Math 166 Spring ~~Fall~~ ~~2013~~ 2014

Practice Test 4

Name Solution

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

A point on the terminal side of angle θ is given. Find the exact value of the indicated trigonometric function.

1) (18, 24) Find cos θ. 1) ________

x y

Use a coterminal angle to find the exact value of the expression. Do not use a calculator.

2) tan -690° 2) ________

Name the quadrant in which the angle θ lies.

3) $\tan \theta > 0, \quad \sin \theta < 0$ 3) ________

Solve the problem.

4) Which of the following trigonometric values are negative? 4) ________

I. sin(-292°)
II. tan(-193°)
III. cos(-207°)
IV. cot 222°

Find the exact value of the indicated trigonometric function of θ.

5) $\sin \theta = -\frac{2}{3}, \quad \tan \theta > 0$ Find sec θ. 5) ________

Write the equation of a sine function that has the given characteristics.

6) Amplitude: 2 6) ________
Period: π
Phase Shift: - 3

Find the exact value of the expression.

7) $\sin^{-1} \frac{\sqrt{2}}{2}$ 7) ________

8) $\sin (\tan^{-1} 2)$ 8) ________

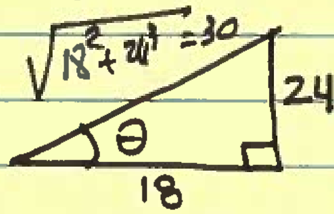
9) $\cos^{-1}\left(\sin \frac{7\pi}{6}\right)$ 9) ________

Write the trigonometric expression as an algebraic expression in u.

10) $\sin (\tan^{-1} u)$ 10) ________

Practice test 4 Sp14
Solution

① (18, 24) $\cos\theta = \frac{18}{30} = \boxed{\frac{3}{5}}$



② $\tan(-690^\circ) = \tan(30) = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

$-690 + 720 = 30^\circ$

③ $\tan\theta > 0$ $\quad$ $\sin\theta < 0$ $\quad$ $\boxed{Q\,III}$

$\downarrow$ $\quad$ $\downarrow$

QI, QIII $\quad$ QIII, QIV

④ I $\quad$ $\sin(-292^\circ)$ posi $\quad$ $-292+360 = 68$ $\quad$ QI.

II $\quad$ $\tan(-193^\circ)$. neg. $\quad$ $-193+360 = 167$ $\quad$ QII.

III $\quad$ $\cos(-207^\circ)$. neg $\quad$ $-207+360 = 153$ $\quad$ QII.

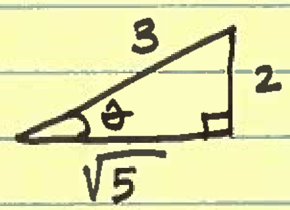
IV $\quad$ $\cot 222^\circ$ posi $\quad$ 222 $\quad$ QIII

~~III~~ $\boxed{\text{I \& IV}}$ positive $\quad$ $\boxed{\text{II \& III}}$ negative.

(5) $\sin\theta = -2/3$ $\quad \tan\theta > 0.$

θ in $\boxed{QIII}$

$\sec\theta = -\frac{3}{\sqrt{5}} = \frac{-3\sqrt{5}}{5}$



(6) $y = a\sin k(x-b).$

$|a| = 2;$ $\quad T = \frac{2\pi}{k} = \pi.$

$b = -3$

$$\boxed{\begin{array}{l} y = 2\sin 2(x+3) \checkmark \\ y = -2\sin 2(x+3) \end{array}}$$

(7) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \theta$ $\quad \boxed{\pi/4}$

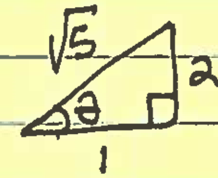
$-\frac{\pi}{2} \leq \theta < \frac{\pi}{2}$

$$\boxed{\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}}$$

(8) $\sin(\tan^{-1}2) = \sin\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

$\theta = \tan^{-1}2$

$\tan\theta = \frac{2}{1}$



(9) $\cos^{-1}\left(\sin \frac{7\pi}{6}\right)$.

$\cos^{-1}\left(\sin\left(\pi+\frac{\pi}{6}\right)\right)$.

$\cos^{-1}\left(-\sin \frac{\pi}{6}\right)$.

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\sin(\pi - x) = \sin x$

$\cos(\pi - x) = -\cos x$

$\sin(\pi + x) = -\sin x$

$\cos(\pi + x) = -\cos x$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\sin\left(\frac{\pi}{2}+x\right) = \cos x$

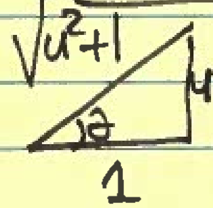
$\cos\left(\frac{\pi}{2}+x\right) = -\sin x$.

$\cos^{-1}\left(\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right) = \cos^{-1}\left(\cos\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}$

(10) $\sin\left(\tan^{-1} u\right) = \sin\theta = \boxed{\frac{u}{\sqrt{1+u^2}}}$

$\theta = \tan^{-1} u$

$\tan\theta = u$



Solve for x; $0 \le x < 2\pi$

$$\sin 5x + \sin 3x = 0$$

$$\downarrow$$

$$2 \sin 4x \cos x = 0$$

$\sin 4x = 0$ or $\cos x = 0$

$4x = n\pi$ $\qquad$ $\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$x = \frac{n\pi}{4}$

$$x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{6\pi}{4}, \frac{7\pi}{4}$$

$$\left\{0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\} \checkmark$$

$\int$ (#31) Reduce the power to 1^{st} power of cosine.

$$\cos^4 x = \cos^2 x \cos^2 x$$

$$\downarrow$$

$$\left(\frac{1+\cos 2x}{2}\right)\left(\frac{1+\cos 2x}{2}\right).$$

$$\frac{1 + 2\cos 2x + \cos^2(2x)}{4} = \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{4}\,\frac{1+\cos(4x)}{2}$$

$$= \frac{1}{4} + \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{1}{8}\cos 4x = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos$$

Solve the equation on the interval $0 \le \theta < 2\pi$.

11) $\sin(4\theta) = \frac{\sqrt{3}}{2}$ — 11) ____________

12) $2\cos(2\theta) = \sqrt{3}$ — 12) ____________

Solve the problem.

13) The function — 13) ____________

$$I(t) = 40\sin\left(60\pi t - \frac{\pi}{2}\right)$$

represents the amperes of current produced by an electric generator as a function of time t, where t is measured in seconds. Find the smallest value of t for which the current is 20 amperes. Round your answer to three decimal places, if necessary.

Use a calculator to solve the equation on the interval $0 \le \theta < 2\pi$. Round the answer to two decimal places.

14) $\sin\theta = 0.25$ — 14) ____________

Solve the equation on the interval $0 \le \theta < 2\pi$.

15) $\cos^2\theta + 2\cos\theta + 1 = 0$ — 15) ____________

16) $\sin^2\theta + \sin\theta = 0$ — 16) ____________

17) $\sin(2\theta) + \sin\theta = 0$ — 17) ____________

Find the exact value of the expression.

18) $\sin\left(-\frac{11\pi}{12}\right)$ — 18) ____________

Complete the identity.

19) $\sin(\alpha+\beta)\cos(\alpha-\beta) = ? \ \frac{\sin(2\alpha)+\sin 2\beta}{2}$ — 19) ____________

Find the exact value of the expression.

20) $\sin\left(\sin^{-1}\frac{2}{3} + \cos^{-1}\frac{1}{3}\right)$ — 20) $\frac{2}{9} + \frac{2\sqrt{10}}{9}$

$\sin(a+b) = \sin a\cos b + \sin b\cos a$

$\sin(\sin^{-1}\tfrac{2}{3})\cos(\cos^{-1}\tfrac{1}{3}) + \sin(\cos^{-1}\tfrac{1}{3})\cos(\sin^{-1}\tfrac{2}{3})$.

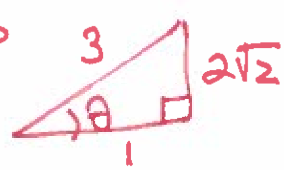
$\frac{2}{3}\cdot\frac{1}{3} + \frac{2\sqrt{2}}{3}\frac{\sqrt{5}}{3}$

$\boxed{\frac{2+2\sqrt{10}}{9}}$

$\sin(\cos^{-1}\tfrac{1}{3}) = ?\frac{2\sqrt{2}}{3}$

$\theta = \cos^{-1}\tfrac{1}{3}$

$\cos\theta = \tfrac{1}{3}$



$\cos(\sin^{-1}\tfrac{2}{3}) = \frac{\sqrt{5}}{3}$

$\theta = \sin^{-1}\tfrac{2}{3}$

$\sin\theta = \tfrac{2}{3}$