

Boosting

13 December 2022 20:38

Boosting

Bagging :-

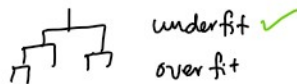
R.F. :-

(Parallel)
Base models + aggregation
[Low bias, Base Models (Deep trees (overfit) high variance) + (Row sampling + Column Sampling) + Agg.] (diversity) (low Variance)

Boosting -

(high bias) Base models + addition (sequence)

Shallow trees



$n(x_i, y_i)$

$x_1, x_2, y, e_0, \hat{y}_0, m_0$ Step 0 (Base model)

-	-	9	-7	16
		15	-1	16
		20	4	16

m_0 - average value of y ($\bar{y} = 16$)



20 4 16 Step 1

m_1, x_1, x_2	\hat{e}_0	\hat{y}_1	e_1
	-7	-3	-4
	-1	0	-1
	4	2	2
	4	1	3

$\hat{y}_1 = m_1 = n_1(x_1, x_2)$

y	m_0	\bar{y}	e_0	\hat{y}_1	e_1
20	16	4	1	3	

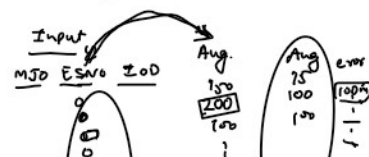
$$m_1 = 16 + 1$$

$$M_1 = m_0 + m_1 = 16 + 1 = 17$$

$$30 \rightarrow 25$$

$$100 + 25 = 125$$

"EL NINO"
"LANINA"



$$M = m_0 + m_1 + m_2 + m_3 + \dots + m_m$$

$$M = M_0 + M_1 + M_2 + M_3 + \dots + M_m$$

$$(x, y) \quad (x, \hat{y}_0) \quad (x, \hat{y}_1)$$

$$\bar{y} \quad \hat{y}_1 \quad \hat{y}_2$$

$$e_0 = y_1 - \bar{y} \quad e_1 = e_0 - \hat{y}_1 \quad e_2 = e_1 - \hat{y}_2 \quad \dots \quad e_m ?$$

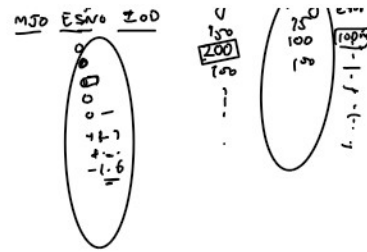
$$M_0 = \bar{y}$$

$$M_1 = \bar{y} + \hat{y}_1$$

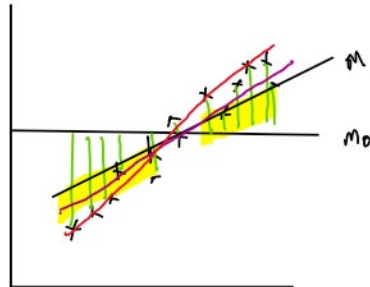
$$M_3 = \bar{y} + \hat{y}_1 + \hat{y}_2$$

$$M = \alpha_0 M_0 + \alpha_1 M_1 + \alpha_2 M_2 + \dots + \alpha_m M_m$$

$$F(x) = \alpha_0 h_0(x) + \alpha_1 h_1(x) + \alpha_2 h_2(x) + \dots + \alpha_m h_m(x)$$



Gradient Boosting Trees
 → Regression ✓
 → Classification



$$\text{Gradient Boosting} \quad (\alpha_0, \alpha_1)$$

y	M_0	M_1	M
9	16	-3	$\alpha_0 16 - \alpha_1 3$
15	16	0	$\alpha_0 16 + \alpha_1 0$
20	16	2	$\alpha_0 16 + \alpha_1 2$
20	16	1	$\alpha_0 16 + \alpha_1 1$

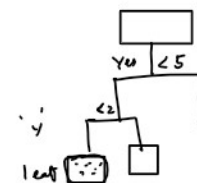
$$\frac{1}{n} \sum \left[(y - (\alpha_0 16 - \alpha_1 3))^2 + 15 - \dots \right] \quad (\alpha_0, \alpha_1)$$

$$\text{Step 3} = \alpha_0 x + h_0(x) + \alpha_1 x + h_1(x) + \alpha_2 x + h_2(x)$$

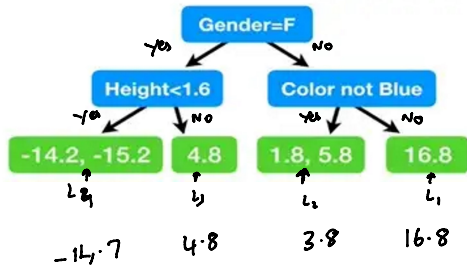
Height (m)	Favorite Color	Gender	Weight (kg)	\bar{y}	e_0	\hat{y}_1
1.6	Blue	Male	88	71.2	16.8	16.8
1.6	Green	Female	76	71.2	4.8	4.8
1.5	Blue	Female	56	71.2	-15.2	-14.7
1.8	Red	Male	73	71.2	1.8	3.8
1.5	Green	Male	77	71.2	5.8	3.8
1.4	Blue	Female	57	71.2	-14.2	-14.7

Step 1

Step 1 $\alpha_0 = 1, \alpha_1 = 1$



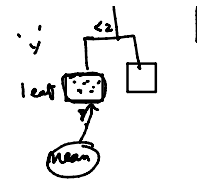
Step 1 Model



Step 1 $\alpha_0 = 1, \alpha_1 = 1$

$$\alpha_0 \cdot m_0 + \alpha_1 \cdot m_1$$

$$71.2 + 16.8 = 88$$



$$\eta = 0.01$$

Learning rate | Shrinkage

$$M = \alpha_0 \cdot m_0 + \eta \cdot \alpha_1 \cdot m_1 + \eta \cdot \alpha_2 \cdot m_2 + \eta \cdot \alpha_3 \cdot m_3$$

$$\eta = 0.1$$

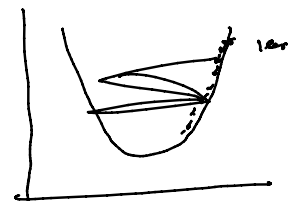
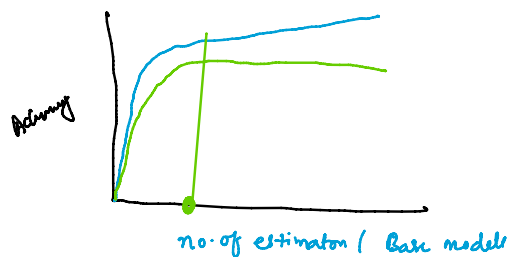
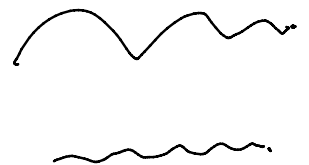
$$\eta = 1 \leftarrow$$

(no. of models)

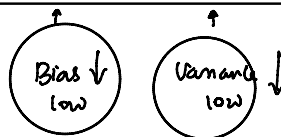
$$M = \alpha_0 \cdot m_0 + 0.1 \cdot (\alpha_1 \cdot m_1)$$

$$= 71.2 + 0.1 (16.8)$$

$$= 71.2 + 1.68 = 72.88$$



$$\text{Model Error} = \text{Bias}^2 + \text{Variance} + \text{unexplainable error}$$



Balance b/w Bias and Variance

Overfitting (Boosting) →

- don't train too many Base models (n -estimators)
- keep the learning rate or shrinkage (low)
- Base models → shallow (high Bias)

Tree Control Parameters

- max depth
- min no. of Sample leaf node
- max no. of leaf nodes

no. of estimator vs Learning rate

more no. of estimator ← (Small) (very minute steps)
 less no. of estimator ← large

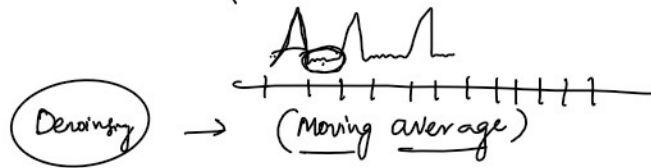
① More no. of estimator, and large learning rate?

What happens? (Rectification)



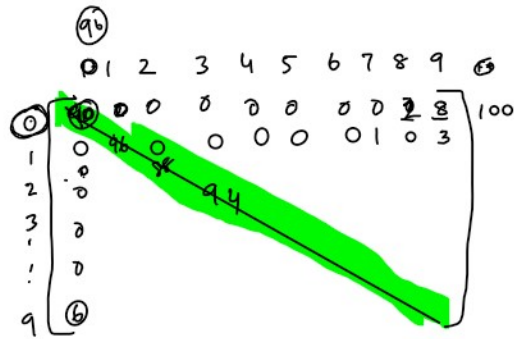
Noise handling:-

(Running) → (Brain Signals) ←



100
1000

Smoothens, ~~the~~ Remove small spikes (diffraction)



$$\frac{90 + 96 + 98 + 94 + \dots}{1000} = 0.95$$

Perm 0

$$\begin{bmatrix} 0 & 0 \\ 90 & 10 \\ 6 & 2 \end{bmatrix}$$