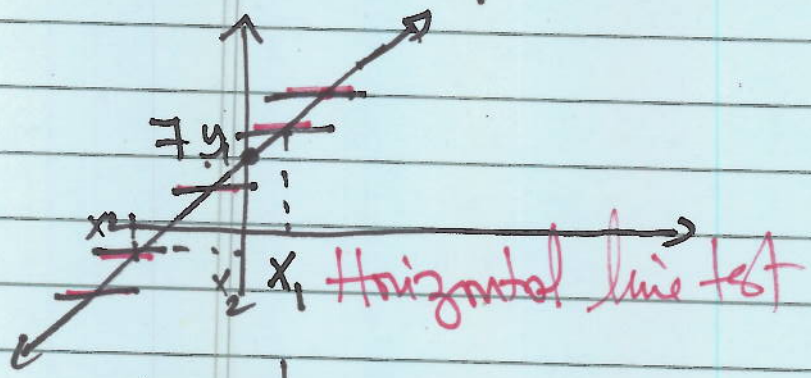
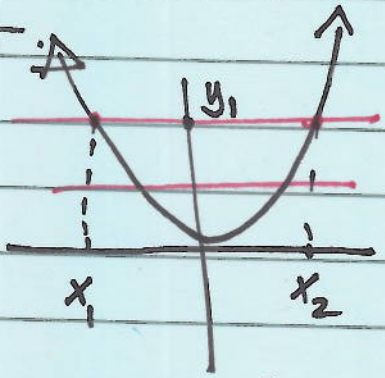
1.7 Inverse functions

$f(x) = 3x + 7$



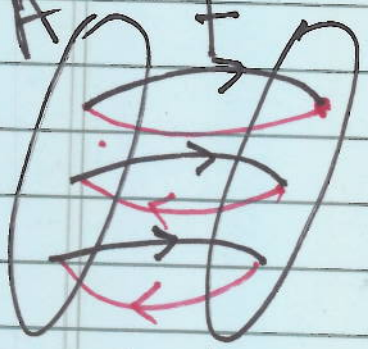
Horizontal line test

one-to-one



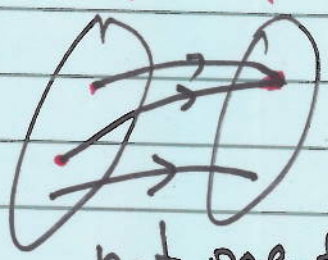
$f(x_1) = f(x_2)$

Given $x_1 \neq x_2$.



one to one

inverse.



not one-to-one.

is not a function

$f(x) = 3x + 7$ one-to-one.

let $y = 3x + 7$ Switch x and y.

$x = 3y + 7$ Solve for y.

$y = \frac{x-7}{3} = \frac{1}{3}x - \frac{7}{3}$. Inverse function

$f^{-1}(x) = \frac{1}{3}x - \frac{7}{3}$

$g(x) = \frac{1}{3}x - \frac{7}{3}$; $f(x) = 3x + 7$

$f \circ g$; $g \circ f$.

$$f(g(x)) = 3g(x) + 7 = 3\left(\frac{1}{3}x - \frac{7}{3}\right) + 7.$$

$$= x - 7 + 7$$

$$\boxed{f(g(x)) = x}.$$

$$(g \circ f)(x) = g(f(x)) = \frac{1}{3}f(x) - \frac{7}{3}.$$

$$= \frac{1}{3}(3x + 7) - \frac{7}{3}.$$

$$= x + \frac{7}{3} - \frac{7}{3}$$

$$(g \circ f)(x) = x = g(f(x)).$$

If $f(g(x)) = g(f(x)) = x$ Then.

$f = g^{-1}$ or $g = f^{-1}$

they are Inverse of each other.

Check if $f(x)$ and $g(x)$ are inverse functions

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(25) $f(x) = -\frac{7}{2}x - 3$ $\quad g(x) = -\frac{2x+6}{7}$

$y = -\frac{7}{2}x - 3.$

Switch. $x = -\frac{7}{2}y - 3$

Solve for y: $x + 3 = -\frac{7}{2}y.$

$y = -\frac{2}{7}(x+3) = -\frac{2x+6}{7}$ ✓

Check $f(g(x)) = g(f(x)) = x$

$f(g(x)) = -\frac{7}{2}g(x) - 3$

$= -\frac{7}{2}\left(-\frac{2x+6}{7}\right) - 3$

$= x + 3 - 3$

$= x$ ✓

$g(f(x)) = -\frac{2f(x)+6}{7}$

$= -\frac{2\left(-\frac{7}{2}x - 3\right) + 6}{7}$

$= -\frac{-7x - 6 + 6}{7}$

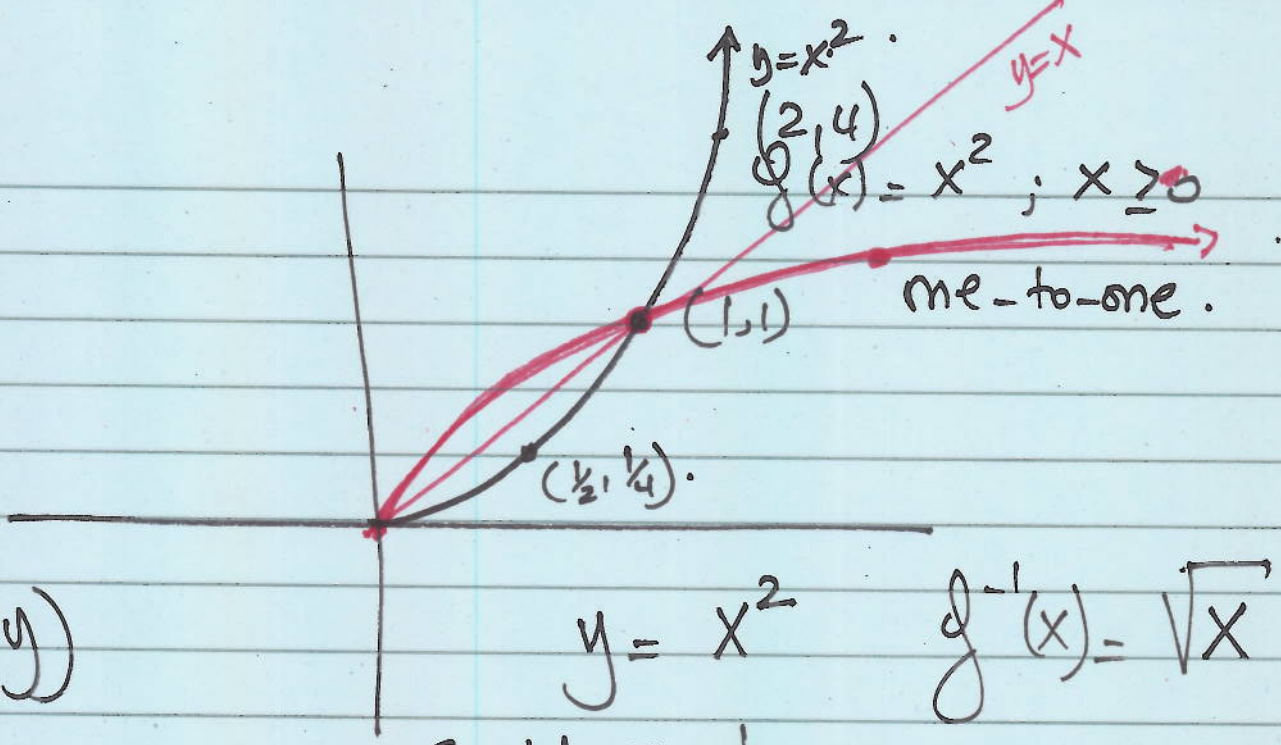
$= x$ ✓

$f(x) = |x|$

NO inverse bc not one-to-one

$g(x) = x^2 \quad x \geq 0$

$g^{-1}(x) = \sqrt{x} \quad x \geq 0$ ✓



(x, y)

$y = x^2$ $\quad g^{-1}(x) = \sqrt{x}$

Switch x and.

$x = y^2$.

$|y| = \sqrt{x}$

$y = \sqrt{x}$ ✓