# Ice Core Dating using Probabilistic Programming

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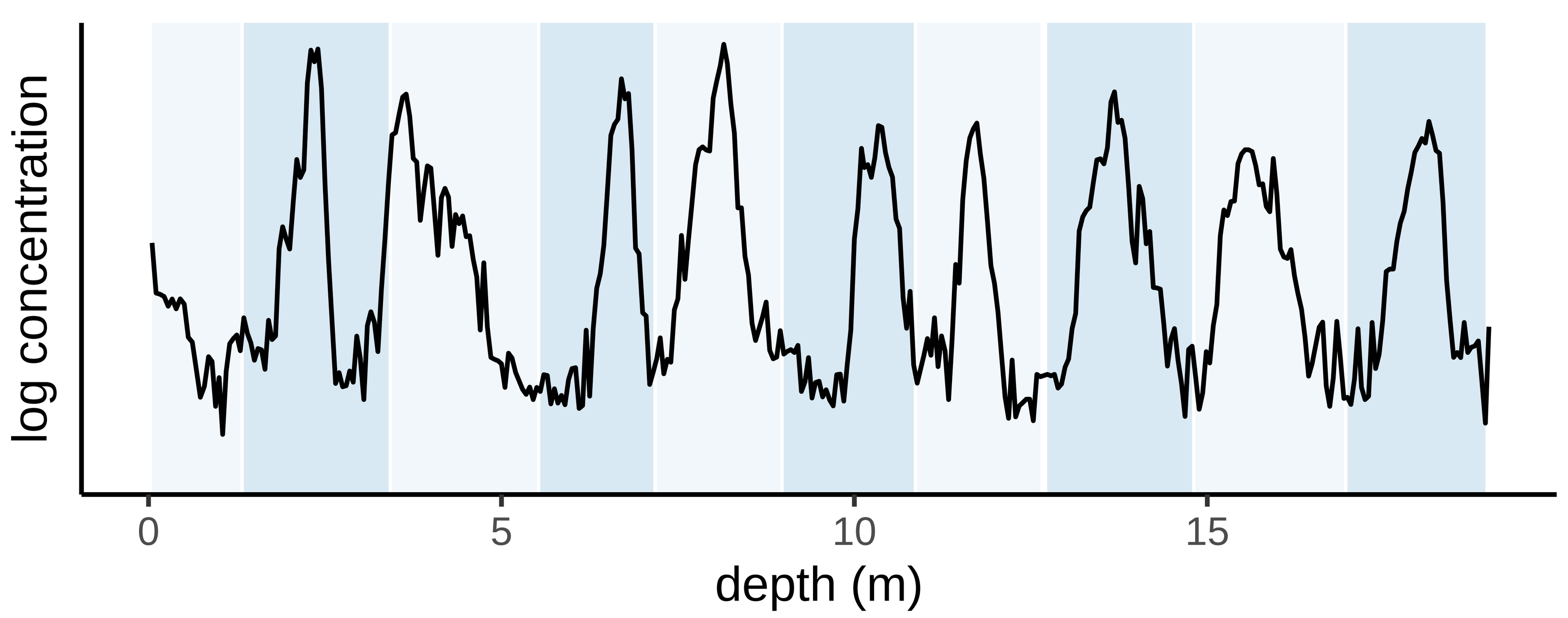
## The Ice Core Dating Problem

1. Climate → atmospheric chemical *proxies*
2. Precipitation on ice sheet → annual layers in proxies
3. Seasonal proxy depth-series → chronology (what we infer)
4. Chronology → past climate conditions (what scientists want)

Manual layer counting is arduous [1] → automate with uncertainty!

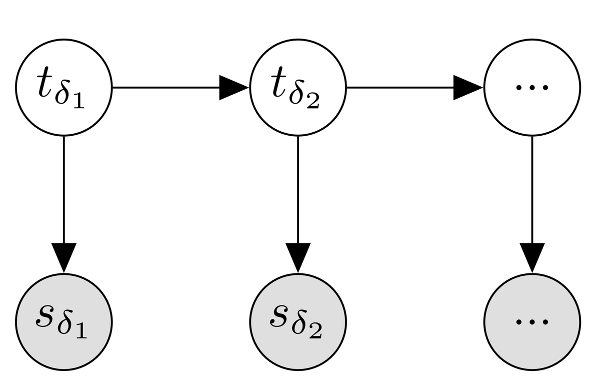



## Our contribution

- A series of models for ice core dating
- Implemented using **probabilistic programming languages (PPLs)** for fully automatic inference

## Graphical model

Hidden Markov model (HMMs) with latent time values, $t_{\delta_i}$, indexed by discrete depth values $\delta_i$




## Discrete domain and index: Classical HMMs

$$\text{Transitions: } t_{\delta_i} | t_{\delta_{i-1}} \sim Categorical(P_{t_{\delta_{i-1}}}) \quad (1)$$

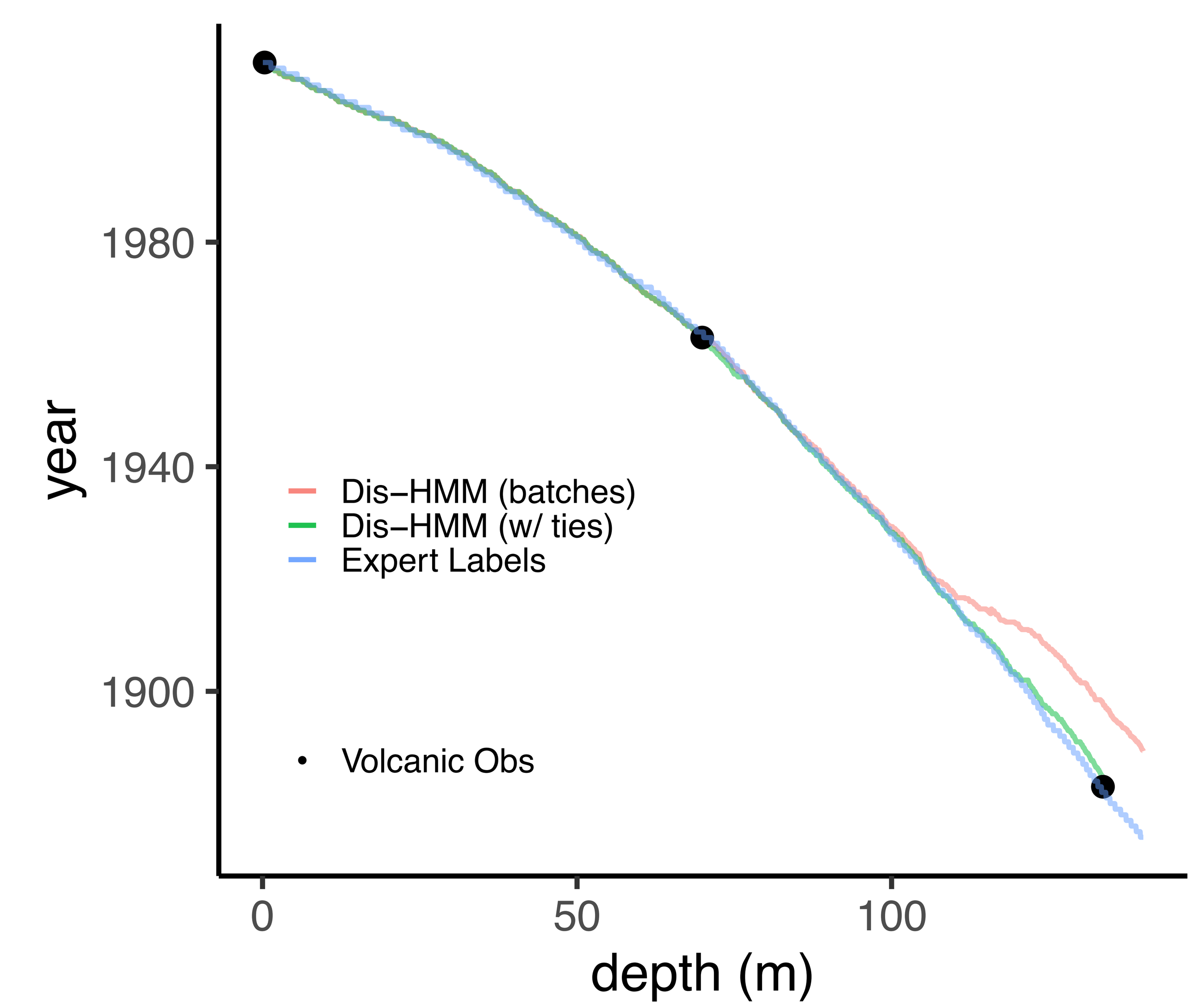
$$\text{Observations: } s_i | t_{\delta_i} \sim \mathcal{N}(a\cos(2\pi t_{\delta_i}) + b, \sigma^2) \quad (2)$$

$$\text{Transition matrix } P = \begin{bmatrix} \times & \times & 0 & \dots \\ 0 & \times & \times & \dots \\ 0 & 0 & \times & \ddots \end{bmatrix}, \quad (3)$$

with $t_{\delta_i} \in \{^1/_{12}, ^2/_{12}, \dots\}$, ensuring $t_{\delta_i}$ is monotonically increasing.

**Hierarchical observation model**

- Allow for parameters $a, b$ in (2) to change with each data point with a hierarchical prior placed over $a_i, b_i$.
- Latent parameters $(a_i, b_i)$ must be marginalized. MCMC is expensive, so we use variational inference (VI).
- Dated volcanic events can be incorporated, constraining the depth-time mapping.

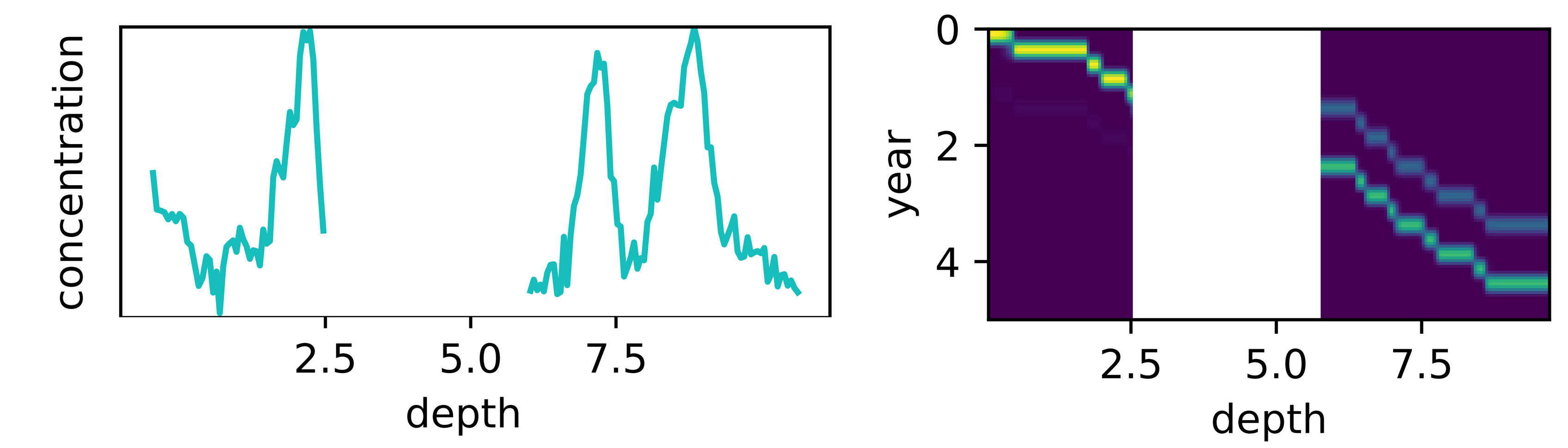



## Discrete domain and continuous index: Cts-HMMs

Continuous depth index $\delta_i$ → continuous index Markov process:

$$\mathbb{P}(t_{\delta_i} | t_{\delta_{i-1}}) = \exp\left((\delta_i - \delta_{i-1})\mathbf{Q}\right).$$

With transition rate matrix $\mathbf{Q}$, can capture uncertainty over a missing data section with *bimodal* latent posterior:




## Future work: An extension to SDEs

Depth & time both continuous → stochastic differential equations. For example, with monotonic sample paths:

$$\begin{bmatrix} dz_\delta \\ dt_\delta \end{bmatrix} = \begin{bmatrix} -\theta z_\delta \\ -\mu^+(z_\delta, t_\delta, \delta) \end{bmatrix} d\delta + \begin{bmatrix} \sqrt{2\theta} \\ 0 \end{bmatrix} d\mathbf{W}_\delta.$$

## The Promise of Probabilistic Programming

Probabilistic programming languages promise to enable automatic inference and fast model prototyping, while ensuring maintainability.

In practice, inference is only possible in limited model classes, and some forms of inference are not scalable.

[1] Mai Winstrup.
A hidden markov model approach to infer timescales for high-resolution climate archives.
In *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence*, AAAI'16, 2016.