Probability Theory.

- Experiments
  - Tossing coin
  - Roll · dice.
- Events
  - (H, T) ✓
  - 7X
  - 8X
- Sample space.
  - H, T ⇒ (2)
  - 1, 2, 3, 4, 5, 6 ⇒ (6)
  - Rain / N Rain ⇒ 2.

E ⊂ S

Probality ⇒ 0 to 1 ✓

- 0 ↓ will not happen
- 1 ↓ will happen.

0.2 - 0.8

0.8994

Experiment: Random & Deterministic. (1 possible O/P)

- Random ⇓ H, T, 1, 2, 3, 4, 5, 6.
- Deterministic ⇓ 2+2 = 4 ≠ 5 ≠ 6

Play a game:

✓ variant 1: $10 ⇒ S.S = 2.

✓ variant 2: $10 ⇒ Sample.S ⇒ 100

P(W) in v.1 ⇒ $\frac{1}{2}$ ⇒ 0.5.

P(ω) in v2 ⇒ $\frac{1}{100}$ ⇒ 0.01

50% 1%

Variant 2 : $10

Variant 9 : $100 ⇒ S.S. 100

V1 ⇒ $\frac{1}{2}$ ⇒ 0.5 ⇒ 50%

V2 ⇒ $\frac{99}{100}$ = 0.99 ⇒ 99%

Toss a coin. P(H) = 1/2. P(T) = 1/2.

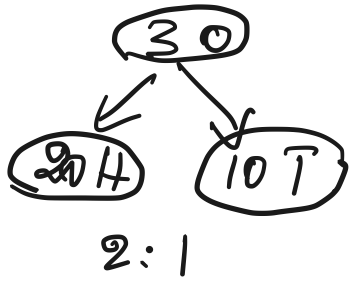
P(H) = 2 * P(T)

- P(H) + P(T) = 1
- 2*P(T) + P(T) = 1

3*P(T) = 1

$P(T) = \frac{1}{3}$ = ~~0.67~~ 0.33

$P(H) = \frac{2}{3}$ = ~~0.33~~ 0.67.



Experiment:	Event	S.S.
Roll die.	getting 5.	1, 2, 3, 4, 5, 6 ⇒ 6.

$$P(5) = \frac{\text{total. faces with 5.}}{\text{All faces.(SS)}} = \frac{1}{6} = \frac{1.}{6.}$$

$P(7) = 0$ 7 ✗ Dice.

What is prob. of S.S?

$$\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} \Rightarrow \frac{6}{6} = 1$$

2 Rules of prob.

$P = 0 - 1.$

$P(SS) = 1.$

$P(5) \Rightarrow \frac{1.}{6} = 0.166$

$\frac{20.}{100} = 0.2.$

$\frac{16}{100} \Rightarrow 0.16$ or $\frac{17}{100} = 0.17.$

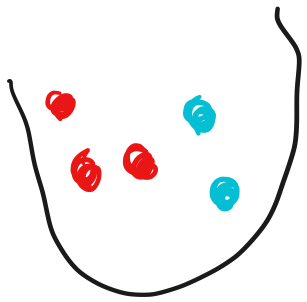
Sample Size = 100.

$n * \frac{1.}{6} = 100 \times \frac{1}{6} = \frac{100}{6.}$

$\frac{20}{100} \Rightarrow$ won.

$\frac{16.}{100}$ or $\frac{17}{100}$

2 $\Rightarrow$ 0.5.



Draw 1 marble
⇒ 300 times.
⇒ with replacement

Ex 1 : 1 ⇒ put back to the bag.
Ex 2 : 1 ⇒ " " "
⋮
Ex 300

$$P(B) = \frac{2}{5}$$

$$B = \frac{2}{5} \times 300 \quad \frac{1}{5} \times 300^{60}$$

$$= 2 \times 60 = 120$$

close to 120 but not exactly 120.

- Marginal probability → one event not affected by other
- Joint " → 2 event happen together
- Conditional " → given one event · prob of 2nd event.
- Independent event
- Disjoint events.

	Age.		
	10-20	20-30	30-40.
Engineer	5	25	15
			45

Profession				
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100

 $\Rightarrow \frac{45}{100} = 0.45$

 $\Rightarrow \frac{31}{100} = 0.31$

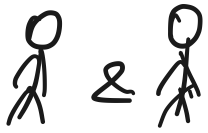
$\frac{34}{100}$

Marginal probability:

Lawyer: $\frac{18}{100} = 0.18$

$10-20 = \frac{35}{100} = 0.35$

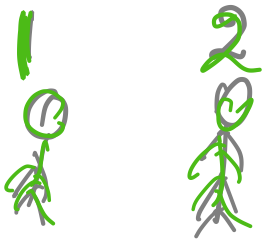
Profession	Age 10-20	20-30	30-40	
Engineer	5	25	15	45
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100

 $\Rightarrow \frac{30}{100} = 0.3$

10-20 Student

Joint probability

Profession \ Age	10-20	20-30	30-40	
Engineer	5	25	15	45
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100



20-30. Engineer.

$P(\text{Engineer} / \text{20-30}) = \frac{25}{34}$

Conditional probality.

$P(\text{10-20} / \text{Student}) = 30/37.$

Profession \ Age	10-20	20-30	30-40	
Engineer	5	25	15	45
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100

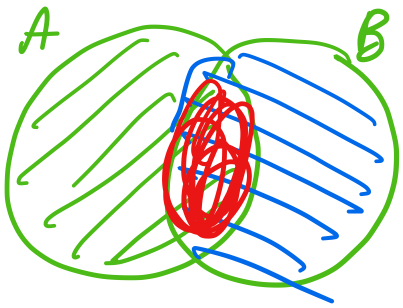
$= \frac{35}{100} \quad \frac{37}{100}$

10-20 Student

$= \frac{35}{100} + \frac{37}{100} = \frac{72}{100}$

$= \frac{35}{100} + \frac{37}{100} - \frac{30}{100} = \frac{42}{100}$

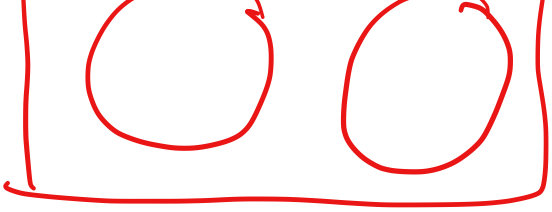
Union :

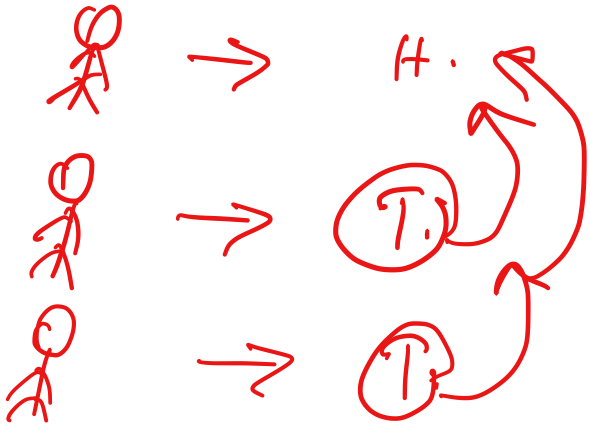


$P(A)+P(B) - P(A \cap B)$

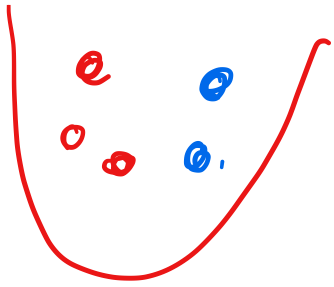
Profession \ Age	10-20	20-30	30-40	
Engineer	5	25	15	45
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100

 ⇒ M.E.





Independence.



$1 = \frac{2}{5}$ $2 = \frac{2}{5}$ with replac — Independent events.

without replacem: $1 = \frac{2}{5}$, $2 = \frac{1}{4}$ — Dependent

	Male	Female	Total
Event 1 → Football	120	75	195
Event 2 → Rugby	100	25	125
Event 3 → Others	50	130	180
	270	230	500

$P(A \cap B) = P(B|A)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A/B) =$ 1 $= \frac{120}{270}$

Condit: prob: $P(\text{others} / \text{male}) = \frac{50}{270}$

(A) (B)

Male, football $= P(B/A) = \frac{P(B \cap A)}{P(A)}$

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$P(A \cap B) = P(B \cap A)$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \Rightarrow$$

$$P(A/B) = \frac{P(A) * P(B)}{P(B)}$$

$P(A/B) = P(A)$

B has no effect

A - V.K.

B - P.W.Q

$P(A/B) = P(A)$

if A and B are independent

$P(A \cap B) = P(A) * P(B)$

1 → 0.5.

2 → 0.5.

$P(H.H.) = P(1) * P(2)$
$= 0.5 \times 0.5.$
$= 0.25.$

A — V.K. playing

B — V.K. injury per day

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Profession \ Age	10-20	20-30	30-40	
Engineer	5	25	15	45
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100

$$P(\text{Lawyer}/\text{Student}) = \frac{P(\text{Law}/\text{Stud})}{P(\text{Stud})} = \frac{P(A) * P(B)}{P(B)}$$

(A = Law, B = Stud)

$$P(\text{law}/\text{st}) = P(A) = P(\text{lawyers}) = \frac{18}{100}$$

9 hearts | 5 spades | 6 hearts | 2 spades | 4 hearts | 7 hearts

$P(A/B) = P(A)$

- $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

$P(A \cap B) = \frac{2}{6}$.

$P(A/B) = \frac{2/6}{4/6} = \frac{2}{4} = \frac{1}{2}$

(A) = even numbered.

(B) = heart.

$P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B) = 4/6 = \frac{2}{3}$.

$P(B/A) = P(B)$

$P(B/A) = \frac{P(B \cap A)}{P(A)}$

$P(A \cap B) = P(B \cap A)$

$P(B \cap A) = 2/6$; $P(A) = 1/2$.

$= \frac{2/6}{1/2} = \frac{2}{6} \times \frac{2}{1} = \frac{4}{6} = \frac{2}{3}$.

$P(B/A) = \frac{2}{3}$.

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8 coins, 3 - unfair, 5 - fair.

unfair	fair
$P(H) = 0.6$	$P(H) = 0.5$
$P(T) = 0.4$	$P(T) = 0.5$.

0 , 1 , 2.

$P(2 \text{ heads}) = P(2 \text{ heads}, \text{fair coin}) + P(2 \text{ H}, \text{unfair coin})$

Case1 : Fair coin

$P(H) = 0.5.$ $\Rightarrow P(H) * P(H) = 0.5 \times 0.5.$

$P(HH / \text{fair coin}) = \frac{5}{8} \times 0.5 \times 0.5$

$= 0.15625.$

Case 2: Unfair coin

$P(H) = 0.6$ $P(H) * P(H) = 0.6 * 0.6.$

$P(HH / \text{unfair coin}) = \frac{3}{8} \times 0.6 \times 0.6$

$= 0.135$

$P(2 \text{ heads}) = 0.15625 + 0.135.$

Profession	Age. 10-20	20-30	30-40.	
Engineer.	5	25	15	45
Lawyer	0	4	14	18
Student	30	5	2	37
	35	34	31	100.


0
1
2
3
4
5
6

H
T

H
T