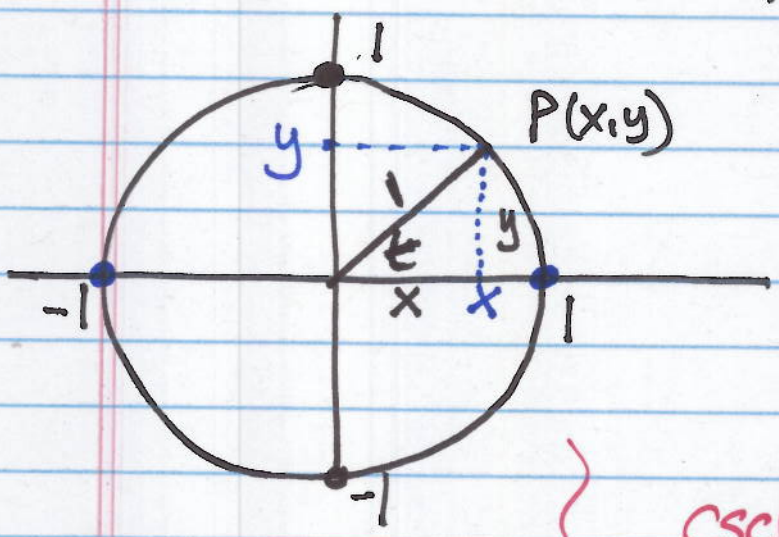
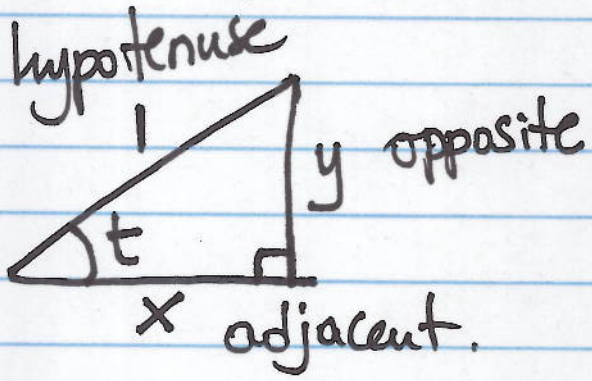
5.2 Trigonometric functions of real numbers.





$\cos(t) = x$

$\sin(t) = y$

$\tan(t) = \frac{y}{x}$

Domain of trigonometric functions

$\sin(x)$ and $\cos(x)$ $(-\infty, \infty)$.

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ and $\sec(x) = \frac{1}{\cos(x)}$

$\sin(t) = \frac{opp}{hyp} = \frac{y}{1}$

$\csc(t) = \frac{1}{\sin(t)} = \frac{1}{y}$

$\cos(t) = \frac{adj}{hyp} = \frac{x}{1}$

$\sec(t) = \frac{1}{\cos(t)} = \frac{1}{x}$

$\tan(t) = \frac{opp}{adj} = \frac{\sin(t)}{\cos(t)} = \frac{y}{x}$

$\cot(t) = \frac{1}{\tan(t)} = \frac{\cos(t)}{\sin(t)} = \frac{x}{y}$.

6 trigonometric functions

$\cos(x) \neq 0$ all real except.

$x \neq \pi/2 + n\pi$

n integer.

$\cos(x) = 0 \therefore x = (2n+1)\frac{\pi}{2}$

$x \neq (2n+1)\frac{\pi}{2}$

odd multiples of $\pi/2$

cot(x) and csc(x)

$$\cot(x) = \frac{\cos(x)}{\sin(x)} \quad \& \quad \csc(x) = \frac{1}{\sin(x)}$$

domain $\sin(x) \neq 0$

$x \neq n\pi$

$\sin(x) = 0 \therefore x = n\pi$ n integer

Even and odd property

Even function $\therefore f(-x) = f(x)$

odd function $\therefore f(-x) = -f(x)$.

$\sin(-x) = -\sin(x)$. Therefore $y = \sin(x)$ odd function

Symmetry with respect to the origin.

$\csc(-x) = -\csc(x)$. odd function

$\cos(-x) = \cos(x)$. Therefore $y = \cos(x)$ and $y = \sec(x)$ are even functions.

$\sec(-x) = \sec(x)$

Symmetry with respect to y-axis

$$y = \tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin(x)}{\cos(x)} = -\tan(x)$$

$$\tan(-x) = -\tan(x)$$

Therefore $y = \tan(x)$ and $y = \cot(x)$ are odd functions

$$\cot(-x) = -\cot(x)$$

Symmetry with respect to the origin

examples: $\sin(-\pi/6) = -\sin(\pi/6) = \boxed{-\frac{1}{2}}$

$$\cos(-\pi/4) = \cos(\pi/4) = \boxed{\frac{\sqrt{2}}{2}}$$

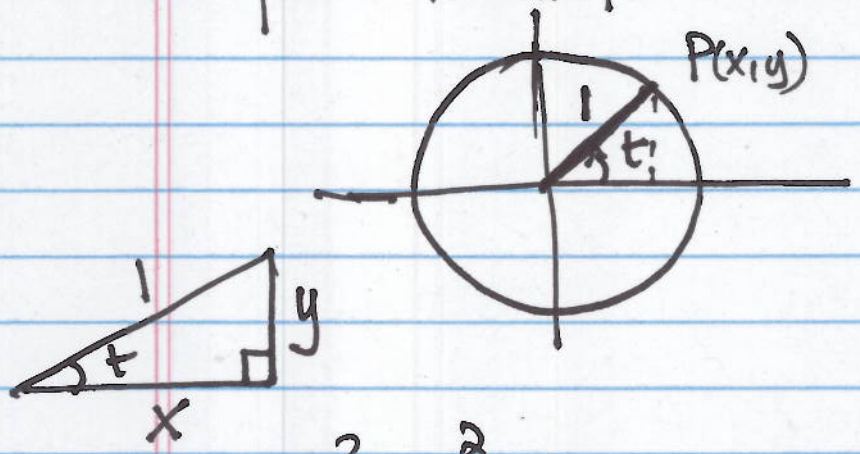
θ	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(\theta)$	$\sqrt{0}/2$	$\sqrt{1}/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	$\sqrt{4}/2$
$\cos(\theta)$	$\sqrt{4}/2$	$\sqrt{3}/2$	$\sqrt{2}/2$	$\sqrt{1}/2$	$\sqrt{0}/2$
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	und
$\cot\theta$					

$\sin \pi/6 = \frac{1}{2}$

$\sin \pi/3 = \frac{\sqrt{3}}{2}$

$\sin \pi/2 = 1$

fundamental identities



$x = \cos(t)$

$y = \sin(t)$

$\frac{y}{x} = \tan(t)$.

$x^2 + y^2 = 1$

$$\cos^2(t) + \sin^2(t) = 1$$

$\cos^2(t) = (\cos t)^2$

true for any t

$\cos^2(\theta) + \sin^2(\theta) = 1$.

$\cos^2(5t) + \sin^2(5t) = 1$

$$\cos^2(t) = 1 - \sin^2(t) \qquad \sin^2(t) = 1 - \cos^2(t)$$

$$\frac{\cos^2(t)}{\cos^2(t)} + \frac{\sin^2(t)}{\cos^2(t)} = \frac{1}{\cos^2(t)}$$

$$\tan^2(t) = \sec^2(t) - 1$$

$$1 + \tan^2(t) = \sec^2(t) \qquad \sec^2(t) - \tan^2(t) = 1$$

$$\frac{\cos^2(t)}{\sin^2(t)} + \frac{\sin^2(t)}{\sin^2(t)} = \frac{1}{\sin^2(t)}$$

$$\cot^2(t) + 1 = \csc^2(t)$$

$$\cot^2(t) = \csc^2(t) - 1 \qquad \csc^2(t) - \cot^2(t) = 1$$