Задание 2

$$\left(tg^2\left(\sin(x)+\cos(2x+3)\right)\right)' =$$

$$= 2\left(\frac{d}{dx}\left(tg\left(\cos(3+2x)+\sin(x)\right)\right)\right) \cdot tg\left(\cos(3+2x)+\sin(x)\right) =$$

$$= \left(\left(\frac{d}{dx}\left(\cos(3+2x)+\sin(x)\right)\right) \cdot \frac{1}{\cos^2\left(\cos(3+2x)+\sin(x)\right)}\right) \cdot$$

$$\cdot 2tg\left(\cos(3+2x)+\sin(x)\right) =$$

$$= \left(\frac{d}{dx}\left(\sin(x)\right)+\left(-\left(\frac{d}{dx}(3+2x)\right)\sin(3+2x)\right)\right) tg\left(\cos(3+2x)+\sin(x)\right) \cdot$$

$$\cdot \frac{2}{\cos^2\left(\cos(3+2x)+\sin(x)\right)} =$$

$$= \frac{2}{\cos^2\left(\cos(3+2x)+\sin(x)\right)} \cdot \left(\frac{d}{dx}\left(\sin(x)\right)-\left(\frac{d}{dx}(3)+2\left(\frac{d}{dx}(x)\right)\right)\sin(3+2x)\right)$$

$$\cdot tg\left(\cos(3+2x)+\sin(x)\right) =$$

$$= \frac{2}{\cos^2\left(\cos(3+2x)+\sin(x)\right)} \cdot \left(\cos(x)-\sin(3+2x)\left(2\left(\frac{d}{dx}(x)\right)+0\right)\right) \cdot$$

$$\cdot tg\left(\cos(3+2x)+\sin(x)\right) =$$

$$= \frac{2}{\cos^2\left(\cos(3+2x)+\sin(x)\right)} \cdot \left(\cos(x)-2\left(\frac{d}{dx}(x)\right)\sin(3+2x)\right) \cdot$$

$$\cdot tg\left(\cos(3+2x)+\sin(x)\right) =$$

$$= \frac{2}{\cos^2\left(\cos(3+2x)+\sin(x)\right)} \cdot \left(\cos(x)-2\sin(3+2x)\right) \cdot$$

$$tg\left(\cos(3+2x)+\sin(x)\right).$$

При $x_0 = 1$:

$$\frac{2}{\cos^2(\cos(3+2)+\sin(2))} \cdot (\cos(1) - 2\sin(3+2)) \cdot$$

$$\cdot \operatorname{tg}(\cos(3+2)+\sin(1)) =$$

$$= \frac{2}{\cos^2(\cos(5)+\sin(1))} \cdot (\cos(1) - 2\sin(5)) \cdot$$

$$\cdot \operatorname{tg}(\cos(5)+\sin(1)) =$$

$$= \frac{\sin^2(2(\sin(1)+\cos(5)))}{\cos(2(\sin(1)+\cos(5)))+1)^2}$$

Задание 5

Найти собственное значение и собственный вектор матрицы

$$\begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \times \begin{pmatrix} x \\ y \end{pmatrix} = \lambda \begin{pmatrix} x \\ y \end{pmatrix} \qquad \begin{pmatrix} x+4y \\ x+y \end{pmatrix} = \begin{pmatrix} \lambda x \\ \lambda y \end{pmatrix}$$

$$\begin{cases} x+4y = \lambda x \\ x+y = \lambda y \end{cases} \Rightarrow \begin{cases} x+4y-\lambda x = 0 \\ x+y-\lambda y = 0 \end{cases} \Rightarrow$$

$$\begin{cases} x(1-\lambda)+4y = 0 \\ y(1-\lambda)+x = 0 \end{cases}$$

$$\begin{vmatrix} 1-\lambda & 4 \\ 1 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 4 = \lambda^2 - 2\lambda + 1 - 4 =$$

$$= \lambda^2 - 2\lambda - 3 = (\lambda - 3)(\lambda + 1)$$

$$\Rightarrow \boxed{\lambda_{1,2} = -1;\ 3}$$

$\lambda = -1$:

$$\begin{cases} x+4y+x = 0 \\ x+y+y = 0 \end{cases} \Rightarrow \begin{cases} 2x+4y = 0 \\ x+2y = 0 \end{cases} \Rightarrow \begin{cases} x+2y = 0 \\ x+2y = 0 \end{cases} \Rightarrow$$

$x = -2y$ $\quad \boxed{\text{Собств. вектор}_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix}}$

$\lambda = 3$:

$$\begin{cases} x+4y-3x = 0 \\ x+y-3y = 0 \end{cases} \Rightarrow \begin{cases} 4y-2x = 0 \\ x-2y = 0 \end{cases} \Rightarrow \begin{cases} 2y-x = 0 \\ x-2y = 0 \end{cases}$$

$x = 2y$

$\boxed{\text{Собств. вектор}_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}}$