Relationship between θ and t_w

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Given θ from the 2d Gaussian model which is between $(0, \pi)$ and is the angle from the positive time axis to the semi-major axis of the ellipse, the drift $d\nu_{obs}/dt_D$ is calculated according to

$$\frac{d\nu_{obs}}{dt_D} = \frac{\nu_{res}}{t_{res}} \tan(\theta), \tag{1}$$

where ν_{res} and t_{res} are the spectral and time resolutions of the burst waterfall, respectively. In order to find a relationship between the model angle and burst duration t_w we substitute into eq (7) of Rajabi et al. and find

$$-\frac{\nu_{res}}{t_{res}} \tan(\theta) = \frac{A}{t_w}, \tag{2}$$

$$\tan(\theta) = \frac{A'}{t_w}. \tag{3}$$

In order to compare θ and t_w directly we can invert the $\tan(\theta)$, however we must first introduce a substitution $\theta = \theta' - \pi/2$, so that the function is on the same range as $\arctan(\theta)$, ie., $(-\pi/2, \pi/2)$. Then,

$$\tan(\theta) = \frac{A'}{t_w},, \tag{4}$$

$$\tan(\theta' - \pi/2) = \frac{A'}{t_w} \tag{5}$$

$$\frac{\sin(\theta' - \pi/2)}{\cos(\theta' - \pi/2)} = \frac{A'}{t_w}, \tag{6}$$

$$-\frac{1}{\tan(\theta')} = \frac{A'}{t_w}, \tag{7}$$

$$\theta' = \arctan(t_w/A''), \tag{8}$$

where the new constant A'' absorbs all previous constants and the negative sign.