Supplementary Materials

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Materials and Methods

This section will describe the process of deriving measurements of the sub-burst drift rate and the burst duration from dynamic spectra of Fast Radio Bursts, and is largely based on the autocorrelation technique described in [1, 2]. In addition, we describe tests to the robustness of the inverse trend between the drift rate and the burst duration through variations of the DM, which show that small variations of the DM serve to rotate the FRB dynamic spectra, and, even though they affect the measured value of the sub-burst drift rate, do not affect the relative trend between bursts.

Dynamic spectra of bursts are obtained from several sources and processed based on the format they are made available in, and are typically dedispersed to the DMs presented in their respective publications. Bursts from the repeater FRB121102 were obtained from [3, 4, 2]. These data were taken as presented in Fig. 2 of [3], Fig. 1 of [4], and Fig. 5 of [2], and range in frequency from ˜600 Mhz to 8 GHz. For all bursts from FRB121102, we used the structure-optimized dedispersed spectra without modification which corresponds to DMs of 565, 559.7, and 563.6 pc/cm^3 from [3, 4, 2], respectively. Data for FRB180916.J0158+65 and FRB180814.J0422+73 are available from the public data archive for the CHIME telescope (chime-frb-open-data.github.io) [5, 6]. Dedispersed bursts from FRB180916.J0158+65 are used at a DM of 348.82 pc/cm^3, though we also investigated the effect of small variations of this DM, as discussed later. Data for FRB180814.J0422+74 is provided at its original resolution and without dedispersion. The range of DMs used for this source in [5] vary between 188.9-190 pc/cm^3. We find that a good fit between this data and our model exists for DMs in between 188.9-189.4 pc/cm^3. However we will tentatively argue that the shape of the FRB180814.J0422+7 bursts suggests too aggressive of a dedispersion and that a slightly lower DM of 188.7-188.8 pc/cm^3 is optimal in terms of the burst shape expected when taken in context with the bursts from the other two sources. In all cases dynamic spectra are downsampled in frequency to increase SNR. When a dynamic spectrum consists of a train of multiple bursts, we separate the components and measure the drift rate and duration of each burst separately, as described in Fig 3. of [7].

Even when from the same source, fast radio bursts can have DMs that differ from each other and that potentially also vary with time [3], which raises questions about which DM is appropriate to use. For this work we choose a single DM per source since multiple bursts from short duration pulse trains (which should have a single canonical DM) obey the inverse relationship between the sub-burst drift rate and burst duration, and a single DM simplifies the analysis. We found that small variations in the DM (on the order of 5%) can increase the spread of the points on a drift vs. duration plot and can translate the points up or down, which clearly affects the fit, but the existence of the relationship in general remains. Additionally, because the DM is primarily a property of the interstellar medium and not the source, any model that requires a fine-tuned selection of DMs adds additional parameters and complexity that require justification. For example, a model that selects DMs based on maximizing structure [3, 1] as a selection criteria brings with it assumptions that components of the burst are emitted (and arrive) simultaneously across a large frequency range [1]. While this criteria advantageously gives a narrow range of DMs to choose from, there is no reason to strictly adhere to it from burst to burst. Therefore, in general, we limit ourselves to the range of DMs found from all the bursts as a whole when considering variations, and for the analysis proper we select a single DM for every burst from a particular source.

The general pipeline that every burst is put through is written in Python and consists of computing the autocorrelation of the signal, fitting a two-dimensional Gaussian to the resulting autocorrelation, and a calculation of the physical quantities of interest from the fit parameters: namely the sub-burst drift rate and the burst duration. The autocorrelation of the spectrum measures the self-similarity of the burst in frequency and time space and for FRBs results in an ellipsoid with an intensity that follows a 2D Gaussian [1]. Before autocorrelation and depending on the source and/or burst, some noise removal is performed. For the bursts from FRB121102 and FRB180916.J0158+65 this is done by subtracting the background from the entire spectrum, which is obtained from a time-average of twenty or so samples taken prior to the burst. For FRB180814.J0422+73, due to the raw format the bursts are provided in, a noise mask was acquired through correspondence and the channels are normalized by the standard deviation of the intensity. We compute the autocorrelation of the dynamic spectrum using an intermediate FFT, which allows us to perform the correlation through a multiplication and is much faster. After multiplication we perform an inverse FFT and the result is the autocorrelation of the burst. The shape of the autocorrelation is an ellipsoid centered about its peak that varies in width and angle. Thus, the next step is to find parameters for the following functional form of a rotate-able two-dimensional Gaussian,

$$G(x,y) = A\exp(-a(x-x_0)^2 - b(x-x_0)(y-y_0) - c(y-y_0)^2), \tag{1}$$

$$a = \frac{\cos^2(\theta)}{2\sigma_x^2} + \frac{\sin^2(\theta)}{2\sigma_y^2}, \tag{2}$$

$$b = \frac{\sin(2\theta)}{2\sigma_x^2} - \frac{\sin(2\theta)}{2\sigma_y^2}, \tag{3}$$

$$c = \frac{\sin^2(\theta)}{2\sigma_x^2} + \frac{\cos^2(\theta)}{2\sigma_y^2}, \tag{4}$$

where the free parameters are $A, x_0, y_0, \sigma_x, \sigma_y$, and θ, which are the amplitude, position of the center, standard deviation along the x and y axes, and angle from the positive x-axis to the semi-major axis of the ellipsoid. To find these parameters we use the `scipy.optimize.curve_fit` package, which performs a non-linear least squares to find a fit. The package also returns a covariance matrix, which we calculate the standard deviation of the parameters from by taking the square root of the diagonal terms. These errors are then scaled by the reduced χ^2 computed from the residual between the autocorrelation and the Gaussian fit. The error calculated this way while useful does not capture the entire error budget which depends more significantly on the error in the DM as well the parts of the burst spectra that have been masked out. With the parameters found, we check which of the two widths σ_x and σ_y is larger and rotate θ by $\pi/2$ to coincide with the definition of

θ we have chosen, which is the angle to the semi-major axis. We then choose to transform θ so that it lies within the range $(0, \pi)$ to simplify later comparisons. It is possible to constrain the solver so that it only searches for solutions where one of the widths is larger or for angles that lie in our preferred range but this often results in no solution being found. With θ, the sub-burst drift rate of the burst is calculated via

$$\frac{d\nu_{\rm obs}}{dt_D} = \frac{\nu_{\rm res}}{t_{\rm res}} \tan\theta, \tag{5}$$

where $\nu_{\rm res}$ and $t_{\rm res}$ are the frequency and time resolutions of the dynamic spectrum. The burst duration measured from the fit parameters is found through

$$t_{\rm w} = t_{\rm res} \frac{\sigma_m \sigma_M}{\sqrt{\sigma_m^2 \cos^2(\theta) + \sigma_M^2 \sin^2(\theta)}}, \tag{6}$$

where σ_m and σ_M are the minimum and maximum of σ_x and σ_y, respectively. This expression is derived by projecting the semi-minor axis of an ellipse rotated through θ onto the time-axis. These expressions are also used to derive error formulae in order to propagate the parameter errors to the the values of $d\nu_{\rm obs}/dt_D$ and $t_{\rm w}$. These are the error bars shown in Fig (??), and do not take into account errors on the DM or other sources of error.

With these measurements of the sub-burst drift rate $d\nu_{\rm obs}/dt_D$ and the burst duration t_w from each burst, we plot their relationship and compare it to the model described by eq. (??) and in [7]. Because of the dependance on the frequency of observation $\nu_{\rm obs}$ on the right-hand-side, we plot $\nu_{\rm obs}^{-1} d\nu_{\rm obs}/dt_D$ vs. $t_{\rm w}$, which provides us a fit parameter that is independent of the observation frequency. We calculate the observation frequency $\nu_{\rm obs}$ of each burst via an intensity-weighted average of the time-averaged frequency series. We used the `scipy.odr.RealData` package, which uses orthogonal distance regression and incorporates the errors on the data to find a fit. This fit differs slightly between sources, and a single fit that includes all sources can be found.

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TODO:

How do you test its robustness with DM variations?

Since a variation in the DM used to de-disperse a burst will translate to a variation in its fit angle θ and thus a variation in the measured sub-burst drift rate, we varied the DM used to test the robustness of the relationship found between the sub-burst drift rate and burst duration. We found that a variation in the DM acts as a rotation of the fit angle θ, which shifts the final fit found but preserves the aforementioned relationship. Using the bursts from FRB˜180916, we repeated the autocorrelation analysis described above while varying the original DM of 348.82 pc/cm^3 with steps of ΔDM =0.5, -1, and -2 pc/cm^3, which is within the range of DMs found by (??) for this source. Values of ΔDM larger than 0.5 pc/cm^3 yielded positive sub-burst drift rates which indicates over-de-dispersion. Figure 1 shows the fit angles θ and durations $t_{\rm w}$ found for the collection of bursts used from FRB˜180916 dedispersed to the different DM values. Using eq. (5), the angle is related to the burst duration through

$$\theta = \arctan(t_{\rm w}/A), \tag{7}$$

where A is a constant and θ is transformed to fit in the range of $\arctan(x)$. Using the fit found with the above form for the bursts at $\Delta\mathrm{DM} = 0$, we find angles that offset this fit to satisfactorily fit the bursts at each of the other DM values. We also note that the subburst durations found largely stay the same under different DMs. This result indicates that even

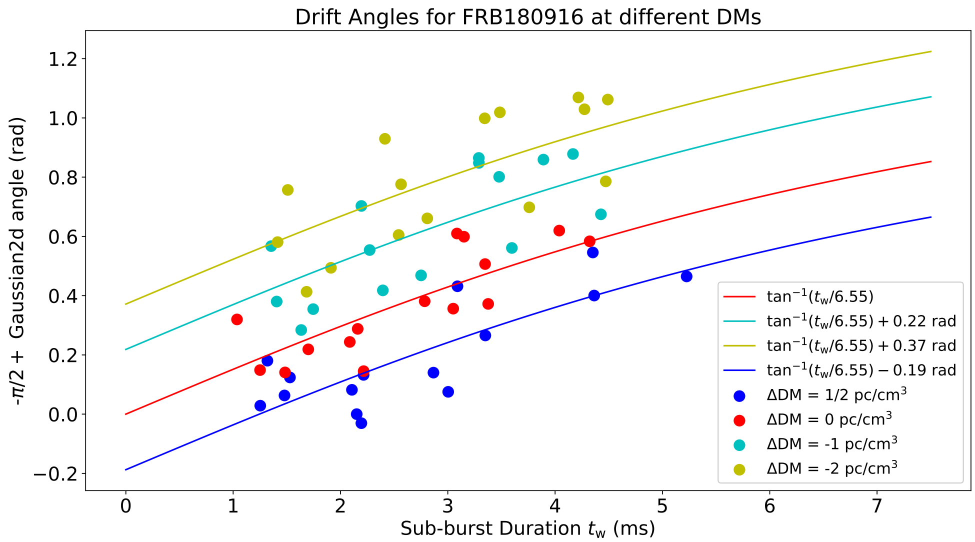

Figure 1: Robustness test: the fit angle θ vs. sub-burst duration t_{w} from bursts dedispersed to small variations in the DM for the source FRB˜180916. Red points are bursts at $\Delta\mathrm{DM} = 0$ which corresponds to a DM of 348.82 pc/cm^3. Blue, cyan, and green points are bursts de-dispersed to $\Delta\mathrm{DM} = 0.5$,-1, and -2 pc/cm^3, respectively. The red line is fit to the red points and is of the form $\arctan(t_{\mathrm{w}}/A)$ derived from the dynamical model described in the main text. Blue, cyan, and green lines are fits found by adding a rotation (adding an angle) to the $\Delta\mathrm{DM} = 0$ model. This plot demonstrates the rotational effect small variations in the DM has on dynamic spectra of FRBs.

though the sub-burst drift rate is quite sensitive to the DM chosen, the relative differences of the drifts between a cohort of bursts is consistent and indeed that the overall inverse trend between the sub-burst drift rate and burst duration exists even at different choices of DM.

How do you optimize DM based on the previous test?

Stampcard

References

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