4.3 Properties of Logarithms

Change of base.

Base b.

$$\log_a x = \frac{\log_b x}{\log_b a}$$

<u>Base 10</u> $\quad \log_a x = \frac{\log_{10} x}{\log_{10} a} = \frac{\log x}{\log a}$

Base e.

$$\log_a x = \frac{\log_e x}{\log_e a} = \frac{\operatorname{Ln} x}{\operatorname{Ln} a}$$

$$\log_4 25 = \frac{\operatorname{Ln} 25}{\operatorname{Ln} 4}$$

→ Product properties

$$\log_a AB = \log_a A + \log_a B.$$

$$\operatorname{Ln} AB = \operatorname{Ln} A + \operatorname{Ln} B.$$

2. Quotient property

$$\log_a \frac{A}{B} = \log_a A - \log_a B$$

$$\ln \frac{A}{B} = \ln A - \ln B.$$

3. Power property

$$\log_a A^n = n \log_a A.$$

$$\ln A^n = n \ln A$$

$$\ln A^3 = \ln(A \cdot A \cdot A) \qquad = 3 \ln A$$

$$= \ln A + \ln A + \ln A = \boxed{3 \ln A}$$

Write each logarithm in terms of ln 2 and ln 3.

$$\ln 6 = \ln(2 \cdot 3) = \ln 2 + \ln 3 \checkmark$$

Express in terms of ln 4 and ln 5

#22 $\ln 500 = \ln(4 \times 125)$

$$= \ln 4 + \ln 125$$

$$= \ln 4 + \ln 5^3$$

$$= \ln 4 + 3 \ln 5 \checkmark$$

(24) $\ln\frac{5}{2} = \ln 5 - \ln 2$.

$= \ln 5 - \ln 4^{1/2}$

$= \ln 5 - \frac{1}{2}\ln 4$ ✓

Use the properties of logarithms to rewrite and simplify the logarithmic expression.

(38) $\log_9 243 = \log_9 3 \cdot 81 = \log_9 3^5 = 5\log_9 3$.

$243 = 3^5$

$\log_9 3 = \frac{\ln 3}{\ln 9} = \frac{\ln 3}{\ln 3^2} = \frac{\cancel{\ln 3}}{2\cancel{\ln 3}}$

$$\boxed{\log_9 3 = \frac{1}{2}}$$

$\log_9 243 = \frac{5}{2}$

$= \frac{\ln 243}{\ln 9} = \frac{5\ln 3}{2\ln 3} = \boxed{\frac{5}{2}}$ ✓

$\log_a x = y \longleftrightarrow a^y = x$

$\log_9 243 = x \longleftrightarrow 9^x = 243$.

$(3^2)^x = 3^5$

$3^{2x} = 3^5 \quad \therefore 2x = 5 \therefore \boxed{x = \frac{5}{2}}$

(42) $\ln 8e^3$

$\ln 8 + \ln e^3$

$\ln 2^3 + 3\cancel{\ln e}_{1}$

$\boxed{3\ln 2 + 3}$

(44) $\ln \frac{e^5}{7} = \ln e^5 - \ln 7$

$= \boxed{5 - \ln 7}$

derivative

Use the properties of Logarithms to Expand the expression as a Sum, difference, and/or. constant multiply

(58) $\log_4 x \cdot y^6 \cdot z^4 = \log_4 x + \log_4 y^6 + \log_4 z^4$

$= \log_4 x + 6\log_4 y + 4\log_4 z$

(63) $\ln\left(\frac{x^4\sqrt{y}}{z^5}\right) = \ln x^4\sqrt{y} - \ln z^5$

$= \ln x^4 + \ln\sqrt{y} - 5\ln z$

$= 4\ln x + \frac{1}{2}\ln y - 5\ln z$

(61) $\ln\left(\frac{x^2-1}{x^3}\right) \quad x > 1$

$\ln(x^2-1) - \ln x^3 = \boxed{\ln(x-1) + \ln(x+1) - 3\ln x}$

$\ln(x+1)(x-1) - 3\ln x$

Condense the expression to the logarithm of a single quantity.
(techniques: Solving Equations).

84 $2\left[\ln x - \ln(x+1) - \ln(x-1)\right]$.

$2\left[\ln x - \ln(x+1)(x-1)\right]$.

$2\left[\ln x - \ln(x^2-1)\right] = 2\left[\ln \frac{x}{(x^2-1)}\right]$

$\ln\left(\frac{x}{(x^2-1)}\right)^2$

$$\boxed{\ln\left(\frac{x^2}{(x^2-1)^2}\right)}$$

122 Prove that $\frac{\log_a x}{\log_{a/b} x} = 1 + \log_a \frac{1}{b}$.

RHS :

$$\frac{\log_a x}{\log_{a/b} x} = \frac{\frac{\ln x}{\ln a}}{\frac{\ln x}{\ln \frac{a}{b}}} = \frac{\ln a/b}{\ln a} = \frac{\ln a - \ln b}{\ln a}$$

$$= \frac{\ln a}{\ln a} - \frac{\ln b}{\ln a} = 1 - \log_a b$$

$$= 1 + \log_a \frac{1}{b} \checkmark$$