Quantum Computing

Guide:

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Entangled State

The entanglement between n qubits means that the state of the system cannot be specified in terms of the state of each of the n qubits individually.

Simple 2 qubit entanglement pairs(EPR) have a few identified applications in quantum computing including Superdense coding,Quantum cryptography,Quantum teleportation.

Schmidt Decomposition

It can be defined as the normal form for any bipartite pure state, which allows us to easily determine if the state is entangled or not. This is called Schmidt decomposition.

Also referred to as the fundamental tool when working with the entangled state.

Theorem:

Let H1 and H2 be the Hilbert space of dimensions n,m respectively. Assume n>=m. For any vector ω in the tensor product H1⊗H2 there exists orthonormal sets { v1,v2,......,vn} ⊂ H1 and { w1,w2,......,wn} ⊂ H2 such that

$\omega = \sum^{d}_{i=1} \mu_i v_i \otimes w_i$ where the scalars μ_i are real,positive and unique.

d = degree of entanglement.

Proof:

Consider the density operator, $\rho v = tr_w(|\Psi><\Psi|)$
ρv is self adjoint (i.e., they are diagonalizable in eigen values)non negative with eigen values $\lambda_1,\lambda_2.....\lambda_n$

Let { v1,v2,......,vn} be the orthonormal basis of eigenvector of ρv
$= \Psi = v1\otimes w1.....+ vn\otimes wn$
$= \rho v = tr_w(|\Psi><\Psi|) = tr_w(\Sigma^m_{i=1} \Sigma^n_{j=1}|vi>|wi>+<vj|<wj|)$
$= \Sigma^n_{j,i=1}|vi><vj|+<wj|wi>)$

We know,
ρv is diagonalizable in these basis i.e.,
$= \rho v = \lambda_1|v1><v1| +......+\lambda_n|vn><vn|$

If $i \neq j$, the scalar product $<wj|wi> = 0$
If $i=j$, $<wi|wi> = \lambda_i$

Assume that $\lambda_1, \lambda_2 ..... \lambda_d > 0$ and $\lambda_{d+1}, \lambda_{d+2} ..... \lambda_n = 0$

For i>d, <wi|wi> = 0, wi = 0

Therefore,

= Ψ = v1⊗w1 + …….+vn⊗wn = v1⊗w1 + ….. + vd⊗wd

For i = 1, …,d

= wi=$(1/\sqrt{\lambda_i})$wi , such that $\langle \bar{w}i | \bar{w}i \rangle = 1$

So Ψ = v1⊗w1 + …….+vd⊗wd can be written as:

= $\sqrt{\lambda_i}$ v1⊗$\bar{w}$1 + ……+$\sqrt{\lambda_i}$ vd⊗$\bar{w}$d

=$\sqrt{\lambda_i} = \mu_i > 0$ ===> Schmidt decomposition

Corollary:

For $\Psi \in v \otimes w$, ρv. ρw have the same set of non- zero eigen values.

= d = 1, unentangled state

= d>1, entangled state

Superdense Coding

Superdense coding is a quantum communications protocol that allows a user to send 2 classical bits by sending only 1 qubit

Superdense coding can:

- Allow the user to send ahead of time half of what will be needed to reconstruct a classical message ahead of time,which let's the user transmit at double speed until the pre-delivered qubits run out.
- Convert high latency bandwidth into low latency bandwidth by sending half of the information over the high latency channel to support the information coming over the low latency channel
- Double classical capacity in one direction of a 2-way quantum channel (eg: converting a 2-way quantum channel with bandwidth B (in both directions) into a 1-way classical channel with bandwidth 2B)

Protocols:

- **Step 1** : **Preparing the Bell pair**

  First a bell pair consisting of two qubits is prepared .Where q0 is the senders qubit and q1 is the receivers qubit.To do this q0 is put into a superposition of states using a Hadamard gate. Then a CNOT operation is performed with q0 being the control and q1 being the target.

- **Step 2 : Encode the information on to q0**

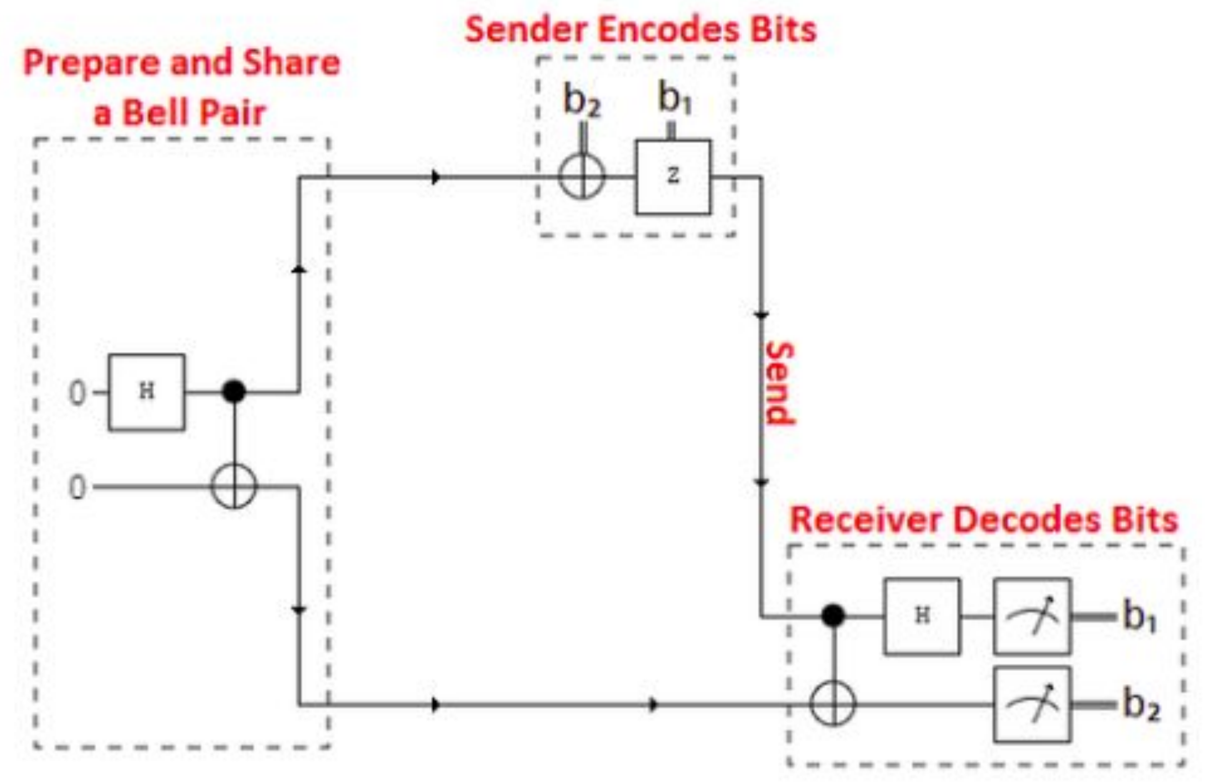
  Next the sender has to encode the information they want to send on to q0 by applying certain operations to it.

  1. If they want to send 00 then they perform no operation
  2. If they want to send 01 then they perform a Pauli-Z operation where q1 's state is flipped.
  3. If they want to send 10 then they apply a Pauli-X gate.
  4. If they want to send 11 then apply a Pauli-Z gate followed by a Pauli-X gate

Protocols:

- **Step 3: Receiver decodes the information**

  Next q0 is send and the receiver has to decode the qubit.This is done by applying a CNOT where the received q0 is the control and q1 is the target.Then they Hadamard gate is applied to q0.



Circuit diagram showing the Superdense coding protocol

Quantum Teleportation

- Quantum teleportation is a technique for transferring quantum information from a sender at one location to a receiver some distance away .
- Quantum teleportation only transfers quantum information .The sender does not have to know the particular quantum state being transferred.Moreover the location of the recipient can be unknown,but to complete the quantum teleportation,classical information needs to be sent from sender to receiver.Because classical information needs to be sent from the sender to receiver .Because the classical information needs to be sent ,quantum teleportation cannot occur faster than speed of light.
- Experimental determinations of quantum teleportation have been made in information content - including photons,atoms,electrons and superconducting circuits - as well as distance with 1400km being the longest distance of successful teleportation by the group of Jian-Wei Pan using the Micius satellite for space-based quantum teleportation.

Protocols:

The resources required for quantum teleportation are a communication channel capable of transmitting two classical bit, a means of generating an entangled Bell state of qubits and distributing two different locations,performing a Bell measurements on one of the Bell state qubits,and manipulating the quantum state of the other qubit from the pair.There must also be some input qubit to be teleported.

The protocol is then as follows:

- A Bell state is generated with one qubit sent to location A and the other sent to location B
- A Bell measurement of the Bell state qubit and the qubit to be teleported ($|\phi>$) is performed at location A.This yields one of four measurement outcomes which can be encoded in two classical bits of information.Both qubits at location A are then discarded.
- Using the classical channel, the two bits are sent from A to B (This is the only potentially time-consuming step after step 1 since information transfer is limited by the speed of light.)

- As a result of the measurement performed at location A, the Bell state qubit at location B is in one of four possible states. Of these four possible state, one is identical to the original quantum state ($|\phi>$),and the other three are closely related. The identity of the state actually obtained is encoded in two classical bits and sent to location B.The Bell state qubit at location B is then modified in one of three ways, or not at all, which results in a qubit identical to $|\phi>$, the state of the qubit that was chosen for teleportation.

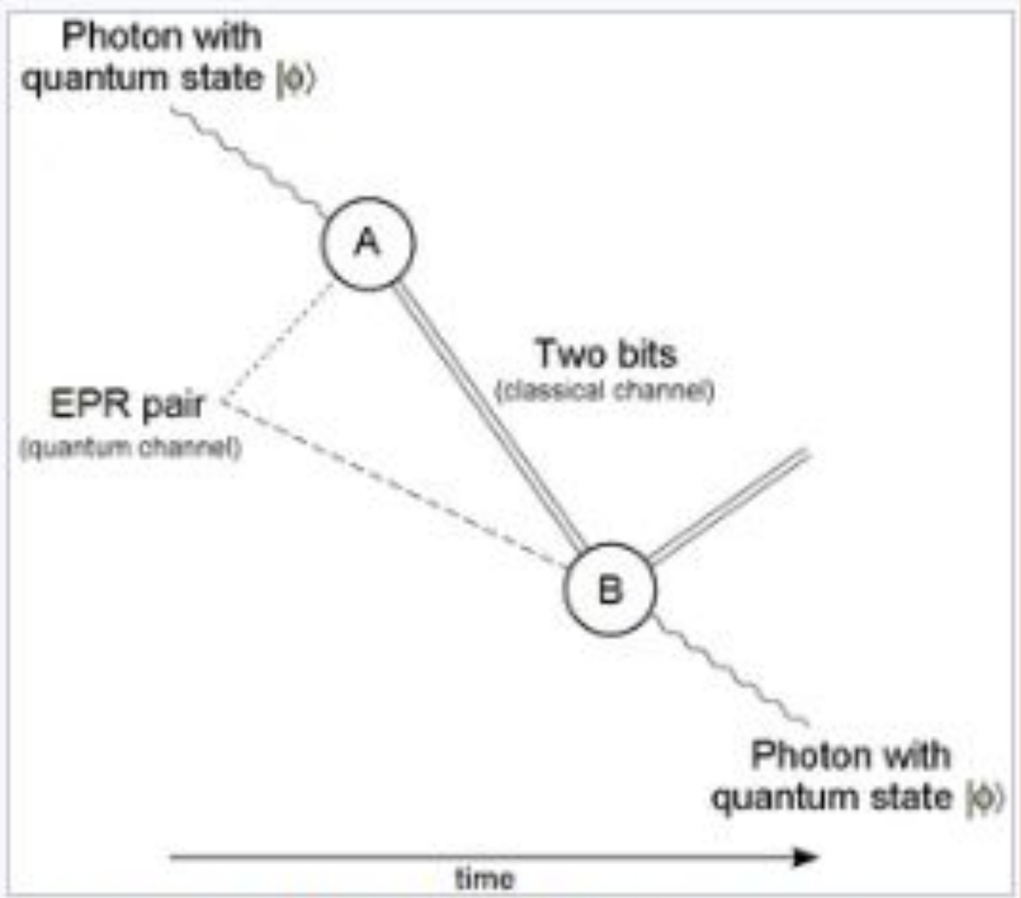


Diagram for quantum teleportation of a photon

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Thank you..