Suppose k events form a partition of the sample space Ω. That is, they are disjoint and

$$\bigcup_{i=1}^{k} A_i = \Omega.$$

Assume that $P(B) > 0$. Prove that if $P(A_1|B) < P(A_1)$ then $P(A_i|B) > P(A_i)$ for some $i \in \{2, \dots, k\}$.

Proof. Suppose to the contrary that $P(A_1|B) < P(A_1)$ and $P(A_i|B) \le P(A_i)$ for each $i \in \{2, \dots, k\}$. This means,

$$P(A_1|B) = \frac{P(A_1B)}{P(B)} < P(A_1)$$

and

$$P(A_i|B) = \frac{P(A_iB)}{P(B)} \le P(A_i)$$

for each $i > 1$. Given $P(B) > 0$, then (for each i)

$$P(A_iB) \le P(A_i)P(B)$$

Consequently, seeing that $P(A_1B) < P(A_1)P(B)$, we must have

$$\sum_{i=1}^{k} P(A_iB) < \sum_{i=1}^{k} P(A_i)P(B)$$
$$\sum_{i=1}^{k} P(A_iB) < P(B)\sum_{i=1}^{k} P(A_i).$$

We know that each A_i is disjoint, so $A_n \cap A_m = \emptyset$ whenever $n \neq m$. Simultaneously, we know that $A_iB \subseteq A_i$ for each i. As such $A_nB \cap A_mB \subseteq A_n \cap A_m = \emptyset$ whenever $n \neq m$. That is to say, each A_iB is disjoint. By the third axiom of probability†,

$$P\left(\bigcup_{i=1}^{k} A_iB\right) < P(B) \cdot P\left(\bigcup_{i=1}^{k} A_i\right).$$

Recall that the collection of each A_i forms a partition of Ω, so $\bigcup_{i=1}^{k} A_i = \Omega$. Furthermore, by the distributive property of set intersection over set union, we have $\bigcup_{i=1}^{k} A_iB = \left(\bigcup_{i=1}^{k} A_i\right)B = \Omega B = B$. Ultimately,

$$P(B) < P(B) \cdot 1$$
$$P(B) < P(B),$$

a contradiction. It must then be the case that at $P(A_i|B) > P(A_i)$ for some $i \in \{2, \dots, k\}$.

† - We assume that σ-additivity implies finite additivity.