Discrete Random Variable

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Introduction

What is a Random Variable?

A random variable (or stochastic variable) is a function which assigns a number to each point of a sample space. It is usually denoted by capital letters such as X and Y.

Mathematically,

$$X : \Omega \to \mathbb{R}$$

The set of values x of random variable X such that $P(X = x) > 0$ is called the 'support' of X.

Types of Random Variable

- Discrete Random Variable - It takes a finite or countably infinite number of values.
- Nondiscrete Random Variable - It can take a noncountably infinite number of values.

Example of Discrete Random Variable

Consider an experiment where we toss a fair coin twice. The sample space consists of four possible outcomes: $S = \{HH, HT, TH, TT\}$. Here are some random variables on this space:

1. Let X be the number of Heads. Then,

$$X(HH) = 2, \quad X(HT) = 1, \quad X(TH) = 1, \quad X(TT) = 0$$

2. Let Y be the number of Tails. In terms of X, $Y = 2 - X$. Or

$$Y(HH) = 0, \quad Y(HT) = 1, \quad Y(TH) = 1, \quad Y(TT) = 2$$

3. Let I be 1 if the first toss lands Heads and 0 otherwise.

$$I(HH) = 1, \quad I(HT) = 1, \quad I(TH) = 0, \quad I(TT) = 0$$

This is an example of what is called an indicator random variable since it indicates whether the first toss lands Heads, using 1 to mean “yes” and 0 to mean “no”.

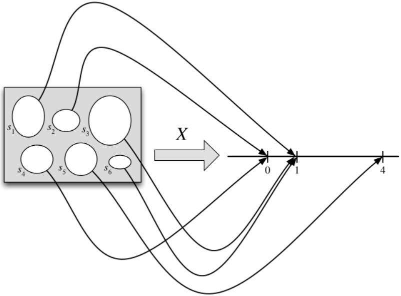

Fig.: A random variable maps the sample space into the real line

Distribution of Random Variable

The distribution of a random variable specifies the probabilities of all events associated with that random variable. For discrete random variables, we generally use the probability mass function (PMF).

The probability mass function (PMF) of a discrete random variable X is the function p_X given by:

$$p_X(x) = P(X = x)$$

Note that this is positive if x is in the support of X, and 0 otherwise.

Here, $X = x$ denotes an event, consisting of all outcomes s to which X assigns the number x. Formally,

$$X = x \equiv s \in S : X(s) = x$$

Example of PMF

Using the previous example of two coin tosses, let us find PMFs of random variables X, Y, and I.

- For X:

$$p_X(0) = P(X = 0) = \frac{1}{4}, \quad p_X(1) = P(X = 1) = \frac{1}{2},$$

$$p_X(2) = P(X = 2) = \frac{1}{4}$$

  and $p_X(x) = 0$ for all other values of x.

- For Y:

$$p_Y(0) = P(Y = 0) = \frac{1}{4}, \quad p_Y(1) = P(Y = 1) = \frac{1}{2},$$

$$p_Y(2) = P(Y = 2) = \frac{1}{4}$$

  and $p_Y(y) = 0$ for all other values of y.

- For I:

$$p_I(0) = P(I = 0) = \frac{1}{2}, \quad p_I(1) = P(I = 1) = \frac{1}{2}$$

and $p_I(i) = 0$ for all other values of i.

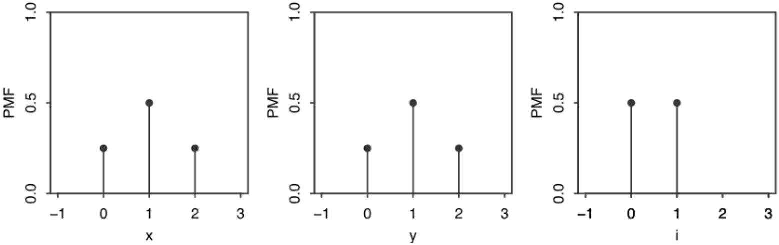

Fig.: Plots of PMF for random variables X, Y and I

Valid PMFs

Let X be a discrete random variable with support $x_1, x_2, \ldots$. The PMF p_X of X must satisfy the following two criteria:

1. Nonnegative: $p_X(x) > 0$ if $x = x_j$ for some j, and $p_X(x) = 0$ otherwise.
2. Sums to 1:

$$\sum_{j=1}^{\infty} p_X(x_j) = 1$$

Bernoulli Distribution

There are only two possible outcomes of an experiment which we often denote by random variable X which maps to only 0 and 1. By convention, probability of success is

$$P(X = 1) = p$$

and of failure is

$$P(X = 0) = 1 - p = q$$

. Here, $0 \leq p, q \leq 1$ and q is probability of failure. Also $p + q = 1$.

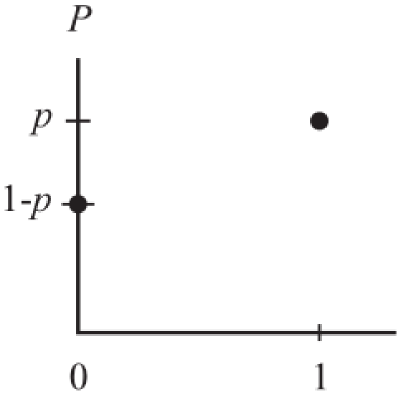

Fig.: Plot of P vs X for Bernoulli

References

1. Blitzstein, Joseph K., and Hwang, Jessica. *Introduction to Probability*. 2nd ed., Chapman and Hall/CRC, 2019.
2. Walrand, Jean. *Probability in Electrical Engineering and Computer Science: An Application-Driven Course*. Springer, 2021.