Final Project

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Final Project for the course ENPM 667 Control of Robotics Systems
Masters of Engineering Robotics

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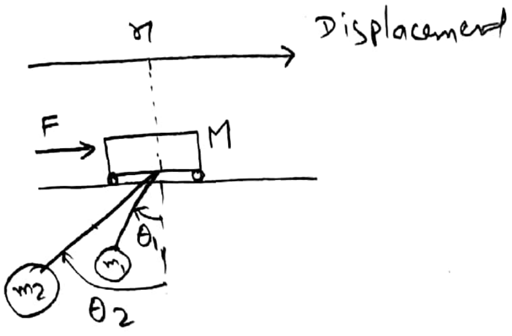
ENPM 667 Final Project

First component

Solution (A)



Using Lagrangian method, we have lagrangian equation as

$$\frac{d}{dt}\left(\frac{\partial La}{\partial \dot{q}_i}\right) - \frac{\partial La}{\partial q_i} = T_i$$

$La = K - P$

q_i = generalized coordinate

$= (q_1 = x)\ (q_2 = \theta_1)\ (q_3 = \theta_2)$

T_i = External force

System kinetic energy = $\frac{1}{2} M \dot{x}^2 + \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$$v_1^2 = v_{x_1}^2 + v_{y_1}^2$$

$$v_2^2 = v_{x_2}^2 + v_{y_2}^2$$

$$v_{x_1} = \dot{x} - l_1 \dot{\theta}_1 \cos\theta_1$$

$$v_{y_1} = l_1 \dot{\theta}_1 \sin\theta_1$$

$$v_{x_2} = \dot{x} - l_2 \dot{\theta}_2 \cos\theta_2$$

$$v_{y_2} = l_2 \dot{\theta}_2 \sin\theta_2$$

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Potential energy, $P = m_1 g l_1 (1-\cos\theta_1) + m_2 g l_2 (1-\cos\theta_2)$

Now, $L_a = K - P$

$$= \frac{1}{2}[M\dot{x}^2 + m_1 v_1^2 + m_2 v_2^2] - m_1 g l_1 (1-\cos\theta_1) - m_2 g l_2 (1-\cos\theta_2)$$

Differentiating L_a with respect to x, we get

$$\frac{\partial L_a}{\partial x} = 0$$

Differentiating L_a w.r.t $\dot{x}$

$$\frac{\partial L_a}{\partial \dot{x}} = M\dot{x} + \frac{\partial}{\partial \dot{x}}\left\{\frac{1}{2} m_1 (v_{x_1}^2 + v_{y_1}^2) + \frac{1}{2} m_2 (v_{x_2}^2 + v_{y_2}^2)\right\}$$

$$\Rightarrow M\dot{x} + \frac{1}{2}[m_1 2\dot{x} - m_1 2 l_1 \dot{\theta}_1 \cos\theta_1] + \frac{1}{2}[m_2 2\dot{x} - m_2 2 l_2 \dot{\theta}_2 \cos\theta_2]$$

$$\Rightarrow \dot{x}(M + m_1 + m_2) - m_1 l_1 \dot{\theta}_1 \cos\theta_1 - m_2 l_2 \dot{\theta}_2 \cos\theta_2$$

Differentiating $\frac{\partial L_a}{\partial \dot{x}}$ w.r.t t

$$\frac{d}{dt}\left(\frac{\partial L_a}{\partial \dot{x}}\right) = \ddot{x}(M + m_1 + m_2) - [m_1 l_1 \ddot{\theta}_1 \cos\theta_1 - m_1 l_1 \dot{\theta}_1^2 \sin\theta_1 + m_2 l_2 \ddot{\theta}_2 \cos\theta_2 - m_2 l_2 \dot{\theta}_2^2 \sin\theta_2]$$

So, equation of motion with respect to x is

$$\frac{d}{dt}\left(\frac{\partial L_a}{\partial \dot{x}}\right) - \frac{\partial L_a}{\partial x} = F$$

$$\Rightarrow \boxed{\ddot{x}(M + m_1 + m_2) + m_1 l_1 (\dot{\theta}_1^2 \sin\theta_1 - \ddot{\theta}_1 \cos\theta_1) + m_2 l_2 (\dot{\theta}_2^2 \sin\theta_2 - \ddot{\theta}_2 \cos\theta_2) = F}$$

$- (1)$

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consider variable θ_1

$$\frac{\partial L_a}{\partial \theta_1} = \frac{\partial}{\partial \theta_1}\left\{ \frac{1}{2}(M\dot{x}^2 + m_1 v_1^2 + m_2 v_2^2) - m_1 g l_1(1-\cos\theta_1) - m_2 g l_2(1-\cos\theta_2) \right\}$$

$$= \frac{\partial}{\partial \theta_1}\left\{ \frac{1}{2} m_1 (\dot{x}^2 + l_1^2 \dot{\theta}_1^2 \cos^2\theta_1 - 2\dot{x} l_1 \dot{\theta}_1 \cos\theta_1 + l_1^2 \dot{\theta}_1^2 \sin^2\theta_1) + m_1 g l_1 \cos\theta_1 \right\}$$

$$= \frac{1}{2} m_1 \left[-2\cos\theta_1 l_1^2 \dot{\theta}_1^2 \sin\theta_1 + 2\dot{x} l_1 \dot{\theta}_1 \sin\theta_1 + l_1^2 \dot{\theta}_1^2 2\sin\theta_1 \cos\theta_1 - g l_1 \sin\theta_1 \right]$$

$$\Rightarrow m_1\{\dot{x} l_1 \dot{\theta}_1 \sin\theta_1 - g l_1 \sin\theta_1\}$$

$$\frac{\partial L_a}{\partial \theta_1} \Rightarrow m_1 \sin\theta_1 (\dot{x} \dot{\theta}_1 l_1 - g l_1)$$

Now, Differentiating with respect to $\dot{\theta}_1$

$$\frac{\partial L_a}{\partial \dot{\theta}_1} = \frac{\partial}{\partial \dot{\theta}_1}\left[\frac{1}{2} m_1 \{ \dot{x}^2 + l_1^2 \dot{\theta}_1^2 \cos^2\theta_1 - 2\dot{x} l_1 \dot{\theta}_1 \cos\theta_1 + l_1^2 \dot{\theta}_1^2 \sin^2\theta_1 \} \right]$$

$$= \frac{1}{2} m_1 \left[2\dot{\theta}_1 l_1^2 \cos^2\theta_1 - 2\dot{x} l_1 \cos\theta_1 + 2\dot{\theta}_1 l_1^2 \sin^2\theta_1 \right]$$

$$\left(\frac{\partial L_a}{\partial \dot{\theta}_1}\right) \Rightarrow m_1 (\dot{\theta}_1 l_1^2 - \dot{x} l_1 \cos\theta_1)$$

Differentiating $\left(\frac{\partial L_a}{\partial \dot{\theta}_1}\right)$ with respect to t, we get

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$$\frac{d}{dt}\left(\frac{\partial L_a}{\partial \dot{\theta}_1}\right) = m_1\left[\ddot{\theta}_1 l_1^2 + \dot{\theta}_1 \dot{x} l_1 \sin\theta_1 - \ddot{x} l_1 \cos\theta_1\right]$$

so, Equation of motion with respect to θ_1 is

$$\frac{d}{dt}\left(\frac{\partial L_a}{\partial \dot{\theta}_1}\right) - \frac{\partial L_a}{\partial x} = 0$$

$$\Rightarrow \quad \ddot{\theta}_1 l_1^2 + \dot{\theta}_1 \dot{x} l_1 \sin\theta_1 - \ddot{x} l_1 \cos\theta_1 - \dot{x}\dot{\theta}_1 l_1 \sin\theta_1 + g l_1 \sin\theta_1 = 0$$

$$\Rightarrow \quad \boxed{\ddot{\theta}_1 l_1 - \ddot{x}\cos\theta_1 + g\sin\theta_1 = 0} \quad -(2)$$

For variable θ_2, similar calculation as for θ_1, will give the equation of motion as:

$$\boxed{\ddot{\theta}_2 l_2 - \ddot{x}\cos\theta_2 + g\sin\theta_2 = 0} \quad -(3)$$

so, equation (1), (2) & (3) are the required equation of motion for the system.

Non-Linear state space

From equation 2 & 3 we have

$$\ddot{\theta}_1 = \frac{\ddot{x}\cos\theta_1 - g\sin\theta_1}{l_1} \quad — (5)$$

and

$$\ddot{\theta}_2 = \frac{\ddot{x}\cos\theta_2 - g\sin\theta_2}{l_2} \quad — (6)$$

Putting these values of $\ddot{\theta}_1$ and $\ddot{\theta}_2$ in eqn 1, we have

$$\Rightarrow \ddot{x}(M+m_1+m_2) + m_1 l_1 \dot{\theta}_1^2 \sin\theta_1 - m_1 l_1 \cos\theta_1 \left(\frac{\ddot{x}\cos\theta_1 - g\sin\theta_1}{l_1} \right)$$
$$+ m_2 l_2 \dot{\theta}_2^2 \sin\theta_2 - m_2 l_2 \cos\theta_2 \left(\frac{\ddot{x}\cos\theta_2 - g\sin\theta_2}{l} \right) = F$$

on simplification, we get

$$\Rightarrow \ddot{x}\{M + m_1(1-\cos^2\theta_1) + m_2(1-\cos^2\theta_2)\} = F - m_1 l_1 \dot{\theta}_1^2 \sin\theta_1$$
$$- m_1 g \sin\theta_1 \cos\theta_1 - m_2 l_2 \dot{\theta}_2^2 \sin\theta_2$$
$$- m_2 g \sin\theta_2 \cos\theta_2$$

or,

$$\boxed{\ddot{x} = \frac{F}{(M+m_1\sin^2\theta_1+m_2\sin^2\theta_2)} - \frac{(m_1 l_1 \dot{\theta}_1^2 \sin\theta_1 + m_1 g\sin\theta_1\cos\theta_1 + m_2 l_2 \dot{\theta}_2^2 \sin\theta_2 + m_2 g \sin\theta_2 \cos\theta_2)}{(M+m_1\sin^2\theta_1+m_2\sin^2\theta_2)}} \quad — (7)$$

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Putting the value of $\ddot{x}$ from eqn (7) in (4) & (5) respectively (6)
we get $\ddot{\theta}_1$ & $\ddot{\theta}_2$ as

$$\ddot{\theta}_1 = \frac{F \cdot \cos\theta_1}{(M+m_1\sin^2\theta_1+m_2\sin^2\theta_2)\cdot l_1} - \frac{\left[(M+m_1)g\sin\theta_1 + m_1 l_1 \dot{\theta}_1^2 \sin\theta_1\cos\theta_1 + m_2 l_2 \dot{\theta}_2^2 \sin\theta_2\cos\theta_1 + m_2 g\sin\theta_2\cos(\theta_1-\theta_2)\right]}{(M+m_1\sin^2\theta_1+m_2\sin^2\theta_2)\cdot l_1} \quad - (8)$$

and

$$\ddot{\theta}_2 = \frac{F\cos\theta_2}{(M+m_1\sin^2\theta_1+m_2\sin^2\theta_2) l_2} - \frac{\left\{ m_1 l_1 \dot{\theta}_1^2 \sin\theta_1\cos\theta_2 + m_1 g\sin\theta_1\cos(\theta_1-\theta_2) + (M+m_2)g\sin\theta_2 + m_2 l_2 \dot{\theta}_2^2 \sin\theta_2\cos\theta_2 \right\}}{(M+m_1\sin^2\theta_1+m_2\sin^2\theta_2)\cdot l_2} \quad - (9)$$

we chose state variable as:

$$x = [x_1, x_2, x_3, x_4, x_5, x_6]^T ;$$

$$x_1 = x$$
$$x_2 = \dot{x}$$
$$x_3 = \theta_1$$
$$x_4 = \dot{\theta}_1$$
$$x_5 = \theta_2$$
$$x_6 = \dot{\theta}_2$$

Nonlinear state equation is of the form

$$\dot{\vec{x}}(t) = f(x(t), \vec{u}(t))$$

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this gives:

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{-\{m_1 l_1 x_4^2 \sin x_3 + m_1 g \sin x_3 \cos x_3 + m_2 l_2 x_6^2 \sin x_5 + m_2 g \sin x_5 \cos x_5\}}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)} + \frac{F}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)}$$

$$\dot{x}_3 = x_4$$

$$\dot{x}_4 = \frac{-\{(M+m_1) g \sin x_3 + m_1 l_1 x_4^2 \sin x_3 \cos x_3 + m_2 l_2 x_6^2 \sin x_5 \cos x_3 + m_2 g \sin x_5 \cos(x_3 - x_5)\}}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)\cdot l_1} + \frac{F}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)\cdot l_1}$$

$$\dot{x}_5 = x_6$$

$$\dot{x}_6 = \frac{-\{m_1 l_1 x_4^2 \sin x_3 \cos x_5 + m_1 g \sin x_3 \cos(x_3 - x_5) + (M+m_2) g \sin x_5 + m_2 l_2 x_6^2 \sin x_5 \cos x_5\}}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)\cdot l_2} + \frac{F}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)\cdot l_2}$$

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(B) Linearized state space

we have state space equation in non-linear form:
To linearize it and get A_F & B_F Matrix, we use the formula

$$A_F = \nabla_{\vec{x}} \Big| f(\vec{x}(t), \vec{U}(t))$$

$$A_F = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} & \frac{\partial f_1}{\partial x_5} & \frac{\partial f_1}{\partial x_6} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & & & \frac{\partial f_2}{\partial x_n} \\ \frac{\partial f_3}{\partial x_1} & \cdots \\ \frac{\partial f_4}{\partial x_1} & \cdots \\ \frac{\partial f_5}{\partial x_1} \\ \frac{\partial f_6}{\partial x_1} & \frac{\partial f_6}{\partial x_2} & \cdots & & & \frac{\partial f_6}{\partial x_6} \end{bmatrix}\right|_{x=0} \quad - (8)$$

we have, $f_1 = x_2$

so, $\frac{\partial f_1}{\partial x_1} = \frac{\partial f_1}{\partial x_3} = \frac{\partial f_1}{\partial x_4} = \frac{\partial f_1}{\partial x_5} = \frac{\partial f_1}{\partial x_6} = 0$

and $\frac{\partial f_1}{\partial x_2} = 1$

$$f_2 = \dot{x}_2 = \frac{-\{m_1 l_1 x_4^2 \sin x_3 + m_1 g \sin x_3 \cos x_3 + m_2 l_2 x_6^2 \sin x_5 + m_2 g \sin x_5 \cos x_5 + F\}}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)}$$

f_2 has no x_1, x_2 terms so,

$$\frac{\partial f_2}{\partial x_1} = \frac{\partial f_2}{x_2} = 0$$

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$$\left.\frac{\partial f_2}{\partial x_3}\right|_{x=0} = \frac{-\left(m_1 g (\cos(0))^2 M - 0\right)}{M^2} = -\frac{m_1 g}{M}$$ (9)

$$\left.\frac{\partial f_2}{\partial x_4}\right|_{x=0} = 0$$

$$\left.\frac{\partial f_2}{\partial x_5}\right|_{x=0} = \frac{-\left(m_2 g (\cos(0))^2 \cdot M\right)}{M^2} = -\frac{m_2 g}{M}$$

$$\left.\frac{\partial f_2}{\partial x_6}\right|_{x=0} = 0$$

$$f_3 = x_4$$

so, $\frac{\partial f_3}{\partial x_1} = \frac{\partial f_3}{x_2} = \frac{\partial f_3}{\partial x_3} = \frac{\partial f_3}{\partial x_5} = \frac{\partial f_3}{\partial f_6} = 0$ and $\frac{\partial f_3}{\partial x_4} = 1$

x_4 $$f_4 = \frac{-\{(M+m_1)g\sin x_3 + m_1 l_1 x_4^2 \sin x_3 \cos x_3 + m_2 l_2 x_6^2 \sin x_5 \cos x_3 + m_2 g \sin x_5 \cos(x_3 - x_5) + F\}}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5)\cdot l_1}$$

again, $\frac{\partial f_4}{\partial x_1} = \frac{\partial f_4}{\partial x_2} = 0$

$$\left.\frac{\partial f_4}{\partial x_3}\right|_{x=0} = \frac{-(M+m_1)g}{M l_1}$$

$$\left.\frac{\partial f_4}{\partial x_4}\right|_{x=0} = 0$$

$$\left.\frac{\partial f_5}{\partial x_5}\right|_{x=0} = \frac{-(m_2 g)}{M l_1}$$

$$\left.\frac{\partial f_6}{\partial x_6}\right|_{x=0} = 0$$

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$f_5 = x_6$

So, $\frac{\partial f_5}{\partial x_1} = \frac{\partial f_5}{\partial x_2} = \frac{\partial f_5}{\partial x_3} = \frac{\partial f_5}{\partial x_4} = \frac{\partial f_5}{\partial x_5} = 0$

and $\frac{\partial f_5}{\partial x_6} = 1$

$$f_6 = \frac{-\{m_1 l_1 x_4^2 \sin x_3 \cos x_5 + m_1 g \sin x_3 \cos(x_3 - x_5) + (M+m_2) g \sin x_5 + m_2 l_2 x_6^2 \sin x_5 \cos x_5 + F\}}{(M + m_1 \sin^2 x_3 + m_2 \sin^2 x_5) \cdot l_2}$$

$\frac{\partial f_6}{\partial x_1} = \frac{\partial f_6}{x_2} = 0$

$$\left.\frac{\partial f_6}{\partial x_3}\right|_{x=0} = -\frac{\{m_1 g \cos(0) \cos(0) - 0\} M l_2}{M^2 l_2^2}$$

$$= -\left(\frac{m_1 g}{M l_2}\right)$$

$\left.\frac{\partial f_6}{\partial x_4}\right|_{x=0} = 0$

$$\frac{\partial f_6}{\partial x_5} = \frac{-g(M+m_2)}{M l_2}$$

$\frac{\partial f_6}{\partial x_6} = 0$

Putting these values in eqn (8) of A_F

we get A_F as:

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$$A_F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\left(\frac{m_1 g}{M}\right) & 0 & -\left(\frac{m_2 g}{M}\right) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{g(M+m_1)}{Ml_1} & 0 & -\left(\frac{m_2 g}{Ml_1}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\left(\frac{m_1 g}{Ml_2}\right) & 0 & -\frac{g(M+m_2)}{Ml_2} & 0 \end{bmatrix} \quad -(9)$$

$$B_F = \nabla_{\vec{U}} \left| f(\vec{x}(t), \vec{U}(t)) \right|$$

$$B_F = \left[\frac{\partial f_1}{\partial u} \; \frac{\partial f_2}{\partial u} \; \frac{\partial f_3}{\partial u} \; \frac{\partial f_4}{\partial u} \; \frac{\partial f_5}{\partial u} \; \frac{\partial f_6}{\partial u} \right]^T_{\Big| x=0} \qquad \{ u = F \}$$

$$\frac{\partial f_1}{\partial u} = 0 \quad , \quad \left.\frac{\partial f_2}{\partial u}\right|_{x=0} = 1 \times \frac{1}{M+0+0} = \frac{1}{M}$$

$$\frac{\partial f_3}{\partial u} = 0 \quad , \quad \frac{\partial f_4}{\partial u} = \frac{1}{Ml_1} \quad , \quad \frac{\partial f_5}{\partial u} = 0 \quad , \quad \frac{\partial f_6}{\partial u} = \frac{1}{ML_2}$$

this gives

$$B_F = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 1/Ml_1 \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} \quad - (10)$$

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Hence, Linearized state space eqn is

$$\dot{x} = A_F x + B_F u$$

where, $\dot{x} = [\dot{x}_1 \ \dot{x}_2 \ \dot{x}_3 \ \dot{x}_4 \ \dot{x}_5 \ \dot{x}_6]^T$

$x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6]^T$

$u = F$

Value of A_F and B_F as given by eqn (9) & (10)

this is the required state space eqn.

The linearized system in terms of system variables can be represented as:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \\ \dot{\theta}_1 \\ \ddot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\left(\frac{m_1 g}{M}\right) & 0 & -\left(\frac{m_2 g}{M}\right) & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{g(M+m_1)}{M l_1} & 0 & -\left(\frac{m_2 g}{M l_1}\right) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -\left(\frac{m_1 g}{M l_2}\right) & 0 & -\frac{g(M+m_2)}{M l_2} & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \\ \theta_1 \\ \dot{\theta}_1 \\ \theta_2 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 1/M l_1 \\ 0 \\ 1/M l_2 \end{bmatrix} F.$$

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(c) To check the controllability we need to check the Rank of $\begin{bmatrix} B & AB & A^2B & A^3B & A^4B & A^5B \end{bmatrix}$

$$B = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 1/Ml_1 \\ 0 \\ 1/Ml_2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1/M \\ 0 \\ \frac{1}{Ml_1} \\ 0 \\ \frac{1}{Ml_2} \end{bmatrix} , \quad A^2B = \begin{bmatrix} 0 \\ \frac{-g(m_2l_1 + l_2m_1)}{M^2l_1l_2} \\ 0 \\ \frac{-g(Ml_2 + l_1m_2 + l_2m_1)}{M^2l_1^2l_2} \\ 0 \\ \frac{-g(Ml_1 + l_1m_2 + l_2m_1)}{M^2l_1l_2^2} \end{bmatrix}$$

$$A^3B = \begin{bmatrix} \frac{-g(l_1m_2 + l_2m_1)}{M^2l_1l_2} \\ 0 \\ \frac{-g(Ml_2 + l_1m_2 + l_2m_1)}{M^2l_1^2l_2} \\ 0 \\ \frac{-g(Ml_1 + l_1m_2 + l_2m_1)}{M^2l_1l_2^2} \\ 0 \end{bmatrix}$$

$$A^4B = \begin{bmatrix} 0 \\ \dfrac{g^2\left(l_1^2 m_2^2 + M l_1^2 m_2 + 2 l_1 l_2 m_1 m_2 + l_2^2 m_1^2 + M l_2^2 m_1\right)}{M^3 l_1^2 l_2^2} \\ 0 \\ \dfrac{g^2\left(M^2 l_2^2 + M l_1^2 m_2 + M l_1 l_2 m_2 + 2 M l_1^2 m_1 + l_1^2 m_2^2 + 2 l_1 l_2 m_1 m_2 + l_2^2 m_1^2\right)}{M^3 l_1^3 l_2^2} \\ 0 \\ \dfrac{g^2\left(M^2 l_1^2 + 2 M l_1^2 m_2 + M l_1 l_2 m_1 + M l_2^2 m_1 + l_1^2 m_2^2 + 2 l_1 l_2 m_1 m_2 + l_2^2 m_1^2\right)}{M^3 l_1^2 l_2^3} \end{bmatrix}$$

$$A^5B = \begin{bmatrix} \dfrac{g^2\left(l_1^2 m_2^2 + M l_1^2 m_2 + 2 l_1 l_2 m_1 m_2 + l_2^2 m_1^2 + M l_2^2 m_1\right)}{M^3 l_1^2 l_2^2} \\ 0 \\ \dfrac{g^2\left(M^2 l_2^2 + M l_2^2 m_2 + M l_1 l_2 m_2 + 2 M l_2^2 m_1 + l_1^2 m_2^2 + 2 l_1 l_2 m_1 m_2 + l_2^2 m_1^2\right)}{M^3 l_1^3 l_2^2} \\ 0 \\ \dfrac{g^2\left(M^2 l_1^2 + 2 M l_1^2 m_2 + M l_1 l_2 m_1 + M l_2^2 m_1 + l_1^2 m_2^2 + 2 l_1 l_2 m_1 m_2 + l_2^2 m_1^2\right)}{M^3 l_1^2 l_2^3} \end{bmatrix}$$

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this gives . the

$$\mathrm{Det}\left[B \;\; AB \;\; A^2B \;\; A^3B \;\; A^4B \;\; A^5B\right]$$

$$= \frac{-(g^6 \lambda_1^2 - 2g^6\lambda_1\lambda_2 + g^6\lambda_2^2)}{M^6 \lambda_1^6 \lambda_2^6}$$

For, Rank to be 6

Det $\neq 0$ { Non-Singular matrix }

Hence,

$$\frac{-(g^6 \lambda_1^2 - 2g^6\lambda_1\lambda_2 + g^6\lambda_2^2)}{M^6 \lambda_1^6 \lambda_2^6} \neq 0$$

$$\Rightarrow \quad (\lambda_1 - \lambda_2)^2 \neq 0$$

$$\Rightarrow \quad \boxed{\lambda_1 \neq \lambda_2}$$ → This is the required condition for system to be controllable

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(D) Given, $M = 1000 kg$, $m_1 = m_2 = 100 kg$, $l_1 = 20 m$, $l_2 = 10 m$

Putting these values in the matrix obtained in question (c)

i.e $\begin{bmatrix} B & AB & A^2B & A^3B & A^4B & A^5B \end{bmatrix}$, we get

$$[C] = 10^{-3} \times \begin{bmatrix} 0 & 1 & 0 & -0.1470 & 0 & 0.1417 \\ 1 & 0 & -0.147 & 0 & 0.1417 & 0 \\ 0 & 0.05 & 0 & -0.0319 & 0 & 0.0227 \\ 0.05 & 0 & -0.0319 & 0 & 0.0227 & 0 \\ 0 & 0.10 & 0 & -0.1127 & 0 & 0.1246 \\ 0.1 & 0 & -0.1127 & 0 & 0.1246 & 0 \end{bmatrix}$$

$$\mathrm{Det}(C) = -1.3841 \times 10^{-20} \neq 0$$

Since, $\mathrm{Det}(C) \neq 0$, contralibility matrix is non-singullar

→ Hence, System is contrallable

LQR Design

LQR Design Done in Matlab using command lqr:

optimized value of Q and R was:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 100 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 150 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1500 \end{bmatrix}, \quad R = 0.0001$$

→ Eigen value of $(A-BK)$ was found to be on left of the plane, i.e stable feedback.

$$eig(A-BK) = (-0.2108 \pm 1.024i), (-0.203 \pm 0.2026i)$$
and
$$(-0.0612 \pm 0.7246i)$$

with gain Matrix K, obtained from LQR

$$K = 10^3 [0.1 \quad 0.5553 \quad 0.4648 \quad 3.12 \quad 2.39 \quad 2.4276]$$

Natural response to Linearized system

Response was plotted for initial condition

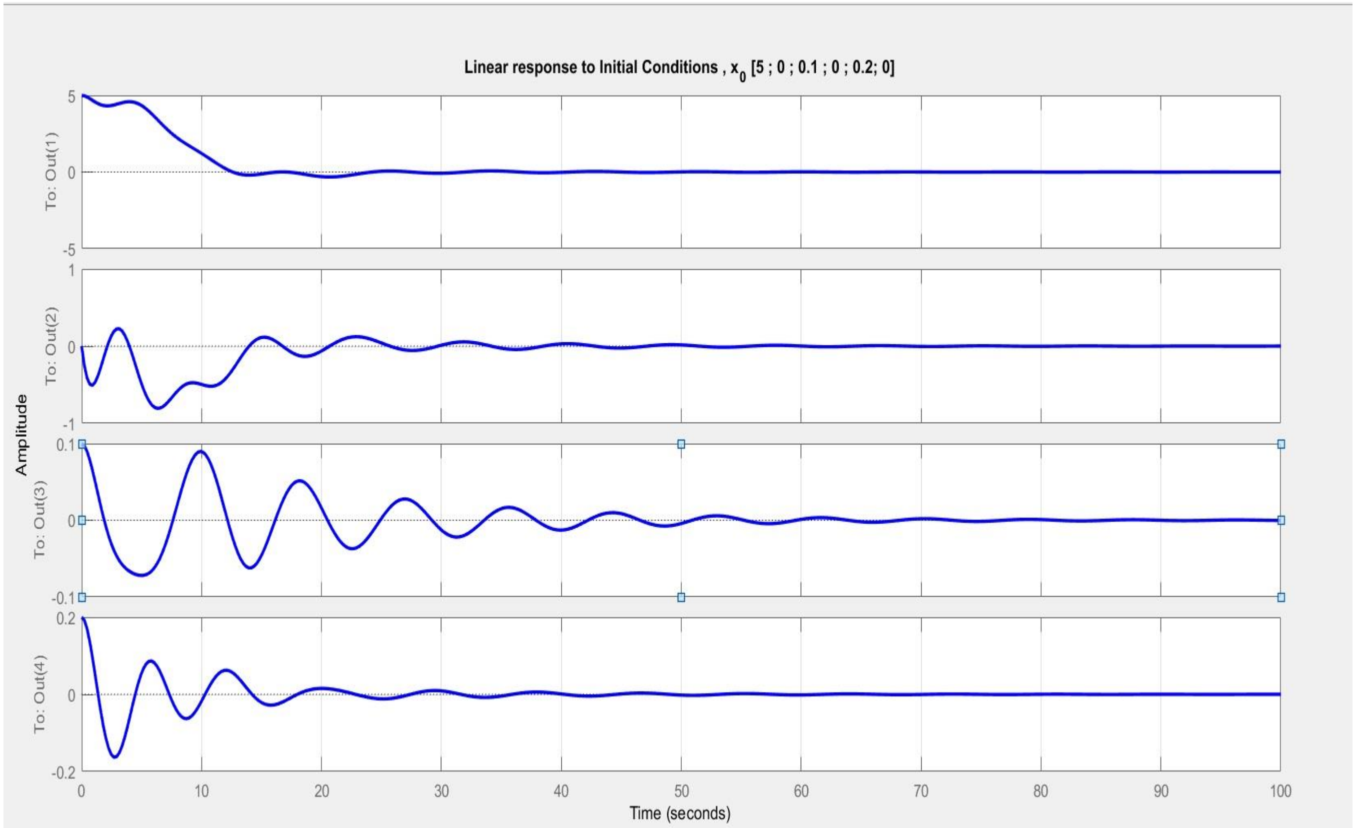
$$X\_0 = \begin{bmatrix} 5 \\ 0 \\ 0.1 \\ 0 \\ 0.2 \\ 0 \end{bmatrix}$$ and output was calculated for $x, \dot{x}, \theta_1, \theta_2$

→ Matlab File: Part1d.m

Non-linear system

Non-linear resp. obtained using ode45, for same initial condition as of linear one.

→ Matlab File: Part1dnonlinear.

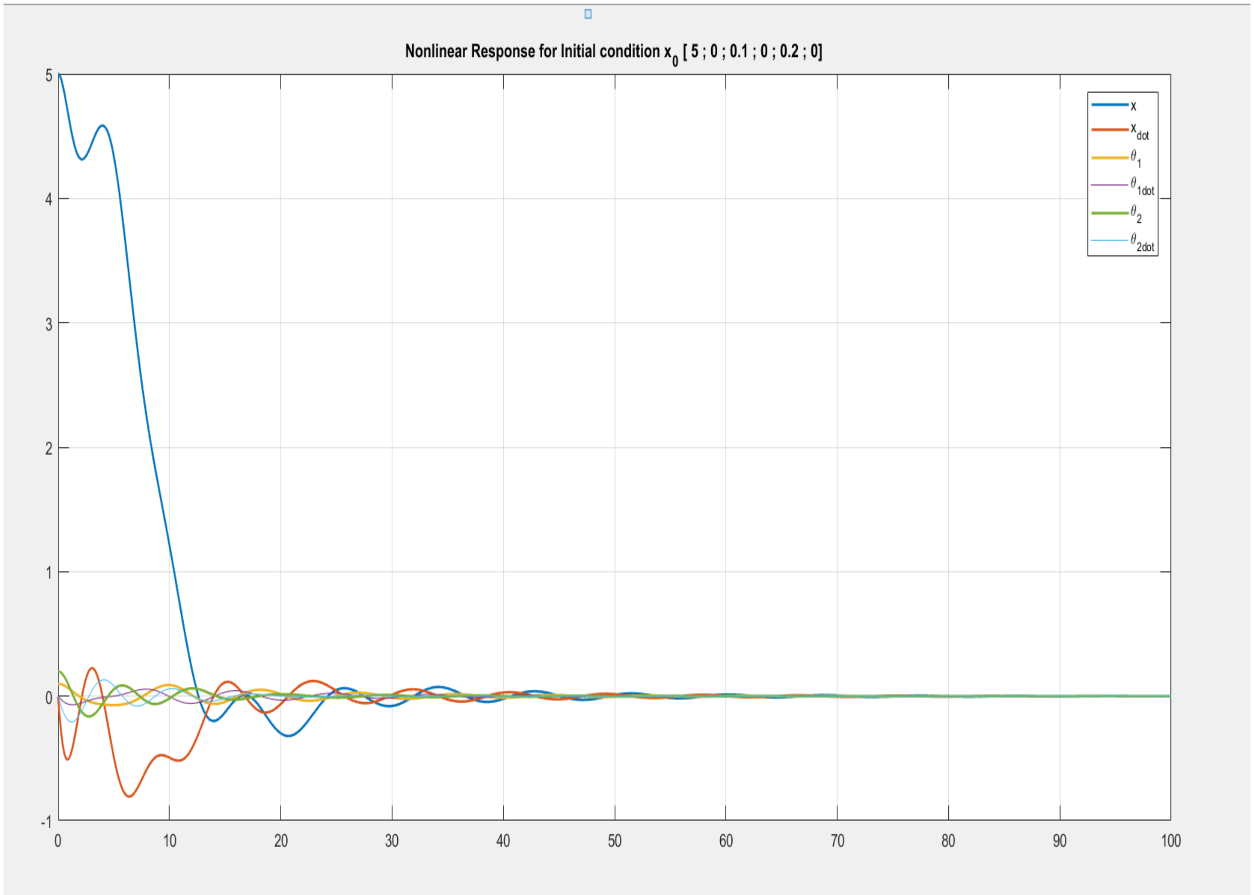


Linear Response to Initial Condition

```
%part 1D Linearized response to initial condition x_0%

m1=100;
m2=100;
M = 1000;
g = 9.8;
l1 =20;
l2 =10;

A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0 ; 0 0 -
(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0 0 -(m1*g)/(M*l2)
0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0;0 1 0 0 0 0;0 0 1 0 0 0; 0 0 0 0 1 0 ]
D=[0]
x_0= [ 5 ; 0; 0.1 ;  0 ; 0.2; 0]
Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0; 0 0 0 0 150
0; 0 0 0 0 0 1500]
R=0.0001
K=lqr(A,B,Q,R)
eig(A-B*K)
sys=ss(A-B*K,B,C,D)
initial(sys,x_0)
```



Non-Linear response to Initial condition

Matlab File Name : part1nonlinear.m

```
%Component1 D , Non linear response to initial condition#

m1=100;
m2=100;
M = 1000;
g = 9.8;
l1 =20;
l2 =10;
x_0= [ 5 ; 0; 0.1 ;  0 ; 0.2; 0]
t=0:0.01:100;%Timestep & Final Time
[t,x]=ode45(@findx,t,x_0);
plot(t,x,'linewidth',1.5);
title('Nonlinear Response');
function dx= findx(t,x)
m1=100;
m2=100;
```

```
M = 1000;
g = 9.8;
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0];
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0];
D=[0];
Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500]
R=0.0001;
K=lqr(A,B,Q,R);
eig(A-B*K);
u = -K*x;
dx = zeros(6,1);
dx(1)= x(2);
dx(2)= -((-u) + m1*l1*x(4)^2*sin(x(3)) +
m1*g*sin(x(3))*cos(x(3))+
m2*l2*x(6)^2*sin(x(5))+m2*g*sin(x(5))*cos(x(5)))/(M+m1*sin(
x(3)^2)+ m2*sin(x(5)^2));
dx(3)= x(4);
dx(4)= -((-u) +(M+m1)*g*sin(x(3)) +
m1*l1*x(4)^2*sin(x(3))*cos(x(3)) +
m2*l2*x(6)^2*sin(x(5))*cos(x(3)) + m2*g*sin(x(5))*cos(x(3)-
x(5)))/(( M+ m1*sin(x(3)^2)+ m2*sin(x(5)^2))*l1);
dx(5)= x(6);
dx(6)= -((-u) + m1*l1*x(4)^2*sin(x(3))*cos(x(5))
+m1*g*sin(x(3))*cos(x(3)-x(5)) +
(M+m1)*g*sin(x(5))+m2*l2*x(6)^2*sin(x(5))*cos(x(5)))/(( M+
m1*sin(x(3)^2)+ m2*sin(x(5)^2))*l2);
end
```

Stability from Lyapunov Indirect method

In Lyapunov Indirect method we check the eigen value of $(A-BK)$.

$eig(A-BK) = (-0.2108 \pm 1.024i), (-0.2050 \pm 0.2026i)$

$(-0.0612 \pm 0.2246i)$ { All real part is -ve }

→ this shows that the linearized system is stable, which means the Non-linear system is locally stable in the neighborhood of the equilibrium point.

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Second component

(E) **For output vector $x(t)$**

In this case $C = [1\ 0\ 0\ 0\ 0\ 0]$

we check the observability by checking the rank of observability matrix, $O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \\ CA^4 \\ CA^5 \end{bmatrix}$

$$CA = [0\ 1\ 0\ 0\ 0\ 0]$$

$$CA^2 = [0\ 1\ -0.98\ 0\ -0.98\ 0]$$

$$CA^3 = [0\ 0\ 0\ -0.98\ 0\ -0.98]$$

$$CA^4 = [0\ 0\ 0.6243\ 0\ 1.1045\ 0]$$

$$CA^5 = [0\ 0\ 0\ 0.6243\ 0\ 1.1045]$$

this gives,

$$O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.98 & 0 & -0.98 & 0 \\ 0 & 0 & 0 & -0.98 & 0 & -0.98 \\ 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \end{bmatrix}$$

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Rank of O is 6, hence it is observable for output $x(t)$

<u>For output $(\theta_1(t), \theta_2(t))$</u>

In this case $C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$$CA = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 0 & 0 & -0.539 & 0 & -0.049 & 0 \\ 0 & 0 & -0.098 & 0 & -1.078 & 0 \end{bmatrix}$$

$$CA^3 = \begin{bmatrix} 0 & 0 & 0 & -0.539 & 0 & -0.049 \\ 0 & 0 & 0 & -0.098 & 0 & -1.078 \end{bmatrix}$$

$$CA^4 = \begin{bmatrix} 0 & 0 & 0.2953 & 0 & 0.0792 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1669 & 0 \end{bmatrix}$$

$$CA^5 = \begin{bmatrix} 0 & 0 & 0 & 0.2953 & 0 & 0.0792 \\ 0 & 0 & 0 & 0.1585 & 0 & 1.1669 \end{bmatrix}$$

this gives,

$$O = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.539 & 0 & -0.049 & 0 \\ 0 & 0 & -0.098 & 0 & -1.078 & 0 \\ 0 & 0 & 0 & -0.539 & 0 & -0.049 \\ 0 & 0 & 0 & -0.098 & 0 & -1.078 \\ 0 & 0 & 0.2953 & 0 & 0.0792 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1669 & 0 \\ 0 & 0 & 0 & -0.2953 & 0 & 0.0792 \\ 0 & 0 & 0 & 0.1583 & 0 & 1.1669 \end{bmatrix}$$

Rank of O in this case is 4.

Hence, system is not ~~obser~~ observable for output $(\theta_1(t), \theta_2(t))$

For output $(x(t), \theta_2(t))$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$CA = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 0 & 0 & -0.9800 & 0 & -0.98 & 0 \\ 0 & 0 & -0.098 & 0 & -1.078 & 0 \end{bmatrix}$$

$$CA^3 = \begin{bmatrix} 0 & 0 & 0 & -0.98 & 0 & -0.98 \\ 0 & 0 & 0 & -0.098 & 0 & -1.078 \end{bmatrix}$$

$$CA^4 = \begin{bmatrix} 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1669 & 0 \end{bmatrix}$$

$$CA^5 = \begin{bmatrix} 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \\ 0 & 0 & 0 & 0.1585 & 0 & 1.1669 \end{bmatrix}$$

$$O = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.98 & 0 & -0.98 & 0 \\ 0 & 0 & -0.98 & 0 & -1.078 & 0 \\ 0 & 0 & 0 & -0.98 & 0 & -0.98 \\ 0 & 0 & 0 & -0.098 & 0 & -1.078 \\ 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1609 & 0 \\ 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \\ 0 & 0 & 0 & 0.1585 & 0 & 1.1609 \end{bmatrix}$$

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Rank of O is 6, so system is observable for output $(x(t), \theta_2(t))$

<u>For output $(x(t), \theta_1(t), \theta_2(t))$</u>

C in this case $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

$$CA = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 0 & 0 & -0.98 & 0 & -0.98 & 0 \\ 0 & 0 & -0.539 & 0 & -0.049 & 0 \\ 0 & 0 & -0.098 & 0 & -1.078 & 0 \end{bmatrix}$$

$$CA^3 = \begin{bmatrix} 0 & 0 & 0 & -0.98 & 0 & -0.98 \\ 0 & 0 & 0 & -0.539 & 0 & -0.049 \\ 0 & 0 & 0 & -0.098 & 0 & -1.078 \end{bmatrix}$$

$$CA^4 = \begin{bmatrix} 0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\ 0 & 0 & 0.2953 & 0 & 0.0792 & 0 \\ 0 & 0 & 0.1585 & 0 & 1.1669 & 0 \end{bmatrix}$$

$$CA^5 = \begin{bmatrix} 0 & 0 & 0 & 0.6243 & 0 & 1.1045 \\ 0 & 0 & 0 & 0.2953 & 0 & 0.0792 \\ 0 & 0 & 0 & 0.1585 & 0 & 1.1669 \end{bmatrix}$$

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$$
O = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & -0.98 & 0 & -0.98 & 0 \\
0 & 0 & -0.539 & 0 & -0.049 & 0 \\
0 & 0 & -0.098 & 0 & -1.078 & 0 \\
0 & 0 & 0 & -0.98 & 0 & -0.98 \\
0 & 0 & 0 & -0.539 & 0 & -0.049 \\
0 & 0 & 0 & -0.098 & 0 & -1.078 \\
0 & 0 & 0.6243 & 0 & 1.1045 & 0 \\
0 & 0 & 0.2953 & 0 & 0.0792 & 0 \\
0 & 0 & 0.1585 & 0 & 1.1669 & 0 \\
0 & 0 & 0 & 0.6243 & 0 & 1.1045 \\
0 & 0 & 0 & 0.2953 & 0 & 0.0792 \\
0 & 0 & 0 & 0.1585 & 0 & 1.1669
\end{bmatrix}
$$

Rank of O is 6, hence system is observable for output $(x(t), \theta_1(t), \theta_2(t))$.

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(F) Luenberger observer for output $x(t)$

The resultant closed loop system with observer is given by

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A-BK & BK \\ 0 & A-LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} F.$$

Poles are placed at $[-2\ -3\ -4\ -5\ -6\ -7]$ which gives observer gain matrix

$$L = 10^4 \begin{bmatrix} 0.0027 \\ 0.0293 \\ -1.5031 \\ -0.5420 \\ 1.3327 \\ 0.0697 \end{bmatrix},$$

K is same as designed for LQR

$$= 10^3 [0.1 \quad 0.5553 \quad 0.4648 \quad 3.12 \quad 2.39 \quad 2.4226]$$

Initial condition used is same as previous one

$$X\_0 = \begin{bmatrix} 5 \\ 0 \\ 0.1 \\ 0 \\ 0.2 \\ 0 \end{bmatrix}$$

observer for output $(x(t), \theta_2(t))$

In this case only difference will be, C

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Rest of the Design Parameters will be same.

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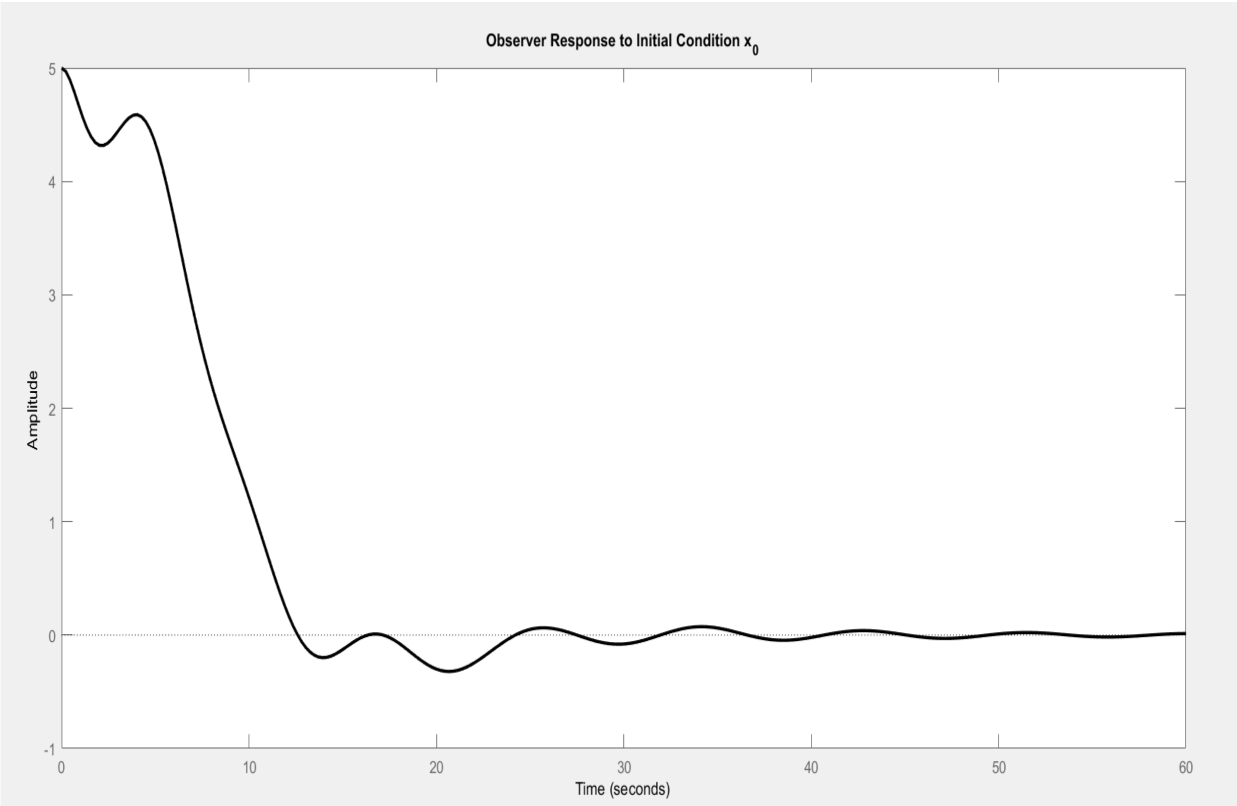
Observer for output $(x(t), \theta_1(t), \theta_2(t))$

In this C matrix will be

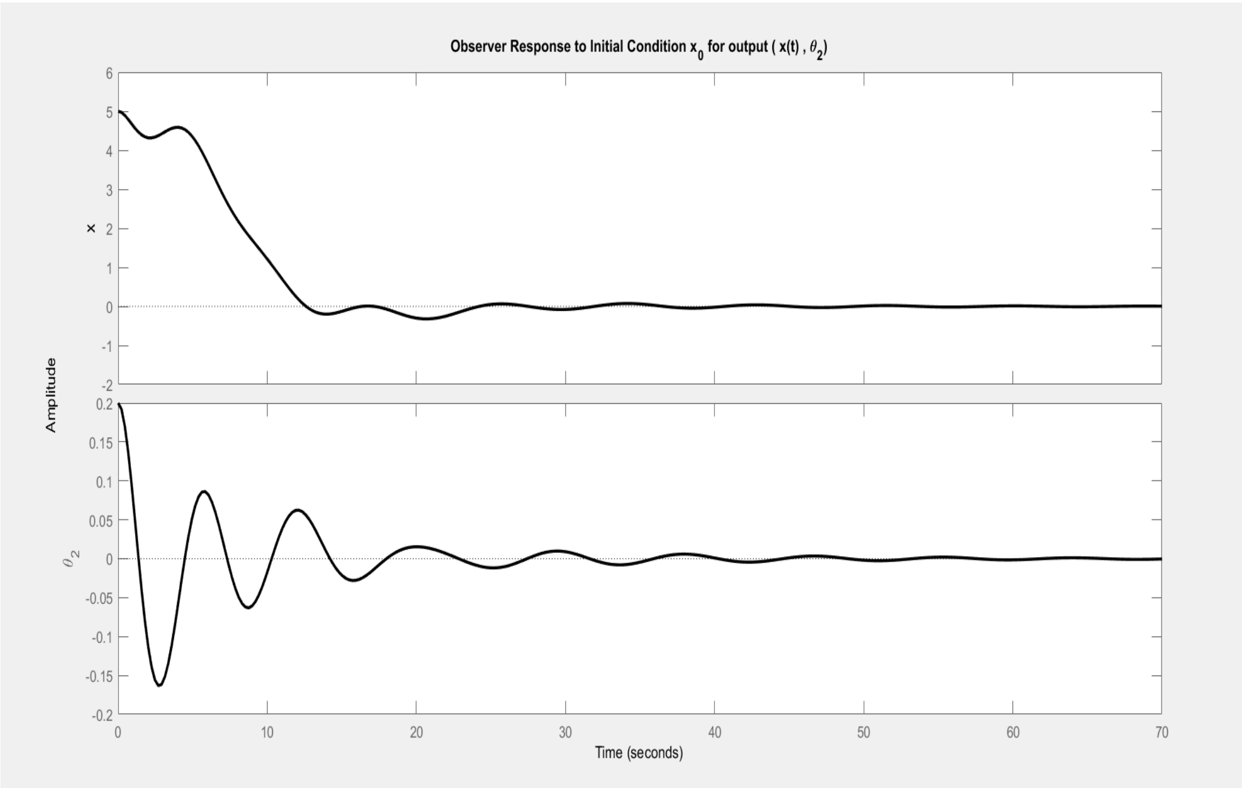
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$



Observer response for Initial condition for output x(t)

```
%second Component (F) , Luenberger observer with output
x(t)%
m1=100;
m2=100;
M = 1000;
g = 9.8;
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0 ]
D=[0]
x_0= [ 5 ; 0 ; 0.1 ; 0 ;0.2 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0]
Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500]
R=0.0001
K=lqr(A,B,Q,R)
```

```
p=[-2 -3 -4 -5 -6 -7 ]
L=place(A',C',p)
L=L'
eig(A-L*C)
Ac=[(A-B*K) (B*K); zeros(size(A)) (A-L*C)];
Bc=[ B ;zeros(size(B))];
Cc= [C zeros(size(C))];
Dc=[0]
sys=ss(Ac,Bc,Cc,Dc)
initial(sys,x_0)
```

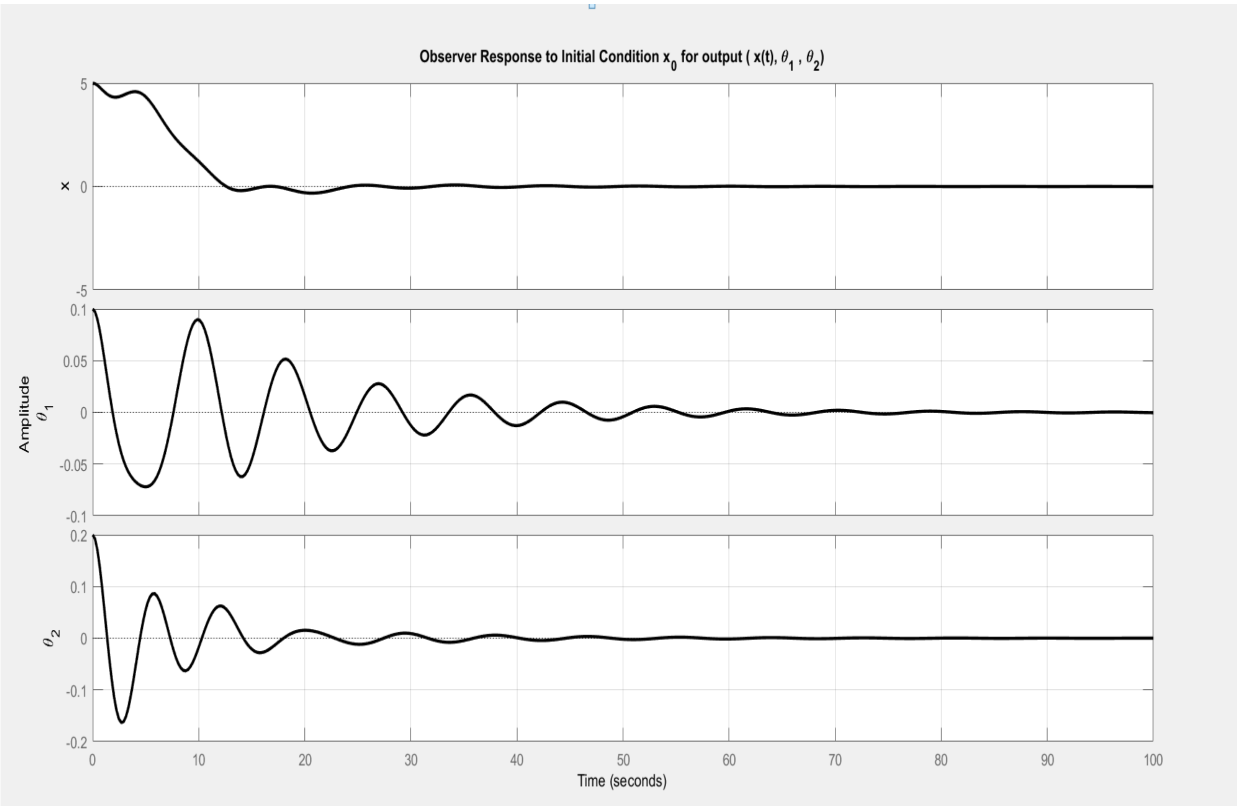


Observer response for Initial condition for output (x(t), theta_2)

```
%second Component (F) , Luenberger observer with output
(x(t),theta_2(t))%

m1=100;
m2=100;
M = 1000;
g = 9.8;
```

```
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0 ; 0 0 0 0 1 0]
D=[0]
x_0= [ 5 ; 0 ; 0.1 ; 0 ;0.2 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0]
 Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500]
R=0.0001
K=lqr(A,B,Q,R)
eig(A-B*K)
p=[-2 -3 -4 -5 -6 -7 ]
L=place(A',C',p)
L=L'
eig(A-L*C)
Ac=[(A-B*K) (B*K); zeros(size(A)) (A-L*C)];
Bc=[ B ;zeros(size(B))];
Cc= [C zeros(size(C))];
Dc=[0]
sys=ss(Ac,Bc,Cc,Dc)
initial(sys,x_0)
grid
```

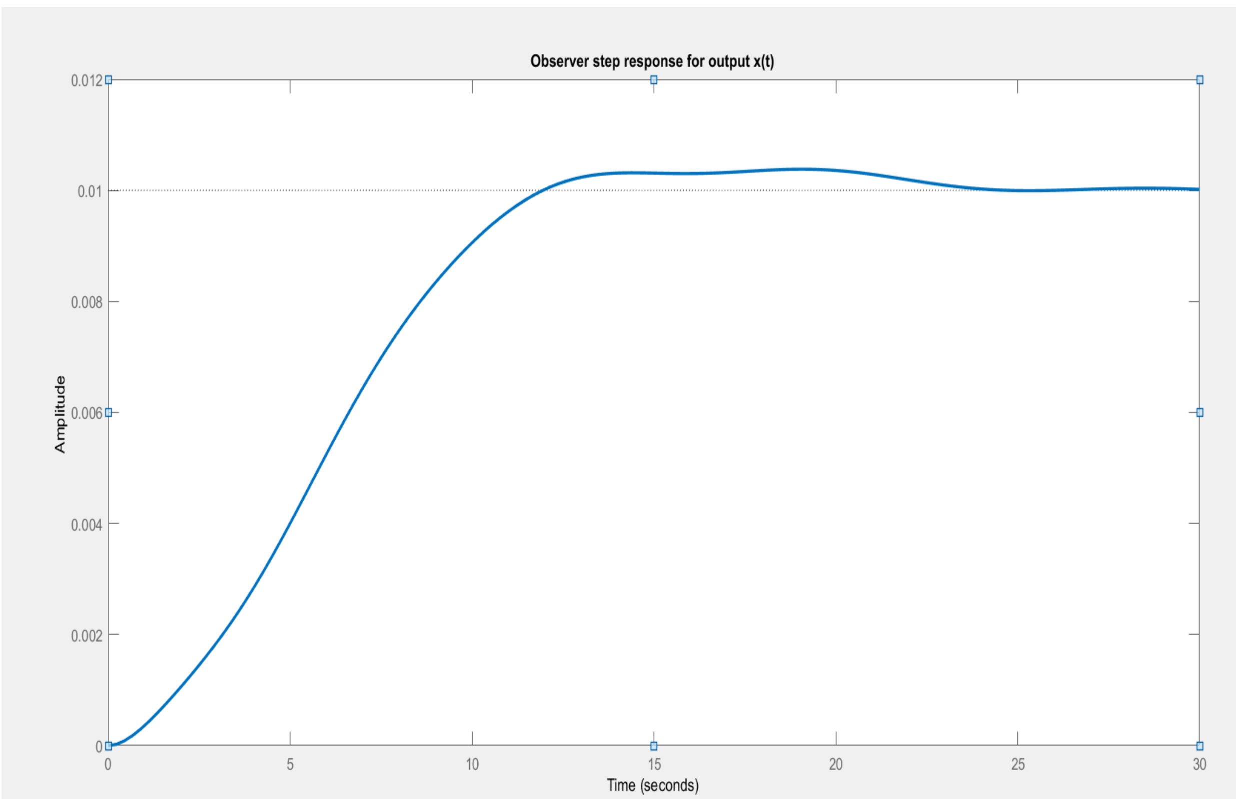


Observer response for Initial condition for output (x(t), theta_1,theta_2)

```
%second Component (F) , Luenberger observer with output
(x(t),theta_1(t),theta_2(t))%

m1=100;
m2=100;
M = 1000;
g = 9.8;
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0 ; 0 0 1 0 0 0; 0 0 0 0 1 0]
D=[0]
x_0= [ 5 ; 0 ; 0.1 ; 0 ;0.2 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0]
 Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500]
R=0.0001
```

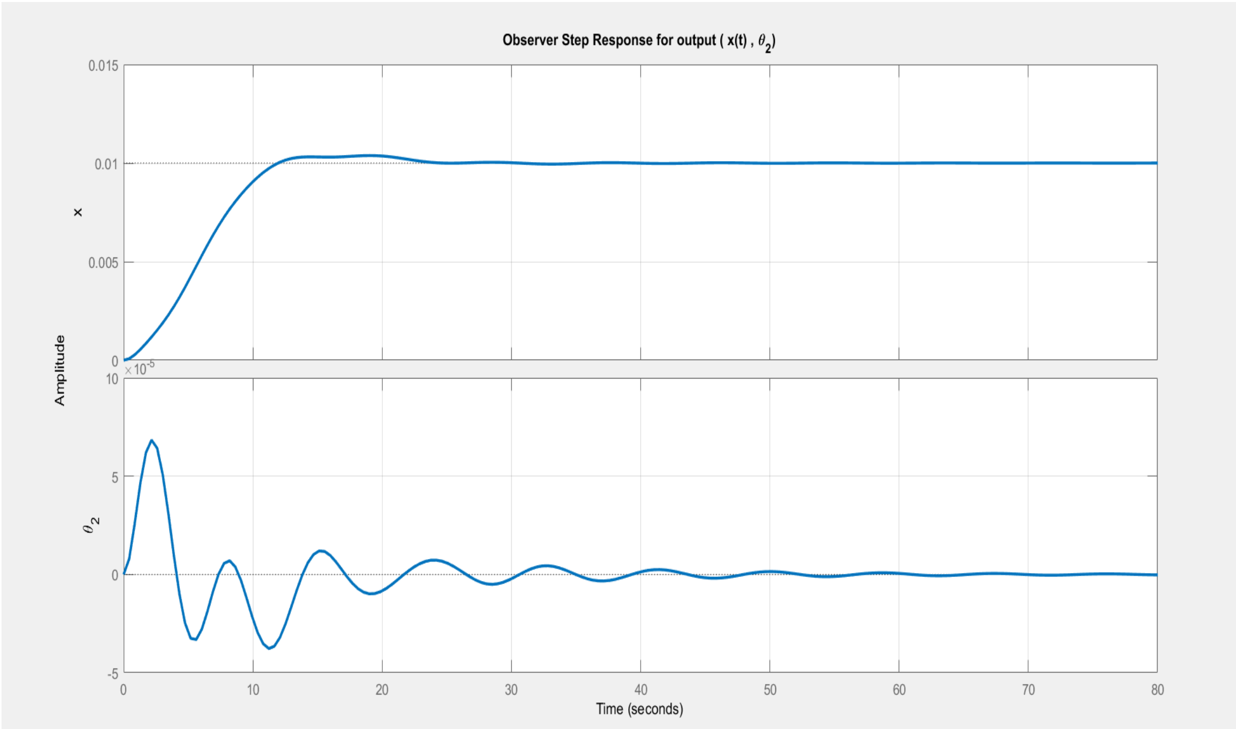
```
K=lqr(A,B,Q,R)
eig(A-B*K)
p=[-2 -3 -4 -5 -6 -7 ]
L=place(A',C',p)
L=L'
eig(A-L*C)
Ac=[(A-B*K) (B*K); zeros(size(A)) (A-L*C)];
Bc=[ B ;zeros(size(B))];
Cc= [C zeros(size(C))];
Dc=[0]
sys=ss(Ac,Bc,Cc,Dc)
initial(sys,x_0)
grid
```



Observer step response for output x(t)

```
%second Component (F), Luenberger observer with output
x(t), step response%
```

```
m1=100;
m2=100;
M = 1000;
g = 9.8;
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0 ]
D=[0]
 Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500]
R=0.0001
K=lqr(A,B,Q,R)
p=[-2 -3 -4 -5 -6 -7 ]
L=place(A',C',p)
L=L'
eig(A-L*C)
Ac=[(A-B*K) (B*K); zeros(size(A)) (A-L*C)];
Bc=[ B ;zeros(size(B))];
Cc= [C zeros(size(C))];
Dc=[0]
sys=ss(Ac,Bc,Cc,Dc)
step(sys)
```

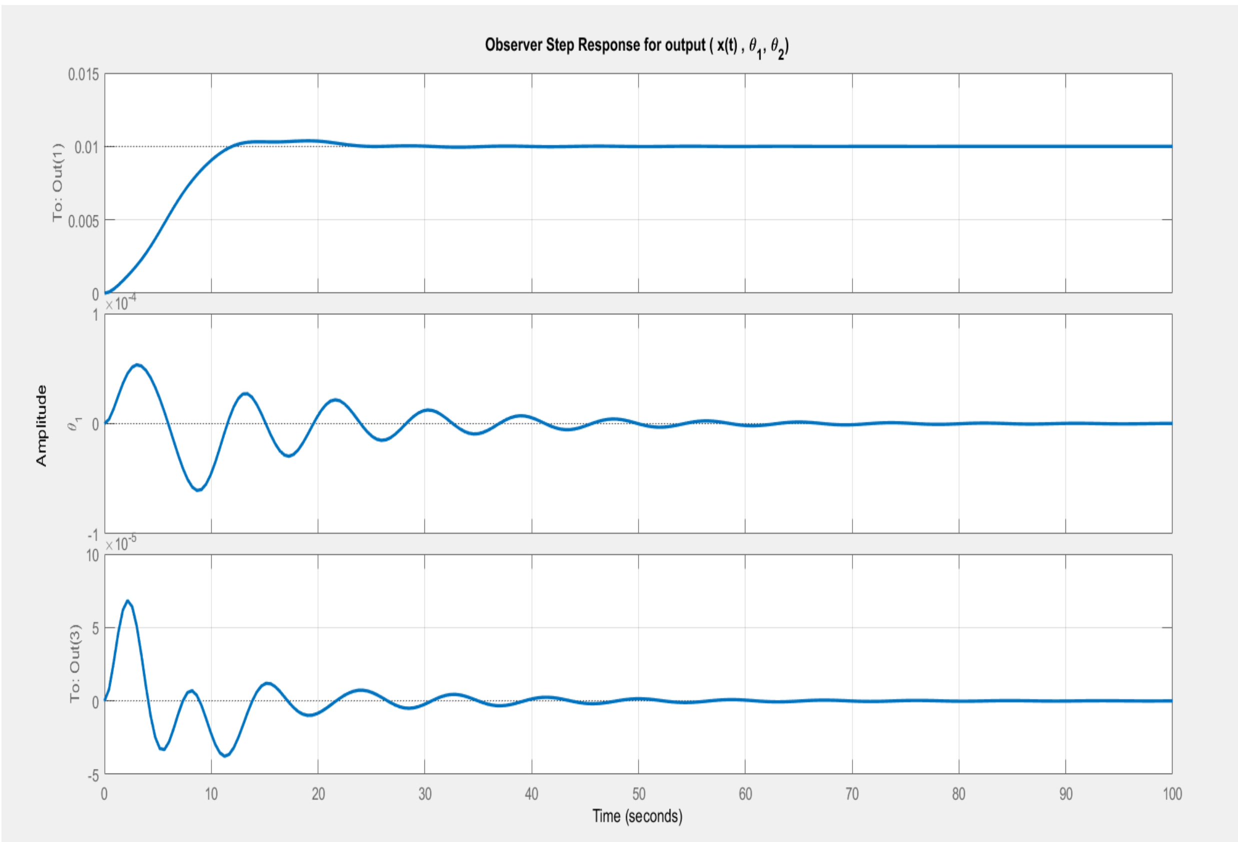


Observer step response for output (x(t), theta_2)

```
%second Component (F) , Luenberger observer with output
(x(t),theta_2(t))%

m1=100;
m2=100;
M = 1000;
g = 9.8;
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0 ; 0 0 0 0 1 0];
D=[0];
Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500];
R=0.0001;
K=lqr(A,B,Q,R);
eig(A-B*K);
p=[-2 -3 -4 -5 -6 -7 ]
L=place(A',C',p)
```

```
L=L'
eig(A-L*C);
Ac=[(A-B*K) (B*K); zeros(size(A)) (A-L*C)];
Bc=[ B ;zeros(size(B))];
Cc= [C zeros(size(C))];
Dc=[0]
sys=ss(Ac,Bc,Cc,Dc)
step(sys)
grid
```



Observer step response for output (x(t), theta_1 , theta_2)

```
%second Component (F) , Luenberger observer with output
(x(t),theta_1(t),theta_2(t))%

m1=100;
m2=100;
M = 1000;
g = 9.8;
```

```
l1 =20;
l2 =10;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C=[1 0 0 0 0 0 ; 0 0 1 0 0 0; 0 0 0 0 1 0]
D=[0]
Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500];
R=0.0001
K=lqr(A,B,Q,R)
eig(A-B*K)
p=[-2 -3 -4 -5 -6 -7 ]
L=place(A',C',p)
L=L'
eig(A-L*C)
Ac=[(A-B*K) (B*K); zeros(size(A)) (A-L*C)];
Bc=[ B ;zeros(size(B))];
Cc= [C zeros(size(C))];
Dc=[0]
sys=ss(Ac,Bc,Cc,Dc)
step(sys)
grid
```

(G) LQG control

Linear quadratic control is simply combination of Kalman filter (linear-quadratic estimator) and linear quadratic regulator.

State space is written as

$$\dot{x} = Ax + BU + Bw$$
$$y = Cx + v$$

v, w are are disturbance and Noise respectively.

$V_d = 0.3$ (covariance of disturbance)

$V_n = 1$ (covariance of Noise)

obtained gain Matrix $L = 10^4 \begin{bmatrix} 0.0027 \\ 0.0293 \\ -1.5031 \\ -0.5420 \\ 1.3377 \\ 0.0697 \end{bmatrix}$

→ response is plotted for initial condition

$$x_0 = \begin{bmatrix} 5 \\ 0 \\ 0.1 \\ 0 \\ 0.2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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→ To plot the response, modeling of the system was Done in Simulink.

Reference Tracking

There is always a steady state error in reference tracking, because we are estimating the states using LQG with consideration of disturbances and noise.
this can be solved using a precompensator. we can multiply the reference with a suitable gain.

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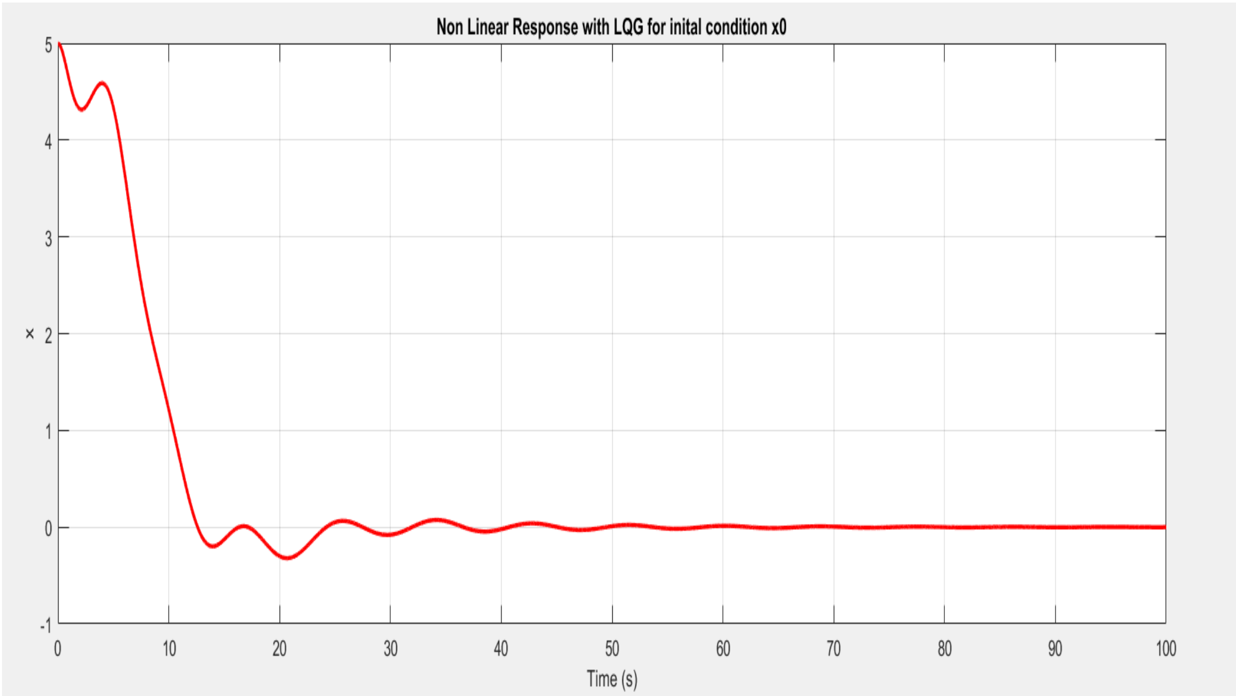

Non-Linear response with LQG for initial condition x0

```
%LQG Control , Part G%

function simulateLQG()
M = 1000;
m1 = 100;
m2 = 100;
l1 = 20;
l2 = 10;
g = 9.80;
A= [0 1 0 0 0 0 ; 0 0 -(m1*g/M) 0 -(m2*g/M) 0 ; 0 0 0 1 0 0
; 0 0 -(g*(M+m1))/(M*l1) 0 -(m2*g)/(M*l1) 0 ;0 0 0 0 0 1; 0
0 -(m1*g)/(M*l2) 0 -(g*(M+m2))/(M*l2) 0]
B= [ 0; 1/M ;0; 1/(M*l1) ;0 ;1/(M*l2)];
C = [1 0 0 0 0 0];
D = 0;
Q=[1 0 0 0 0 0;0 1 0 0 0 0; 0 0 100 0 0 0; 0 0 0 1000 0 0;
0 0 0 0 150 0; 0 0 0 0 0 1500]
R=0.0001
K = lqr(A,B,Q,R);
sys_1 = ss(A,[B B],C,[zeros(1,1) zeros(1,1)]);
vd = 0.3;
vn = 1;
```

```
sen = [1];
known = [1];
[~,L,~] = kalman(sys_1,vd,vn,[],sen,known)
states =
{'x','x_dot','theta1','theta1_dot','theta2','theta2_dot','e
_1','e_2','e_3','e_4','e_5','e_6'};
inputs = {'F'};
outputs = {'x'};

Ac = [A-B*K B*K;zeros(size(A)) A-L*C];
Bc = zeros(12,1);
Cc = [C zeros(size(C))];
sys_cl_lqg = ss(Ac,Bc,Cc,D,
'statename',states,'inputname',inputs,'outputname',outputs)
;
x0= [ 5 ; 0 ; 0.1 ; 0 ;0.2 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0]
t = 0:0.01:100;
F = zeros(size(t));
[Y,~,X] = lsim(sys_cl_lqg,F,t,x0);
figure
plot(t,Y(:,1),'b');
u = zeros(size(t));
for i = 1:size(X,1)
   u(i) = K * (X(i,1:6))';
end
Xhat = X(:,1) - X(:,6);
figure
subplot(3,1,1), plot(t,Xhat), hold on, plot(t,X(:,1),'r')
end
```

Simulink Model