

Semester Awal 2024/2025

Matriks

(Week 1)

Aljabar Linear

Notasi

- Matriks berukuran $m \times n$ (m baris dan n kolom):

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Jika $m = n$ maka dinamakan matriks persegi (*square matrix*) orde n

- Contoh matriks A berukuran 3×4 :

$$A = \begin{bmatrix} 3 & 2 & 4 & 6 \\ 7 & 0 & 8 & -12 \\ 13 & 11 & -1 & 0 \end{bmatrix}$$

- **Diagonal utama** matriks persegi berukuran $n \times n$:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

Penjumlahan Matriks

- Penjumlahan dua buah matriks $C_{m \times n} = A_{m \times n} + B_{m \times n}$

Misal $A = [a_{ij}]$

$B = [b_{ij}]$

maka $C = A + B = [c_{ij}]$, $c_{ij} = a_{ij} + b_{ij}$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$

- Pengurangan matriks: $C = A - B = [c_{ij}]$, $c_{ij} = a_{ij} - b_{ij}$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$
- Algoritma penjumlahan dua buah matriks:

for $i \leftarrow 1$ to m do

 for $j \leftarrow 1$ to n do

$c_{ij} \leftarrow a_{ij} + b_{ij}$

 end for

end for

- Contoh:

$$A = \begin{bmatrix} 2 & 1 & 0 & 3 \\ -1 & 0 & 2 & 4 \\ 4 & -2 & 7 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -4 & 3 & 5 & 1 \\ 2 & 2 & 0 & -1 \\ 3 & 2 & -4 & 5 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

- Maka,

$$A + B = \begin{bmatrix} -2 & 4 & 5 & 4 \\ 1 & 2 & 2 & 3 \\ 7 & 0 & 3 & 5 \end{bmatrix} \quad \text{and} \quad A - B = \begin{bmatrix} 6 & -2 & -5 & 2 \\ -3 & -2 & 2 & 5 \\ 1 & -4 & 11 & -5 \end{bmatrix}$$

Note: $A + C$, $B + C$, $A - C$, $B - C$ tidak dapat dihitung.

Perkalian Matriks

- Perkalian dua buah matriks $C_{m \times n} = A_{m \times r} \times B_{r \times n}$

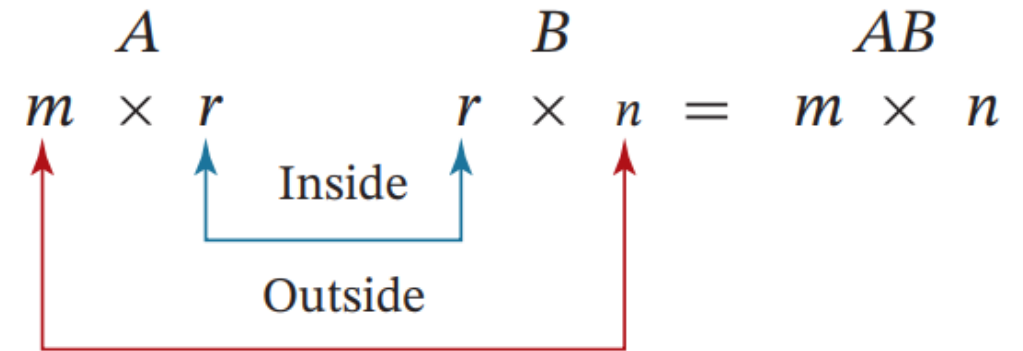
Misal $A = [a_{ij}]$ dan $B = [b_{ij}]$

maka $C = A \times B = [c_{ij}]$, $c_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + \dots + a_{in}b_{nj}$

syarat: jumlah kolom A sama dengan jumlah baris B

- Algoritma perkalian dua buah matriks $C_{m \times n} = A_{m \times r} \times B_{r \times n}$

```
for i ← 1 to m do
  for j ← 1 to n do
     $c_{ij} \leftarrow 0$ 
    for k ← 1 to r do
       $c_{ij} \leftarrow c_{ij} + a_{ik} * b_{kj}$ 
    end for
  end for
end for
```



- Contoh:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

Maka AB dapat dihitung

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & \square \\ \square & \square & 26 & \square \end{bmatrix}$$

$$(2 \cdot 4) + (6 \cdot 3) + (0 \cdot 5) = 26$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} \square & \square & \square & 13 \\ \square & \square & \square & \square \end{bmatrix}$$

$$(1 \cdot 3) + (2 \cdot 1) + (4 \cdot 2) = 13$$

• Contoh:

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix}$$

$$(1 \cdot 4) + (2 \cdot 0) + (4 \cdot 2) = 12$$

$$(1 \cdot 1) - (2 \cdot 1) + (4 \cdot 7) = 27$$

$$(1 \cdot 4) + (2 \cdot 3) + (4 \cdot 5) = 30$$

$$(2 \cdot 4) + (6 \cdot 0) + (0 \cdot 2) = 8$$

$$(2 \cdot 1) - (6 \cdot 1) + (0 \cdot 7) = -4$$

$$(2 \cdot 3) + (6 \cdot 1) + (0 \cdot 2) = 12$$

$$AB = \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}$$

Perkalian Matriks dengan Skalar

- Misal $A = [a_{ij}]$ dan c adalah skalar
maka

$$cA = [ca_{ij}] \quad , i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

- Contoh: Misalkan $A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 2 & 7 \\ -1 & 3 & -5 \end{bmatrix}$, $C = \begin{bmatrix} 9 & -6 & 3 \\ 3 & 0 & 12 \end{bmatrix}$

maka

$$2A = \begin{bmatrix} 4 & 6 & 8 \\ 2 & 6 & 2 \end{bmatrix}, \quad (-1)B = \begin{bmatrix} 0 & -2 & -7 \\ 1 & -3 & 5 \end{bmatrix}, \quad \frac{1}{3}C = \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 4 \end{bmatrix}$$

dan $2A - B + \frac{1}{3}C = 2A + (-1)B + \frac{1}{3}C$ ← Kombinasi linear A, B, dan C dengan koefisien 2, -1, dan 1/3

$$= \begin{bmatrix} 4 & 6 & 8 \\ 2 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -2 & -7 \\ 1 & -3 & 5 \end{bmatrix} + \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 2 & 2 \\ 4 & 3 & 11 \end{bmatrix}$$

Kombinasi Linear Matriks

- Perkalian matriks dapat dipandang sebagai kombinasi linear
- Misalkan:

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \quad \text{and} \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

maka

$$A\mathbf{x} = \begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \end{bmatrix} = x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \cdots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

- Contoh: perkalian matriks

$$\begin{bmatrix} -1 & 3 & 2 \\ 1 & 2 & -3 \\ 2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}$$

dapat ditulis sebagai kombinasi linear

$$2 \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} - 1 \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ -3 \end{bmatrix}$$

- Contoh lain: perkalian matriks

$$AB = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \end{bmatrix} \begin{bmatrix} 4 & 1 & 4 & 3 \\ 0 & -1 & 3 & 1 \\ 2 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 12 & 27 & 30 & 13 \\ 8 & -4 & 26 & 12 \end{bmatrix}$$

dapat dinyatakan sebagai kombinasi linear

$$\begin{bmatrix} 12 \\ 8 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 6 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 30 \\ 26 \end{bmatrix} = 4 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + 3 \begin{bmatrix} 2 \\ 6 \end{bmatrix} + 5 \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 27 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \end{bmatrix} + 7 \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 13 \\ 12 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \end{bmatrix} + 2 \begin{bmatrix} 4 \\ 0 \end{bmatrix}$$

Transpose Matriks

- Transpose matriks, $B = A^T$

$$b_{ji} = a_{ij} \quad i = 1, 2, \dots, m; j = 1, 2, \dots, n$$

- Algoritma transpose matriks:

```
for i ← 1 to m do
  for j ← 1 to n do
     $b_{ji} \leftarrow a_{ij}$ 
  end for
end for
```

- Contoh: Transpose matriks

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \\ 5 & 6 \end{bmatrix}, \quad C = [1 \quad 3 \quad 5], \quad D = [4]$$

$$A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \\ a_{14} & a_{24} & a_{34} \end{bmatrix}, \quad B^T = \begin{bmatrix} 2 & 1 & 5 \\ 3 & 4 & 6 \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \quad D^T = [4]$$

- Untuk matriks persegi A berukuran $n \times n$, transpose matriks A dapat diperoleh dengan mempertukarkan elemen yang simetri dengan diagonal utama:

$$A = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 7 & 0 \\ -5 & 8 & 6 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 3 & 7 & 0 \\ -5 & 8 & 6 \end{bmatrix} \rightarrow A^T = \begin{bmatrix} 1 & 3 & -5 \\ -2 & 7 & 8 \\ 4 & 0 & 6 \end{bmatrix}$$

↑
Interchange entries that are symmetrically positioned about the main diagonal.

- Sifat-sifat transpose matriks

Theorem 1.4.8

If the sizes of the matrices are such that the stated operations can be performed, then:

$$(a) \quad (A^T)^T = A$$

$$(b) \quad (A + B)^T = A^T + B^T$$

$$(c) \quad (A - B)^T = A^T - B^T$$

$$(d) \quad (kA)^T = kA^T$$

$$(e) \quad (AB)^T = B^T A^T$$

Trace sebuah Matriks

- Jika A adalah matriks persegi, maka *trace* matriks A adalah jumlah semua elemen pada diagonal utama, disimbolkan dengan $\text{tr}(A)$.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$$

$$\text{tr}(A) = a_{11} + a_{22} + a_{33}$$

$$\text{tr}(B) = -1 + 5 + 7 + 0 = 11$$

- Jika A bukan matriks persegi, maka $\text{tr}(A)$ tidak terdefinisi.

Sifat-sifat Operasi Aritmetika Matriks

Theorem 1.4.1

Properties of Matrix Arithmetic

Assuming that the sizes of the matrices are such that the indicated operations can be performed, the following rules of matrix arithmetic are valid.

- (a) $A + B = B + A$ [Commutative law for matrix addition]
- (b) $A + (B + C) = (A + B) + C$ [Associative law for matrix addition]
- (c) $A(BC) = (AB)C$ [Associative law for matrix multiplication]
- (d) $A(B + C) = AB + AC$ [Left distributive law]
- (e) $(B + C)A = BA + CA$ [Right distributive law]
- (f) $A(B - C) = AB - AC$
- (g) $(B - C)A = BA - CA$
- (h) $a(B + C) = aB + aC$
- (i) $a(B - C) = aB - aC$
- (j) $(a + b)C = aC + bC$
- (k) $(a - b)C = aC - bC$
- (l) $a(bC) = (ab)C$
- (m) $a(BC) = (aB)C = B(aC)$

Matriks Nol

- Matriks nol: matriks yang seluruh elemennya bernilai nol

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad [0]$$

- Matriks nol dilambangkan dengan **0** atau $\mathbf{0}_{m \times n}$.
- Sifat-sifat matriks nol:

Theorem 1.4.2

Properties of Zero Matrices

If c is a scalar, and if the sizes of the matrices are such that the operations can be performed, then:

(a) $A + 0 = 0 + A = A$

(b) $A - 0 = A$

(c) $A - A = A + (-A) = 0$

(d) $0A = 0$

(e) If $cA = 0$, then $c = 0$ or $A = 0$.

Matriks Identitas

- Matriks identitas: matriks persegi yang semua elemen bernilai 1 pada diagonal utamanya dan bernilai 0 pada posisi lainnya.

$$[1], \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Matriks identitas disimbolkan dengan I .

- Perkalian matriks identitas dengan sembarang matriks menghasilkan matriks itu sendiri:

$$AI = IA = A$$

$$AI_3 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = A$$

$$I_2A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = A$$

Matriks Balikan (Invers Matriks)

- Matriks balikan (*invers*) dari sebuah matriks A adalah matriks B sedemikian sehingga

$$AB = BA = I$$

- Kita katakan A dan B merupakan balikan matriks satu sama lain

- Contoh: Misalkan $A = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

maka $AB = \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$BA = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

- Balikan matriks A disimbolkan dengan A^{-1}
- Sifat: $AA^{-1} = A^{-1}A = I$
- Untuk matriks A berukuran 2×2 , maka A^{-1} dihitung sebagai berikut:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \longrightarrow \quad A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

dengan syarat $ad - bc \neq 0$

- Nilai $ad - bc$ disebut *determinan*. Jika $ad - bc = 0$ maka matriks A tidak memiliki balikan (*not invertible*).

- Contoh:

$$A = \begin{bmatrix} 6 & 1 \\ 5 & 2 \end{bmatrix} \longrightarrow A^{-1} = \frac{1}{7} \begin{bmatrix} 2 & -1 \\ -5 & 6 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ -\frac{5}{7} & \frac{6}{7} \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix} \longrightarrow \text{Tidak memiliki invers, sebab } (-1)(-6) - (3)(2) = 0.$$

Matriks Eselon

Matriks Eselon Baris (REF)

- Matriks eselon baris (*row echelon form*) atau REF adalah matriks yang memiliki **1 utama** pada setiap baris, kecuali baris yang seluruhnya nol.
- Berbentuk:

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & * & * & * & * & * & * & * & * \\ 0 & 0 & 0 & 1 & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 1 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * \end{bmatrix}$$

Keterangan: * adalah sembarang nilai

- Contoh-contoh matriks eselon baris:

$$\begin{bmatrix} 1 & 2 & 7 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 2 & 6 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Bukan matriks eselon baris:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 1 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

- Ciri-ciri matriks eselon baris: memiliki nol-nol di bawah 1 utama

Matriks Eselon Baris Tereduksi (RREF)

- Matriks eselon baris tereduksi (*reduced row echelon form*) berbentuk:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & * & 0 & 0 & 0 & * & * & 0 & * \\ 0 & 0 & 0 & 1 & 0 & 0 & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 1 & 0 & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 1 & * & * & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & * \end{bmatrix}$$

- Ciri-ciri: memiliki nol-nol di bawah dan di atas 1 utama

- Contoh-contoh matriks eselon baris tereduksi:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Bukan matriks eselon baris tereduksi:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 8 \end{bmatrix}$$

Latihan 1

Dari sejumlah matriks di bawah ini, tentukan mana yang matriks eselon baris (REF), eselon baris tereduksi (RREF), keduanya, atau bukan sama sekali.

a. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

f. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

g. $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

Jawaban:

- (a) Keduanya (REF dan RREF)
- (b) Keduanya
- (c) Keduanya
- (d) Keduanya
- (e) Keduanya
- (f) Keduanya
- (g) REF

Latihan 2

Dari sejumlah matriks di bawah ini, tentukan mana yang matriks eselon baris (REF), eselon baris tereduksi (RREF), keduanya, atau bukan sama sekali.

a. $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 5 & -3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

f. $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 0 & 7 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

g. $\begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

Referensi

- Anton, H., & Rorres, C. (2019). Elementary Linear Algebra: Applications Version. John Wiley & Sons.
- <https://informatika.stei.itb.ac.id/~rinaldi.munir/>