7.1 Law of Sines.
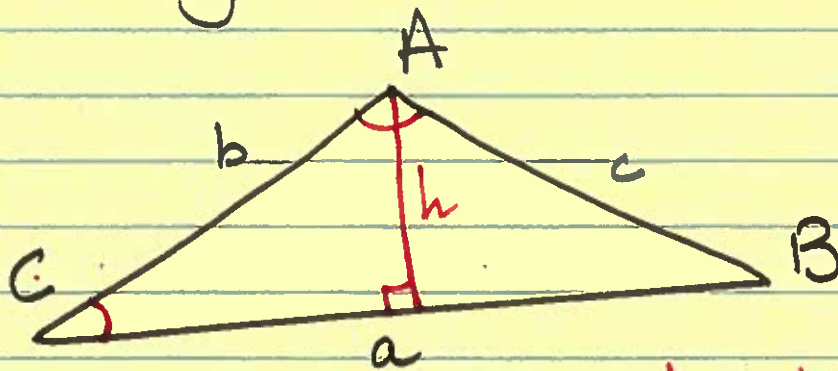

$$Area = \frac{1}{2} ah$$

$$Sin C = \frac{h}{b}$$

$$h = b Sin C$$

$$\boxed{Area = \frac{1}{2} ab Sin C}$$

$$\frac{a}{Sin A} = \frac{b}{Sin B} = \frac{c}{Sin C}$$

$$\downarrow$$
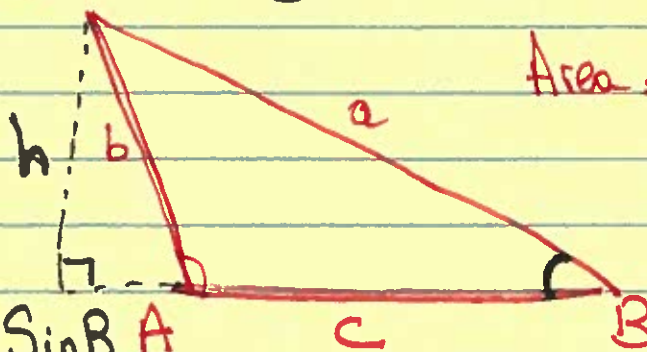
$$\frac{Sin A}{a} = \frac{Sin B}{b} = \frac{Sin C}{c}$$

Area of triangle:

$$\frac{1}{2} Base \times height$$

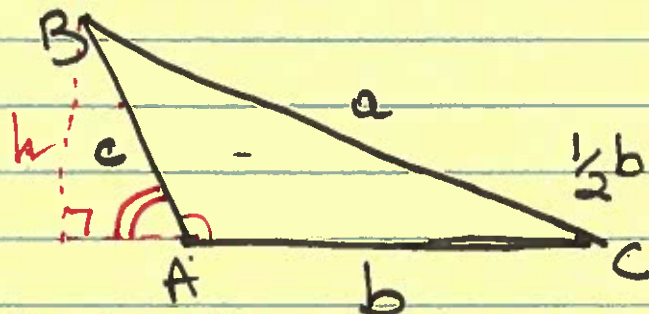Example:

$$\frac{1}{2} ab Sin C = \frac{1}{2} ac Sin B$$

$$Area = \frac{1}{2} c h = \boxed{\frac{1}{2} ca Sin B}$$

$$Sin B = \frac{h}{a}$$

$$h = a Sin B$$
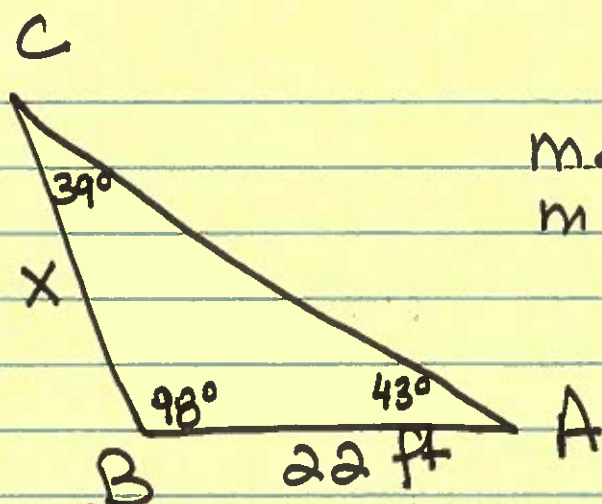

$$\frac{1}{2} b \cdot h = \boxed{\frac{1}{2} bc Sin A}$$

$$Sin C = \frac{h}{a}$$

$$Sin (\pi - A) = \frac{h}{c}$$

$$\boxed{Sin A = \frac{h}{c}}$$

$$\frac{\frac{1}{2} bc Sin A}{\frac{1}{2} abc} = \frac{\frac{1}{2} ca Sin B}{\frac{1}{2} abc} = \frac{\frac{1}{2} ab Sin C}{\frac{1}{2} abc}$$

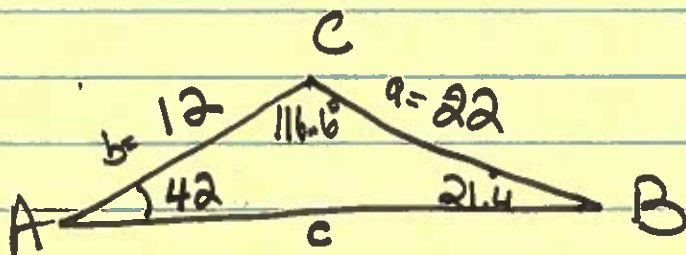$$\boxed{\frac{Sin A}{a} = \frac{Sin B}{b} = \frac{Sin C}{c}}$$

$$\frac{\sin 43}{x} = \frac{\sin 39}{22}$$

$$x = \frac{\sin 43}{\sin 39} \cdot 22$$

$$\boxed{x = 23.84 \text{ ft}}$$
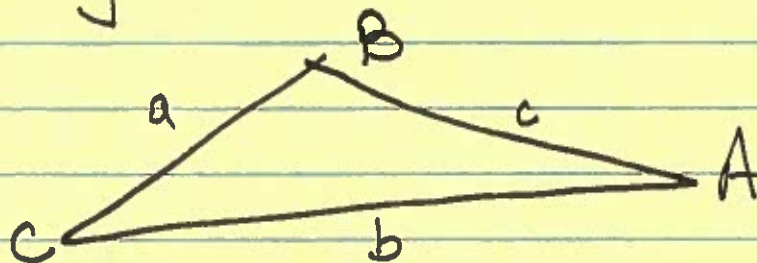

Find the remaining side and angles.

$$\boxed{\frac{\sin 42}{22} = \frac{\sin B}{12}} \longrightarrow \sin B = \frac{12}{22} \sin 42$$

$$\frac{\sin 42}{22} = \frac{\sin 116.6°}{c} \qquad B = \sin^{-1}\left(\frac{12}{22} \sin 42\right)$$

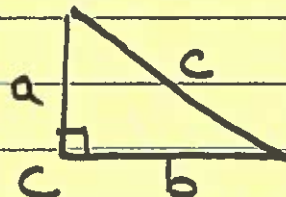29.38 $c = \frac{\sin 116.6}{\sin 42} \cdot 22$ $B = 21.4°$
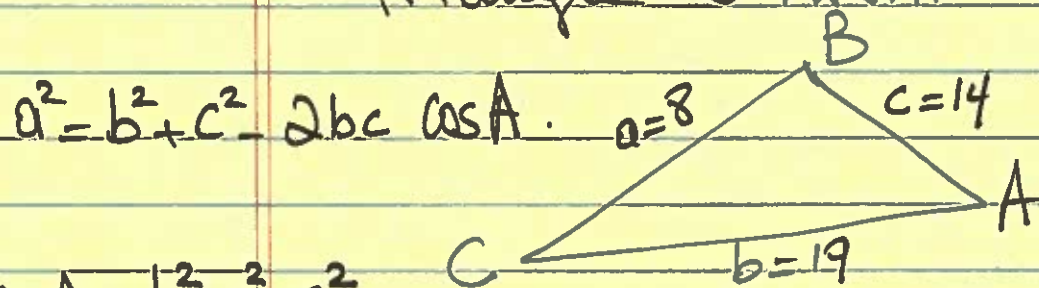
7.2 Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Find the three angles of the triangle shown.

$$a^2 = b^2 + c^2 - 2bc \cos A. \qquad a = 8$$
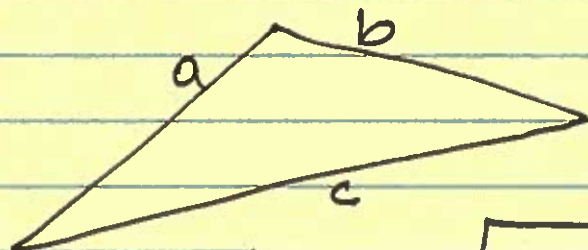

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \boxed{A = 22.08°}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} < 0 \qquad B = 116.8°$$

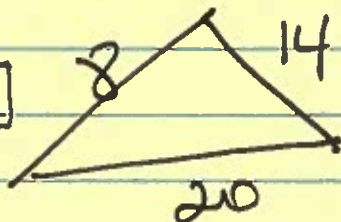$$C = 180 - 116.8 - 22.08 = \boxed{41.12}$$

Heron's Area formula

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)}$$
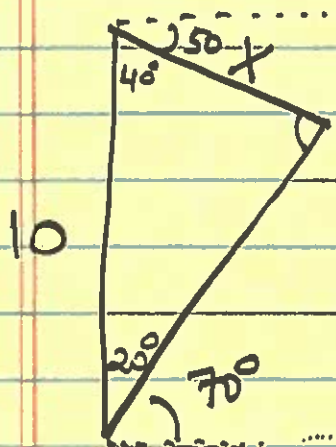
$$S = \frac{1}{2}[a+b+c]$$

Find the area of the ~~previous~~ triangle.

$$S = \frac{1}{2}[8+14+20] = \boxed{21}$$

$$\text{Area} = \sqrt{21(21-8)(21-14)(21-20)} \checkmark$$

$$\boxed{43.715 \text{ unit}^2}$$

$$\frac{\sin 20}{x} = \frac{\sin 120}{10}$$

$$x = \frac{\sin 20}{\sin 120} 10$$