CSCI 241, Project #2, Raytracing Optimization

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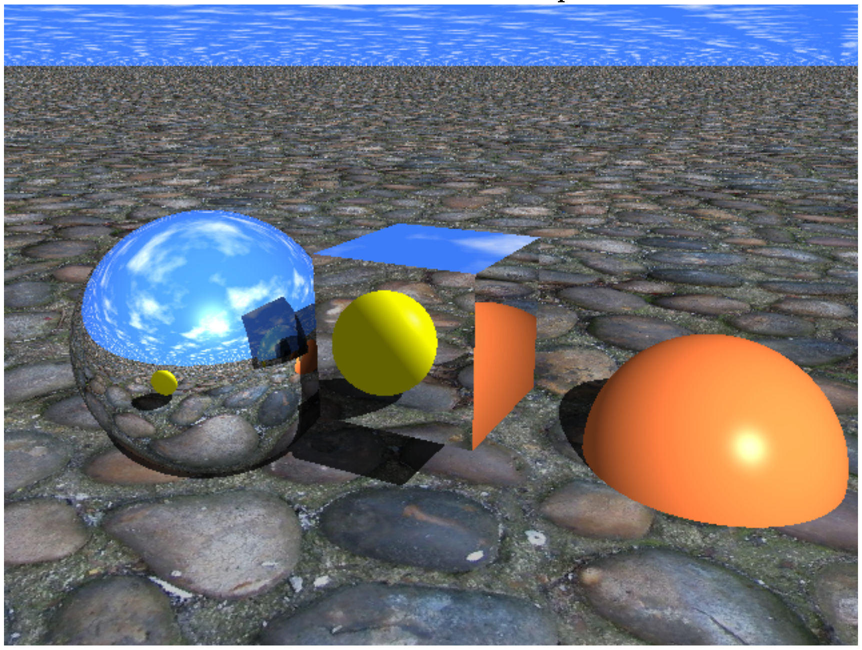
Note: This is still incomplete.



Due date: Midnight, May 12.

Turn in: Be sure to **zip** all your source files together before submission. Canvas downloads mangle the names, and java likes to have the names of the files the same as the public class in the file.

To make it easier for me to write scripts to unpack, compile, and run your file:

- Use **zip**, not tar or 7z or any other archiving software.
- Zip the folder, not the individual files in the folder.
- Name the zip file `lab02.zip`.
- Nmae the file (and class) with your `main` method in it `lab02.java`.

Ray tracing: Ray tracing is a fairly direct method of making spectacularly realistic images on the computer. The image above is one I made to demonstrate what was possible for the first project in CSCI 480 one year. One of the major problems with raytracing is that it is very slow.

We won't be including all the features that made the above image possible, we will only be rendering simple spheres. But we will use a tree structure to improve the speed of the rendering.

The world: The world we need to represent (see Figure 1), will be represented by a Java object, consisting of the following things:

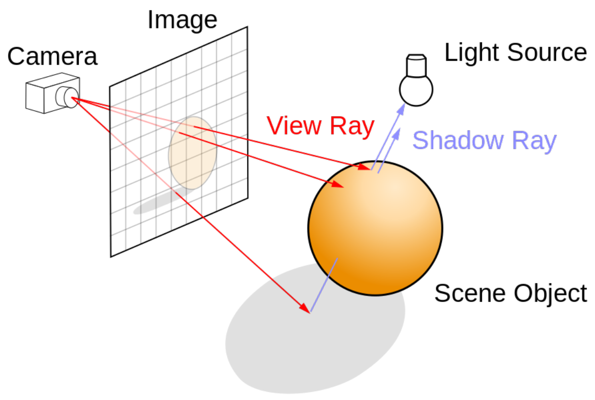


Figure 1: Basic world for ray tracing. We will not be using shadow rays. (By Henrik - Own work, GFDL, `https://commons.wikimedia.org/w/index.php?curid=3869326`)

1. **A camera:** This is just a point in space, p. To make the math a little simpler, we will assume that this point is on the z axis, at point $p = (0, 0, d)$. In the program, the distance d will be the instance variable `cameraDistance`, and we will assume $d > 0$.
2. **An image plane:** This is a flat rectangular region of space in front of the camera. Each grid point on the image plane will determine a color for a pixel in the image. To simplify the math, again, we will assume our image plane is a square centered in the (x, y) plane, so that its corners are at $(\pm s, \pm s, 0)$. In the program, the size s will be the instance variable `imagePlaneSize`.
3. **A light source:** We will assume only one light source, and we will assume that it is so distant, that it appears to be in the same direction no matter where you are in the world. It will be represented, therefore, by a single 3d vector, $\ell = (\ell_x, \ell_y, \ell_z)$. convenience, we assume the vector is *normalized*, that is, it's length is one: $\sqrt{\ell_x^2 + \ell_y^2 + \ell_z^2} = 1$. For example, one light source could be $\ell = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$. In the program, the light will be represented by the instance variable `lightDirection`.
4. **Objects:** Objects in the scene to be rendered. For our purposes, the only objects we will render will be *spheres*, which are represented by a data structure that holds a 3D point for the center, c, and a real number r, for the radius. We will want to include many spheres, so you will need to use some kind of a container for the spheres. A linear structure is fine, since we will just be iterating over them. In the program, the spheres will be represented by the instance variable `objectList`. Spheres will also have a *color*, which will be a vector of three floats giving the red, green, and blue components of the color of the sphere.

Pseudocode:

```
for each pixel in the image {
  find the ray from the camera
  for each sphere in the world {
    intersect the ray with the sphere
    keep the closest sphere and intersection point (largest z-value)
  }
  if there were no intersections {
    put the background color in the pixel
  } else {
    color the pixel with Lambertian shading of the intersection point
  }
}
```

Explanation of pseudocode:

Line 1. We open our Java image with a size of `WIDTH` $= w$ and `HEIGHT` $= h$, and we iterate over the pixels in our image with the variables u and v, with $u = 0 \ldots w-1$ and $v = 0 \ldots h-1$.

Line 2. In order to make a 2D image from our 3D world, we need to map pixels in the 2D image to points in the image plane. The integer pixel (u, v) will be mapped to the following 3D point on the image plane, with $s =$ `imagePlaneSize`:

$$\begin{aligned} \text{image\_plane}(u, v) &= (s(2u/w - 1), -s(2v/h - 1), 0) \\ &= q \end{aligned}$$

The **ray** from the camera point p through a point q on the image plane, is a data structure that consists of the camera point and a vector, (p, v), where v is the *normalized* vector

$$v = \frac{q - p}{|q - p|}$$

Line 3. Let us assume that, as we loop through the sphere objects, (c, r) is the current sphere we're talking about.

Line 4. Given a ray, (p, v) and a sphere (c, r), we can find the intersection point as follows (refer to Figure 2).

The math to follow will be much simpler if we assume that the sphere is centerd on the origin. We can do that if we move both the ray and the sphere by the same translation. If we subtract c from each, c is moved to the origin, c is moved to $c - c = (0, 0, 0)$, and p is moved to

$$q = p - c$$

Note in the math below that we use q, not p (in the picture, imagine p replaced with q).

We need to solve the equation, for some value of t (skip the math if you're not interested):

$$|q + tv|^2 = r^2$$

We can do this as follows.

$$\begin{aligned} |q + tv|^2 &= (q + tv) \cdot (q + tv) \\ &= \sum_i (q_i + tv_i)(q_i + tv_i) \\ &= \sum_i \left(q_i^2 + 2q_i v_i t + v_i^2 t^2\right) \\ &= \sum_i q_i^2 + 2\sum_i q_i v_i t + \sum_i v_i^2 t^2 \\ &= (q \cdot q) + 2(q \cdot v)t + (v \cdot v)t^2 \end{aligned}$$

So, in the quadratic $at^2 + bt + c = 0$,

$$\begin{aligned} a &= v \cdot v && (= 1, \text{why?}) \\ b &= 2q \cdot v \\ c &= q \cdot q - r^2 \end{aligned}$$

The two solutions to this quadratic will give you two values of t to plug into the expression

$$p + tv$$

where (p, v) is your ray. Note that when we plug t back into the ray, we use p, not q!

This will give you the two intersection points on the sphere, as in Figure 2. Pick the one closest to the camera; it will have the smallest t value (why?).

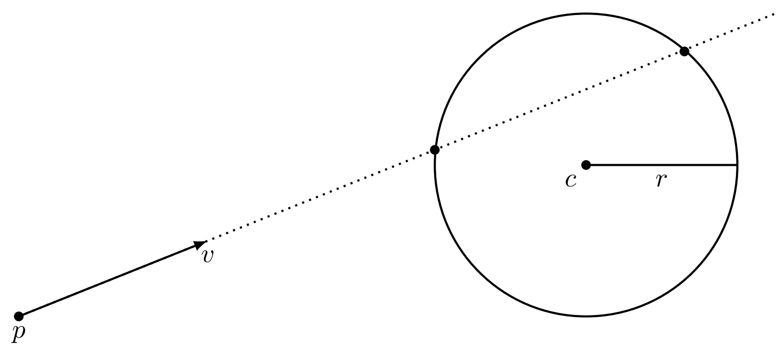


Figure 2: Intersecting a ray and a sphere.

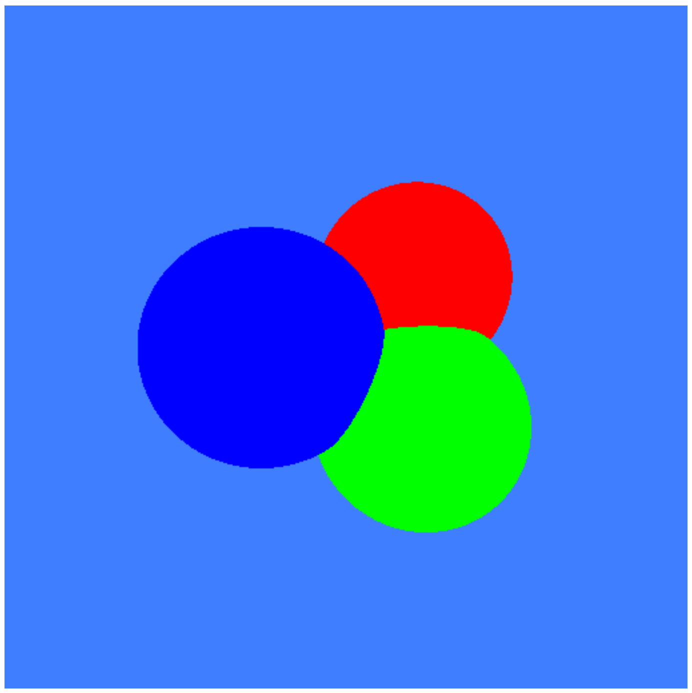


Figure 3: Three spheres, flat shading.

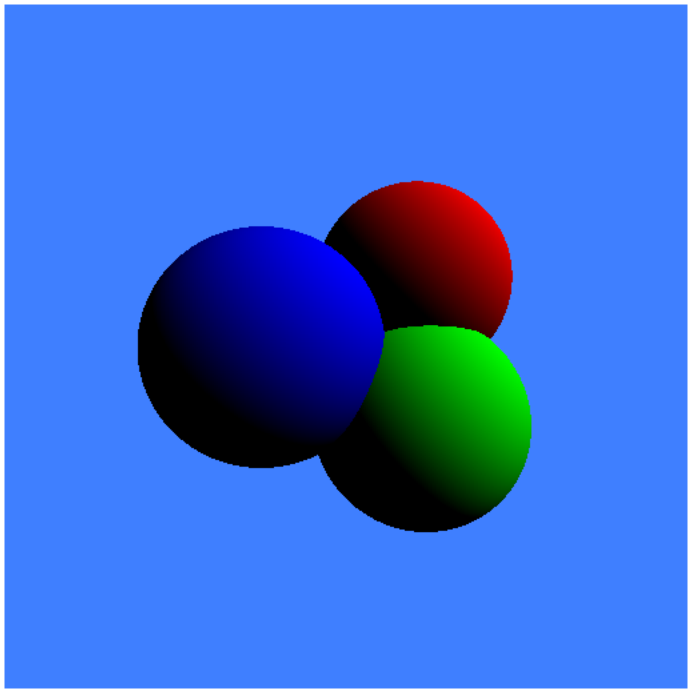


Figure 4: Three spheres, Lambertian shading.

Line 5. The closest intersection point of all the spheres will have the largest z value (why?).

Line 10. If we use the sphere's color for this pixel, we get a figure like the one in Figure 3. This is not very exciting. We would like to shade the spheres as in Figure 4.

We will use *Lambertian shading* for the spheres, a very simple shader. We just have to multiply the color of the sphere by the cosine of the angle between the surface normal, n, and the light vector, ℓ. This is easier than it sounds.

To find the surface normal, n, of a sphere, (c, r) at any point, p, simply subtract the center from the point and normalize the resulting vector:

$$n = \text{normalize}(p - c)$$

To find the cosine of the angle between two normalized vectors, simply take the dot product.

$$\cos(\text{angle}(n, \ell)) = n \cdot \ell$$

In this case, if the cosine is negative, we want the color to be black, so any result that is negative is replaced with zero. We then multiply the color of the sphere by this value (between 0 and 1), to get the shading seen in Figure 4.

If you want to get a little bit of color in the shadows (instead of pitch black), you can truncate the cosine at a small number, say 0.1, instead of 0, to add some *diffuse* shading.

Note that this only works if both n and ℓ are normalized.

Java code for images: I found the simplest Java code I could on the web to create an image and color its pixels. This file, called `Example01.java` is on the website. It should be self-explanatory.

Given the pseudocode above and the Java example, you should be able to make some pretty amazing pictures (such as the ones at the end of this document).

Don't put the camera too close to the scene, unless you want extremely exaggerated perspective. Place all the spheres near the image plane (in the $(\pm s, \pm s, \pm s)$ cube), or you won't see them in the image.

World files: Input to this program will be the description of a world. Output will simply be the graphical display.

The world description will be in a file, which will give the camera position, the light direction, and the spheres. Here is an example world file, it puts the camera at 20 units along the z axis, the light at normalized(1,1,1), and a red sphere of radius 3 at $(2, 2, -1)$, a green sphere of radius 3 at $(2, -2, 0)$, and a blue sphere of radius 2 at $(-2, 0, 1)$.

myworld.wrl

```
camera: 20
light: 0.577 0.577 0.577
sphere:  2.0  2.0 -1.0 3.0 1.0 0.0 0.0
sphere:  2.0 -2.0  0.0 3.0 0.0 1.0 0.0
sphere: -2.0  0.0  1.0 2.0 0.0 0.0 1.0
```

Design your program so that the world file is a command line parameter, the name of the world file. For example:

```
>  javac *.java
>  java lab02 myworld.wrl
```

Optimizing the ray tracing: This is done in two phases, calculating the bounding boxes for the spheres, and then inserting the spheres into a tree.

Bounding boxes: The method described so far is easy to implement and will create stunning images of spheres. However, it is not very efficient. We need to cast thousands of rays. Even a relatively small image of size 256×256 will have 65536 rays. Each of these rays is intersected with *every* sphere in the world. We can do better.

Let's see if we can tell in advance whether or not a ray might hit a sphere, and eliminate some of these expensive tests.

Our strategy is to find a rectangular region in the image plane such that, if the ray does *not* go through this region, then it's impossible for the ray to intersect the sphere. We call this region the *bounding box* for the sphere. It will consist of an extent in x, from x_1 to x_2, and an extent in y, from y_1 to y_2. If we assume $x_1 < x_2$ and $y_1 < y_2$ then we can represent the rectangle by its corners: (x_1, y_1) and (x_2, y_x).

Let's consider the y extent, first. Figure 5 is a picture of a typical sphere viewed in the (y, z) plane. The camera is located at point p, the sphere at point c with radius r. We want to find the numbers y_1 and y_2.

We can find the numbers y_1 and y_2 with a little trigonometry. a is the distance in the (y, z) plane between $p = (0, 0, p_z)$ and $c = (c_x, c_y, c_z)$, which is

$$a = \sqrt{c_y^2 + (c_z - p_z)^2}$$

And then, since the radii of the circle are perpendicular to it, we have

$$\tan(\theta) = \frac{r}{a}$$

from which we can find θ. If we let $\psi = \theta + \phi$, then

$$\sin(\psi) = \frac{c_y}{a}$$

from which we can get ψ, and $\phi = \psi - \theta$. It's then easy to conclude that

$$\begin{aligned} y_1 &= p_z \tan(\phi) \\ y_2 &= p_z \tan(\phi + 2\theta) \end{aligned}$$

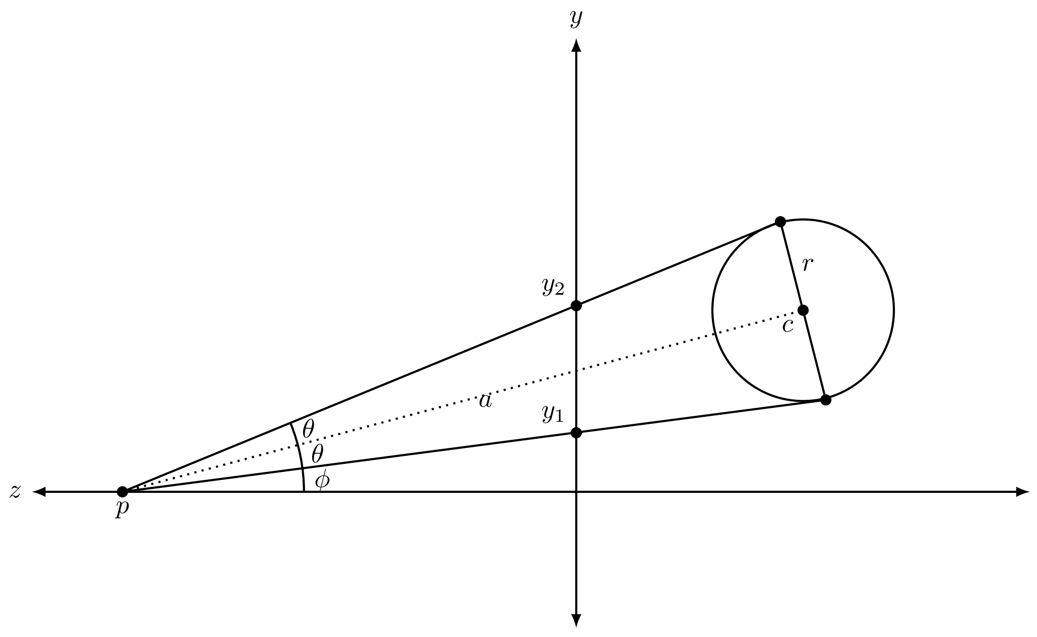


Figure 5: Calculating the image plane bounding box of a sphere. All rays from the camera at p that intersect the sphere at c with radius r must pass through the image plane between y_1 and y_2. A figure for the (x, z) plane is similar.

We can do the same thing in the (x, z) plane, getting x_1 and x_2.

As in the figure, we let $x_1 < x_2$ and $y_1 < y_2$.

For each sphere, then, we have limits on the rays that might hit it. Each ray that hits the sphere must hit the image plane in the rectangle between (x_1, y_1) and (x_2, y_2). If the ray does not go through this rectangle, we do not have to test the ray against this sphere for intersection.

When we trace a ray, we find the point at which it intersects the image plane. If this point is not in a sphere's bounding box, then the ray cannot hit it. We need a way to quickly find all spheres with bounding boxes that intersect this ray.

Preprocessing the spheres: Prior to beginning our raytrace, we insert each sphere into a quadtree. We divide the image plane up into four quadrants: *ul*, upper left, *ur* upper right, *ll* lower left, and *lr* lower right. We build a tree to represent the entire image plane ($\pm s$ in the (x, y) plane), and its four children represent the four parts of the image plane.

Clearly, we can then divide each of the four children into four parts (*ul, ur, ll, lr*), getting a tree with 16 leaves. If we do this one more time, we get a tree with 64 leaves. This will be our optimization tree. Each 1/64th of the image plane will be called a *cell.* Each cell stores its extent in the (x, y) plane.

Before we begin the raytracing, we insert each sphere into the quadtree. We insert a sphere in a subtree when its bounding box intersects the extent of the subtree. If we begin inserting at the top, we can reach a leaf in only $\log_2 64 = 4$ steps! Bear in mind, however, that a circle may intersect more than one quadrant at each level. Simply insert it into every quadrant its bounding box intersects.

When we construct a ray, we know which point of the image plane it hits. We can quickly traverse the tree to find the cell that the ray hits. We retrieve its accompanying list of spheres and we test the ray for intersection with each one of these, instead of the entire list of spheres.

What kind of speedup can we expect? If there are large empty areas of your scene (as in many scenes), the speedup will be huge. Any ray in these areas will not be tested against *any spheres.* As the spheres get more dense, or larger, the speedup may not be so great.

We can, of course, use deeper trees (with 256, or 1024 leaves) to try to get more speedup. This is an optional exercise, but if you designed your tree well, it should be an easy modification.

Vector operations needed:

$$\begin{aligned}
v + w &= (v_x + w_x, v_y + w_y, v_z + w_z) && \text{sum of two vectors} \\
av &= (av_x, av_y, av_z) && \text{product of a scalar and a vector} \\
v \cdot w &= v_x w_x + v_y w_y + v_z w_z && \text{dot product of two vectors} \\
|v| &= \sqrt{v \cdot v} && \text{length of a vector} \\
\text{normalized}(v) &= \frac{v}{|v|} && \text{vector in the same direction with unit length}
\end{aligned}$$

Sample images using only spheres.