State Regulation

First goal is to design a control objective to drive all states to 0, $x(t) \underset{n\to\infty}{\to} 0$ - closed loop system is stable

This is called state regulation - different from Reference Tracking where we wish to drive the output to a known function of time $y(t) \underset{t\to\infty}{\to} y_r(t)$

FSFB Control law:

$$u(t) = -Kx(t), K \in \mathbb{R}^{n\cdot m}$$

Or

$$\begin{aligned}
&u_1 = -k_{11}x_1 - k_{12}x_2 + \cdots - k_{1n}x_n \\
&\cdots \\
&u_m = -k_{m1}x_1 - k_{m2}x_2 + \cdots - k_{mn}x_n
\end{aligned}$$

The closed loop state equation is then

$$\begin{aligned}
\underline{\dot{x}} &= A\underline{x} - BK\underline{x} \\
&= (A - BK)\underline{x}
\end{aligned}$$

where $A - BK$ is a new matrix which can be modified to change poles, transient response etc. and is a new $n \cdot m$ state matrix of the CL system

Arbitrary pole placement:
Find $K \in \mathbb{R}^{m\cdot n} \ni \det(\lambda I - (A - BK)) = (\lambda - \lambda_1^d)(\lambda - \lambda_n^d)\ldots(\lambda - \lambda_n^d)$
where $\lambda_1^D, \lambda_2^D .., \lambda_n^D$ are the desired pole locations of the CL system. Allowing for complex conjugate pairs

To move poles from (x_1, x_2) to (y_1, y_2) solve $A - BK$ where $A =$ matrix which (x_1, x_2) is the eigenvalues of,

$$B = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

and $K = [k_1, k_2]$ (gains) and solve $K \in \mathbb{R}^{m\cdot n} \ni \ldots$

Cannot always place poles where desired. There needs to be $K \in u - K\underline{x}$ to be possible.

DEF:
The linear system $\underline{\dot{x}} = A\underline{x} + B\underline{u}$ is *Controllable* means that there exists a control signal $\underline{u}(t)$ to drive the system from any initial state $\underline{x}_0$ to any final state $\underline{x}_f$ in finite time
Abbreviated as (A, B) Controllable

To test for controllability can use the rank test
$(A_{n \cdot n}, B_{n \cdot m})$ controllable $\equiv$ rank$(M_c) = n$
where $M_c = [B, AB, A^2B, \ldots, A^{n-1}B]$
For single input, B is a column vector and M_c is square so we can use det(M_c) to test rank$(M_c) = n$

See relevant MATLAB Functions for methods of solving

Notes

1. If a system is controllable, we are able to arbitrarily place poles when $\underline{u} = 0K\underline{x}$. However if a system is uncontrollable, we are still able to move the poles, the location just cannot be arbitrary
2. Controlling Nonlinear Systems with a linear controller requires consideration of the Equilibrium Point about which the linear model is defined.