Practice Final Exam

Name Solution

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

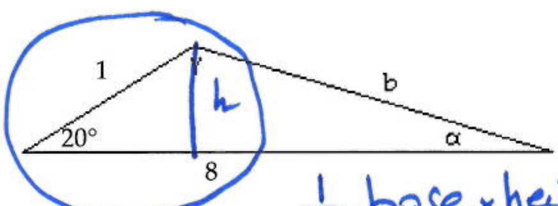
Complete the identity.

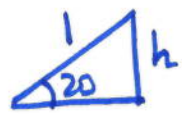
1) $\tan(\pi - \theta) = ?$ $\dfrac{\sin(\pi-\theta)}{\cos(\pi-\theta)} = \dfrac{\sin\theta}{-\cos\theta} = -\tan\theta$

1) $\boxed{-\tan\theta}$

Find the area of the triangle. If necessary, round the answer to two decimal places.

2)





$\sin 20 = \frac{h}{1} \therefore h = \sin 20$.

$\frac{1}{2}\text{base} \times \text{height} = \frac{1}{2} 8\, h = \frac{1}{2}(8)\sin 20$

2) $4\sin 20 = \boxed{1.37}$

Find the exact value of the expression.

3) $\cot\left[\sin^{-1}\frac{\sqrt{2}}{2}\right] = \cot(\pi/4) = 1$

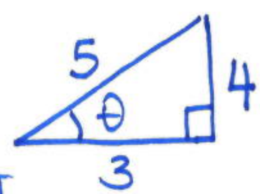
3) $\boxed{1}$

4) $\sin\left[2\cos^{-1}\left(-\frac{3}{5}\right)\right] = \sin(2\theta) = 2\sin\theta\cos\theta = 2\,\frac{4}{5}\left(-\frac{3}{5}\right)$

Let $\theta = \cos^{-1}(-3/5)$

$\boxed{\cos\theta = -3/5}$

θ in QII



4) $\boxed{\dfrac{-24}{25}}$

5) $\sin\frac{\pi}{12}$

$\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{6}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2}$ $\boxed{\dfrac{\sqrt{6}-\sqrt{2}}{4}}$

5) $\dfrac{\sqrt{2}(\sqrt{3}-1)}{4}$

Find the exact value of the expression. Do not use a calculator.

6) $\sin^{-1}\left[\sin\left(\frac{3\pi}{5}\right)\right] = \sin^{-1}\left(\sin\left(\pi - \frac{2\pi}{5}\right)\right) = \sin^{-1}\left(\sin\left(\frac{2\pi}{5}\right)\right)$

6) $\boxed{2\pi/5}$

Key $\therefore$ $\sin^{-1}(\sin x) = x \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

$\frac{3\pi}{5} = \pi - \frac{2\pi}{5}$

$\cos^{-1}(\cos y) = y \quad 0 \le y \le \pi$

Graph the function. Show at least one period.

7) $y = -3 \sin\left(5x + \frac{\pi}{2}\right)$ 7) ________

$y = -3\sin 5\left(x + \frac{\pi}{10}\right)$

Amplitude $|-3| = 3$

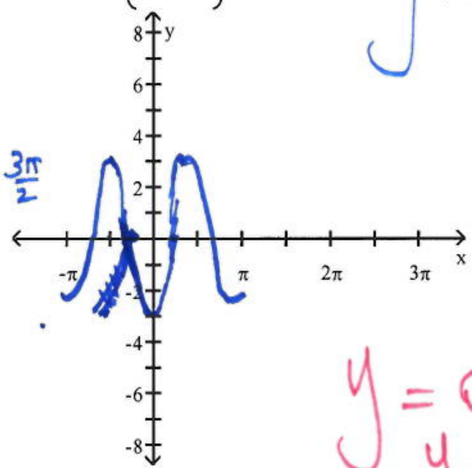
Period $T = \frac{2\pi}{5}$

Phase Shift $= \frac{\pi}{10}$

$5x + \frac{\pi}{2} = \frac{3\pi}{2}$

$5x = \pi$

$\boxed{x = \frac{\pi}{5}}$



$y = a\sin k(x - b)$ $T = \frac{2\pi}{k}$

$y = a\cos k(x - b)$ $|a|$; b phase shift.

In the problem, sin θ and cos θ are given. Find the exact value of the indicated trigonometric function.

8) $\sin\theta = \frac{2\sqrt{2}}{3}$, $\cos\theta = \frac{1}{3}$ Find $\csc\theta$. $= \frac{1}{\sin\theta} = \frac{3}{2\sqrt{2}} = \boxed{\frac{3\sqrt{2}}{4}}$ 8) $\boxed{\frac{3\sqrt{2}}{4}}$

θ in QI

In the problem, t is a real number and P = (x, y) is the point on the unit circle that corresponds to t. Find the exact value of the indicated trigonometric function of t.

9) $(\frac{3}{7}, -\frac{2\sqrt{10}}{7})$ Find csc t. $= \frac{1}{y} = \frac{7}{-2\sqrt{10}} = \boxed{\frac{-7\sqrt{10}}{20}}$ $x = 1\cos\theta$; $y = 1\sin\theta$ 9) $\boxed{\frac{-7\sqrt{10}}{20}}$

$r^2 = x^2 + y^2 = \frac{9}{49} + \frac{40}{49} = 1$ unit circle.

-3
4
5

10) $(-\frac{\sqrt{65}}{9}, -\frac{4}{9})$ Find sin t. 10) $\boxed{-4/9}$

$\sin t = y$

$r^2 = \frac{65}{81} + \frac{16}{81} = \frac{81}{81} = 1$

$\boxed{\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}}$

$\left(\frac{3}{4}, \frac{7}{8}\right)$ $\sin t = \frac{7/8}{\sqrt{7}}$

Simplify the expression.

11) $\frac{\cos\theta}{1 + \sin\theta} + \tan\theta = \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta}$ 11) $\boxed{\frac{1}{\cos\theta} = \sec\theta}$

$\frac{\cos^2\theta + \sin^2\theta + \sin\theta}{(1+\sin\theta)\cos\theta} = \frac{\cancel{(1+\sin\theta)}}{(1+\sin\theta)\cos\theta}$

$r^2 = \frac{9}{16} + \frac{49}{64}$.

$r = \sqrt{?}$

Simplify the trigonometric expression by following the indicated direction.

12) Multiply and simplify: $\frac{(\cot\theta + 1)(\cot\theta + 1) - \csc^2\theta}{\cot\theta}$ 12) $\boxed{2}$

$\frac{\cot^2\theta + 2\cot\theta + 1 - \csc^2\theta}{\cot\theta} = \frac{2\cot\theta}{\cot\theta}$

$\cot^2\theta + 1 = \csc^2\theta$

Solve the equation on the interval $0 \le \theta < 2\pi$.

13) $\tan(2\theta) - \tan\theta = 0$ $\rightarrow$ $\tan 2\theta = \tan\theta$

$\frac{\tan\theta + \tan\theta}{1 - \tan^2\theta} - \tan\theta = 0$

$2\theta = \theta + n\pi$ $\quad \frac{2\tan\theta}{1-\tan^2\theta} = \tan\theta$

$\boxed{\theta = n\pi}$

13) $\boxed{0, \pi}$

14) $\sin^2\theta - \cos^2\theta = 0$

$-\cos 2\theta = 0$ $\boxed{\cos 2\theta = 0}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$2\tan\theta = \tan\theta - \tan^3\theta$

$\tan^3\theta + \tan\theta = 0$

$\tan\theta[\tan^2\theta + 1] = 0$

$\tan\theta = 0$; $\boxed{\theta = 0, \pi}$

$2\theta = (2n+1)\frac{\pi}{2}$

$\theta = (2n+1)\frac{\pi}{4}$

14) $\boxed{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

15) $\sin(2\theta) + \sin\theta = 0$

15) __________

Solve the equation. Express irrational answers in exact form and as a decimal rounded to 3 decimal places.

16) $\left(\frac{9}{7}\right)^x = 5^{1-x}$

$x \ln \frac{9}{7} = (1-x)\ln 5 \rightarrow x \ln\frac{9}{7} + x\ln 5 = \ln 5$

$x[\ln 9 - \ln 7 + \ln 5] = \ln 5$

16) $\boxed{x = \frac{\ln 5}{\ln 9 - \ln 7 + \ln 5}}$

17) $\ln x + \ln(x + 6) = 2$

Compress $\ln x(x+6) = 2 \rightarrow x^2 + 6x = e^2$

$x^2 + 6x - e^2 = 0$ $\quad x = -3 - \sqrt{9+e^2}$, $\boxed{x = -3 + \sqrt{9+e^2}}$

17) $x = -3 + \sqrt{9+e^2}$

Solve the problem.

18) If $\sin\theta = \frac{1}{8}$, find $\csc\theta$. $= \boxed{8}$

18) $\boxed{8}$

19) An airplane is sighted at the same time by two ground observers who are 5 miles apart and both directly west of the airplane. They report the angles of elevation as 15° and 25°. How high is the airplane?

19) Quiz Solution

Solve the problem. Leave your answer in polar form.

20) $z = 10(\cos 30° + i \sin 30°)$

$w = 5(\cos 10° + i \sin 10°)$

Find zw.

$= 50[\cos 40 + i\sin 40]$.

20) __________

(14) $\sin^2\theta - \cos^2\theta = 0$

$\sin^2\theta = \cos^2\theta$

$\theta = (2n+1)\frac{\pi}{2}$

$1 - \cos^2\theta - \cos^2\theta = 0$

$\frac{\sin^2\theta}{\cos^2\theta} = 1$

$1 - 2\cos^2\theta = 0$

$\sqrt{\tan^2\theta} = \sqrt{1}$

$\cos^2\theta = \frac{1}{2}$

$|\tan\theta| = 1$

$\tan\theta = 1$

QI, QIII

$\frac{\pi}{4}$, $\frac{\pi}{4} + \pi$

$\cos\theta = \pm\frac{\sqrt{2}}{2}$

$\boxed{\tan\theta = \pm 1}$

$\tan\theta = -1$

QII, QIV.

$\pi - \frac{\pi}{4}$; $2\pi - \frac{\pi}{4}$

$\sin^2\theta - \cos^2\theta = 0$

$\sin^2\theta - (1 - \sin^2\theta) = 0$

$2\sin^2\theta - 1 = 0$

$\bar{\theta} = \frac{\pi}{4}$

$\sin^2\theta = \frac{1}{2}$. $\sin\theta = \pm\frac{\sqrt{2}}{2}$

$\sin\theta \sin\theta$

#15 $\sin(2\theta) + \sin\theta = 0$

$\downarrow$

$2\sin\theta\cos\theta + \sin\theta = 0$

$\sin\theta[2\cos\theta + 1] = 0$

$\frac{\tan \cancel{x}}{\cancel{x}}$

QII, QIII

$\sin\theta = 0$ $\qquad$ $\cos\theta = -\frac{1}{2}$ $\qquad$ $\bar{\theta} = \frac{\pi}{3}$

$\tan$

$\theta = n\pi$ $\qquad$ $\theta = \pi - \frac{\pi}{3} = \boxed{\frac{2\pi}{3}}$

$\boxed{\theta = 0, \pi}$ $\qquad$ $\theta = \pi + \frac{\pi}{3} = \boxed{\frac{4\pi}{3}}$

$\frac{\cancel{a} + b}{\cancel{a} + c}$

21) $z = 5(\cos 200° + i \sin 200°)$
$w = 4(\cos 50° + i \sin 50°)$
Find $\frac{z}{w}$. $= \frac{5}{4}\left[\cos 150 + i \sin 150\right].$

21) ______

Use the given zero to find the remaining zeros of the function.

22) $f(x) = x^4 - 12x^2 - 64$; zero: $-2i$, $2i$

22) $\boxed{2i, \pm 4}$

$$x^2+4 \overset{x^2-16}{\overline{\big)\, x^4 - 12x^2 - 64}}$$
$$-x^4 \quad +4x^2$$
$$-16x^2 - 64$$
$$\boxed{0}$$

$f(x) = (x-2i)(x+2i)\,Q(x) = (x^2+4)\,Q(x)$

Use the information given about the angle θ, 0 ≤ θ ≤ 2π, to find the exact value of the indicated trigonometric function.

23) $\sin \theta = \frac{5}{13}$, $0 < \theta < \frac{\pi}{2}$ Find $\cos(2\theta)$.

23) $\frac{169-50}{169} = \boxed{\frac{119}{169}}$

$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\frac{25}{169}$

$\cos 2\theta = 2\cos^2\theta - 1$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$

Write the complex number in polar form. Express the argument in degrees, rounded to the nearest tenth, if necessary.

24) $1 - \sqrt{3}i$

24) $\boxed{2\left[\cos 300 + i \sin 300\right]}$

(r, θ)

QIV $\tan\theta = \frac{-\sqrt{3}}{1}$

$r = \sqrt{1+3} = 2$

$\bar{\theta} = \frac{\pi}{3} \longrightarrow \theta = 2\pi - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}} = \boxed{300}$

Write the expression in the standard form a + bi.

25) $[2(\cos 15° + i \sin 15°)]^3 = 2^3\left[\cos 45 + i \sin 45\right].$

25) $\boxed{4\sqrt{2} + i4\sqrt{2}}$

De Moivre theorem

$8\left[\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right].$

$\boxed{4\sqrt{2} + i4\sqrt{2}}$