3.4 Fundamental theorem of ~~Calculus~~ Algebra.

Any polynomial of degree n

has n complex roots.

Let $f(x) = x^3 - 1$

$x^3 - 1 = 0$; $x^3 - 1 = 0$

$x^3 = 1$

$x = \sqrt[3]{1}$

$x = 1$

$(x-1)(x^2+x+1) = 0$

$x - 1 = 0 \therefore \boxed{x=1}$

$x^2 + x + 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1-4(1)(1)}}{2} = \dfrac{-1 \pm i\sqrt{3}}{2}$

$f(x) = (x-1)^3$

$x = 1 , \underbrace{-\frac{1}{2} + \frac{\sqrt{3}}{2} i} ; \underbrace{-\frac{1}{2} - \frac{\sqrt{3}}{2} i}$

$\sqrt{x^4} = \sqrt{1}$

$|x^2| = 1$

$\sqrt{x^2} = \sqrt{1}$ or $x^2 = -1$

$|x| = 1$ $\qquad$ $|x| = i$ $\quad x = i$, $x = -i$

$\boxed{x = \pm 1}$ $\qquad$ $\boxed{x = \pm i}$

Let $f(x)$ be a polynomial function

If $a+bi$ $(b \neq 0)$

is a zero, then the conjugate $a-bi$

is also a zero of $f(x)$.

Find a Polynomial of degree 4

that has $\underline{2}$, $\underline{1}$, and $\underline{3i}$ as zeros.

$-3i$

$$P(x) = a(x-2)(x-1)(x-3i)(x+3i)$$

$$= a(x-2)(x-1)(x^2+9)$$

$$(a+bi)(a-bi) = a^2+b^2$$

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$\boxed{45\text{-}50}$ #46

$\underline{3, 4i, -4i}$ $P(x) = a(x-3)(x-4i)(x+4i)$

$P(x) = a(x-3)(x^2+16)$

(48) $-1, -1, 2+5i, 2-5i$

$$P(x) = a(x+1)^2(x-(2+5i))(x-(2-5i))$$

$$= a(x+1)^2((x-2)-5i)((x-2)+5i)$$

$$= a(x+1)^2((x-2)^2+25)$$

$$P(x) = a(x+1)^2(x^2-4x+29)$$

(50) $0, 4, 1+\sqrt{2}i, 1-\sqrt{2}i$

$$P(x) = ax(x-4)(x-(1+\sqrt{2}i))(x-(1-\sqrt{2}i))$$

$$P(x) = ax(x-4)((x-1)-\sqrt{2}i)((x-1)+\sqrt{2}i)$$

$$P(x) = ax(x-4)((x-1)^2+2)$$

$$= ax(x-4)(x^2-2x+3)$$

(52) degree 4 ; $-1, 2, i$ $\quad f(1) = 8$

$$f(x) = a(x+1)(x-2)(x+i)(x-i)$$

$$f(x) = a(x+1)(x-2)(x^2+1)$$

$$f(1) = a(2)(-1)(2) = 8$$

IVP

$$\boxed{a = -2}$$

$$\therefore f(x) = -2(x+1)(x-2)(x^2+1)$$

#36 $\quad h(x) = x^4 + 6x^3 + 10x^2 + 6x + 9$

$$p/q = \pm 1, \pm 3, \pm 9, \quad = (x+3)(x^3+3x^2+x+3)$$

$$= (x+3)^2(x^2+1)$$

-3	1	6	10	6	9
		-3	-9	-3	-9
~~9~~	1	3	1	3	$\boxed{0}$

~~-3 | 1 6 10~~

		~~-9~~	~~54~~
	~~1~~	~~-6~~	~~55~~

~~Roots~~ Zeros

$$\boxed{-3, -3, i, -i}$$

-3	1	3	1	3
		-3	0	-3
	1	0	1	$\boxed{0}$

HW 12, 26, 30, 34, 40, 54
64 $\quad$ p 286-287.