Method	Num Beams	Num Tokens	Avg Reward	Safety Rate	Inference Time (s)
InferenceGuard	32	64	$8.30 (\pm 1.53)$	99.21%	2.02
InferenceGuard	64	64	$8.53 (\pm 1.73)$	99.60%	4.05
InferenceGuard	32	32	$9.12 (\pm 1.48)$	99.54%	4.74

Table 2. Win-rate Percentage Comparison on PKU-SafeRLHF evaluated by 'Deepseek-r1-distill-qwen-32b'

Helpfulness Win Rate (%)	Harmlessness Win Rate (%)		
72	76.8		
66.8	76.6		
68.6	75		
61.4	62.0		
67	60.6		
14.2	52.2		
52	50.4		
50.6	49.2		
51.2	47.6		
	72 66.8 68.6 61.4 67 14.2 52 50.6		

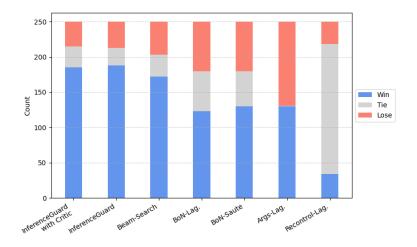


Figure 1. Win, tie, and loss counts of alignment methods compared against responses generated by Alpaca-7B on the PKU-SafeRLHF dataset, using <code>Deepseek-r1-distill-qwen-32b</code> as the judge model. **InferenceGuard** achieves the highest win counts while maintaining low tie and loss rates.

Table 3. Performance Comparison of InfGuard w.r.t. Alpaca-7B on Dataset PKU-SafeRLHF using Different d, N, and K and fixed D=128

	Method	K	Average Reward	Average Cost	Safety Rate	Inference Time (s)
		64	6.6 (± 2.5)	-0.72	94.07%	28.45
	InferenceGuard $N = 128, d = 16$	32	$7.14 (\pm 2.75)$	-0.84	94.3%	27.15
		16	$7.64 (\pm 2.85)$	-0.81	94.33%	25.66
		64	5.98 (± 2.5)	-0.86	95.65%	14.70
	InferenceGuard $N = 128, d = 32$	32	$6.39 (\pm 2.7)$	-0.94	96.3%	13.38
		16	$6.66 (\pm 2.74)$	-0.89	96.05%	14.22
		64	5.5 (± 2.46)	-0.98	96.97%	7.82
	InferenceGuard $N = 128, d = 64$	32	$5.71 (\pm 2.5)$	-0.92	96.84%	7.85
		16	$5.82 (\pm 2.61)$	-0.94	96.97%	5.84
		128	$6.83 (\pm 2.5)$	-0.88	96.18%	42.24
	InferenceGuard $N = 256, d = 16$	64	$7.56 (\pm 2.81)$	-0.98	97.1%	37.77
		32	$7.73 (\pm 2.93)$	-1	98.55%	36.92
		128	$6.19 (\pm 2.51)$	-0.99	98.15%	22.66
Alpaca-7B	InferenceGuard $N = 256, d = 32$	64	$6.67 (\pm 2.73)$	-0.94	96.97%	22.38
		32	$6.99 (\pm 2.90)$	-1.03	98.15%	22.77
		128	$5.82 (\pm 2.6)$	-0.98	98.42%	5.82
	InferenceGuard $N = 256, d = 64$	64	$5.92 (\pm 2.63)$	-1.05	99.34%	9.89
		32	$6.08 (\pm 2.72)$	-1.04	97.5%	11.28
		32	$6.76 (\pm 2.46)$	-0.5	86.56%	16.44
	InferenceGuard $N = 64, d = 16$	16	$7.28 (\pm 2.59)$	-0.65	89.2%	15.32
		8	$7.45 (\pm 2.69)$	-0.6	89.06%	15.29
		32	5.95 (± 2.42)	-0.65	90.38%	12.09
	InferenceGuard $N=64, d=32$	16	$6.48 (\pm 2.5)$	-0.63	90.0%	11.58
		8	$6.64 (\pm 2.63)$	-0.67	90.8%	28.45 27.15 25.66 14.70 13.38 14.22 7.82 7.85 5.84 42.24 37.77 36.92 22.66 22.38 22.77 5.82 9.89 11.28 16.44 15.32 15.29 12.09
		32	$5.67 (\pm 2.41)$	-0.73	91.17%	6.17
	InferenceGuard $N = 64, d = 64$	16	$5.79 (\pm 2.46)$	-0.76	91.57%	6.23
		8	$5.81 (\pm 2.45)$	-0.75	90.6%	3.93

Table 4. Performance Comparison using Vicuna-7B on Datasets HEx-PHI and HH-RLHF using N=128

	Dataset	Method	Average Reward	Average Cost	Safety Rate	Inference Time (s)
Vicuna-7B	HEx-PHI	Base	4.69 (± 1.36)	-1.77	48%	1.8
		RECONTROL	$4.75 (\pm 1.31)$	-1.93	49.33%	2.37
		RECONTROL + Lagrangian multiplier	$4.65 (\pm 1.33)$	-2.07	50.7%	2.62
		Best-of-N + Lagrangian multiplier	$5.22 (\pm 1.39)$	-4.05	79.3%	36.32
		Best-of-N + Augmented safety	$6.46 (\pm 1.51)$	-2.69	92.6%	40.17
		Beam search + Lagrangian multiplier	$5.70 (\pm 1.57)$	-4.32	83%	28.8
		Beam search + Augmented safety	$7.57 (\pm 1.67)$	-2.78	89.33%	46.53
		ARGS $\omega = 2.5$	$5.67 (\pm 1.45)$	-0.98	47%	95.53
		ARGS $\omega = 2.5$ + Lagrangian multiplier	$1.72 (\pm 1.96)$	-1.85	93.33%	138.75
		ARGS $\omega = 2.5$ + Cost Model	$0.07 (\pm 1.60)$	-2.21	96%	97.11
		InferenceGuard	$6.90 (\pm 2.08)$	-2.86	96.67%	44.04
		InferenceGuard with Critic	$6.99 (\pm 2.1)$	-2.72	96.67%	53.15
Vicuna-7B	HH-RLHF	Base	5.82 (± 1.56)	-2.72	95%	1.77
		RECONTROL	$5.9 (\pm 1.55)$	-2.72	95.13%	2.17
		RECONTROL + Lagrangian multiplier	$5.85 (\pm 1.50)$	-2.73	95.4%	3.14
		Best-of-N + Lagrangian multiplier	$6.97 (\pm 2.54)$	-3.53	97.24%	33.27
		Best-of-N + Augmented safety	$8.33 (\pm 1.95)$	-2.84	98.36%	34.29
		Beam search + Lagrangian multiplier	$8.05 (\pm 2.25)$	-3.73	97.54%	45.13
		Beam search + Augmented safety	$9.61 (\pm 2.10)$	-2.88	98.23%	47.37
		ARGS $\omega=2.5$	$6.83 (\pm 1.83)$	-2.73	96.2%	109.04
		ARGS $\omega = 2.5$ + Lagrangian multiplier	$2.02 (\pm 1.79)$	-3.6	97.54%	129.04
		ARGS $\omega = 2.5$ + Cost Model	$0.46 (\pm 1.73)$	-3.89	98.96%	102.2
		InferenceGuard	$9.49 (\pm 2.16)$	-2.89	98.97%	45.89
		InferenceGuard with Critic	$9.48 (\pm 2.16)$	-2.89	98.95%	46.15

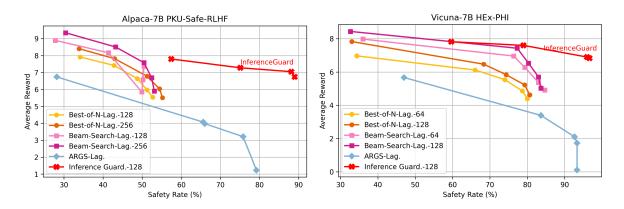


Figure 2. Pareto curves showing the safety-reward trade-offs for Lagrangian-based decoding methods on (1) Alpaca-7B with PKU-SafeRLHF and (2) Vicuna-7B with HEx-PHI. Each curve corresponds to a  $\lambda$  ablation, tracing the approximate Pareto front. The red **InferenceGuard** curve represents its performance across safety budget d and consistently **dominates all baseline method fronts** in both settings.

Table 5. Performance Comparison of Lagrangian Multiplier-Based Methods on Dataset PKU-SafeRLHF using Different  $\lambda$  and N

	Method	λ	Average Reward	Average Cost	Safety Rate	Inference Time (s)
	InferenceGuard. $N=128$	-	$7.08 (\pm 2.49)$	-0.63	88.14%	65
		0	$7.92 (\pm 1.43)$	1.35	34.12%	32.1
		1	$7.42 (\pm 1.72)$	0.20	42.82%	31.8
	Best-of-N Lag. $N = 128$	2.5	$6.64 (\pm 1.89)$	-0.60	48.75%	35.5
		5	$5.97 (\pm 1.82)$	-0.96	51.25%	38.5
		10	$5.54 (\pm 1.64)$	-1.12	52.70%	35.5
		0	8.41 (± 1.45)	1.37	33.86%	58.1
		1	$7.82 (\pm 1.75)$	0.11	42.95%	54.9
	Best-of-N Lag. $N=256$	2.5	$6.78 (\pm 2.01)$	-0.87	51.25%	55.1
		5	$6.04 (\pm 1.85)$	-1.26	54.41%	52.6
		10	$5.51 (\pm 1.69)$	-1.40	55.20%	59.9
		0	$8.90 (\pm 1.71)$	1.65	27.80%	34.88
Alpaca-7B		1	$8.17 (\pm 2.10)$	0.12	41.37%	35
	Beam Search Lag. $N = 128$	2.5	$7.37 (\pm 2.22)$	-0.71	50.46%	35.08
		5	$6.58 (\pm 1.95)$	-1.02	50.19%	32
		10	$5.85 (\pm 1.79)$	-1.19	49.93%	34.76
		0	$9.35 (\pm 1.83)$	1.61	30.43%	57.24
		1	$8.51 (\pm 2.17)$	-0.01	43.21%	58.52
	Beam Search Lag. $N=256$	2.5	$7.58 (\pm 2.25)$	-0.92	50.46%	58.6
		5	$6.69 (\pm 2.08)$	-1.28	52.43%	60.1
		10	$5.90 (\pm 1.82)$	-1.48	53.10%	59.04
		0	$6.74 (\pm 1.70)$	1.47	28.19%	82
		1	$4.07 (\pm 1.64)$	-0.04	65.6%	109
	ARGS Lag.	2.5	$3.98 (\pm 1.61)$	-0.12	66.0%	122
		5	$3.21 (\pm 1.59)$	-0.85	75.8%	111
		10	$1.23 (\pm 1.63)$	-1.76	79.2%	107
	InferenceGuard. $N=128$	-	$10.26~(\pm~1.42)$	-2.96	99.7%	39
		0	$8.68 (\pm 1.37)$	-2.82	77.07%	27.2
		1	$8.47 (\pm 1.45)$	-3.28	81.69%	26.6
	Best-of-N Lag. $N = 64$	2.5	$7.95 (\pm 1.64)$	-3.60	85.11%	29.3
		5	$7.06 (\pm 1.77)$	-3.86	87.48%	28.0
		10	$6.22 (\pm 1.69)$	-4.00	88.14%	26.6
		0	$9.15 (\pm 1.32)$	-2.76	76.82%	47.2
		1	$8.92 (\pm 1.43)$	-3.29	81.69%	48.1
	Best-of-N Lag. $N = 128$	2.5	$8.35 (\pm 1.64)$	-3.66	84.19%	46.3
		5	$7.30 (\pm 1.80)$	-3.96	87.20%	47.5
		10	$6.31 (\pm 1.76)$	-4.12	87.62%	46.9
		0	$11.02 (\pm 1.34)$	-2.72	74.70%	20.72
Beaver-7B-v3		1	$10.64 (\pm 1.37)$	-3.47	82.35%	23.6
	Beam Search Lag. $N = 64$	2.5	$9.99 (\pm 1.58)$	-3.95	87.62%	20.04
		5	$9.84 (\pm 1.4)$	-2.93	95.38 %	22.15
		10	$7.60 (\pm 1.82)$	-4.47	89.20%	19.88
		0	$10.54 (\pm 1.29)$	-2.75	74.44%	35.68
		1	$10.25 (\pm 1.41)$	-3.47	82.74%	36.2
	Beam Search Lag. $N=128$	2.5	$9.57 (\pm 1.60)$	-3.93	87.10%	38.4
		5	$10.31 (\pm 1.37)$	-2.94	97.36%	39
		10	$7.34 (\pm 1.86)$	-4.41	88.41%	39.6
		0	$6.72 (\pm 1.83)$	-2.59	78.5%	94
		1	$4.01 (\pm 1.61)$	-2.22	80.9%	102
	ARGS Lag.	2.5	$3.67 (\pm 1.61)$	-2.10	80.65%	119
	-	5	$2.26~(\pm~1.56)$	-1.64	81%	127
		10	$0.95 (\pm 1.67)$	-2.84	90.8%	110

Table 6. Performance Comparison of Lagrangian Multiplier-Based Methods on Datasets HEx-PHI and HH-RLHF using Different  $\lambda$  and N (Vicuna-7B-v1.5)

Dataset	Method	λ	Average Reward	Average Cost	Safety Rate	Inference Time (
	InferenceGuard. $N=128$	-	$6.80  (\pm  2.13)$	-2.71	96.33%	44.04
		0	$6.97 (\pm 1.27)$	-0.58	34.33%	26.0
		1	$6.12 (\pm 1.52)$	-3.27	66%	25.6
	Best-of-N Lag. $N = 64$	2.5	$5.54 (\pm 1.55)$	-3.69	74%	27.4
		5	$4.86 (\pm 1.52)$	-3.91	78.66%	26.5
		10	$4.39 (\pm 1.43)$	-4.02	80%	28.2
		0	$7.83 (\pm 1.19)$	-0.36	33%	31.68
		1	$6.47 (\pm 1.43)$	-3.39	68.33%	33.82
	Best-of-N Lag. $N = 128$	2.5	$5.84 (\pm 1.51)$	-3.85	74.33%	36
		5	$5.22 (\pm 1.39)$	-4.05	79.3%	36.32
		10	$4.62 (\pm 1.37)$	-4.05	80.67%	35.33
		0	$7.98 (\pm 1.62)$	-0.58	36%	27.2
HEx-PHI		1	$6.96 (\pm 1.67)$	-3.58	76.33%	29.14
	Beam Search Lag. $N = 64$	2.5	$6.28 (\pm 1.66)$	-3.90	79.33%	30.26
		5	$5.36 (\pm 1.42)$	-4.18	83%	29.36
		10	$4.91 (\pm 1.49)$	-4.29	84.67%	28.42
		0	8.44 (± 1.64)	-0.27	32.67%	47.3
		1	$7.43 (\pm 1.76)$	-3.57	77.3%	47.2
	Beam Search Lag. $N = 128$	2.5	$6.53 (\pm 1.75)$	-4.09	80.33%	46.2
	· ·	5	$5.70 (\pm 1.57)$	-4.32	83%	45.13
		10	$5.02 (\pm 1.51)$	-4,46	83.67%	45.65
		0	5.67 (± 1.45)	-0.98	47%	95.53
		1	$3.39 (\pm 1.6)$	-1.05	83.67%	126.71
	ARGS Lag.	2.5	$2.1 (\pm 1.73)$	-1.49	92.67%	132.13
		5	$1.72 (\pm 1.96)$	-1.85	93.33%	138.75
		10	$0.11 (\pm 1.59)$	-2.09	93.33%	140.6
	InferenceGuard. $N = 128$	-	9.49 (± 2.16)	-2.89	98.97%	45.89
		0	$9.14 (\pm 1.99)$	-2.86	95.33%	22.48
	Best-of-N Lag. $N=64$	1	$7.80 (\pm 1.89)$	-3.02	95.98%	24.29
		2.5	$7.44 (\pm 2.09)$	-3.24	96.53%	22.85
		5	$6.67 (\pm 2.48)$	-3.45	97.10%	27.33
		10	$5.44 (\pm 2.69)$	-3.63	97.24%	26.55
		0	9.42 (± 2.01)	-2.93	95.6%	32.02
	Best-of-N Lag. $N=128$	1	$8.32 (\pm 1.90)$	-3.03	95.84%	34.8
		2.5	$7.85 (\pm 2.14)$	-3.24	96.87%	32.96
		5	$6.97 (\pm 2.54)$	-3.53	97.24%	33.27
		10	$5.47 (\pm 2.88)$	-3.77	97.5%	35.15
		0	9.14 (± 1.99)	-2.86	95.33%	30.43
HH-RLHF		1	$8.87 (\pm 2.29)$	-3.16	96.59%	27.66
	Beam Search Lag. $N = 64$	2.5	$8.47 (\pm 2.37)$	-3.41	96.96%	28.17
		5	$7.88 (\pm 2.20)$	-3.88	96.6%	32.08
-		10	$6.80 (\pm 2.42)$	-3.79	97.51%	36.25
		0	$9.42 (\pm 2.01)$	-2.93	95.6%	45.6
		1	$9.33 (\pm 2.36)$	-3.18	96.4%	44.22
	Beam Search Lag. $N = 128$	2.5	$8.89 (\pm 2.51)$	-3.47	97.44%	46.59
	Deam Search Eag. 17 — 120	5	$8.05 (\pm 2.25)$	-3.73	97.54%	45.13
		10	$6.85 (\pm 2.56)$	-3.92	97.77%	49.02
		0	$6.83 (\pm 2.30)$ $6.83 (\pm 1.83)$	-2.73	96.2%	109.04
		1	$3.98 (\pm 1.79)$	-2.73	90.2% 97%	111.66
	ARGS Lag.	2.5	$2.65 (\pm 1.79)$	-2.88 -3.34	97% 96.4%	135.9
	AKOS Lag.	2.3 5			90.4% 97.54%	
		J	$2.02 (\pm 1.79)$	-3.6	71.34%	129.04