

**LABVIEW Project report**

**The inverted pendulum stabilized by a flywheel, a  
under-actuated non-linear system**

**prepared by**

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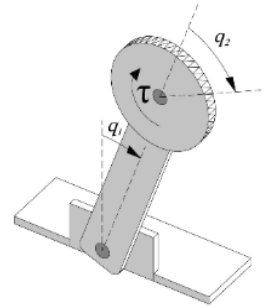
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# 1 Introduction

## 1.1 Scope of the Project:

The Inertia Wheel Pendulum is a benchmark for nonlinear control of under actuated mechanical systems consisting of a physical pendulum with a symmetric disk attached to the tip, which is free to spin about an axis parallel to the axis of rotation of the pendulum. Apart from that, the dynamics of inverted pendulum resembles many real-time systems such as under-actuated robot manipulators, gymnast robots, undersea vehicles, aircrafts, some mobile robots and humanoid robots. In control theory, inertia wheel pendulum is being treated as one of the most attractive systems to test emerging Control strategies due to its distinct features of instability, nonlinearity and under actuation.



## 1.2 Motivation goals:

From the last few decades, inertia wheel pendulums have become a benchmark problem in dynamics and control theory. Due to their inherent nature of nonlinearity, instability and under actuation. The dynamics of inertia wheel pendulum systems resemble many real-world systems such as Atlas Humanoid robot, it is primarily developed by Boston Dynamics, with funding and oversight from the Defence Advanced Research Projects Agency (DARPA). This 1.88 m and 82 kg model is designed for various researches and rescue tasks, it can carry packages, open doors but also stand up when it is pushed or unbalanced. It is designed to push the limits of whole-body mobility and to demonstrate human-level agility.



## 1.3 presentation of the benchmark:

### 1.3.1 Inertia wheel pendulum figure

The Reaction Wheel Pendulum is shown schematically in Fig. 1. It is a physical pendulum with a symmetric disk attached to the end which is free to spin about an axis parallel to the axis of rotation of the pendulum. The disk is actuated by a DC-motor and the coupling torque generated by the angular acceleration of the disk can be used to actively control the system.

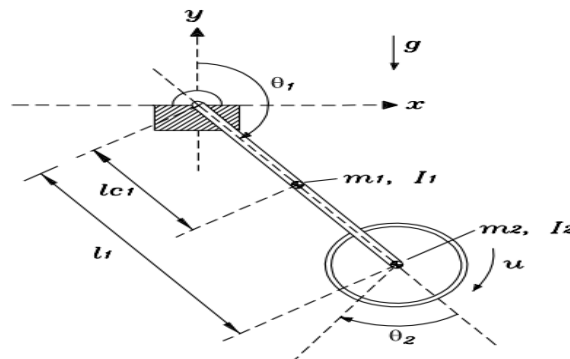


Fig. 1. Inertia wheel pendulum

### 1.3.2 Tab1.Description of dynamic parameters of the inertia wheel inverted pendulum

Notation	Description	value
l1	Length of the pendulum	.....m
lc1	Distance at the center of mass lc1 m of the pendulum	.....m
m1	Mass of the pendulum	.....kg
m2	Mass of the disc	....kg
I1	Moment of inertia of the pendulum	....kg.m2
I2	Moment of inertia of the wheel	....kg.m2
g	Gravity acceleration	.....m/s2

### 1.3.3 Tab2.Description of dynamic variables of the inertia wheel inverted pendulum

Variables	Description	Unit
$\theta_1$	Pendulum angular position	rad
$\dot{\theta}_1$	Pendulum angular velocity	Rad/s
$\ddot{\theta}_1$	Pendulum angular acceleration	Rad/s <sup>2</sup>
$\theta_2$	Wheel angular position	rad
$\dot{\theta}_2$	Wheel angular velocity	Rad/s
$\ddot{\theta}_2$	Wheel angular acceleration	Rad/s <sup>2</sup>
u	Motor torque	Nm

## 2 Theoretical study:

### 2.1 modelling hypothèses

- ✓ Hypothesis 1: The masses of the pendulum and the inertia wheel are considered to be point masses located at their centres of gravity.
- ✓ Hypothesis 2: The study of the dynamics of the inertia wheel pendulum is carried out by neglecting the friction mechanical phenomenal.
- ✓ Hypothesis 3: The dynamics of the actuator motor associated with the wheel is not taken into account in the modelling of the system

### 2.2 Dynamic model of the system using Lagrange model :

The nonlinear model of the inertia wheel pendulum can be also obtained using Lagrange equation. This method consist of the calculation of the Lagrangian. The Lagrange formalism is based on the equation:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = Q$$

Where  $Q = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot u$ ,  $q = (\theta_1 \quad \theta_2)^T$ , the Lagrangian is given by  $L=K-V$  where K is the kinetic energy and is the potential energy. For the nonlinear systems these energies can be written as the following:

$\begin{cases} K = \frac{1}{2} \dot{q}^T \cdot M \cdot \dot{q} \\ v = \beta \cos(\theta_1) \end{cases}$  Where  $M$  is the system matrix given by  $M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix}$ , the different coefficients are:

$\begin{cases} m_{11} = m_1 \cdot l_1^2 + m_2 l_1^2 + I_1 + I_2 \\ m_{12} = m_{21} = m_2 l_1 \end{cases}$  So we can tell the the expression of the kinetic energy is

$$K = \frac{1}{2} (m_1 \cdot l_1^2 + I_1) \dot{\theta}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{\theta}_2^2 + \frac{1}{2} (\dot{\theta}_1 + \dot{\theta}_2)^2$$

The  $\beta$  coefficient in the potential energy is given by  $\beta = (m_1 \cdot l_1 + m_2 \cdot l_1) \cdot g$ , so the potential energy is:

$$V = (m_1 \cdot l_1 + m_2 \cdot l_1) \cdot g \cdot \cos(\theta_1)$$

In result the Lagrangian expression is:

$$L = \frac{1}{2} (m_1 \cdot l_1^2 + I_1) \dot{\theta}_1^2 + \frac{1}{2} m_2 l_1^2 \dot{\theta}_2^2 + \frac{1}{2} (\dot{\theta}_1 + \dot{\theta}_2)^2 - (m_1 \cdot l_1 + m_2 \cdot l_1) \cdot g \cdot \cos(\theta_1)$$

We begin by calculating the first term which is  $\frac{\partial L}{\partial q} = \begin{pmatrix} \frac{\partial L}{\partial \theta_1} \\ \frac{\partial L}{\partial \theta_2} \end{pmatrix} = \begin{pmatrix} (m_1 \cdot l_1 + m_2 \cdot l_1) g \sin(\theta_1) \\ 0 \end{pmatrix}$

Then we calculated the second term which is

$$\frac{\partial L}{\partial \dot{q}} = \begin{pmatrix} \frac{\partial L}{\partial \dot{\theta}_1} \\ \frac{\partial L}{\partial \dot{\theta}_2} \end{pmatrix} = \begin{pmatrix} (m_1 \cdot l_1^2 + m_2 \cdot l_1^2 + I_1) \cdot \dot{\theta}_1 + I_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ I_2 (\dot{\theta}_1 + \dot{\theta}_2) \end{pmatrix}$$
 the last term that need to be

calculated now is

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) = \begin{pmatrix} \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_1} \right) \\ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_2} \right) \end{pmatrix} = \begin{pmatrix} (m_1 \cdot l_1^2 + m_2 \cdot l_1^2 + I_2 + I_1) \cdot \ddot{\theta}_1 + I_2 \cdot \ddot{\theta}_2 \\ I_2 (\ddot{\theta}_1 + \ddot{\theta}_2) \end{pmatrix}$$

So the Lagrangian equation can be written as:

$$\begin{pmatrix} (m_1 \cdot l_1^2 + m_2 \cdot l_1^2 + I_2 + I_1) \cdot \ddot{\theta}_1 + I_2 \cdot \ddot{\theta}_2 - (m_1 \cdot l_1 + m_2 \cdot l_1) g \sin(\theta_1) \\ I_2 (\ddot{\theta}_1 + \ddot{\theta}_2) \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot u$$

Finally, the dynamics that characterized the inertia wheel pendulum is

$$\begin{cases} (m_1 \cdot l_1^2 + m_2 \cdot l_1^2 + I_2 + I_1) \cdot \ddot{\theta}_1 + I_2 \cdot \ddot{\theta}_2 - (m_1 \cdot l_1 + m_2 \cdot l_1) g \sin(\theta_1) = 0 \\ I_2 (\ddot{\theta}_1 + \ddot{\theta}_2) = u \end{cases}$$

### 2.3 PID theory:

For the control of the stability, we used the Proportional-Integral-Derivative (PID) control which is the most common control algorithm used in industry and has been universally accepted in industrial control. The popularity of PID controllers can be attributed partly to their robust performance in a wide range of operating conditions and partly to their functional simplicity, which allows engineers to operate them in a simple, straightforward manner. As the name suggests, PID algorithm consists of three basic coefficients; proportional, integral and derivative which are varied to get optimal response. Closed loop systems, the

theory of classical PID and the effects of tuning a closed loop control system are discussed in this paper. The PID toolset in Labview and the ease of use of these VIs is also discussed.

### 2.3.1 Proportional Response

The proportional component depends only on the difference between the set point and the process variable. This difference is referred to as the Error term. The *proportional gain* ( $K_c$ ) determines the ratio of output response to the error signal. In general, increasing the proportional gain will increase the speed of the control system response. However, if the proportional gain is too large, the process variable will begin to oscillate. If  $K_c$  is increased further, the oscillations will become larger and the system will become unstable and may even oscillate out of control.

### 2.3.2 Integral Response

The integral component sums the error term over time. The result is that even a small error term will cause the integral component to increase slowly. The integral response will continually increase over time unless the error is zero, so the effect is to drive the Steady-State error to zero. Steady-State error is the final difference between the process variable and set point. A phenomenon called integral windup results when integral action saturates a controller without the controller driving the error signal toward zero.

### 2.3.3 Derivative Response

The derivative component causes the output to decrease if the process variable is increasing rapidly. The derivative response is proportional to the rate of change of the process variable. Increasing the *derivative time* ( $T_d$ ) parameter will cause the control system to react more strongly to changes in the error term and will increase the speed of the overall control system response. Most practical control systems use very small derivative time ( $T_d$ ), because the Derivative Response is highly sensitive to noise in the process variable signal. If the sensor feedback signal is noisy or if the control loop rate is too slow, the derivative response can make the control system unstable.

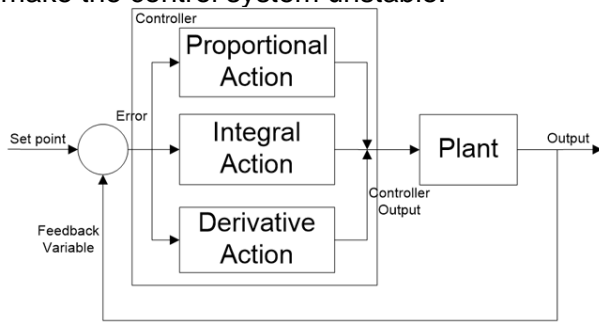


Fig.2 Block diagram of a basic PID control algorithm.

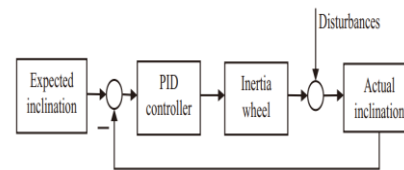


Fig3. Closed loop using a PID

## 3 Experimental study:

### 3.1 Hardware Conception:

#### 3.1.1 Mechanical Hardware

It is composed of a support which supports the pendulum through a passive link (pivot). The body of the pendulum supports a flywheel operated by a direct current motor. The rotation of

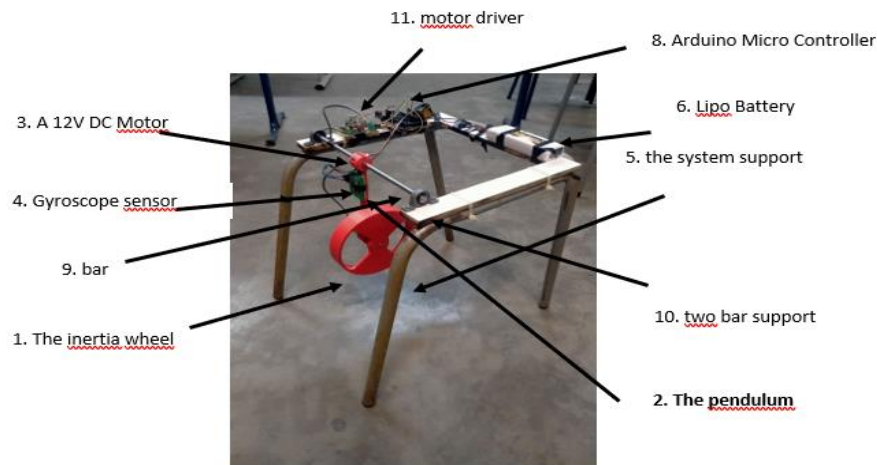
Flywheel causes by the dynamic effects, which it induces, the rotation of the pendulum around of its passive bond. These are the main components that we worked with:

- the support
- the pendulum
- the Inertia wheel
- bar
- 2 bars supports

### 3.1.2 Electrical Hardware:

- Arduino Mega
- 12v DC Motor :
- Lipo Battery
- L298 power card
- Three axis gyroscope
- Electrical wires.
- 

### 3.1.3 real model :

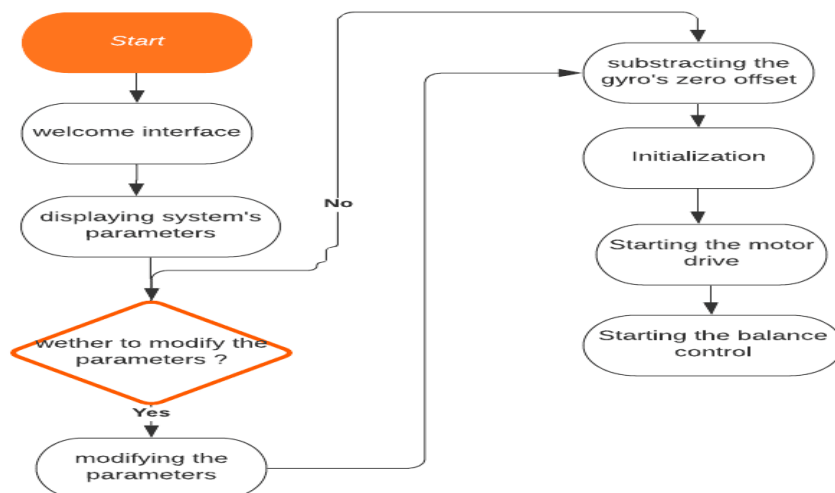


## 3.2 Software conception:

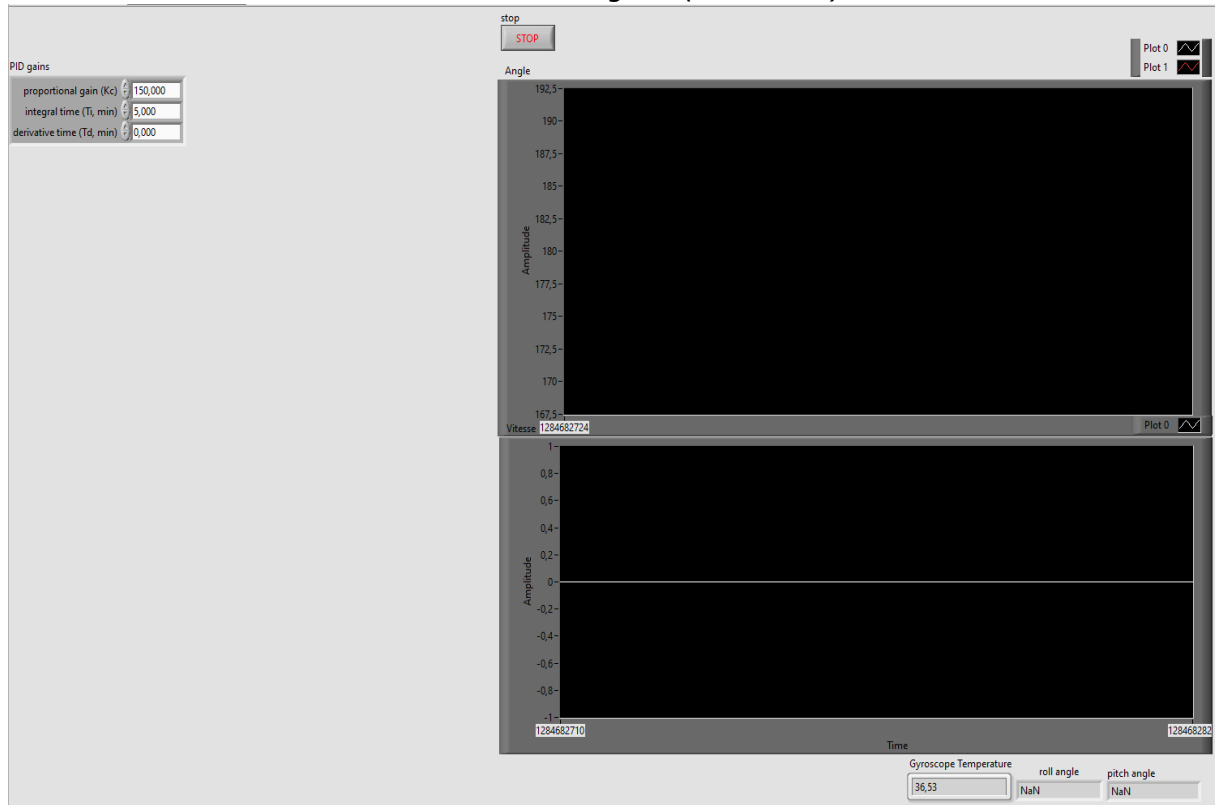
### 3.2.1 Labview:

#### 3.2.1.1 Flow chart Diagram:

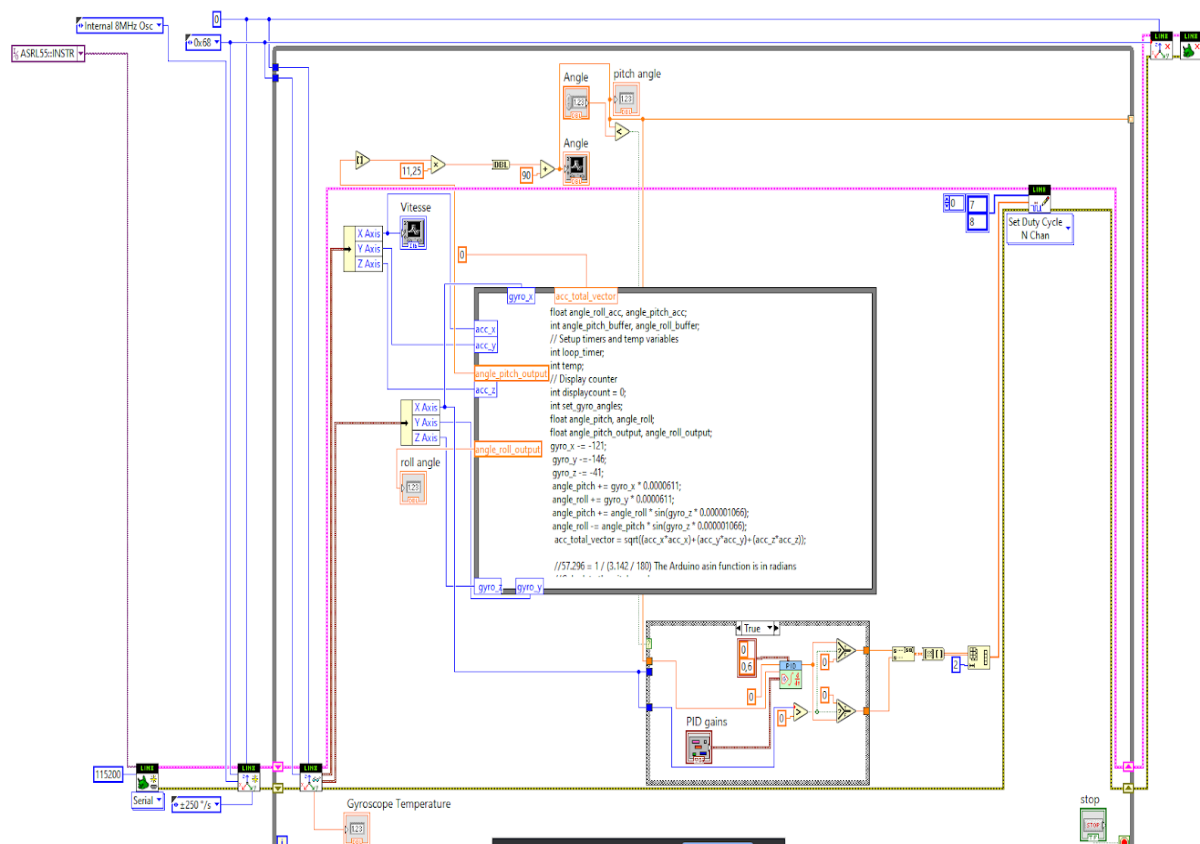
This flow chart diagram showed below describes the process of controlling the inertia wheel pendulum on Labview:



### 3.2.1.2 Front Panel Diagram (Dashboard):



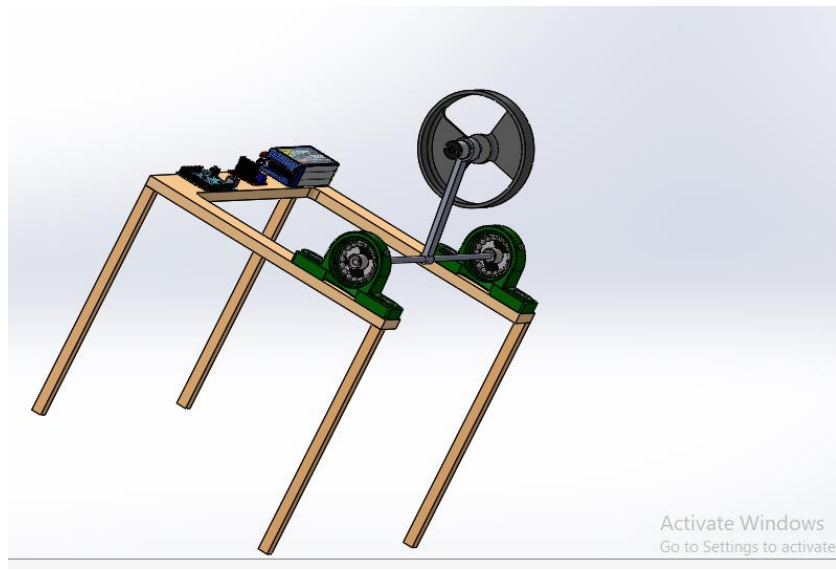
### 3.2.1.3 Bloc Diagram:





### 3.2.2 Solidworks:

The conception of our model was realized on the Solidworks Software. The screen capture below presents most of the material that we've used in this project:



### 3.3 communication technology

**Bluetooth:** It is an unwired method that use bluetooth module to ensure the communication between the labview and the model

=> No real time results, Problem of latency

**Serial:** It is a wired method that uses a cable to communicate the software with the model

## 4 Results:

This chart presents the variation of the pitch angle, it takes values between 0 and 180 degrees where 180 represents the stable equilibrium point (uppoint) and the 0 represents the instable equilibrium point (downpoint)

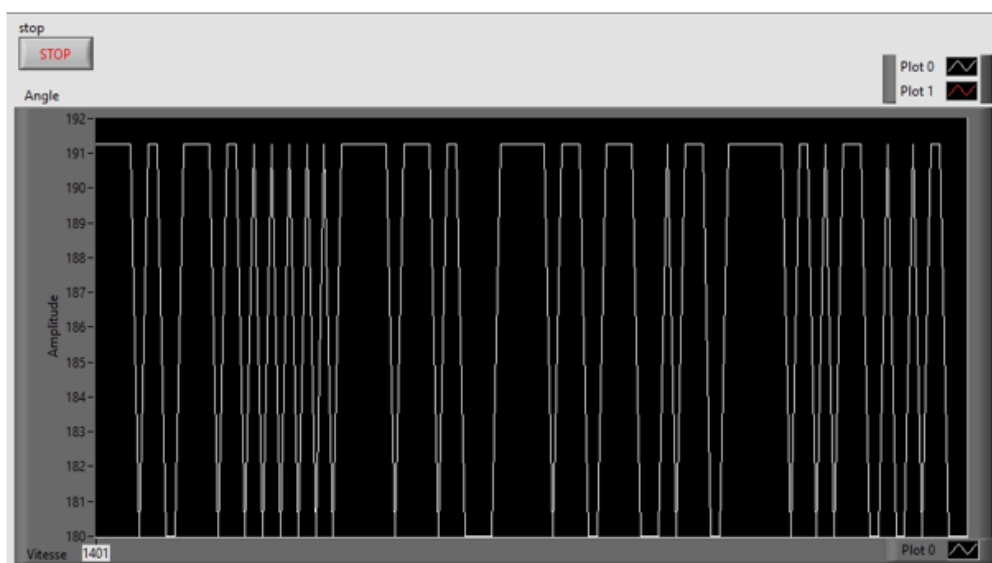


Chart1: pitch angle variation (the angle with respect to the x axis)

This chart represents the acceleration with respect to x axis, it takes positive values if the wheel turns on the clockwise and negative values it turns on indirect clockwise.

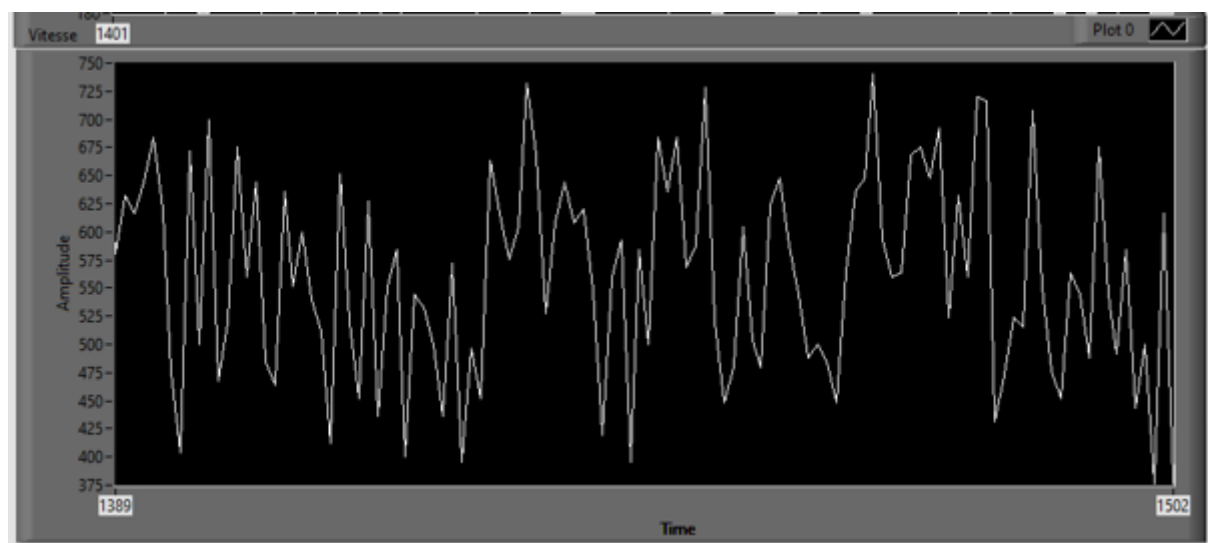


Chart 2: the acceleration of the wheel with respect the x axis

these three indicators represent respectively the gyroscope intern temperature, the roll angle and the pitch angle.

