$ for the low types, and hence the maximum expected quality generated by any such mechanism is at most ε.

An integrated mechanism designer can do better, as follows. All producers are offered a payment of ε to produce at $q = \frac{1}{2}$, a payment of 2ε to produce at $q = 1$, or a payment of 0 to produce 0. All producers are indifferent between these options, so we can suppose an ε fraction produce at $q = 1$, a $(\frac{1}{2} - 2\varepsilon)$ fraction produce at $q = \frac{1}{2}$, and a $\frac{1}{2}$ fraction produce at $q = 0$, for a total cost of $\varepsilon(\frac{1}{2} - 2\varepsilon) + 2\varepsilon^2 = \frac{\varepsilon}{2}$. Next, all consumers are offered a choice between purchasing $q = 0$ at price 0, $q = \frac{1}{2}$ at price ε, or $q = 1$ at price $\frac{1}{2} - \varepsilon$. The low consumer types are indifferent between $q = 0$ and $q = \frac{1}{2}$, and the high types are indifferent between $q = \frac{1}{2}$ and $q = 1$. So we can imagine that all high types purchase $q = 1$, and a $(\frac{1}{2} - \varepsilon)$ mass of low types purchase $q = \frac{1}{2}$, for a total revenue of $\varepsilon(\frac{1}{2} - \varepsilon) + (\frac{1}{2} - \varepsilon)(\varepsilon) = \varepsilon - \varepsilon^2$. As long as $\varepsilon < \frac{1}{2}$, this mechanism is budget-balanced. And the total quality generated is $\frac{\varepsilon}{2} + (\frac{1}{2} - \varepsilon)(\frac{1}{2}) = 1/4$. Taking ε arbitrarily small, the relative improvement grows large.

The result of optimality of subsidies lies in the fact that a large fraction of the market does not have positive gains from trade by themselves, but if their trade happened, positive quality would arise, which is the objective of the certifier. Subsidies allow to include these negative-surplus transactions to occur.

4 Necessity of Certification Design

We first briefly consider what the optimal market for market participants looks like. We call this *gains-from-trade maximization.* For this, we consider, the quantity GFT \E"$[v(q;\phi) - c(q;\psi) - t(q)]$, the value of the buyer net of the cost for the producer, minus the transaction for the certificate. As it will depend on the menu M of certificates that are offered, we will write $\mathrm{GFT}(M)$ for the gains from trade that are realized with menu M. The first result is that larger menus are better for gains from trade, a second that to maximize gains from trade, all certificates should be offered at no cost. This will be our benchmark to compare to when discussing greenwashing, and contrast with more general principal objectives, in which certification design does not become trivial.

Theorem 4.1. *Consider two certification menus M and M' with $M \subseteq M'$. Then $\mathrm{GFT}(M) \leq \mathrm{GFT}(M')$.*

We will comment about this result in our next section, which also shows how it can be proved.

Corollary 4.2. *The GFT-optimal menu of certificates offers every possible certification level $q > 0$ at a cost of $\underline{c}$, and level 0 at a cost of 0.*

Proof. If all quality levels were available, the GFT-maximizing outcome is would be for each producer ψ to trade with consumer $\phi(\psi)$ at whichever quality level q maximizes their gains from trade $f(q;\phi(\psi)) - g(q;\psi)$. However, since quality levels are hidden, producers and consumers can trade at a positive level of quality only if the cost of verification is paid. So if the maximum gains from trade is less than c, then it is preferable to trade at level 0. However, we observe that this is precisely the outcome implemented at equilibrium from the proposed certification menu, so it must be GFT-optimal over all possible menus. $\square$

In many applications, however, such as the application of voluntary carbon markets we consider in this paper, the gains from trade from market participants may not align with the objectives of a certifier. There may be a beneficial trade between producers certifying low-quality (that is, low carbon mitigation) goods to companies that further sell these, compare Greenfield 2023; Rodemeier 2023. This is not in the interest of a certifier with an interest for the climate. The next section considers such objectives.

5 Certification as Selling Surplus to Market Participants

In this section we describe the market outcome that will occur for any given menu of certificates offered by the certifier. We show that it is without loss of generality for the certifier to restrict to offering threshold certificates that guarantee that a product is at least a certain level of quality. We characterize the unique Walrasian market equilibrium allocation that results from any assignment of such certificates to producers. We then use that characterization to solve for each producer's utility-maximizing choice of certificate from the certifier's menu, which will also be unique. Finally, we use this characterization of producer behavior to interpret the certifier's menu-selection task as a non-linear screening problem.

5.1 Certifications as Minimum Quality Thresholds

A first simple observation is that since producer costs are increasing in quality level, and since goods at different quality levels but with the same certification are indistinguishable to consumers (and hence must sell at the same price), a producer who is assigned certificate σ will always choose to produce at the minimum quality level eligible for that certificate.

Observation 5.1. *Fix any certifier menu M and any production and certification strategy of the producers. Then for any producer ψ, selecting certificate σ and producing at quality $q > \min \sigma$ is dominated by selecting certificate σ and producting at quality $\min \sigma$.*

Given this observation, we know that for any menu M, any equilbrium strategy Γ for the producers, and any certificate σ, the marginal distribution over quality Γ_σ will be a point mass at $\min \sigma$. In particular, any two certificates with the same minimum will induce the same equilibrium beliefs over quality and hence have indistinguishable value to all consumers. It is therefore without loss of generality for the certifier to only offer certificates of the form $\sigma_q = [q, 1]$; i.e., certificates that are differentiated only with respect to their minimum values. If a producer selects certificate σ_q, then that producer's chosen quality level at equilibrium will necessarily be q. Any given certification menu M therefore reduces to a (possibly infinite) collection of quality levels in $[0, 1]$ to certify.

Motivated by this observation, we will assume for the remainder of the paper that all certificates are of the form $[q, 1]$, and associate each $\sigma = [q, 1]$ with its quality threshold q. We can then think of a certification menu M as a collection of pairs $\{(q_i, t_i)\}$, where q_i is a quality threshold and t_i is a corresponding price for certifying that quality is at least q_i.

5.2 Uniqueness and Assortativity of Competitive Market Allocations

We now turn to an analysis of the competitive market outcome that will result given the strategies of the certifier and producers. Recall that we can restrict attention to certificates of the form $\sigma_q = [q, 1]$ and that any good with certificate σ_q will have quality q with probability 1, so for the remainder of the section we will think of a market outcome as an allocation x and prices p of quality levels. That is, $x(\phi) \in [0, 1]$ for all consumers ϕ, and for each $q \in [0, 1]$ in menu M there is an associated market price p_q. We emphasize that x is a mapping from consumers to the certified goods they buy at market, whereas Γ is a mapping from producers to the certificates that they choose from the certifier.

The following lemma shows that for any choice of certification menu M and production and certification strategy Γ of the producers, all competitive market equilibria in the resulting market have the same uniquely-determined allocation. This allocation will be assortative, with higher-type consumers purchasing the higher-quality certificates. We note that such analysis is standard in the literature on market equilibrium; we include it here for completeness.

Lemma 5.2. *Fix any certifier menu M and any strategy Γ of the producers. Then in every competitive market outcome (x, p), the allocation x satisfies $x(\phi_1) \leq x(\phi_2)$ and $p_{x(\phi_1)} \leq p_{x(\phi_2)}$ for all $\phi_1 \leq \phi_2$.*

Proof. Since consumers are unit-demand and each producer has a single unit of good, a Walrasian equilibrium (x, p) of the market is guaranteed to exist. By the first welfare theorem, any such equilibrium must maximize the total welfare,

$$\int_\phi f(x(\phi); \phi) dF(\phi).$$

Suppose there exist types $\phi_1 < \phi_2$ with $x(\phi_1) > x(\phi_2)$. By the single-crossing condition, we have that

$$f(x(\phi_1); \phi_2) - f(x(\phi_2); \phi_2) > f(x(\phi_1); \phi_1) - f(x(\phi_2); \phi_1)$$

and hence

$$f(x(\phi_1); \phi_2) + f(x(\phi_2); \phi_1) > f(x(\phi_1); \phi_1) + f(x(\phi_2); \phi_2)$$

which contradicts the supposed welfare optimality of allocation x.

We now turn to prices. Fix any consumer types $\phi_1 < \phi_2$, so in particular we know $x(\phi_1) \leq x(\phi_2)$, and suppose for contradiction that $p_{x(\phi_1)} > p_{x(\phi_2)}$. By monotonicity of the value function f, we must have $f(x(\phi_1); \phi_1) \leq f(x(\phi_2); \phi_1)$. But this then means $f(x(\phi_2); \phi_1) - p_{x(\phi_2)} > f(x(\phi_1); \phi_1) - p_{x(\phi_1)}$, which violates the competitive market condition that consumer type ϕ_1 is choosing her most-preferred good. We therefore conclude that $p_{x(\phi_1)} \leq p_{x(\phi_2)}$, as claimed. □

5.3 Uniqueness and Assortativeness of Certificate Selection

Given that market outcomes are well-defined, we next turn to the equilibrium choices of the producers when selecting quality levels and their corresponding certifications. We again show that for any menu M of certificates offered, the quality choices of producers are unique at equilibrium. Moreover, higher-type producers will always select (weakly) higher certificates. In the Lemma 5.2, we saw that higher-type consumers also purchase (weakly) higher certificates. As we discuss later, this means the matching in any equilibrium will be assortative and hence constrained-efficient given the available certificates.

Recall that a producer strategy Γ is a mapping from producer type ψ to a choice of certification and quality, which we know from will always coincide. We will therefore write $\Gamma(\psi) = q$ to mean that producer ψ produces at quality level q and purchases certificate σ_q. In particular, we must have $\Gamma(\psi) \in M$ for all ψ.

Lemma 5.3 shows that in the (unique) equilibrium of our market, the producer strategy is monotone, meaning that producers of higher types purchase higher levels of certification and produce at higher quality. Our analysis of the equilibrium producer strategy Γ employs ideas from the classical analysis of Walrasian equilibrium under single-crossing conditions: stronger producer types (i.e., those with lower marginal cost of production) can achieve higher relative benefit from higher levels of certification, so certification choices will be monotone. Moreover, since there is monotonicity of both supply and demand for certificates, for any given producer we can solve for the marginal consumer type that will determine that producer's ability to extract revenue from their choice of certificate. This allows us to solve for the market equilibrium explicitly, establishing uniqueness, by considering the first-order equilibrium conditions on both sides of the market. We defer further details of the proof of Lemma 5.3 to Appendix A.

Lemma 5.3. *Fix any certification menu M offered by the certifier. Then there is a unique equilibrium strategy Γ for the producers, and $\Gamma(\psi)$ is weakly increasing in ψ.*

An immediate corollary of Lemma 5.2 and Lemma 5.3 is that the equilibrium outcome for a given menu M is not only essentially unique (up to the choice of market-clearing prices), but also has a natural assortative interpretation. Each producer in the market has a corresponding consumer with whom they will always trade. The producer will select whichever certification level maximizes the gains from trade between themselves and their partner consumer, less the price of the certification.

Corollary 5.4. *For any menu $M = \{(q_i, t_i)\}$ of the certifier, the resulting equilibrium market outcome has producer ψ trade with consumer $\phi = \phi(\psi)$ where $G(\psi) = F(\phi)$. The level of quality at which ϕ and ψ trade maximizes $f(q_i; \phi) - g(q_i; \psi) - t_i$, and is weakly increasing in ψ.*

5.4 Reduction to Sale of Surplus

To this point we have characterized the producer choices and downstream outcomes that result from the certifier's choice of menu. Given this understanding, we can now relate the certifier's design problem to a certain non-linear pricing problem between a single seller and a single buyer.

The non-linear pricing problem is as follows. The buyer seeks to buy a perfectly divisible good. The seller may commit to a menu of quantities and prices. If a buyer of type θ purchases quantity $q \in [0, 1]$ of the good at a total price of t, then the buyer utility is

$$u((q, t); \theta) = v(q; \theta) - t,$$

where v is concave in quantity q but not necessarily non-decreasing, and $v(0; \theta) = 0$ for all θ. The valuations v satisfy a single-crossing condition, which is that for $\theta_1 < \theta_2$ and $q_1 < q_2$, we have

$$v(q_2; \theta_2) - v(q_1; \theta_2) > v(q_2; \theta_1) - v(q_1; \theta_1).$$

The principal faces a constant cost of production $c > 0$ for any non-zero quantity, and seeks to design a menu M over quantity-price pairs given a prior over buyer types. The principal receives a payoff that is determined by the menu item selected and the buyer's utility.

We claim that the problem faced by a certifier choosing a menu of certificates is equivalent to the problem faced by the seller in a corresponding instance of the non-linear pricing problem. Denote by $\mathcal{E}$ an instance of our market economy with a certifier, and denote by $\mathcal{E}'$ an instance of the non-linear pricing problem described above.

Proposition 5.5. *For any instance $\mathcal{E}$ of a market with certification there is a corresponding instance $\mathcal{E}'$ of the non-linear pricing problem described above such that*

- *there is a one-to-one mapping between buyer types in $\mathcal{E}'$ and pairs of producer/consumer types in $\mathcal{E}$;*
- *there is a one-to-one mapping between a menu M chosen by the certifier in $\mathcal{E}$ and a menu M' chosen by the seller in $\mathcal{E}'$;*
- *the outcomes chosen from menu M by a producer type correspond to the outcomes chosen from menu M' by the corresponding buyer type, and the resulting producer/consumer gains from trade in $\mathcal{E}$ correspond to buyer surplus in $\mathcal{E}'$.*

Proof. The certifier's problem is to design a certificate menu $M = \{(q_i, t_i)\}$. By Corollary 5.4, given menu M, each producer ψ will purchase whichever certificate q_i maximizes $f(q_i; \phi(\psi)) - g(q_i; \psi) - t_i$. For $q \in [0, 1]$, we can interpret ψ as a buyer type and define valuation function $v(q; \psi) = f(q_i; \phi(\psi)) - g(q_i; \psi)$. Then since f and g both satisfy single-crossing with respect to their corresponding types, valuation function v does as well. Moreover, v is concave and $v(0; \psi) = 0$. By definition, the producers' choices of certificates from menu M corresponds precisely to the buyer's choice of quantity when facing the same menu, interpreting each certificate quality threshold as a quantity. Thus the outcomes (menu item chosen and transfers made to the seller/certifier) in the two settings are equivalent. Finally, the gains from trade between the producer and consumer is precisely (by definition) the buyer utility under these equivalent outcomes. □

Note that an immediate implication of Proposition 5.5, given Corollary 5.4, is that for any menu M chosen by the seller, the choice of quantity purchased by the buyer is weakly increasing in buyer type.[1] Moreover, any menu-design problem faced by the certifier with an objective determined by revenue, quality level sold, and/or gains from trade is equivalent to a corresponding optimization problem faced by the seller in the non-linear screening scenario.

[1] Alternatively, this is a direct consequence of the single-crossing condition on valuation functions v.

6 A Model of Greenwashing

In this final section, we will compare the quantity $\hat{q}(\theta)$ that a producer-buyer pair would choose in the laissez-faire solution, and compare it to the solution that maximizes an objective of the principal of the form $\mathbb{E}[\lambda q(\theta) + t(\theta)]$ for some $\lambda \in (0, \infty)$, $q^*(\theta)$. If the principal is benevolent, then λ is the social cost of carbon.

We will show under which properties we have that $\hat{q}(\theta) \leq q^*(\theta)$ globally, that is, when is the quality that the certifier chooses higher than the

To solve this issue, we can use generalizations of mechanism design techniques for problems of the type we consider (Toikka 2011). The optimal mechanism is a point-wise maximizer of the following objective function:

$$J(q, \theta) = \lambda q + u(q, \theta) - \frac{1 - F(\theta)}{f(\theta)} u_\theta(q; \theta)$$

The first summand corresponds to the valuation of the principal, the second to the valuation of the buyer of surplus (that is, producer-buyer pairs, see Section 5), and the last term is an information rent. We will sign this result. This formulation allows us to reason about

Theorem 6.1. *Fix θ. If $\frac{\lambda f(\theta)}{1-F(\theta)} \geq u_{q\theta}(q; \theta)$ for all $q \in [0, 1]$, then $q^*(\theta) \leq \hat{q}(\theta)$. If $\frac{\lambda f(\theta)}{1-F(\theta)} \leq u_{q\theta}(q; \theta)$ for all $q \in [0, 1]$, then $q^*(\theta) \geq \hat{q}(\theta)$.*

This result means that if the designer has sufficiently little value for actual mitigation, they will depress the quality of certification even below the quality of the laissez-faire choice. This allows the certifier to sell more certificates, while lowering the absolute quality of the activities. In particular, observe that for $\lambda = 0$, by our assumption that u has increasing differences, it must always be that $q^*(\theta) \leq \hat{q}(\theta)$: A revenue-maximizing principal will depress the quality compared to the laissez-faire solution.

Proof Sketch. For a fixed $\theta \in \mathbb{R}_+$, define the auxiliary function

$$l(q, \tau) = \tau\lambda q + u_q(q, \theta) - \tau \frac{1 - F(\theta)}{f(\theta)} u_\theta(q, \theta)$$

Evaluated at $\tau = 0$, this function gives us $\hat{q}$. Evaluated at $\tau = 1$ it gives us q^*. If we show that l has increasing differences, then by Topkis' Theorem (Topkis 1998), we find that $\hat{q}(\theta) \leq q^*(\theta)$. The second claim follows similarly. Observe that increasing differences is equivalent to

$$l_{q\tau}(q, \tau) \geq 0.$$

That is, $\lambda - \frac{1-F(\theta)}{f(\theta)} u_{q\theta}(q, \theta) \geq 0$. Algebra shows that this is equivalent to the assumption. Showing that the function has decreasing differences under the opposite assumption shows the converse result. □

7 Discussion and Policy Implications

The designer of a financing system for negative emissions activities may consider more complex mechanism beyond certification. Subsidies for production might be necessary. As we show in a model of vertical heterogeneity: certification is enough. Certification is also understood as selling surplus to producer-buyer pairs. We show that for small preference for quality by the principal, the quality chosen by producers is uniformly *lower* than in a comparable laissez-faire solution. For high preference for money, uniformly *higher* qualities are chosen.

We can view instances like the Green Claims Initiative Commission 2025 and by the Integrity Council for the voluntary carbon markets (`https://icvcm.org/`) can be viewed as restricting the availability of certificates of a low quality. If these are a sign of sufficiently high preference for quality, they will lead to certification that is higher than under availability of arbitrary certificates.

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A Omitted Proofs

Proof of Theorem 4.1. If producer ψ selects option (q, p) from certification menu M, then the gains from trade generated for the producer ψ and corresponding consumer ϕ, less the revenue raised by the certifier, is $f(q;\phi(\psi)) - g(q;\psi) - p$. By Corollary 5.4, producer ψ purchases precisely whichever menu item from M maximizes this quantity. Providing additional items can therefore only increase the gains from trade jointly enjoyed by producer type ψ and corresponding consumer $\phi(\psi)$. As this holds point-wise for every ψ, it holds in aggregate over all types as well. □

Proof of Lemma 5.3. First recall the statement of the lemma. We fix any certification menu M offered by the certifier. Then we claim that there is a unique equilibrium strategy Γ for the producers, and $\Gamma(\psi)$ is weakly increasing in ψ.

Fix strategy Γ, which implies the measure of certificates chosen by the collection of producers. Let (x, p) denote a Walrasian equilibrium in the resulting competitive market, and recall that x is uniquely determined.

We first show that Γ is weakly increasing is ψ. Assume for contradiction that there exist $\psi_1 < \psi_2$ with $q_1 = \Gamma(\psi_1)$ and $q_2 = \Gamma(\psi_2)$ with $q_2 < q_1$. Then by the single-crossing condition for producers, we have $g(q_1;\psi_1) - g(q_2;\psi_1) > g(q_1;\psi_2) - g(q_2;\psi_2)$. But then, if we let p_{q_1} and p_{q_2} denote the Walrasian equilibrium prices of q_1 and q_2 given Γ, we have

$$(p_{q_1} - g(q_1;\psi_1)) + (p_{q_2} - g(q_2;\psi_2)) < (p_{q_1} - g(q_1;\psi_2)) + (p_{q_2} - g(q_2;\psi_1))$$

which means that either

$$p_{q_1} - g(q_1;\psi_1) < p_{q_2} - g(q_2;\psi_1)$$

or

$$p_{q_2} - g(q_2;\psi_2) < p_{q_1} - g(q_1;\psi_2).$$

In other words, either producer ψ_1 or ψ_2 (or both) would strictly improve their utility by switching their choice of quality and certification. As such a swap has measure zero and does not influence the competitive equilibrium, this would be an improving deviation for the producer(s), violating the assumption that Γ is an equilibrium strategy for the producers.

We have shown that Γ is weakly increasing in ψ. On the other hand, we know from Lemma 5.2 that the market allocation of quality levels to consumers is weakly increasing in ϕ. This means that any equilibrium outcome of production and trade is equivalent to one in which consumers and producers are matched assortatively, with higher-type consumers trading with higher-type producers. In other words, for any producer type ψ, there is a consumer type $\phi = \phi(\psi)$ such that ψ always trades with $\phi(\psi)$. Specifically, $\phi(\psi)$ is such that $F(\phi(\psi)) = G(\psi)$ (treating F and G as cumulative distribution functions).

Given this, we claim that $\Gamma(\psi)$, the certification selected by producer ψ at equilibrium, will always be a certificate q_i from the certifier's menu $M = \{(q_i, t_i)\}$ that maximizes $f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i$. To see why, suppose for contradiction that the producer instead chooses some other certificate q' at price t' such that $f(q';\phi(\psi)) - g(q';\psi) - t' < f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i - \varepsilon$ for some $\varepsilon > 0$, and sells to consumer $\phi(\psi)$ at an assumed market-clearing price $p_{q'}$.[2] Then, the producer ψ could instead deviate to purchasing q_i at a price of t_i, and offering it on the competitive market at a price of $p_{q'} + g(q_i;\psi) - g(q';\psi) + (t_i - t') + \varepsilon/2$. Note that if consumer $\phi(\psi)$ were to purchase from producer ψ at this price, then her utility would be

$$\begin{aligned}
&f(q_i;\phi(\psi)) - [p_{q'} + g(q_i;\phi(\psi)) - g(q';\phi(\psi)) + (t_i - t') + \varepsilon/2] \\
&= (f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i - \varepsilon) + (g(q';\psi) + t') + \varepsilon/2 - p_{q'} \\
&> f(q';\phi(\psi)) - (g(q';\psi) + t') + (g(q';\psi) + t') - p_{q'} + \varepsilon/2 \\
&> f(q';\phi(\psi)) - p_{q'}.
\end{aligned}$$

But since $(q', p_{q'})$ is the most-demanded offering to consumer $\phi(\psi)$ in the market equilibrium, this means that the offering of q_i at the proposed price would be the most-demanded offering to consumer $\phi(\psi)$ under

[2]Note that as we showed Γ is weakly increasing in ψ, the deviation can not change the ordering of firms in terms of the certificate they purchase and hence does not change the consumer to which they sell.

this deviation. In particular this means that *some* consumer would want to purchase q_i at the suggested price, and therefore in the adjusted market equilibrium after this proposed deviation the price of q_i must be at least this high.

We conclude that the utility of producer ψ under this deviation is at least

$$\begin{aligned}[p_{q'} + (g(q_i;\psi) - g(q';\psi) + (t_i - t') + \varepsilon/2] - g(q_i;\psi) - t_i &= p_{q'} - g(q';\psi) - t' + \varepsilon/2 \\ &> p_{q'} - g(q';\psi) - t'\end{aligned}$$

and hence this deviation is strictly utility-improving for the producer, contradicting the assumption that Γ is an equilibrium.

We therefore conclude that at equilibrium, each producer ψ chooses whichever certificate q_i from the menu maximizes $f(q_i;\phi(\psi)) - g(q_i;\psi) - t_i$. The choice of each producer is therefore unique, up to tie-breaking on sets of measure zero. □