Practice Test 1

Name Solution

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Solve the problem.

1) If $f(x) = 9x^3 + 2x^2 - x + C$ and $f(-2) = 1$, what is the value of C?

$9(-8) + 2(4) - (-2) + C = 1$

1) $C = -53$

2) If a rock falls from a height of 100 meters on Earth, the height H (in meters) after x seconds is approximately

$H(x) = 100 - 4.9x^2$.

When does the rock strike the ground? Round to the nearest hundredth, if necessary.

$H = 0 \quad \therefore \quad 100 - 4.9x^2 = 0$

2) $x = \sqrt{\frac{100}{4.9}} = 4.52$

Find the domain of the function.

3) $f(x) = \frac{x}{x^2 + 3}$

3) all real numbers. $(-\infty, \infty)$

4) $h(x) = \frac{x - 1}{x^3 - 64x}$

$x^3 - 64x = 0 \quad \therefore \quad x(x^2 - 64)$

4) $\{x \mid x \neq 0, -8, 8\}$. $(-\infty, -8) \cup (-8, 8) \cup (0, 8) \cup (8, \infty)$

Determine algebraically whether the function is even, odd, or neither.

5) $f(x) = 4x^3$

odd function ∴ Symmetry with respect to the origin (graph).

5) _____

6) $f(x) = \frac{x}{x^2 - 5}$

$f(-x) = -f(x)$ $-x$, ✗ Both in the domain

6) _____

For the function, find the average rate of change of f from 1 to x:

$\frac{f(x) - f(1)}{x - 1}, x \neq 1$

7) $f(x) = \frac{4}{x + 3}$

$$\frac{\frac{4}{x+3} - 1}{x - 1} = \frac{\frac{4 - x - 3}{x+3}}{x - 1} = \frac{\frac{1 - x}{x+3}}{\frac{x-1}{1}} \quad \frac{(1-x) - 1}{(x+3)(x-1)}$$

$\frac{-1}{x+3}$

Find the average rate of change for the function between the given values.

8) $f(x) = 8x^3 - 7x^2 - 7$; from -6 to 5

$$\frac{f(5) - f(-6)}{5 - (-6)} = \frac{808 - (-1958)}{11}$$

8) $\frac{2805}{11}$

(5) $f(x) = 4x^3$ $D_f = (-\infty, \infty)$.

$f(-x) = 4(-x)^3 = -4x^3 = -f(x)$

$f(-x) = -f(x)$ $\therefore$ f is odd function

(6) $f(x) = \dfrac{x}{x^2-5}$ $D_f = \{x \mid x \neq \sqrt{5}, -\sqrt{5}\}$.

odd $\therefore$ $f(-x) = -f(x)$

$x^2 - 5 \neq 0$ $\therefore$ $x^2 - 5 = 0$

$\sqrt{x^2} = \sqrt{5}$

$|x| = \sqrt{5}$

$x = \pm\sqrt{5}$

$f(-x) = \dfrac{-x}{(-x)^2 - 5}$

$= -\dfrac{x}{x^2-5} = -f(x)$ $\therefore$ f is odd ✓

Example: (odd, even, or neither).

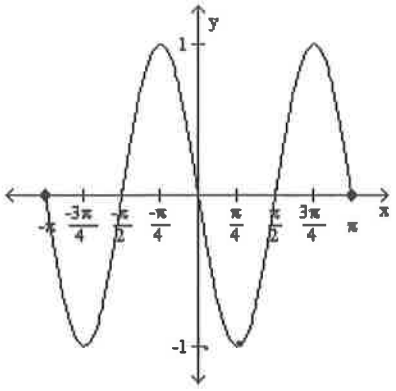
even $f(-x) = f(x)$

$f(x) = |x|$ $D = (-\infty, \infty)$.

$f(-x) = |-x| = |x| = f(x)$ $\therefore$ f is even.

Determine whether the graph is that of a function. If it is, use the graph to find its domain and range, the intercepts, if any, and any symmetry with respect to the x-axis, the y-axis, or the origin.

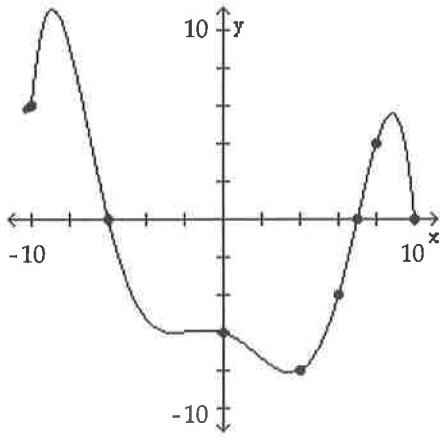
9) 9) _____



a function
Domain $[-\pi, \pi]$
Range $[-1, 1]$
y-intcpt $(0,0)$
x-int $(-\pi, 0)$; $(-\frac{\pi}{2}, 0)$, $(0,0)$, $(\frac{\pi}{2}, 0)$, $(\pi, 0)$
Symmetry with respect to origin (odd)

The graph of a function f is given. Use the graph to answer the question.

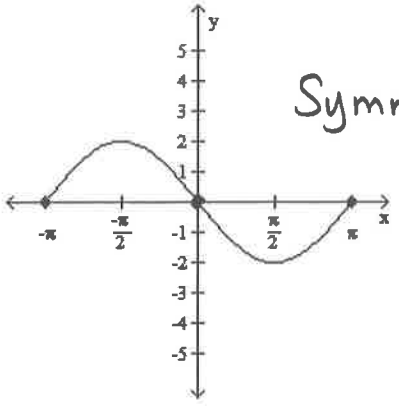
10) For what numbers x is f(x) > 0? 10) _____



$[-10, -6) \cup (7, 10]$

The graph of a function is given. Decide whether it is even, odd, or neither.

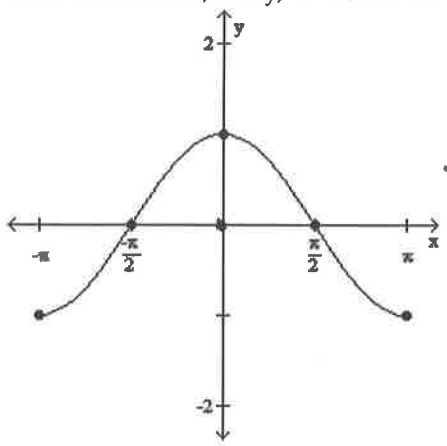
11) 11) odd



Symmetry with respect to the origin (odd)

The graph of a function f is given. Use the graph to answer the question.

12) Find the numbers, if any, at which f has a local maximum. What are the local maxima? 12) _____



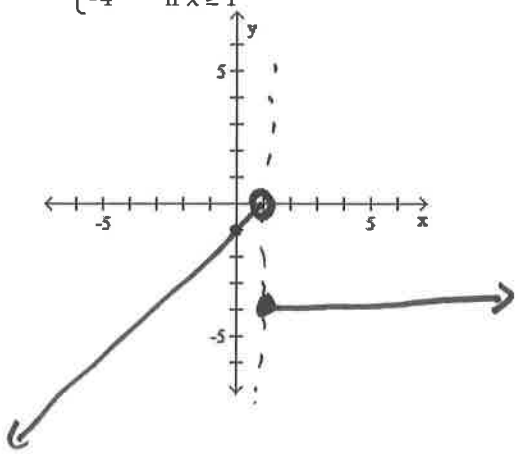
at x=0 there is a max

the max is 1

Graph the function.

13) 13) _____

$$f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ -4 & \text{if } x \geq 1 \end{cases}$$



14) 14) _____

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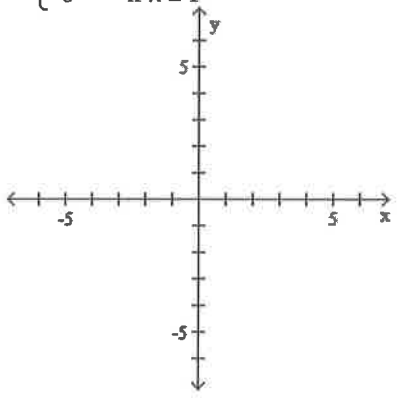


Sec 2.4 $y = ax^2 + bx + c$

$y = a(x-h)^2 + k$

axis of Symmetry. $x = h$

or $x = \frac{-b}{2a}$

Vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

or (h, k)

max if $a<0$ or min if $a>0$

#18 Sec 2.4 $f(x) = x^2 - 4x$

$a = 1$
$b = -4$
$c = 0$

axis of Symmetry.

$x = -\frac{(-4)}{2(1)} = \frac{4}{2} = \boxed{2}$

Vertex $(2, -4)$

Complete the Square. $(a+b)^2$ or $(a-b)^2$

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

$x^2 - 4x + 4 - 4 = (x-2)^2 + 4$ ✓

$a = x$

$2ab = 4x$

$2b = 4$

$b = 2$

$y = x^2 - 4x$

$y = (x-2)^2 + 4$

$(2, -4)$

(#19) $f(x) = 3x^2 + 6x - 9$

axis of symmetry $x = \frac{-6}{2(3)} = -1$

Vertex $(-1, -12)$

$f(x) = 3x^2 + 6x - 9$

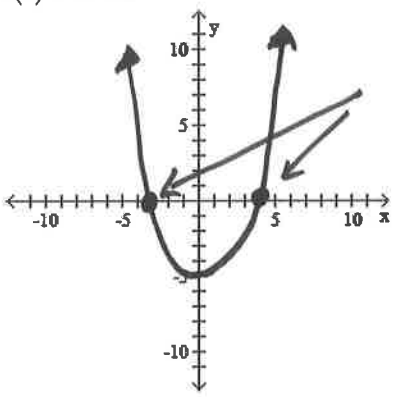
$= 3(x^2 + 2x + 1^2) - 9 - 3$

$= 3(x+1)^2 - 12$ ✓

$(-1, -12)$

Graph the function by starting with the graph of the basic function and then using the techniques of shifting, compressing, stretching, and/or reflecting.

15) $f(x) = x^2 - 5$ 15) _____

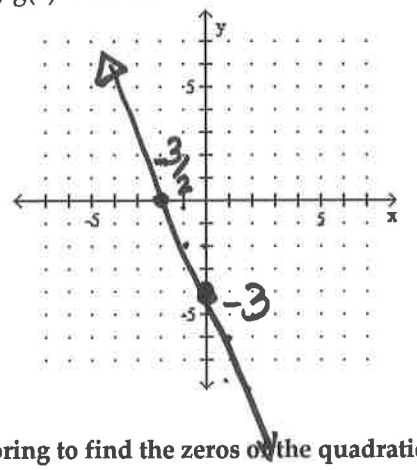


x-intercepts $(\sqrt{5}, 0)$
$(-\sqrt{5}, 0)$

$x^2 - 5 = 0$
$x^2 = 5 \therefore x = \pm\sqrt{5}$

Use the slope and y-intercept to graph the linear function.

16) $g(x) = -2x - 3$ 16) _____



Use factoring to find the zeros of the quadratic function. List the x-intercepts of the graph of the function.

17) $f(x) = x^2 + 5x - 50$ 17) ~~$(-10, 0)$ & $(5, 0)$~~

$f(x) = (x+10)(x-5)$

Find the vertex and axis of symmetry of the graph of the function.

18) $f(x) = x^2 - 4x$ 18) _____

19) $f(x) = 3x^2 + 6x - 9$ 19) _____

Determine, without graphing, whether the given quadratic function has a maximum value or a minimum value and then find that value.

20) $f(x) = -2x^2 + 6x$ 20) _____

Quadratic functions $y = ax^2 + bx + c$ $a \neq 0$

parabola → axis of symmetry: $x = a$

$y = a(x-h)^2 + k$ Standard form
Vertex form Vertex (h, k)

Quadratic Equations

$ax^2 + bx + c = 0$ or $a(x-h)^2 + k = 0$

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant: $b^2 - 4ac \geq 0$

transformation

$y = x^2$ → Shift h units to the Right

$y = (x-h)^2$

$y = a(x-h)^2$ stretching Vertically by a

Shift up by k units.

$y = a(x-h)^2 + k$.

Sect 2.4

(20) $f(x) = -2x^2 + 6x$

Vertex $\left(\frac{3}{2}, \frac{9}{2}\right)$ max

max is $\frac{9}{2}$ at $x = \frac{3}{2}$

$$x = \frac{-6}{2(-2)} = \boxed{\frac{3}{2}}$$

$$f\left(\tfrac{3}{2}\right) = -2\left(\tfrac{3}{2}\right)^2 + 6\cdot\left(\tfrac{3}{2}\right)$$

$$= -\cancel{2}\cdot\frac{9}{\cancel{2}4} + 9 = \frac{9}{2}$$

$$f(x) = -2\left(x^2 - 3x + \left(\tfrac{3}{2}\right)^2\right) + 2\cdot\tfrac{9}{4}$$

$$= -2\left(x - \tfrac{3}{2}\right)^2 + \tfrac{9}{2}$$

$\left(\frac{3}{2}, \frac{9}{2}\right)$ ✓