1 C - curve

Given

1. coordinate of inital point A (x, y)
2. length of inital line $= len$
3. angle α x-axis with initial line
4. order n of the C curve

Now if $n = 0$ we draw line segment from point A (x, y) of length len at angle α with x - axis. That's our 0^{th} degree C curve.
Otherwise if $n > 0$ then we draw lines of equal length from each endpoints of line segment of $(n-1)^{th}$ degree C curve making angle $\frac{\pi}{2}$.

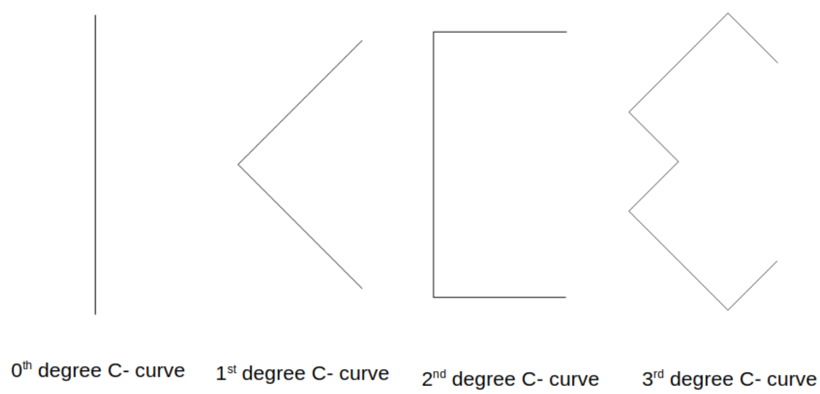

Figure 1: Various C curves

1.1 Derivations

We have an $\triangle ABC$ whose side AB make α with x-axis and $AC = BC$. We make following claims and then prove it.

i. $\angle CAM = \angle CBM = \frac{\pi}{4}$

ii. $AC = BC = \frac{len}{\sqrt{2}}$

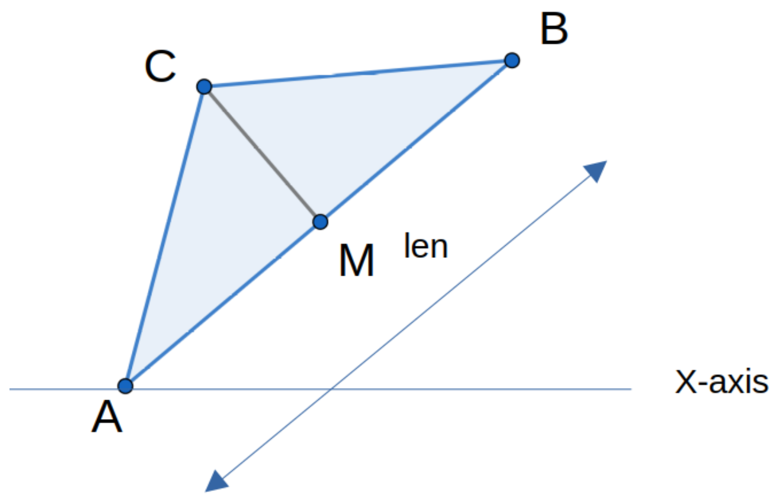

Figure 2: For proving elementry properties

For i.
We join line from C to M.
$\triangle ABC$ is an isosceles as $AC = BC$.
$\Rightarrow \angle CAM = \angle CBM$
Also, in $\triangle ABC$

$$\begin{aligned}
\angle CAM + \angle CBM + \angle ACB &= \pi \\
2\angle CAM + \angle ACB &= \pi \\
2\angle CAM &= \frac{\pi}{2} \left[\text{Since } \angle ACB = \frac{\pi}{2} \right] \\
\angle CAM &= \frac{\pi}{4} = \angle CBM
\end{aligned}$$

i. is proved.
For ii.
We draw a line from C to mid point M of AB.
Consider $\triangle CAM$ and $\triangle CBM$.
Clearly,

$$\begin{aligned}
CA &= CB \\
\angle CAM &= \angle CBM \\
AM &= BM
\end{aligned}$$

By SAS criterion,

$$\triangle CAM \cong \triangle CBM$$
$$\Rightarrow \angle ACM = \angle BCM$$

But,

$$\begin{aligned}
\angle ACB &= \frac{\pi}{2} \\
\angle ACM + \angle BCM &= \frac{\pi}{2} \\
2\angle ACM &= \frac{\pi}{2} \\
\angle ACM &= \frac{\pi}{4} = \angle BCM
\end{aligned}$$

So,
$\triangle ACM$ and $\triangle BCM$ are also isosceles.
$\Rightarrow AM = MC = MB = \frac{len}{2}$
Consider $\triangle ACM$,

$$\begin{aligned}
AM^2 + CM^2 &= AC^2 \\
\frac{len}{4}^2 + \frac{len}{4}^2 &= AC^2 \\
AC &= \frac{len}{\sqrt{2}} = BC
\end{aligned}$$

ii. is also proved.
Now, we make few more claims and then prove them.

iii. $B(a,b) \equiv (x + len\cos\alpha, y + len\sin\alpha)$

iv. $C(x', y') \equiv \left(x + \frac{len}{\sqrt{2}}\cos(\alpha + \frac{\pi}{4}), y + \frac{len}{\sqrt{2}}\sin(\alpha + \frac{\pi}{4})\right)$

v. $B(a,b) \equiv \left(x' + \frac{len}{\sqrt{2}}\cos(\alpha - \frac{\pi}{4}), y' + \frac{len}{\sqrt{2}}\sin(\alpha - \frac{\pi}{4})\right)$

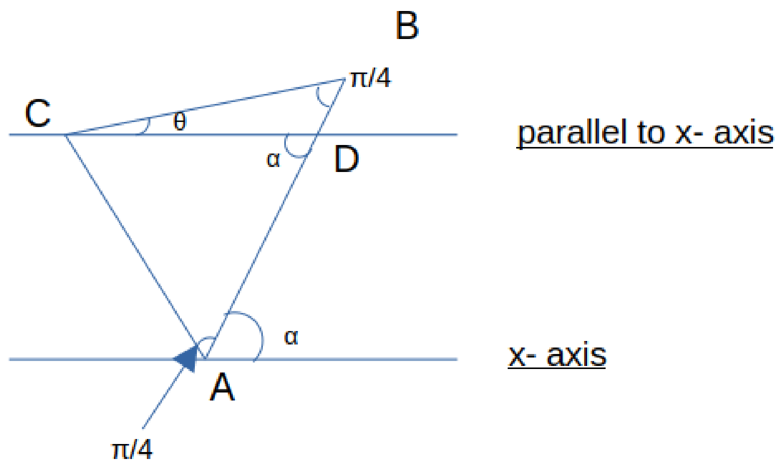

Figure 3: For proving core properties

For iii. We have,

$$\cos\alpha = \frac{a-x}{len}$$
$$a = x + len\cos\alpha$$

Similarly,

$$\sin\alpha = \frac{b-y}{len}$$
$$b = y + len\sin\alpha$$

Thus,

$$(a,b) \equiv (x + len\cos\alpha, y + len\sin\alpha)$$

For iv. We can clearly see that angle made by AC with x-axis is $\frac{\pi}{4}$. By same procedure as above, we arrive at

$$\cos\left(\alpha + \frac{\pi}{4}\right) = \frac{x' - x}{\frac{len}{\sqrt{2}}}$$
$$x' = x + \frac{len}{\sqrt{2}}\cos\left(\alpha + \frac{\pi}{4})\right)$$

And,

$$\sin\left(\alpha + \frac{\pi}{4}\right) = \frac{y' - y}{\frac{len}{\sqrt{2}}}$$
$$y' = y + \frac{len}{\sqrt{2}} \sin\left(\alpha + \frac{\pi}{4}\right)$$

Thus,

$$(x', y') \equiv \left(x + \frac{len}{\sqrt{2}} \cos(\alpha + \frac{\pi}{4}), y + \frac{len}{\sqrt{2}} \sin(\alpha + \frac{\pi}{4})\right)$$

For v. We will only show that $\theta = \alpha - \frac{\pi}{4}$ which is quite clear from fig. 3 and rest of the procedure is left to the reader as it is similar.

$$\angle CDA = \alpha \, [\text{ Since, CD is parallel to x-axis}]$$

So,

$$\angle BCD + \angle CBD + \angle CDB = \pi$$
$$\theta + \frac{\pi}{4} + (\pi - \alpha) = \pi$$
$$\theta = \alpha - \frac{\pi}{4}$$

By ditto procedure, we get

$$(a, b) \equiv \left(x' + \frac{len}{\sqrt{2}} \cos(\alpha - \frac{\pi}{4}), y' + \frac{len}{\sqrt{2}} \sin(\alpha - \frac{\pi}{4})\right)$$

1.2 Pseudocode

C-curve $(x \, y \, len \, \alpha \, n)$

1. if $n = 0$
   (a) draw line between (x, y) and $(x + len \cos\alpha, y + len \sin\alpha)$ i.e from A to B
2. if $n > 0$
   (a) C-curve $(x \, y \, \frac{len}{\sqrt{2}} \, (\alpha + \frac{\pi}{4}) \, (n-1))$ i.e. draw C curve from A toward C, reducing degree by 1

(b) Compute $C(x', y') \equiv \left(x + \frac{len}{\sqrt{2}}\cos(\alpha + \frac{\pi}{4}), y + \frac{len}{\sqrt{2}}\sin(\alpha + \frac{\pi}{4})\right)$

(c) C-curve $(x'\, y'\, \frac{len}{\sqrt{2}}\, (\alpha - \frac{\pi}{4})\, (n-1))$ i.e. draw C curve from C toward B reducing degree by 1

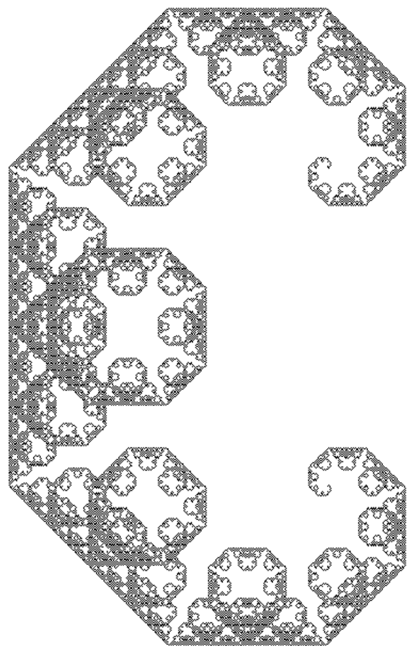

Figure 4: 15^{th} degree C curve