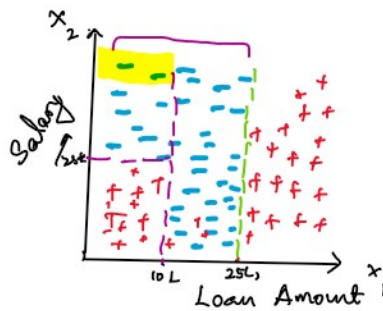


Decision Trees

→ Classification (Binary)

ML → $X \rightarrow Y$ relation
Cost function
Optimization

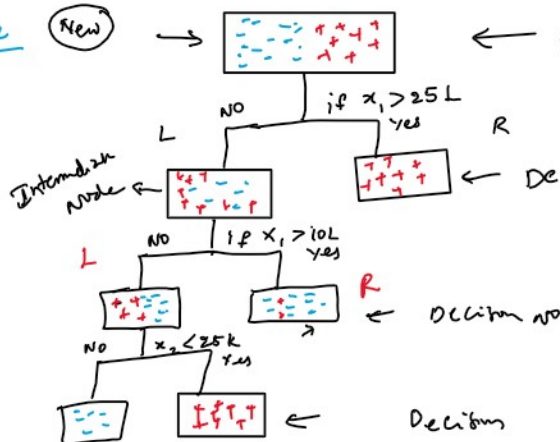
Principle:- "Divide and Conquer"



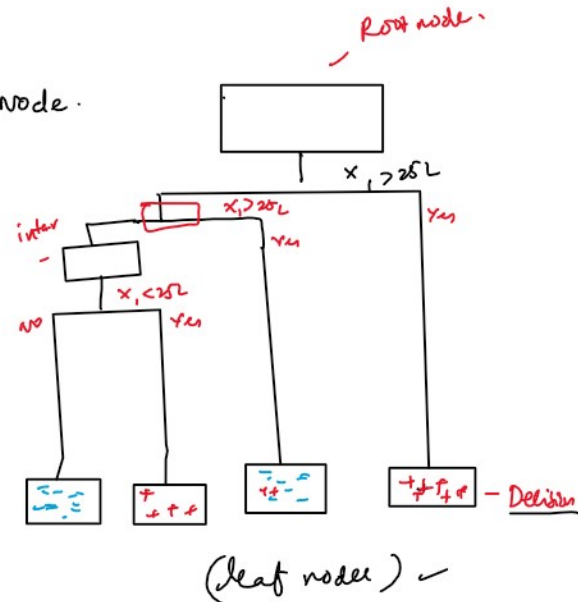
+ - case (defaulted)
- paid up ✓

if $x_1 > 25L \rightarrow +ve$ ✓
else if $x_1 > 10L \rightarrow -ve$ cases
else if $x_2 < 25K \rightarrow +ve$

Tree (New)



Root node.



\underline{Y} - Price (100 - 5000)

Norm $\underline{X} \leftarrow \underline{Y}$ [100 - 5000]
[0 - 1]

$\hat{Y} = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_m x_m$
[0 - 1] w_0 - high value, w_1, w_2, \dots, w_m
100, 300, ...

Norm \underline{Y} [0 - 1] $\hat{Y} = w_0 + w_1 x_1 + w_2 x_2 + \dots + w_m x_m$

$\underline{Y} - (\mu, \sigma)$ $w_0, w_1, w_2, \dots \{0.1, 0.2\}$

$Z = \frac{x_i - \mu}{\sigma}$ $Z \cdot \sigma + \mu = x_i$

(\hat{Y}) $Y [0 - 1] \rightarrow Y [100 - 5000]$

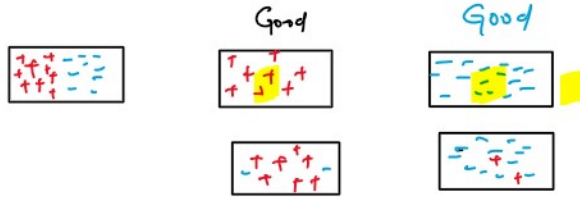
$Z = \frac{y_i - \mu}{\sigma}$

$y_i = (\sigma \cdot Z + \mu)$ ✓

1. Where should we split the data? (for a given variable)
2. Which variable should split the data first?

Learning (Training)

Cost



$$-y_i \log_2(y_i) - (1-y_i) \log_2(1-y_i)$$

Metric

50% 50%

Entropy: $-\sum_{i=1}^K P_i \log_2(P_i)$

P_i is the probability of i th class
 K is the number of classes

(Impurity)

$$\rightarrow -\frac{5}{10} \log_2\left(\frac{5}{10}\right) - \frac{5}{10} \log_2\left(\frac{5}{10}\right)$$

[K=2]

$$\rightarrow -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right)$$



$$-\frac{10}{10} \log_2\left(\frac{10}{10}\right) - \frac{0}{10} \log_2\left(\frac{0}{10}\right)$$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} = 1$$

$$\Rightarrow 0$$

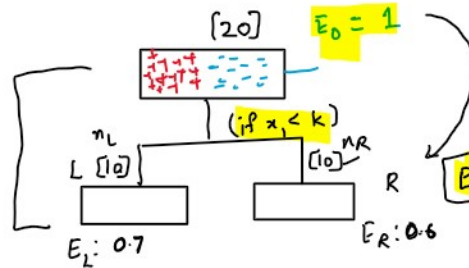
Cost (0-1)

Parent (Children)

$$E_0 - E_1$$

Information gain

Split



$$E_0 = 1 \quad E_1 = 0.65$$

$$IG = 1 - 0.65$$

$$IG = 0.35$$

$$\rightarrow \left(\frac{n_L}{n} \times E_L \right) + \left(\frac{n_R}{n} \times E_R \right)$$

$$\left[\frac{10}{20} \times 0.7 + \frac{10}{20} \times 0.6 \right] -$$

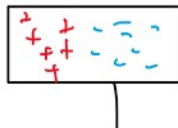
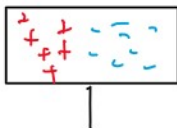
$$X_1 < k$$

$$X_1 < Q$$

$$X_1 < R$$

Best split ✓

$$X_1 \quad E_0 = 1$$

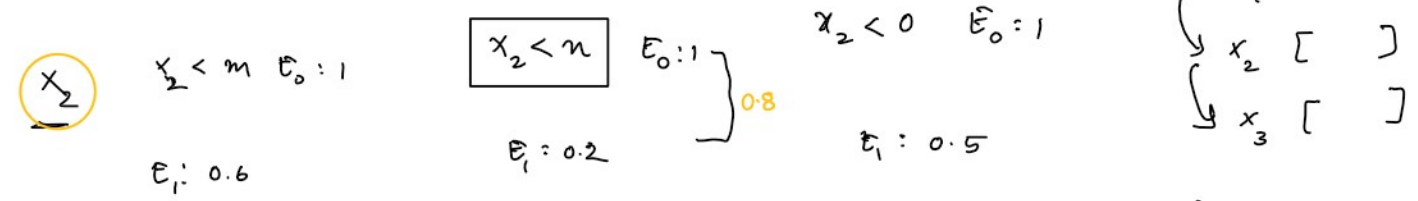
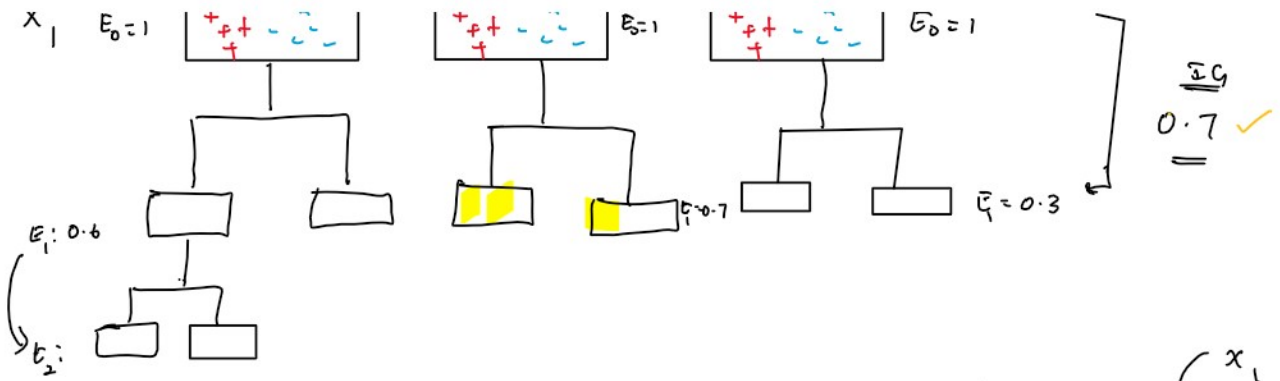


$$E_0 = 1$$

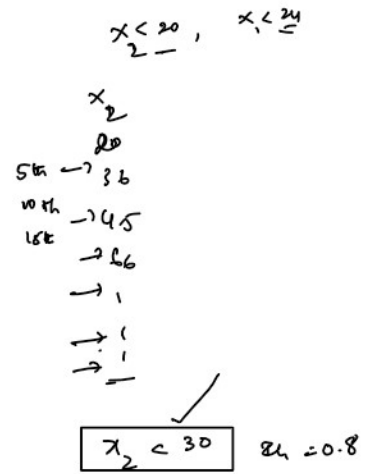
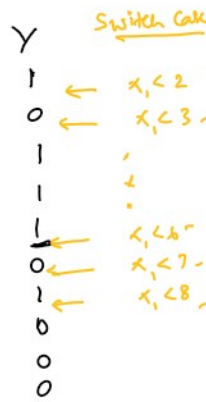
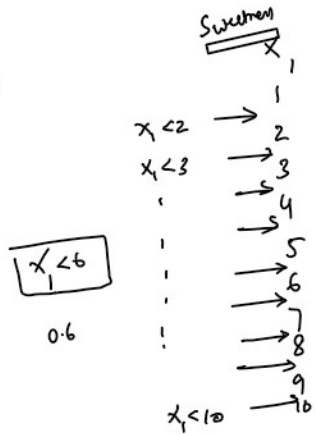


$$E_0 = 1$$

IG

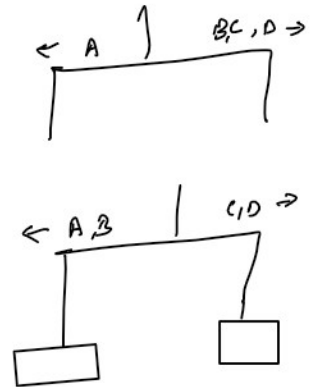


Split points

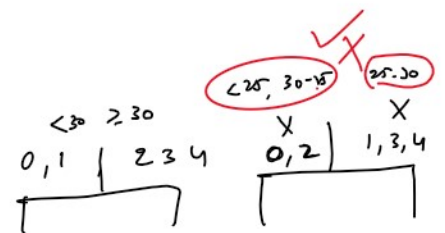


break 10:28

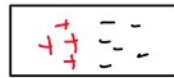
$X_1 \in \{A, B, C, D\}$
 $\rightarrow A \mid B, C, D$
 $\rightarrow A, B \mid C, D$
 $\rightarrow A, C \mid B, D$
 $\rightarrow A, D \mid B, C$



Ordinal variable \rightarrow $[0 \ 1 \ 2 \ 3 \ 4]$
 \rightarrow $0 \mid 1 \ 2 \ 3 \ 4$



3, 5 E - 1
 (6, 4) (E = 0.8)



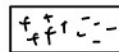
Entropy = $\sum_{i=1}^K p_i \log_2(p_i)$

$$= -\frac{4}{10} \log_2\left(\frac{4}{10}\right) - \frac{6}{10} \log_2\left(\frac{6}{10}\right)$$

$$= (0.8)$$

Gini Index : Measure of Impurity

Gini → $1 - \sum_{i=1}^K (p_i)^2$ p - prob of a class
 i - number of classes



$$= 1 - \left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \right]$$

$$= 1 - [1^2 + 0^2]$$

$$= 1 - 1$$

$$= 0 \text{ Best}$$

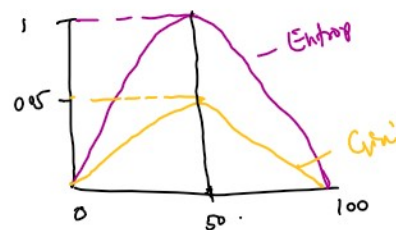
$$= 1 - \left[\frac{1}{4} + \frac{1}{4} \right]$$

$$= 1 - \frac{1}{2} = 0.5$$

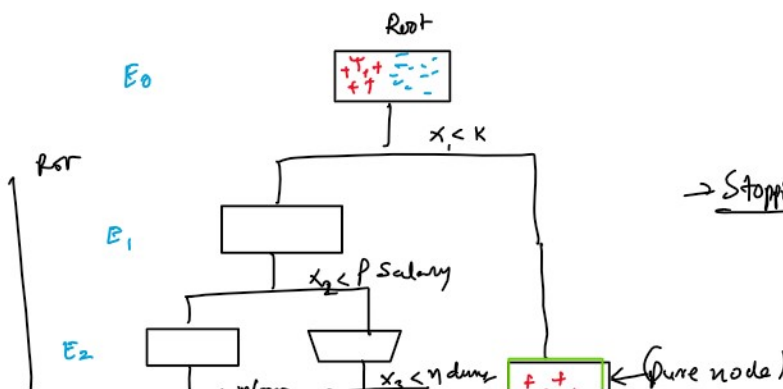
$$\text{Gini} = [0 - 0.5]$$

(1985)

Gini - Computationally faster

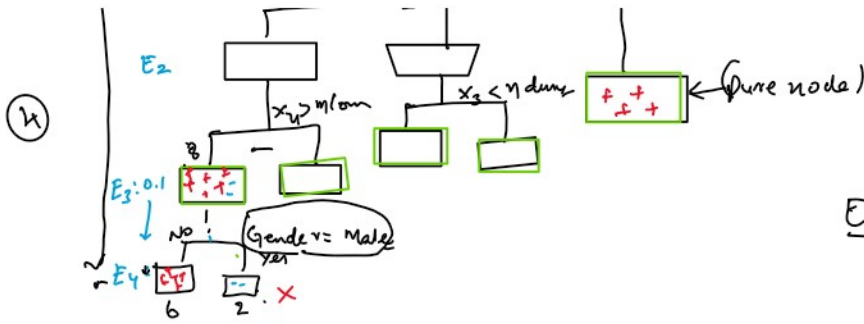


①



→ Stopping :-

- ① When we have a pure node.
- ② When there is no further I.G.



(2) When there is no further split

Early Stopping Criteria
(Pruning) →

- ① → Highly prone to overfitting ✓
→ Greedy

✓✓ ① Depth of the tree ✓
Max-depth = 3

→ ✓ ② min no. of observation to split,

(10) x

→ ✓ ③ min no. of observations in leaf node = (5)

Entropy



P_+ P_-

(0-1)

$$P_+ = P$$

$$P_- = 1-P$$

$$-y_i \log_2(P_i) - (1-y_i) \log_2(1-P_i)$$

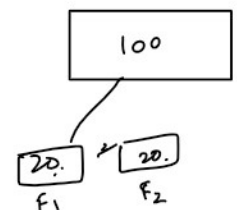
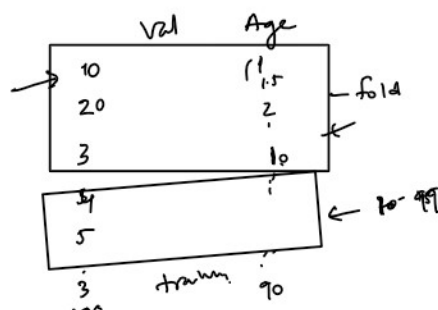
$$-P \log_2(P) - (1-P) \log_2(1-P) \leftarrow$$

$$\text{Entropy} = -\sum_{i=1}^K P_i \log_2(P_i)$$

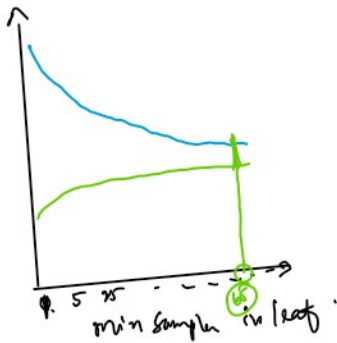
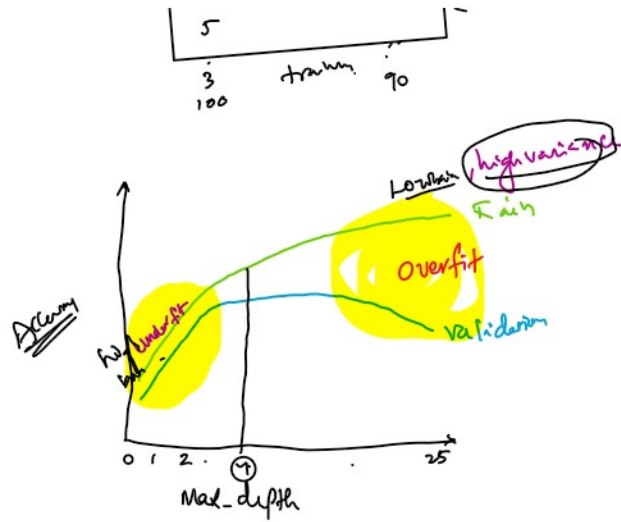
$$\text{Gini} = 1 - \sum_{i=1}^K (P_i)^2$$

Start : 9:05

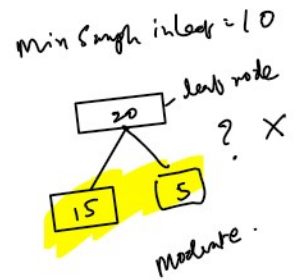
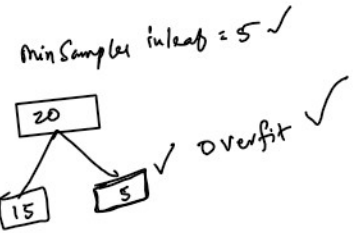
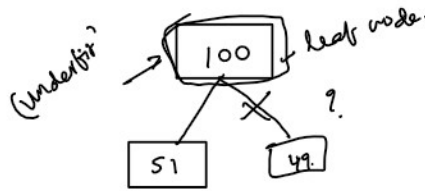
① folds = KFold (n_splits = 5, Shuffle : True, random_state = 1)



Plot



min sample in leaf = 50



→ Minimum Samples to Split

high (?) - underfit

low (?) - overfit

min sample = 50

min = 100

min split = 200

100

120

120

deeper

stopping underfitting

max feature = 4

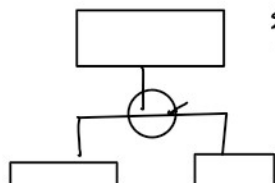
Split points

'best' approach

'Random' approach

age { 10 - 99 }

$\{x_1, x_2, \dots, x_{10}\}$



$\{x_1, x_3, x_8, x_9\}$

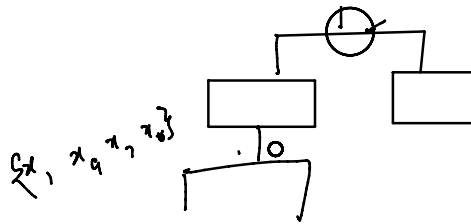
age < 21

age < 36

age < 61

age < 20

age < 25



age < 61
1
..

max. features = 'auto'

Classification

Regres

$$\sqrt{m}$$

m no. of features

$$\frac{m}{3}$$

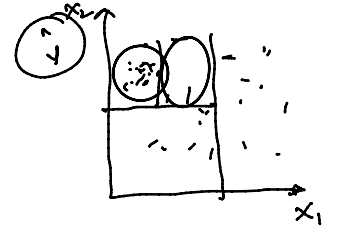
m no. of features

Classification

Regression?

Entropy } for regres?
Gini }

$$-\sum p_i \log_2(p_i)$$

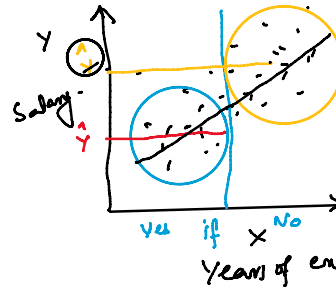


if $x < 5$ $\hat{y} = \text{mean}(y_1)$

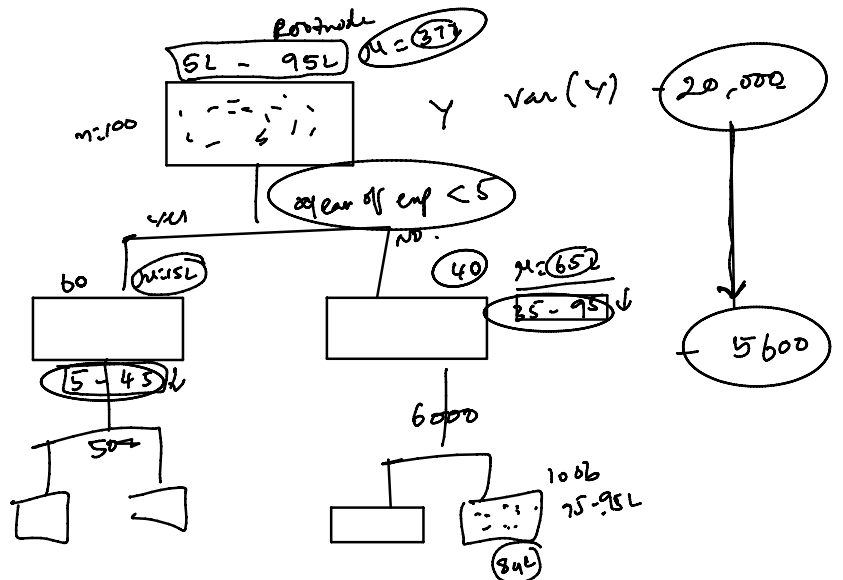
if $x > 5$ $\hat{y} = \text{mean}(y_2)$

MSE

if years < 5 avg sal 15L
> 5 avg sal 30L



if $x < 5$



Multiclass Classification - Entropy, Gini work?

$$- \sum_{i=1}^K p_i \log_2(p_i)$$

K-classes

10 20 30

$$- \frac{10}{60} \log_2\left(\frac{10}{60}\right) - \frac{20}{60} \log_2\left(\frac{20}{60}\right) - \frac{30}{60} \log_2\left(\frac{30}{60}\right)$$

(Binary)
Classification and Regression Trees (CART)

→ Ensemble