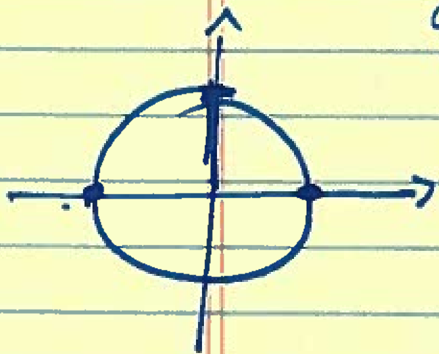
6.3 Solving trigonometric Equations



$\sin x = 0 \qquad x = 0, \pi, 2\pi, 3\pi, \ldots$

Global Solution

$\sin x = 0 \qquad x = n\pi$

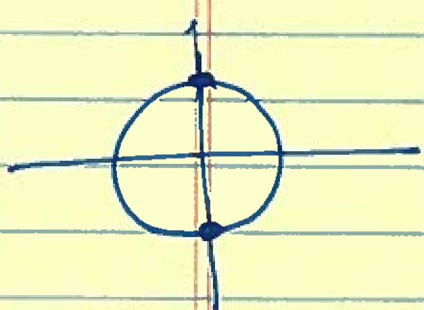
n integer.

$\sin(4x) = 0$

$4x = n\pi$

$$\boxed{x = n\frac{\pi}{4}}$$

$\cos x = 0 \qquad x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$



$$\boxed{x = (2n+1)\frac{\pi}{2}}$$ n integer.

$\cos 3x = 0 \quad \therefore \; 3x = (2n+1)\frac{\pi}{2}.$

Solution $[0, 2\pi)$

$n=0$; $x = \pi/6$ ✓

$n=1$; $x = \frac{3\pi}{6} = \pi/2$. ✓

$n=2$; $x = \frac{5\pi}{6}$. ✓

$n=3$; $x = 7\pi/6$. ✓

$n=4$; $x = 9\pi/6$. ✓

$n=5$; $x = 11\pi/6$. ✓

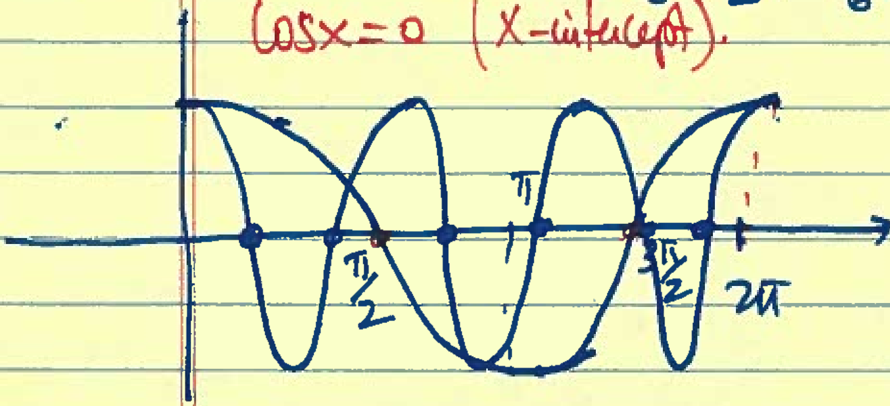
$$\boxed{x = (2n+1)\frac{\pi}{6}}$$

Global Solution

$\cos 3x = 0$

$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{9\pi}{6}, \frac{11\pi}{6}$

$\cos x = 0$ (x-intercept).



FM

AM

$T = \frac{2\pi}{3}$

Solve for x in $[0, 2\pi)$

$\sin x \cos x - \cos x = 0$

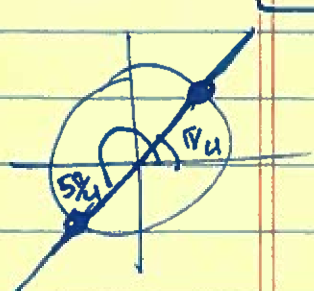
$\cos x (\sin x - 1) = 0$

$\cos x = 0$ or $\sin x - 1 = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$; $\sin x = 1$

$\boxed{x = \pi/2}$

$\boxed{x = \pi/2, 3\pi/2}$



$\sin x - \cos x = 0$ $\qquad 0 \le x < 2\pi$

$\sin x = \cos x$

$\tan x = 1$ $\qquad x = \pi/4, \frac{5\pi}{4}$

$$2\sin x \tan x - \tan x = 1 - 2\sin x$$

$$\tan x(2\sin x - 1) = (1 - 2\sin x)$$

$$\tan x(2\sin x - 1) - (1 - 2\sin x) = 0$$

$$\tan x(2\sin x - 1) + (2\sin x - 1) = 0$$

$$(2\sin x - 1)(\tan x + 1) = 0$$

$2\sin x - 1 = 0$ or $\tan x + 1 = 0$

$\sin x = 1/2$ $\qquad$ $\tan x = -1$

$x = \pi/6, \frac{5\pi}{6}$ $\qquad$ $x = 3\pi/4, 7\pi/4$

Solution $\{\pi/6, 5\pi/6, 3\pi/4, 7\pi/4\}$.

$\cos^3 x = \cos x \qquad 0 \leq x < 2\pi$

$\cos^3 x - \cos x = 0 \longrightarrow$ Solution $\left\{ \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi \right\}$

$\cos x (\cos^2 x - 1) = 0$, $\cos x(-\sin^2 x) = 0$

$\cos x = 0$ or $-\sin^2 x = 0$

$\cos x(\cos x - 1)(\cos x + 1) = 0 \quad \frac{\pi}{2}, \frac{3\pi}{2}, \quad 0, \pi$

$\cos x = 0 \qquad \cos x = 1 \qquad \cos x = -1$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = 0 \qquad x = \pi$

$\cos x = 0$; $\cos^2 x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$; $|\cos x| = 1 \quad \cos x = 1$ or $\cos x = -1$

Global Solution

$$\sin 4x = 1$$

$$(4x) = \frac{\pi}{2} + 2n\pi$$

$$\boxed{x = \frac{\pi}{8} + \frac{n\pi}{2}}$$ n integer.

Global Solution

general

Solution $0 \leq x < 2\pi$

$n=0$; $\pi/8$ $\quad$ ~~$n=4$~~

$n=1$; $5\pi/8$

$n=2$; $9\pi/8$

$n=3$; $13\pi/8$

$\sin(2x) = -\frac{\sqrt{3}}{2}$ reference $\frac{\pi}{3}$

location QIII, QIV

$\pi + \pi/3$

$2x = \frac{4\pi}{3} + 2n\pi$

$2\pi - \pi/3$

$2x = \frac{5\pi}{3} + 2n\pi.$

$\bar{\theta}$ reference.

QI. $\theta = \bar{\theta}$

QII $\theta = \pi - \bar{\theta}$

QIII; $\theta = \pi + \bar{\theta}$

QIV $\theta = 2\pi - \bar{\theta}$

$$\boxed{X = \frac{2\pi}{3} + n\pi \quad \text{or} \quad X = \frac{5\pi}{6} + n\pi}$$

$0 < x < 2\pi$

$n=0$; $n=1$

$\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}$

Solve for x; $0 \le x < 2\pi$

$\tan^2 x - 6\tan x + 5 = 0$

$(\tan x - 1)(\tan x - 5) = 0$

$\tan x = 1$ or $\tan x = 5$ QI, III

$\boxed{x = \pi/4, 5\pi/4}$

$x = \arctan(5)$

$x = 1.373$

$x = 4.515$

$y^2 - 6y + 5 = 0$

$(y-1)(y-5) = 0$

$\cos^2 x - 2\cos x - 1 = 0$

$(\cos x \quad)(\cos x \quad) = 0$

Let $y = \cos x$

$y^2 - 2y - 1 = 0$

$y = 1 \pm \sqrt{2}$ — $1+\sqrt{2}$, $1-\sqrt{2}$.

$\cancel{\cos x = 1+\sqrt{2}}$ or $\cos x = 1-\sqrt{2} < 0$

No Solution $\quad \cos x = -0.4142$

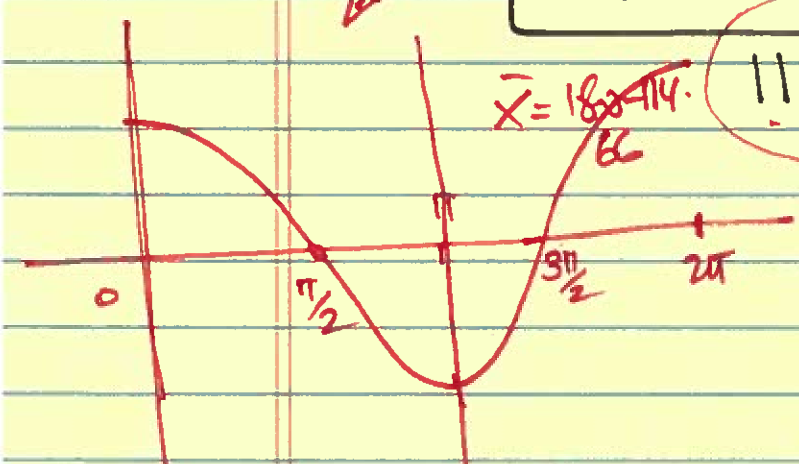
Q II, Q III

$x = \cos^{-1}(1-\sqrt{2})$

$\boxed{x = 1.99786 \text{ rad}}$ $\quad X = 246$

$114^\circ.4689$

$\bar{x} = 180 - 114.66$



$[0, \pi] \xrightarrow[\text{one to one}]{\cos x} [-1, 1]$; $[\pi, 2\pi] \xrightarrow[\text{one-to-one}]{\cos x} [-1, 1]$.