Theorem. Let Ω be a sample space and $A \subseteq \Omega$ be an event. If $P(A) = 0$ or $P(B) = 1$ then A is independent of every other event.

Proof. Let $B \neq A$ be an event of Ω. In the case $P(A) = 0$, we have $P(A)P(B) = 0 \cdot P(B) = 0$. We also know that $AB \subseteq A$, so $P(AB) \leq P(A)$. That is, $P(AB) \leq 0$. Seeing that $AB \subseteq A \subseteq \Omega$, then AB is also an event of Ω. By the first axiom of probability, $P(AB) \geq 0$. So, $P(AB) = 0$. Overall,

$$P(A)P(B) = P(AB).$$

That is to say, A and B are independent events. Given $B \neq A$ was chosen arbitrarily, then A is independent of every other event of Ω.

In the case $P(A) = 1$, we have $P(B) = 1 \cdot P(B) = P(A)P(B)$. Additionally, $AB \subseteq B$, so $P(AB) \leq P(B) = P(A)P(B)$. Suppose that $P(AB) < P(B)$. Then $P(AB) = P(A) + P(B) - P(A \cup B) < P(B)$. That is $1 - P(A \cup B) < 0$, or $1 < P(A \cup B)$, a contradiction to the first axiom of probability. So, $P(AB) = P(B) = P(A)P(B)$, meaning A and B are independent events