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Advanced Extragalactic Astrophysics **2023**

Homework #1

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1 Exercise with simulated SED

A) Point out the differences in the SED determined by age and convert the $F_\lambda \rightarrow F_\nu$

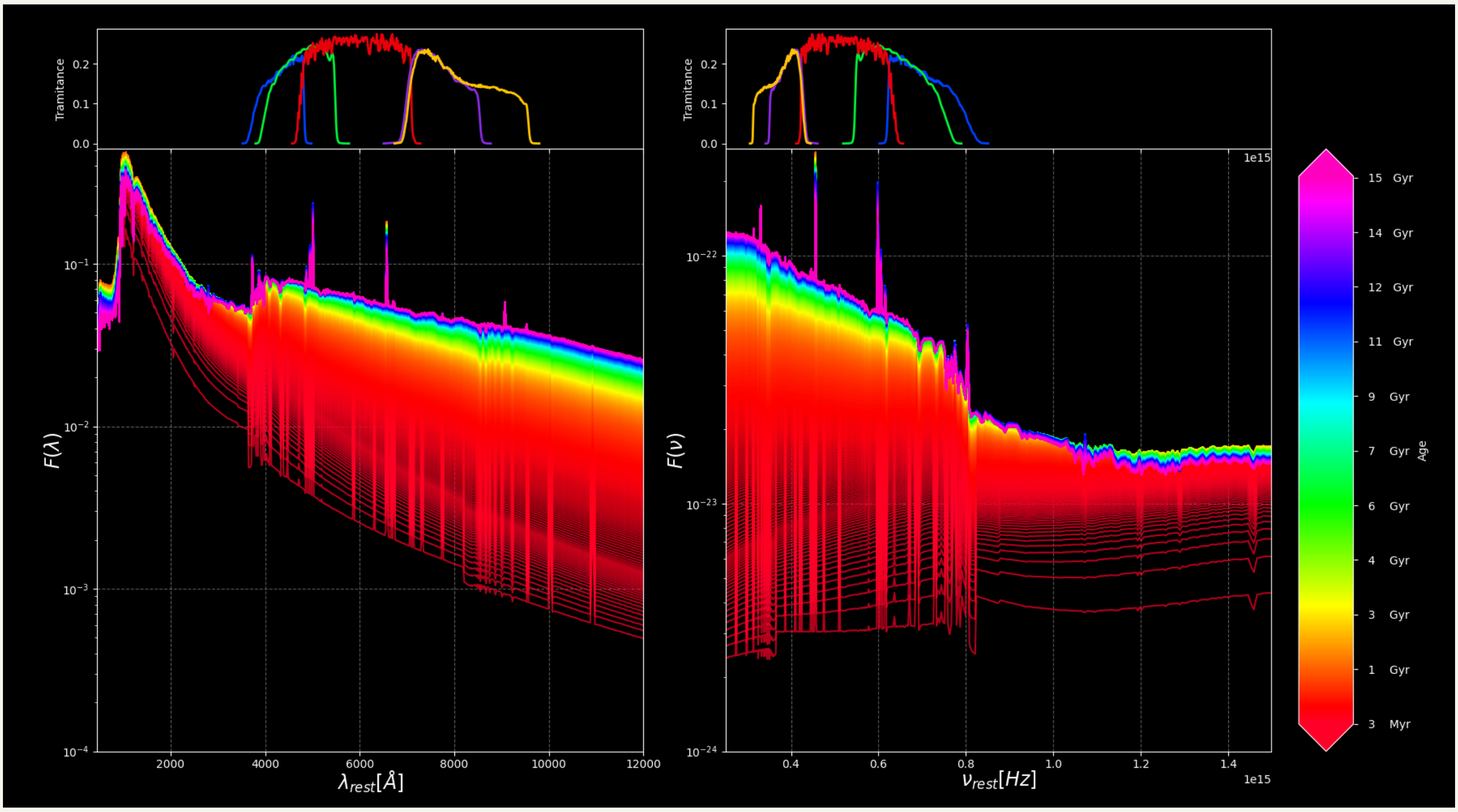


Figure 1: Flux density per wavelength and frequency, colored by age, also in top panel are showed the filters of HST F435W (blue), F475W (green), F606W (red), F775W (purple), F814W (orange). (the figures does not span the same range due visualization purposes, $F_\nu \in [2000, 12000]$Å while $F_\lambda \in [450, 12000]$Å)

As we can see in fig (1) the main behaviour that we can see is the amount of red light that contributes to the total flux is increasing with age, another behaviour that we can see is that the blue peak slowly decreases with the age, besides the general behaviour we can also notice some emission and absorption lines that changes with age (e.g. the most characteristic emission line of $H_\alpha \approx 6562.8$Å, which is decreasing with time) , we are also able to notice the Lyman break (912Å) and the Balmer Break (3645Å) in these spectra.

The conversion of the wavelength to frequency was done with the relation $\nu \cdot \lambda = c$ and the according relation for the flux $\lambda F_\lambda = \nu F_\nu \implies F_\nu = \frac{\lambda^2}{c} F_\lambda$ (Take in account that the documentation of GALEV (Kotulla et al. (2009)) indicates that the F_λ comes normalized at a distance of 10pc in units of $erg \cdot s^{-1} \cdot cm^{-2} \cdot \AA^{-1}$ hence the F_ν have units of $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$)

B) Look at the absolute magnitudes in the 5 different filters for the different dust contents (E(B-V)=0,0.5,1,1.5,2 for the Calzetti law). Comments on the effect of the dust in the simulated galaxy. Transform the absolute magnitudes in luminosities

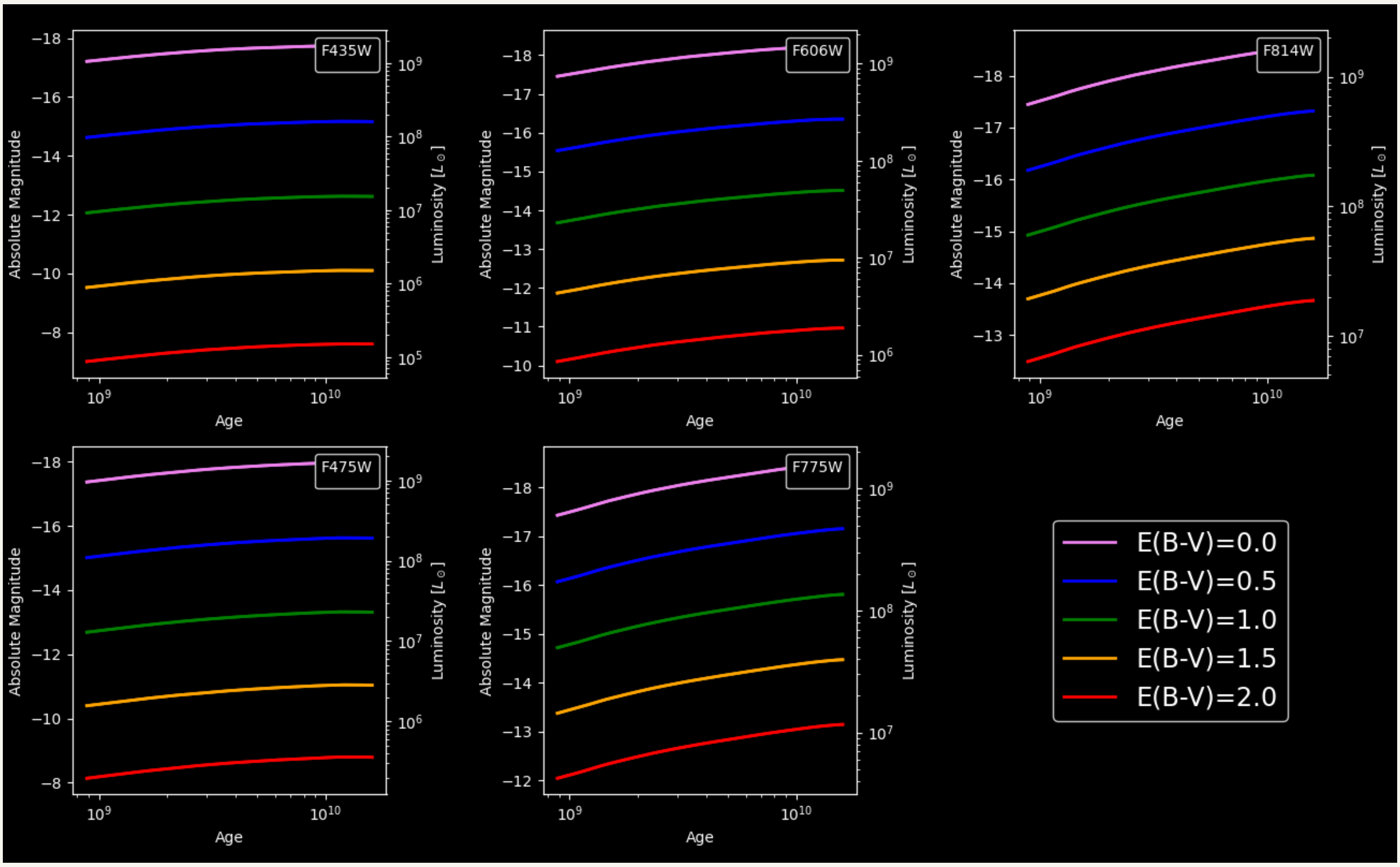


Figure 2: Magnitudes Per filter in each file of {Model}-Color-(abs).dat as function of the Age (rest frame spectra), also overploted their respective luminosity in solar luminosity units

We can clearly see the deep impact of the reddening due Dust content, this affect transversely all the filters making it less bright, losing more than one Mag per each E(B-V)=0.5 at any epoch of it's life.

The luminosity's in each filter were computed with respect to solar values of the filters of HST F435W=5.35, F475W=5.09, F606W=4.72, F775W=4.52, F814W=4.52, were obtained from Willmer (2018).

Hence the respective luminosity's in each X filter were calculated through the equation

$$L_{\text{galaxy},X} = 10^{0.4\cdot(M_{\odot,X}-M_{\text{galaxy}.X})} L_\odot \tag{1.1}$$

C) Consider the F_ν spectra from A), convolve them with the HST filter transmission curves and obtain the apparent magnitudes in the 5 filters. Compare with the magnitudes provided within the outputs for the age-steps/redshifts chosen in A)

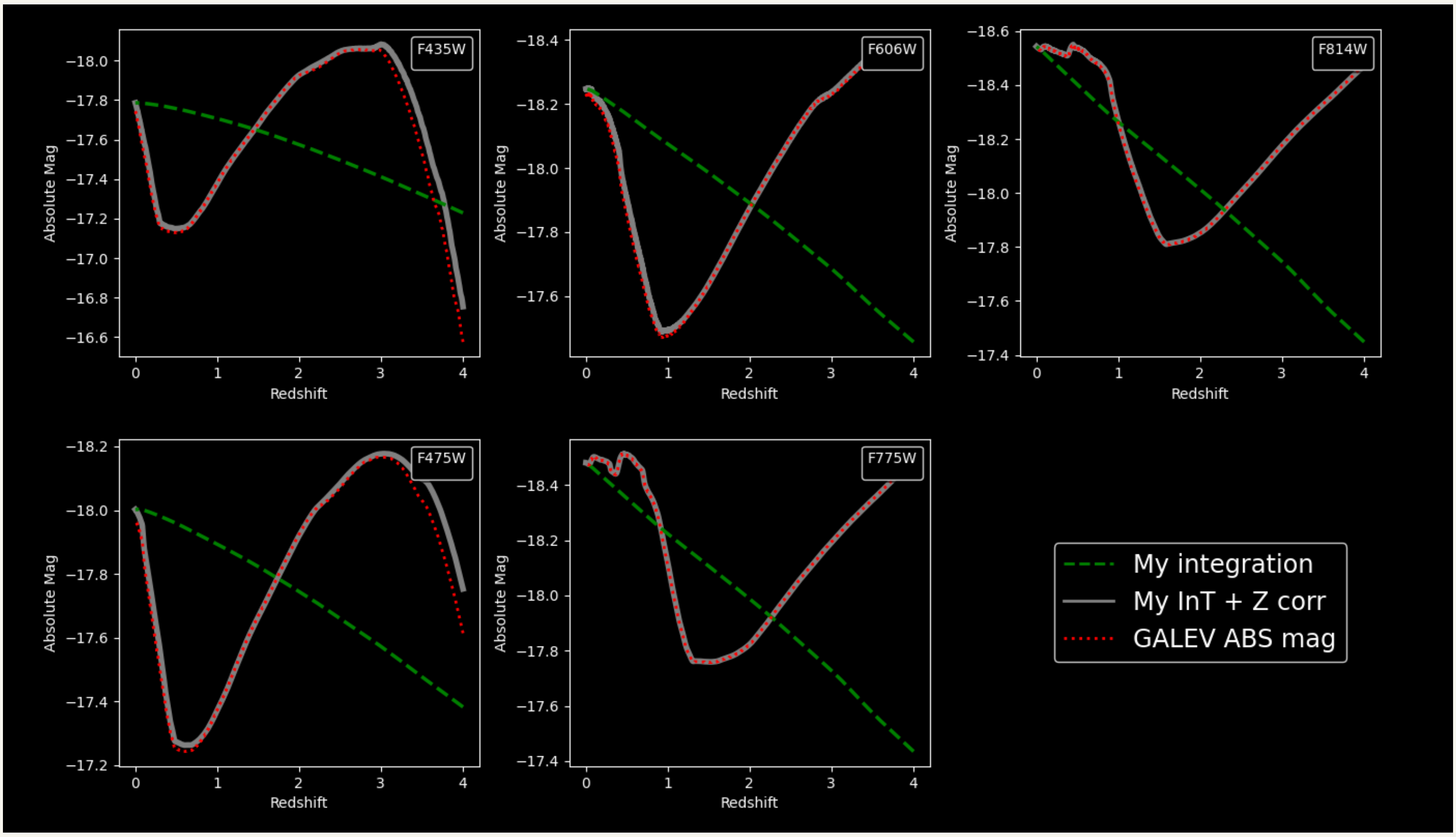


Figure 3: Absolute Magnitudes Per filter computed for me and for GALEV as function of redshit

Due the spectra in the file (model)-spec.dat comes normalized at a distance of 10pc (Kotulla et al. (2009)), if we directly integrate this considering the response of the filters we will get the Absolute Magnitude in those filters, but with the galaxy as is always could be measured in rest frame, so we have to consider the corresponding shift and attenuation due z, The spectrum now has to be redshifted by a factor (1+z) and the flux has to reduced by the same factor (1+z). (in terms of frequency this is applied as $\nu = \nu_{rest}/(1+z)$ and $F_\nu = F_{\nu_{rest}} \cdot (1+z)$.

The behaviour seen in the absolute magnitude in fig(3) can be easily understood due that depending of the redshift the filter will be getting more or less flux due the elongation and shift of the SED (this can be more easily seen in the fig 1 of Muzzin et al. (2013)).

Either way, the exercise ask to directly convolve, so both calculations can be seen in figure (3), where we can compare my integration vs the GALEV Absolute mag (we can notice that there exist some discrepancies between my estimation and the reported by GALEV I presume that this is due I did not apply the e and k correction or by differences in the resolution of the integration).

The integration and computation of the magnitude was done as follows (using AB system of magnitudes)

$$M_X = -2.5\log_{10}\left(\frac{Flux_\nu}{d\nu}\right) - 48.6 \quad \text{with ,} \quad \frac{Flux_\nu}{d\nu} = \frac{\sum_{i=0}^{N-1}\left(\frac{1}{2}(F_{\nu_i}+F_{\nu_{i+1}})\right)\cdot T\left(\frac{1}{2}(\nu_i+\nu_{i+1})\right)\cdot(\nu_{i+1}-\nu_i)\cdot\left(\frac{1}{2}(\nu_i+\nu_{i+1})\right)^{-1}}{\sum_{i=0}^{N-1} T\left(\frac{1}{2}(\nu_i+\nu_{i+1})\right)\cdot(\nu_{i+1}-\nu_i)\cdot\left(\frac{1}{2}(\nu_i+\nu_{i+1})\right)^{-1}}$$

with $\frac{Flux_\nu}{d\nu}$ a flux density defined for the AB system , the limits of ν_0 and ν_N correspond to the edges of transmittance function in the filters , $T(\nu)$ is the transmittance at the specific wavelength/frequency, and as we can see in the equation we just made an addition of bins considering mid height and width , with the specific transmittance and frequency at the center of each bin (the ν and F_ν were always treated in Hz and in $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$ respectively)

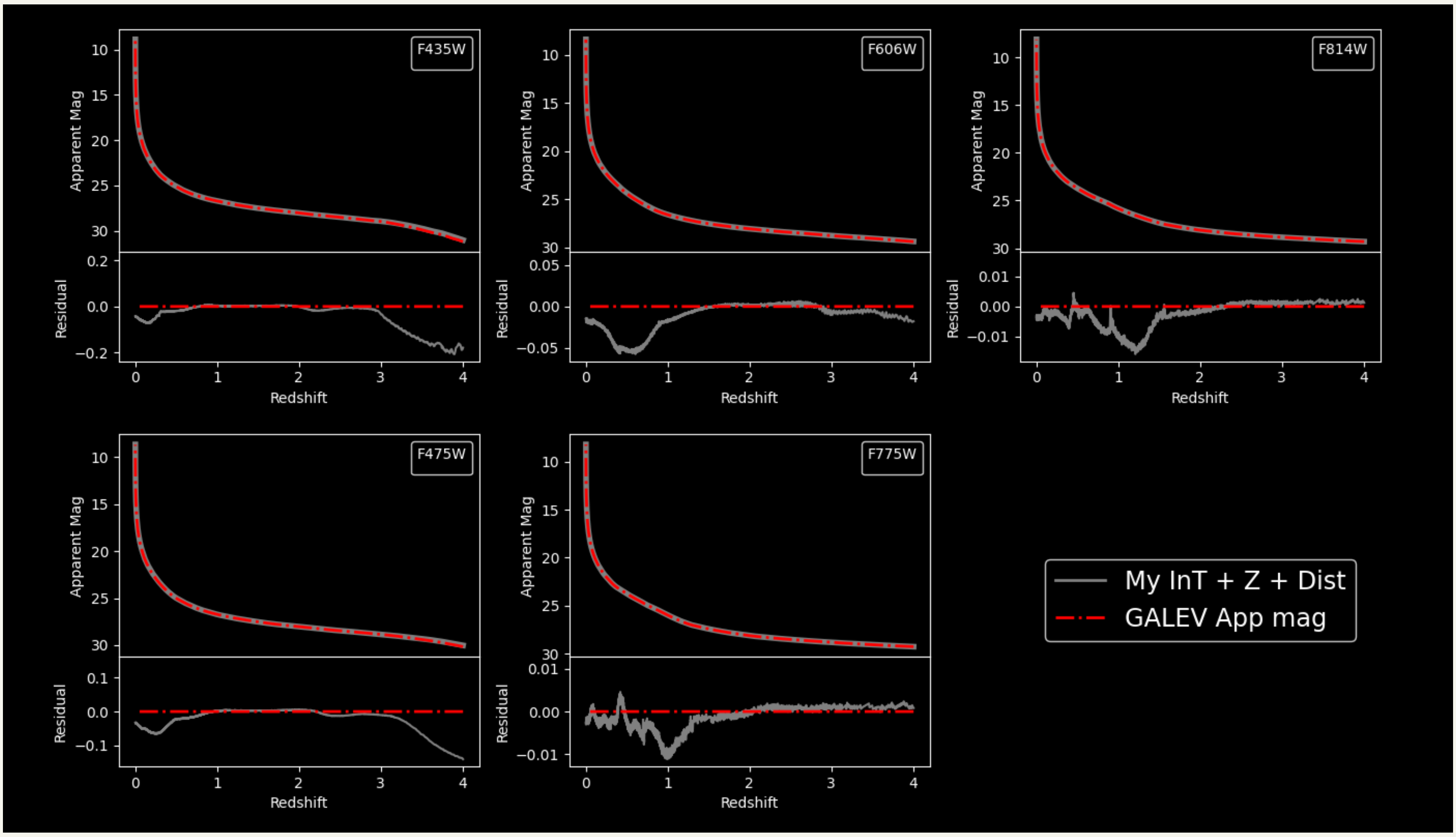


Figure 4: Apparent Magnitudes Per filter computed for me and for GALEV as function of redshit

but the exercise require us to compare Apparent Magnitudes , this means that we have to consider the distance to the galaxy and this depend of the cosmology that we set in the model, fortunately GALEV report the Bolometric distance modulus (μ) in the file (model)-rstat.dat , with this information we can calculate easily the Apparent Magnitude since $m - M = \mu$, so in figure (4) we can see our integration vs the one reported by GALEV, also I included the residuals in order to compare more clearly the values. We can see that my integration is a pretty good approximation to the GALEV values, I associate the discrepancies to the resolution of the integration (more points in the spectra will result in a better integration) and maybe to that I didn't include the e and/or k correction.

D) Assuming that this Sd galaxy has a surface brightness of 28 $mag_{F606W}/arcsec^2$, what is its angular size in the F606W band?

To do this the most simple way it is just use the basic definition of surface brightness

$$S = m + 2.5\log_{10}(A) \quad \Longrightarrow \quad A = 10^{0.4\left(S-m\right)}$$

in this case $S = 28 mag_{F606W}/arcsec^2$ and in the past item we get the Apparent magnitude of the galaxy, therefore in the fig (5) we have an estimation of the angular size as function of redshift

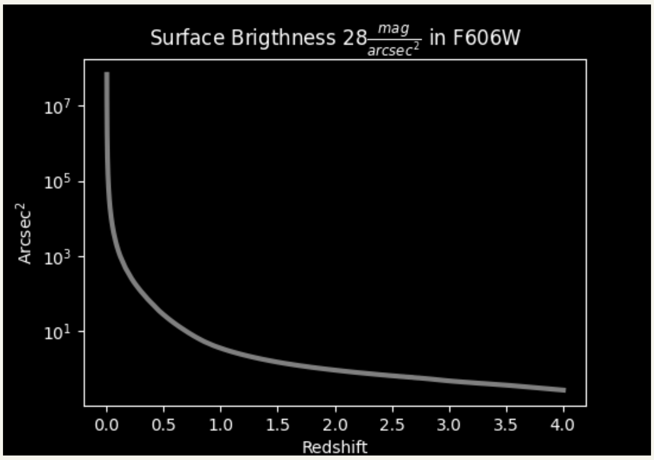


Figure 5: Angular size as function of redshift according to the given value of surface brightness

E) Repeat the exercise for a different star-formation history, like option E (==exponential declining) and comment on the similarities/differences with the Sd choice, such as blue flux, red flux, emission line, accumulated stellar mass/consumed gas mass

Due technical issues with the SED generator we were able to study a Sa Galaxy in this item.

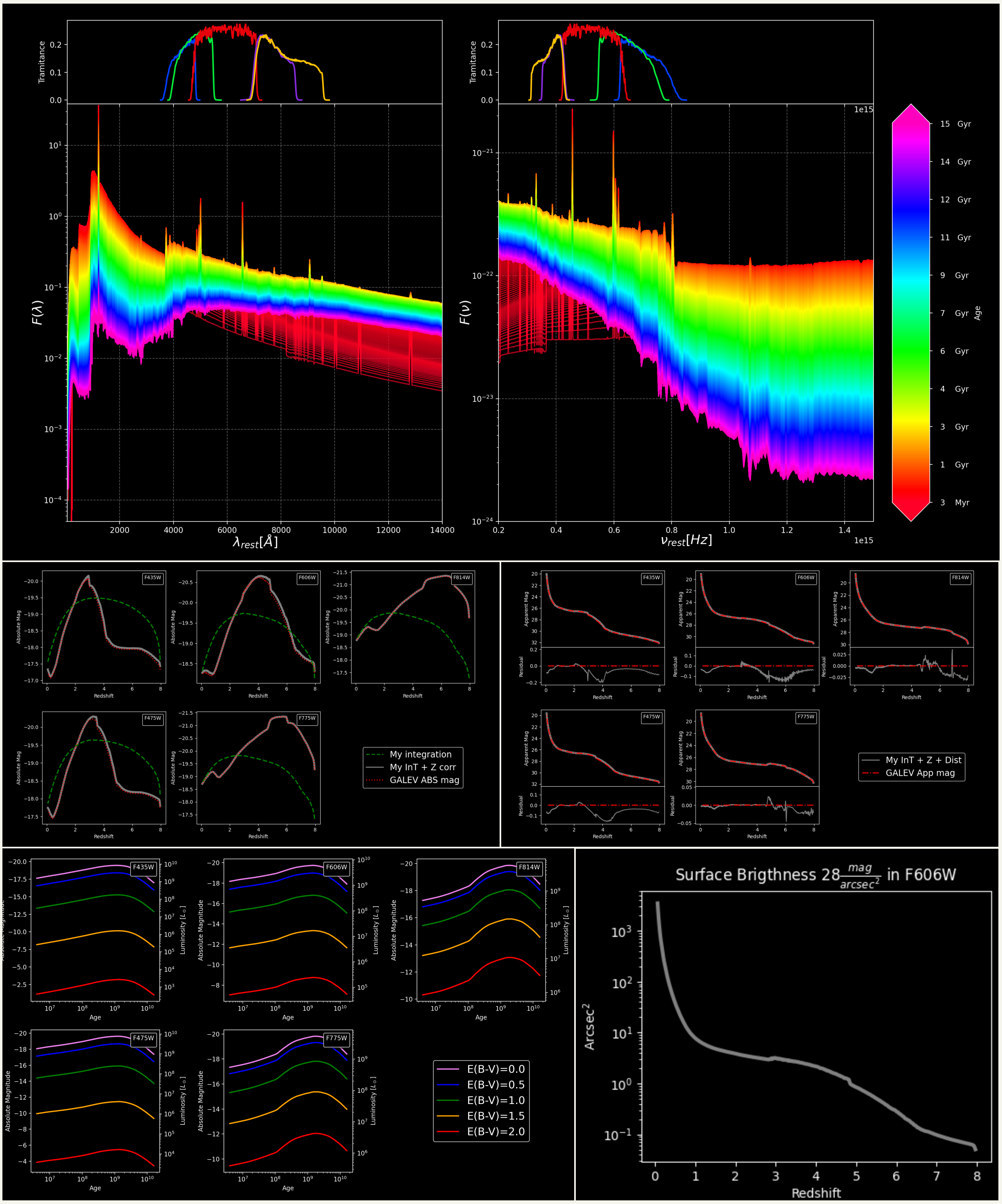


Figure 6: Sa Galaxy plots: From a comparison with previous plots we can see that still we have some similarities with the Sd galaxy , for example the amount of red light increase with time while the blue decreases, the H_α line just decreases with time and becomes almost nonexistent by the end of it's life

F) Repeat the exercise for a Sd but with a different initial mass function, that of Kroupa [0.1- 100 solar masses] and comment on the similarities/differences, such as blue flux, red flux, emission line, accumulated stellar mass/consumed gas mass

Due technical issues with the SED generator we were able to study a E Galaxy in this item.

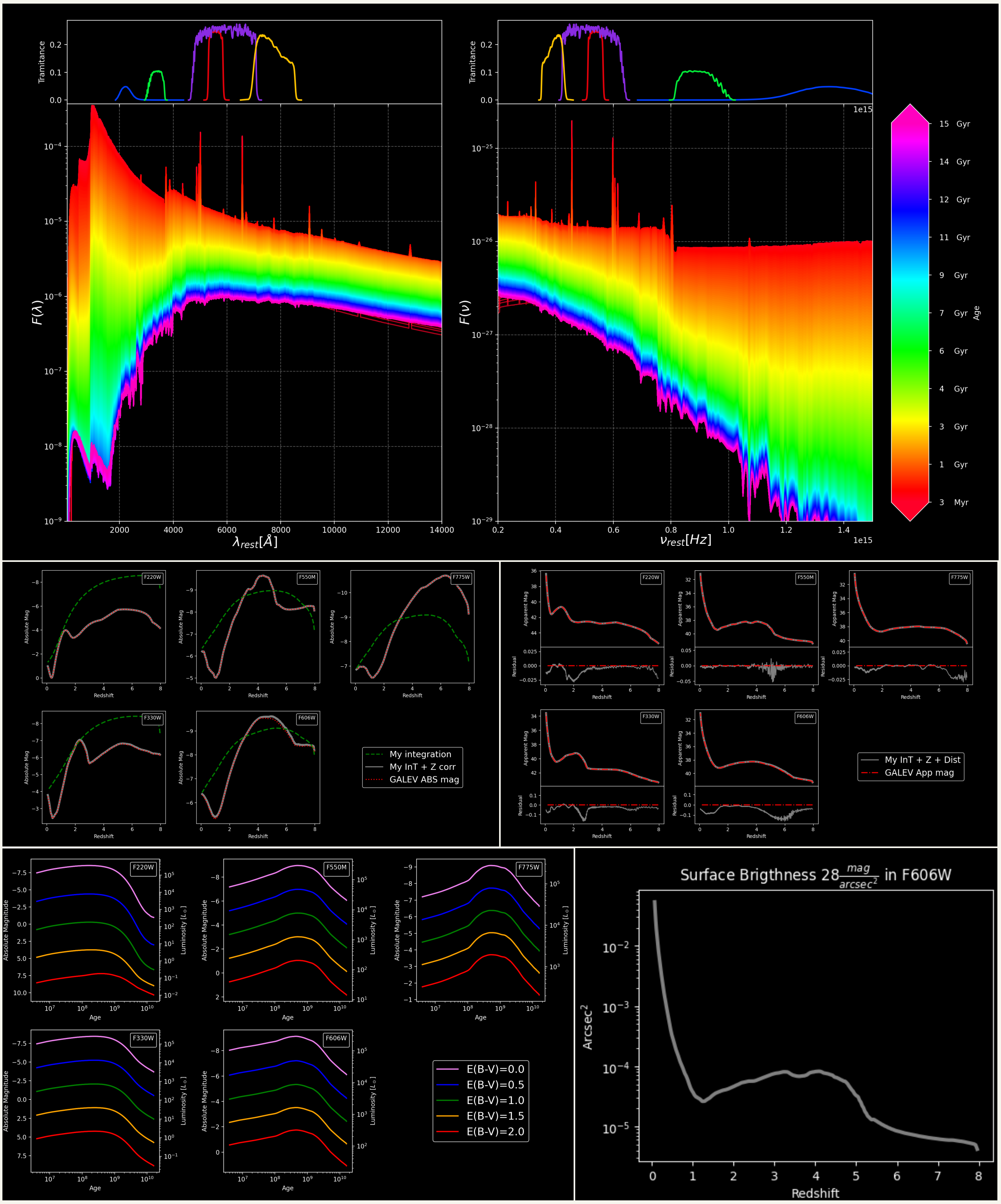


Figure 7: E Galaxy plots: From a comparison with previous plots, we can see that at difference of the Sd galaxy this one starts from the maximum flux at each λ and decreases along the whole spectra as it becomes older, the H_α line quickly disappear, and the blue part it is complete depleted of flux at the end. This galaxy is much fainter than the others.

2 Consider a dark matter halo with mass equal to 1E+10 solar masses at z=3 in virial equilibrium.

$M_{Halo} = 1E+10[M_\odot]$

$z = 3$

A) What is its virial radius at z=3? In which epoch of the Universe we are at z=3?

Assuming a ΛCDM universe and a standard cosmology (which means that $z = 3 \implies \Omega(3) \approx 1$)

Considering the equation,

$$\Delta_{vir} \simeq \frac{18\pi^2 + 82x - 39x^2}{x+1} = \frac{18\pi^2 + 82(0) - 39(0)^2}{0+1} = \frac{18\pi^2}{1} \approx 177.65$$

with, $x = \Omega(z) - 1$

with this value we can get the r_{vir} through,

$$\begin{aligned} r_{vir} &\simeq 163h^{-1}kpc\left[\frac{M_{vir}}{10^{12}h^{-1}M_\odot}\right]^{\frac{1}{3}}\left[\frac{\Delta_{vir}}{200}\right]^{-\frac{1}{3}}\Omega_{m,0}^{-\frac{1}{3}}(1+z)^{-1} \\ &\simeq 163h^{-1}kpc\left[\frac{1E+10M_\odot}{10^{12}h^{-1}M_\odot}\right]^{\frac{1}{3}}\left[\frac{177.65}{200}\right]^{-\frac{1}{3}}(0.3)^{-\frac{1}{3}}(1+3)^{-1} \\ &\simeq 17.31\,kpc \end{aligned}$$

Link to cosmological calculator

and the age at redshift z=3 was 2.171 Gyr since the Big-Bang and this imply that we are at the matter dominated era, just going out from Early Galaxy and Quasar era and getting into the Cosmic noon.

(I would like to thank Benjamin on this exercise because it made me realize that I made a mistake when I copied the equation for r_{vir} at the time I was creating this file and then I used the wrongly written formula to calculate the values.)

B) Can you estimate the Hubble constant at z=3 given all this info about the dark matter halo?

Yes, with this information we are able to infer the Hubble constant (as long as we assume a certain cosmological model), we can use

$$\frac{3M_{vir}}{4\pi r_{vir}^3} = \Delta_{vir}\Omega_m(z)\frac{3H^2(z)}{8\pi G} \implies H(z) = \sqrt{\frac{M_{vir}}{r_{vir}^3}\cdot\frac{2G}{\Delta_{vir}\Omega_m(z)}}$$

if we solve this with the previous values we get an approximate value of $305.67[km/s/Mpc]$.

C) The gas mass fraction in this halo is 20% and the gas is initially in hydrostatic equilibrium with the dark matter halo. The gas is primordial (assume fully ionized) and it is distributed in a spherical volume of 523.6 $[kpc^3]$. Is the gas cloud able to collapse? is it also able to cool? What are the processes that could cool a gas cloud at the T calculated here?[1]

$M_{gas} = \frac{M_{Halo}}{5}$

$M_{gas} = 2E+9[M_\odot]$

$Gas = H_{II}$

$V_g = 523.6[kpc^3]$

$r_g = \sqrt[3]{\frac{3}{4\pi}V} \approx 5[kpc]$

to know if the gas is able to collapse we have to compare with the Jean's mass (which $M_J \propto T^{3/2} \cdot \rho^{-1/2}$) therefore to use this equation we have to estimate the temperature, since the gas it is in the innermost part of the halo , we can say that it is dominated by it's own gravitation and due is in hydrostatic equilibrium that the virial theorem applies in this case, hence we can derive the temperature as follows

$$T_{vir} = \frac{MG\mu m_p}{5k_B R} \implies \frac{2\times 10^9[M_\odot]\cdot G\cdot \mu \cdot m_p}{5k_B\cdot(5[kpc])} \approx 20800K \tag{2.1}$$

for a fully ionized hydrogen gas $\mu = 0.5$

with this information and since we have information about the cosmology context where the halo it is placed we can derive the Jean's Mass as follows

for virialized halo $\delta = 200$

$$\begin{aligned} M_{J,gas} &\approx \left(6.1\times 10^{13}\right)\cdot T_6^{3/2}\cdot f_{gas}\left(1+\delta\right)^{-1/2}\left(\Omega_{m,0}h^2\right)^{-1/2}\left(1+z\right)^{-3/2}[M_\odot] \\ &\approx \left(6.1\times 10^{13}\right)\cdot\left(\frac{208000\,K}{10^6K}\right)^{3/2}\cdot(0.2)\left(1+200\right)^{-1/2}\left(0.3\cdot(0.7^2)\right)^{-1/2}\left(1+3\right)^{-3/2}[M_\odot] \\ &\approx 8.44\times 10^8 M_\odot \end{aligned}$$

which is less than the mass of the gas, this means that the collapse criterion ($M_{gas} > M_J$) is satisfied.

Now to know if it is able to cool we have to compare with the free fall time and the hubble time, if we assume that the density it is constant ($\rho_{gas} = \frac{M_{gas}}{\frac{4}{3}\pi r_g^3}$) we can just get the free-fall time through

$$t_{ff} = \sqrt{\frac{3\pi}{32G\rho_{gas}}} = \sqrt{\frac{\pi^2 r_g^3}{8GM_{gas}}} = \pi\sqrt{\frac{5kpc^3}{8G(2\times 10^9 M_\odot)}} = 130.92\,Myr$$

and from the cosmological calculator we get the Age of the universe at z=3 (2.171 Gyr), so now we have to calculate the cool time , we can do this through

$$t_{cool} \approx 3.3\times 10^9\frac{T_6}{n_{-3}\Lambda_{-23}(T)}\,yr$$

where $\Lambda_{-23}(T)$ is the cooling rate and can be obtained from figure (8), $T_6 = T[K]/10^6[K]$ and

$$n_{-3} \approx 1.9\times 10^{-2}\cdot f_{gas}\left(1+\delta\right)\left(\Omega_{m,0}h^2\right)\left(1+z\right)^3$$

using these equations (and values of$\Lambda_{-23}(T) \approx 13$) we get a value of 5.3 Myr which will imply that the cloud is able to cool efficiently
(since $t_{cool} < t_{ff} < t_H$).

At these temperatures the most important process of cooling are the Radiative ones (free-Bound, bound-bound, etc)

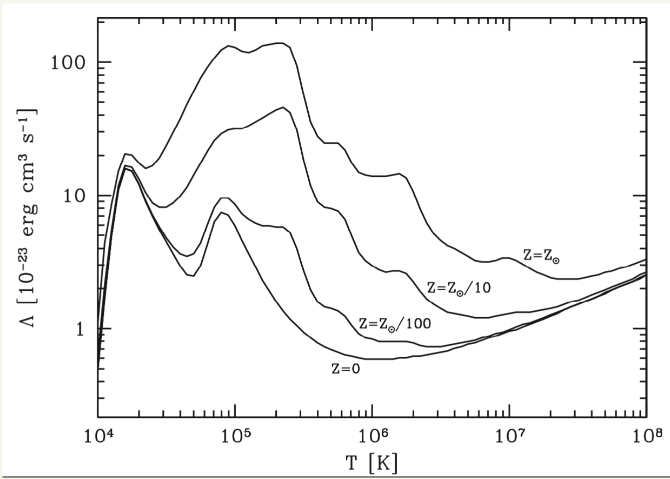


Figure 8: Figure taken from Class 8 (cooling rate from Mo et al. (2010))

[1](on this question I discussed with Benjamin and Mamta because I was a bit confused about which of the expressions I should use to infer a temperature and the mass of Jean, since there are two versions one that includes cosmology and one that does not).

3 2 point correlation function

1.a) What is a 2-point correlation function?

The the 2-point correlation function it is defined as the excess likelihood of finding two objects at a given pair of locations over what would be expected for a random distribution. It is commonly used to explore the universe's large-scale structure, such as the distribution of galaxies and the clustering of dark matter.

1.b) What is the expected value of the 2-point correlation function of dark matter halos at z=3, at scales of 20 kpc, 200 kpc, 2000 kpc?, and what are the expected values at z=0?

The approximate values for the Two point correlation function at redshift 3 are:

- $\xi(20kpc) \approx 230$
- $\xi(200kpc) \approx 23$
- $\xi(2000kpc) \approx 0.53$

While the approximate values for the Two point correlation function at redshift 0 are:

- $\xi(20kpc) \approx 5300$
- $\xi(200kpc) \approx 700$
- $\xi(2000kpc) \approx 13$

these values can be obtanied from the figure (9), where the two point correlation function was calculated depending of the redshift

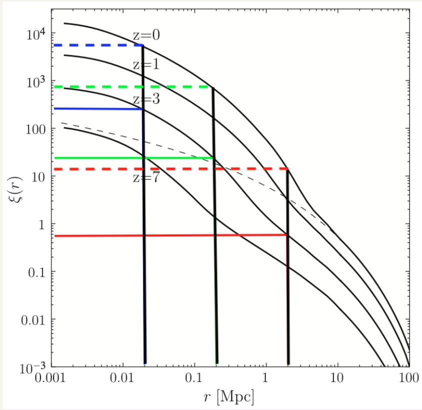
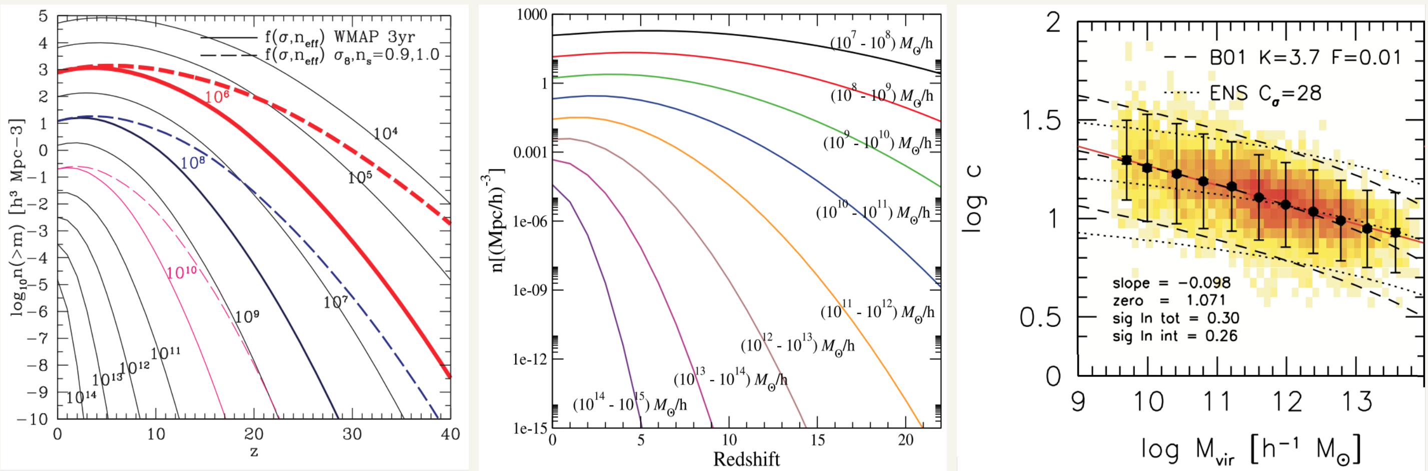


Figure 9: Figure taken from Class 7 (numerical simulations from Boylan-Kolchin et al. (2009))

2) What is the expected concentration of dark matter halos of 1E+10 solar masses at z=0? (Comment)



(a) Representative Halo Mass Function from a Warren mass function. (Reed et al., 2007)

(b) *Representative Halo Mass Function from numerical simulations. (Lukić et al., 2007)*

(c) *Representative C parameter from numerical simulations. (Macciò et al., 2007)*

Figure 10: About "Concentration" of Dark matter halos

If we are talking about number of Halos with this mass per unit of volume in the universe (Halo Mass Function), we should expect a large number of them relative to more massive ones , however this is highly dependant of the cosmology an the model used in figs (10a,10b) it is showed some numerical simulations, from these I could say that at least we expect around 1 DM halo of this mass per Mpc^3 at redshift 0.

On the other hand if we are talking about the concentration of the halo itself (concentration parameter) , since our current model of galaxy formation and evolution consist in a hierarchical model we expect that these relative "less massive" halos are more concentrated than the larger ones as we can see in fig (10c), and from this we can infer a value of $\log(c) \approx 1.3$

3.1 Calculate the 2-point correlation function between the following points and infer at which redshift the points could be

A) between P1 and P2 at r=20kpc where P1 is at x with density=100 times the average density and P2 is at x+r with density 50 the average density.

Considering this form of the two point correlation function

$$\xi(r) = \frac{<(\rho(x) - <\rho>)(\rho(x+r) - <\rho>)>}{<\rho>^2} \tag{3.1}$$

and the figure (9)

$$\begin{aligned}
\xi(20kpc) &= \frac{<(\rho(x) - <\rho>)(\rho(x+20kpc) - <\rho>)>}{<\rho>^2} \\
&= \frac{<(100<\rho> - <\rho>)(50<\rho> - <\rho>)>}{<\rho>^2} \\
&= \frac{<(99 \cdot 49) \cdot \cancel{(<\rho>)^2}>}{\cancel{<\rho>^2}} \\
\xi(20kpc) &= 4851
\end{aligned}$$

with this value at a r of 20 kpc , we can look where this values are matched in fig (9) and we estimate an approximate value of $z \sim 0.1$

B) between P1 and P2 at r=0.2Mpc where P1 is at x with density=3 times the average density and P2 is at x+r with density 1.5 the average density

$$\begin{aligned}
\xi 200kpc) &= \frac{<(\rho(x) - <\rho>)(\rho(x+200kpc) - <\rho>)>}{<\rho>^2} \\
&= \frac{<(3<\rho> - <\rho>)(1.5<\rho> - <\rho>)>}{<\rho>^2} \\
&= \frac{<(2 \cdot 0.5) \cdot \cancel{(<\rho>)^2}>}{\cancel{<\rho>^2}} \\
\xi(200kpc) &= 1
\end{aligned}$$

with this value at a r of 200 kpc , we can look where this values are matched in fig (9) and we estimate an approximate value of $z \sim 7.5$

C) between P1 and P2 at r=2Mpc where P1 is at x with density=500 times the average density and P2 is at x+r with density 1.1 the average density

$$\begin{aligned}
\xi 2000kpc) &= \frac{<(\rho(x) - <\rho>)(\rho(x+2000kpc) - <\rho>)>}{<\rho>^2} \\
&= \frac{<(500<\rho> - <\rho>)(1.1<\rho> - <\rho>)>}{<\rho>^2} \\
&= \frac{<(499 \cdot 0.1) \cdot \cancel{(<\rho>)^2}>}{\cancel{<\rho>^2}} \\
\xi(2000kpc) &= 49.9
\end{aligned}$$

with this value at a r of 2000 kpc , we can look where this values are matched in fig (9) and we estimate an approximate value of $z \sim 0$ (in this projection the value is over the line of redshift 0 probably this is a extreme scenario that can not be well reproduced by the general behaviour, or we are trying with values that can only be achivied in the future)

References

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