Handout 4G1.I) Brownian particle in a harmonic potential

$$
\begin{aligned}
0 &= -\zeta\dot{r} - kr + fr \\
0 &= -\zeta\dot{r}\cdot r - kr\cdot r + fr\cdot r \\
0 &= -\zeta\langle\dot{r}r\rangle - k\langle r^2\rangle + \langle r\rangle\langle fr\rangle \\
\langle fr\rangle = 0 \implies 0 &= -\zeta\langle\dot{r}r\rangle - k\langle r^2\rangle \\
\langle\dot{r}r\rangle = \left\langle\frac{1}{2}\cdot\frac{dr^2}{dt}\right\rangle \implies 0 &= -\zeta\left\langle\frac{1}{2}\cdot\frac{dr^2}{dt}\right\rangle - k\langle r^2\rangle \\
0 &= -\frac{\zeta}{2}\left\langle\frac{dr^2}{dt}\right\rangle - k\langle r^2\rangle \\
0 &= -\frac{\zeta}{2}\cdot\frac{d\langle r^2\rangle}{dt} - k\langle r^2\rangle \\
Y = \langle r^2\rangle \implies 0 &= -\frac{\zeta}{2}\cdot\dot{Y} - kY \\
Y &= -\frac{\zeta}{2k}\dot{Y} \\
Y &= e^{-\frac{\zeta}{2k}t}\cdot C \\
\langle r^2\rangle &= e^{-\frac{\zeta}{2k}t}\cdot C
\end{aligned}
$$

Because $t = r = 0$, by necessity, $C = 0$. This means the particle does not move, which seems to make sense, as at any given time, the force on the particle is always net 0.