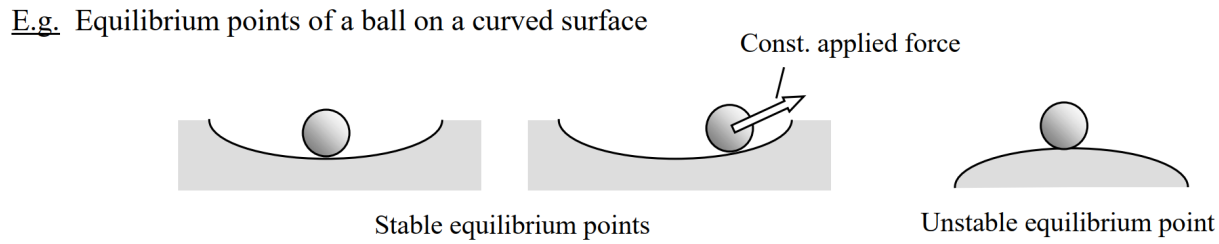
Equilibrium Point

Equilibrium points (a.k.a. stationary points, operating points, fixed points, critical points, steady-state points, trim points) of a nonlinear system are the values of $\underline{x}^*, \underline{y}^*, \underline{u}^*$ when the nonlinear system is in equilibrium.

Equilibrium implies that $\underline{\dot{x}} = \underline{0} \implies \underline{x}$ is constant.



A state x^* is a equilibrium point (or state) of the system $\dot{x} = f(x)$ if once $x(t)$ is equal to x^*, it remains equal to x^* for all future time. Given a EP x^* there always exists a coordinate transformation $y^* = x - x^*$ such that

$$\begin{cases} \dot{y} = \dot{x} = g(x) = g(y + x^*) = f(x) \\ f(0) = 0 \end{cases}$$

Mathematically this means the vector x^* satisfies

$$0 = f(x^*)$$

Equilibrium points can be found by solving the above nonlinear equation.

State-Space Models

Considering the nonlinear State-Space Models the equilibrium equations satisfy

$$\begin{cases} \underline{0} = \underline{f}(\underline{x}^*, \underline{u}^*) \\ \underline{y}^* = \underline{h}(\underline{x}^*, \underline{u}^8) \end{cases}$$

There may be more than one equilibrium point for a particular nonlinear system. To exactly define a particular equilibrium point, typically we have some information on $(\underline{x}^*, \underline{y}^*, \underline{u}^*)$ but need to find the rest by solving the above equations. The process of finding all the information for an equilibrium point is often called Trimming, from its use in vehicle flight dynamics

There is MATLAB Functions to find unknown equilibrium point from a non linear model `trim`

An EP is closely related to stability, see Types of Stability