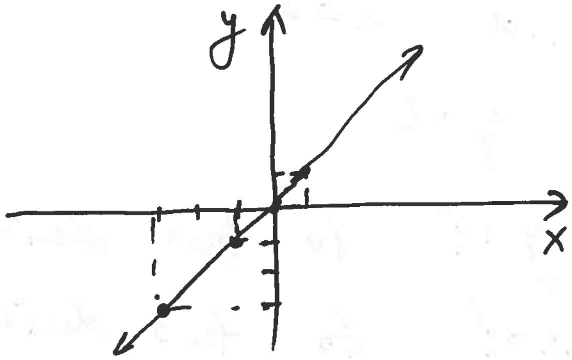
1.1 Graphs and Models

Given $y = \frac{1}{30}x(39 - 10x^2 + x^4)$

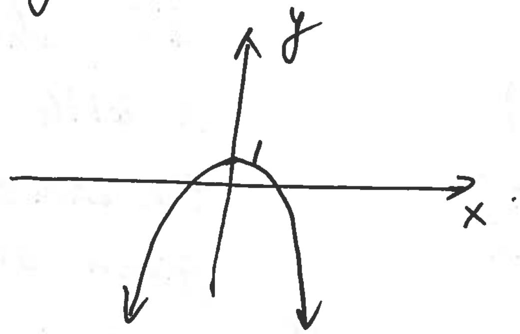
$\left(\begin{array}{c} y^2 = x \\ y = \pm\sqrt{x} \\ \text{(not a function)} \end{array} \right)$

x	y
-3	-3
-1	-1
0	0
1	1



not the correct graph!

Graph: $y = -x^2 + 1$ - parabola



Intercepts of a Graph.

x-intercepts - where the graph crosses the x-axis. To find it, set $y=0$ and solve for x.

y-int. - where the graph crosses the y-axis. Set $x=0$ and solve for y.

Ex.: $y = x^2 - 5x$. x-int = ?

y-int = ?

$y = x^2 - 5x$

x-int : $y=0$ $\quad x^2 - 5x = 0$

$x(x-5) = 0$

$x=0 \quad x=5$

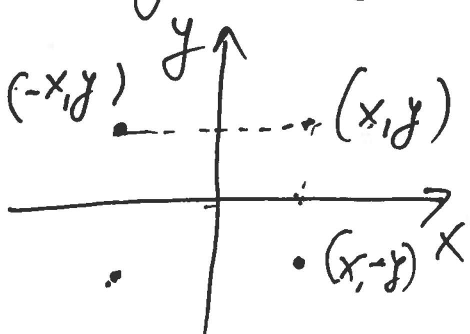
y-int: $x=0 \quad y=0$

a) $(-0.5, y)$ $\quad y = ?$ to two decimal places

b) $(x, -4)$ $\quad x = ?$ to two decimal places.

a) $y \approx 2.75$

Symmetry of a Graph.



If when you substitute x with $-x$, you get the same equation, as the given one, then the graph is symmetric about the y-axis.

If when $x \Leftrightarrow -x$, you get $y = -y$, then the graph is symmetric about the origin.

If when $y \Leftrightarrow -y$ and you get the same equation, the graph is symmetric about the x-axis.

Ex: $y = x^3 + x$ (explicit form of an equation) - check for symmetry

1) $x \Leftrightarrow -x$

$$\underline{\underline{y}} = (-x)^3 + (-x) = -x^3 - x = -\underbrace{(x^3+x)}_{y} = \underline{\underline{-y}}$$

symmetric about the origin

2) $xy - \sqrt{4-x^2} = 0$ — Implicit form of an equation

$x \Leftrightarrow -x$

$-xy - \sqrt{4-(-x)^2} = 0$

$-xy - \sqrt{4-x^2} = 0$ — no symmetry about the y-axis

$y \Leftrightarrow -y$

$-xy - \sqrt{4-x^2} = 0$ — no symmetry about the x-axis.

$x \Leftrightarrow -x$ $\quad y \Leftrightarrow -y$

$(-x)(-y) - \sqrt{4-(-x)^2} = 0$

$xy - \sqrt{4-x^2} = 0$, so symmetric about the origin.

Points of Intersection

Ex: (#60 p. 8) Given $\begin{cases} x = 3 - y^2 \\ y = x - 1 \end{cases} \leftarrow$

Use substitution: $x = 3 - (x-1)^2$

$x = 3 - (x^2 - 2x + 1)$

$x = 3 - x^2 + 2x - 1$

$x^2 - x - 2 = 0$

$(x+1)(x-2) = 0$

$x = -1 \quad x = 2$

Points of intersection

$(-1, -2)$

$(2, 1)$

1.2 Linear Models and Rates of Change.

Point-slope : $y - y_1 = m(x - x_1)$ where (x_1, y_1) is a given point and m is a given slope.

Given (x_1, y_1) and (x_2, y_2) ; then the slope m is $\frac{y_1 - y_2}{x_1 - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$.

Ex) Given $(-5, -2)$ m is undefined

Equation is ? $x = -5$

2) Given $(-2, 4)$ $m = -\frac{3}{5}$. Write an equation in

a) slope-intercept form ($y = mx + b$)

b) General form ($Ax + By + C = 0$, where A, B, and C are integers)

a)
$$y - 4 = -\frac{3}{5}(x + 2)$$
$$y = 4 - \frac{3}{5}x - \frac{6}{5}$$
$$y = -\frac{3}{5}x + \frac{14}{5}$$

b)
$$y = -\frac{3}{5}x + \frac{14}{5} \quad (* \text{ 5 both sides})$$
$$5y = -3x + 14$$
$$3x + 5y - 14 = 0$$

Ratios and Rates of Change.

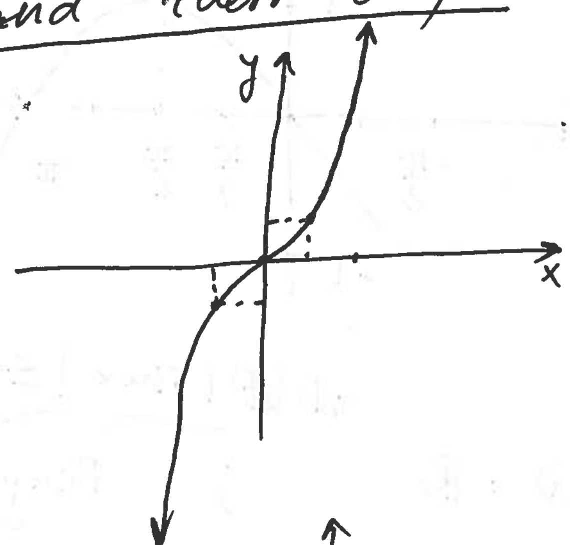
Slope = $\dfrac{\text{Change in } y}{\text{Change in } x}$. If the units are the same, the slope can be seen as a ratio. If the units a different, it is a rate.

Sketching a line

Ex: $y = 3x + 7$

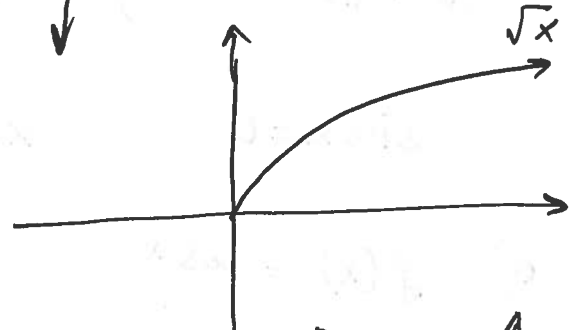
1.3 Functions and their Graph

1) $y = f(x) = x^3$



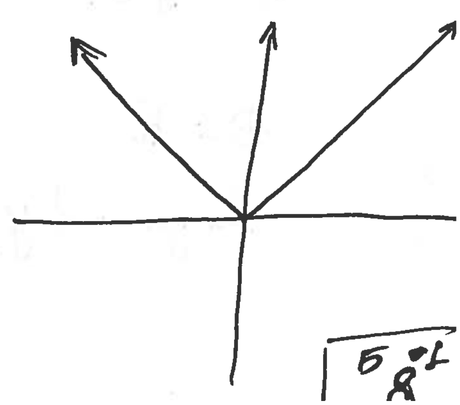
$D = \mathbb{R}$
Range $= \mathbb{R}$

2) $y = f(x) = \sqrt{x}$

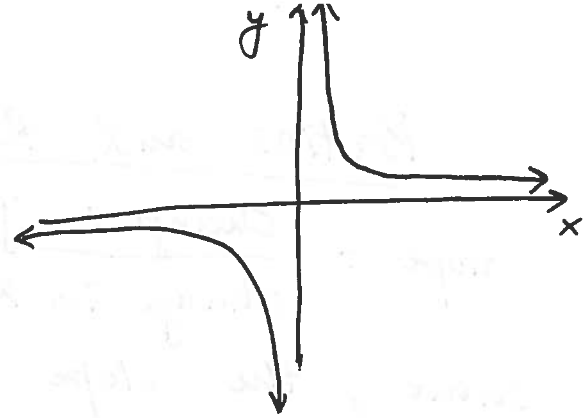


$D = \{x \,/\, x \geq 0\}$
Range $= \{y \,/\, y \geq 0\}$

3) $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$



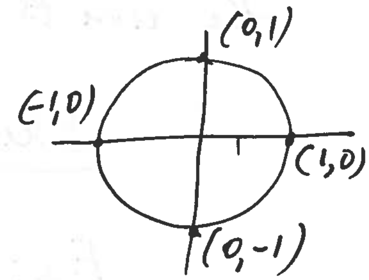
4) $y = f(x) = \frac{1}{x}$



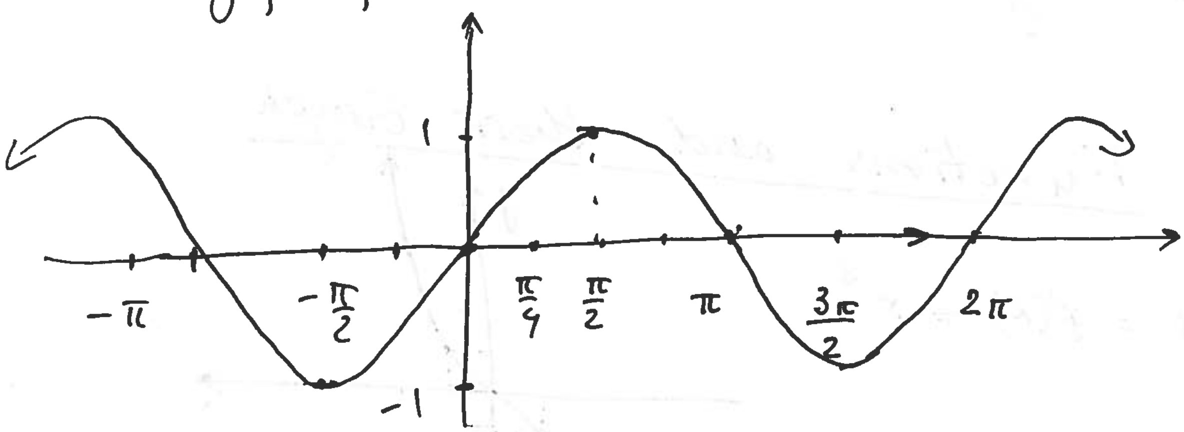
$D = \{x \,/\, x \neq 0\}$

Range $= \{y \,/\, y \neq 0\}$

5) $f(x) = \sin x$



x	$-\pi$	$-\pi/2$	0	$\pi/4$	$\pi/2$	π	$3\pi/2$	2π
y	0	-1	0	$\frac{\sqrt{2}}{2}$	1	0	-1	0

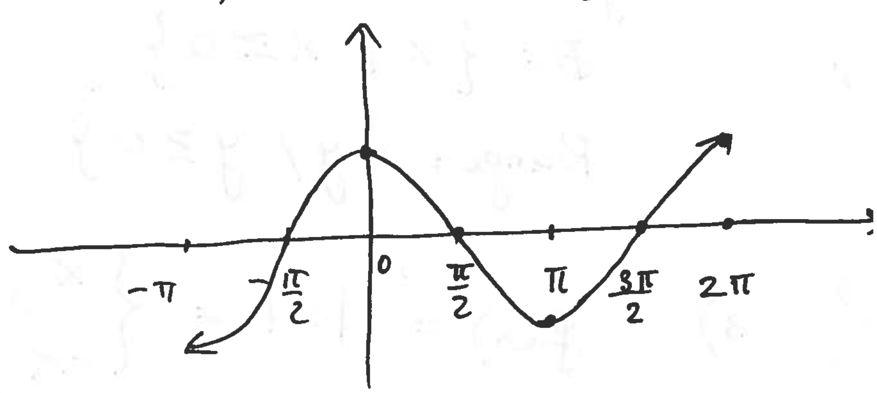


$\underbrace{|\sin x| \leq 1}$ $\qquad$ $|y| \leq 1$

$D = \mathbb{R}$; Range $= \{y \,/\, -1 \leq y \leq 1\}$

$\sin x = 0 \qquad x = n\pi$, n is an integer;

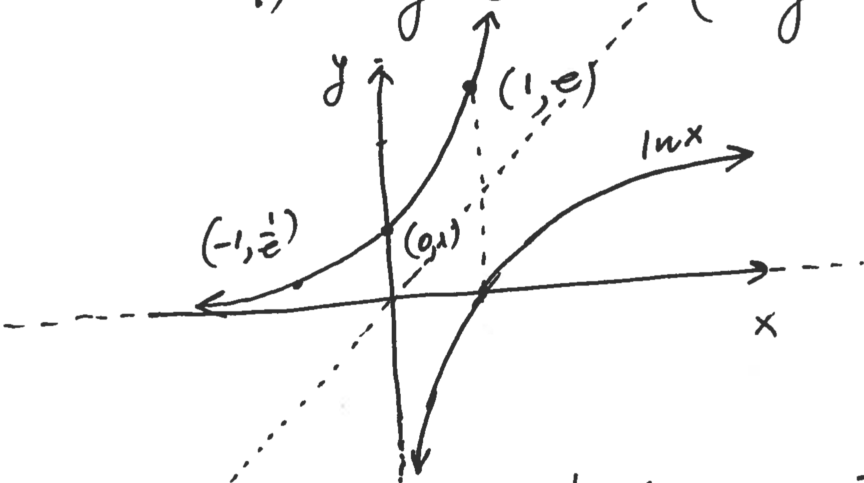
6) $f(x) = \cos x$



$D = \mathbb{R}$

Range $= \{y \,/\, |y| \leq 1\}$

7) $y = e^x$ $\quad (y = a^x \,,\; a > 0 \,,\; a \neq 1 \}$



$D = \mathbb{R}$

Range $= \{ y \,/\, y > 0 \}$

8) $f(x) = \ln x$

$D = \{ x \,/\, x > 0 \}$

Range $= \mathbb{R}$

Ex: $f(x) = -x^2 - 3$ - given

a) $f(2a) = -(2a)^2 - 3 = -4a^2 - 3$

b) $f(b-1) = -(b-1)^2 - 3 = -(b^2 - 2b + 1) - 3 = -b^2 + 2b - 4$

c) $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} =$

$$= \frac{-(x+\Delta x)^2 - 3 - (-x^2 - 3)}{\Delta x} = \frac{-\cancel{x^2} - 2x\cdot\Delta x - (\Delta x)^2 - \cancel{3} + \cancel{x^2} + \cancel{3}}{\Delta x}$$

$$= \frac{\cancel{\Delta x}(-2x - \Delta x)}{\cancel{\Delta x}} = -2x - \Delta x$$

Composition of Two Functions

$f \circ g(x) = f(g(x))$

$\neq$ $g \circ f(x) = g(f(x))$

Ex: $f(x) = \sin x$

$g(x) = \pi x$

Find: a) $f(g(\frac{1}{2})) = \sin(g(\frac{1}{2})) = \sin(\frac{\pi}{2}) = 1$

b) $g(f(0)) = \pi(f(0)) = \pi(\sin 0) = \pi \cdot 0 = 0$

c) $f(g(2)) = \sin(g(2)) = \sin(2\pi) = 0$

d) $g(f(\frac{\pi}{4})) = \pi(\sin\frac{\pi}{4}) = \pi \cdot \frac{\sqrt{2}}{2} = \frac{\pi\sqrt{2}}{2}$

e) $f(g(x)) = \sin(\pi x)$

f) $g(f(x)) = \pi \cdot f(x) = \pi \cdot \sin x$

Transformations

If $f(x)$ is a parent function, then to graph: 1) $f(x) + C$, you need to add C to each y-value; keep x the same;

This is vertical shift up if $c > 0$
~~horizontal~~ vertical shift down shift if $c < 0$

2) $f(x + c)$, horizontal shift

If $c > 0$, shift to the left

If $c < 0$, shift to the right.