Chap 6
Analytic Trigonometry.

6.1 Trig Identities.

$\cos^2 x + \sin^2 x = 1$ $\quad \cos^2(4x) + \sin^2(4x) = 1$

$\tan^2 x + 1 = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

even - odd identities $\quad \cos(-x) = \cos x$ even.

$\sin(-x) = -\sin x$ odd.

$\tan(-x) = -\tan x$ odd.

Co-function identities

$\sin(\pi/2 - x) = \cos x$

$\cos(\pi/2 - x) = \sin x$

$\tan(\pi/2 - x) = \cot x$.

$\sin(x + \pi/2) = \sin(\pi/2 + x)$

$= \sin(\pi/2 - (-x))$

$= \cos(-x) = \cos x$

$\boxed{\sin(x + \pi/2) = \cos x}$

$\tan(\pi/2 + x) = -\cot x$

$\cos(\pi/2 + x) = \cos(\pi/2 - (-x)) = \sin(-x) = -\sin x$

$\boxed{\cos(\pi/2 + x) = -\sin x}$

Simplify the trig expressions

(1) $\cos^3 x + \sin^2 x \cos x$

$\cos x \left[\cos^2 x + \sin^2 x\right] = \boxed{\cos x}$ ✓

(2) $\dfrac{\tan(x)}{\sec(-x)} = \dfrac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \boxed{\sin x}$ ✓

even

(3) $\dfrac{\sec x - \cos x}{\tan x} = \dfrac{\frac{1}{\cos x} - \cos x}{\frac{\sin x}{\cos x}} = \dfrac{\frac{1-\cos^2 x}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \dfrac{1-\cos^2 x}{\sin x} = \dfrac{\sin^2 x}{\sin x}$

$= \boxed{\sin x}$

(4) $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = \dfrac{\sin x}{\frac{1}{\sin x}} + \dfrac{\cos x}{\frac{1}{\cos x}} = 1$

$= \sin^2 x + \cos^2 x = \boxed{1}$ ✓

(5) $\tan x \cos x \csc x = \dfrac{\sin x}{\cos x} \cdot \cos x \cdot \dfrac{1}{\sin x} = \boxed{1}$ ✓

(6) $$\frac{1+\cot x}{1+\sec x} = \frac{1+\frac{\cos x}{\sin x}}{1+\frac{1}{\cos x}} = \frac{\frac{\sin x+\cos x}{\sin x}}{\frac{\cos x+1}{\cos x}}$$

$$= \frac{(\sin x+\cos x)}{\sin x} \cdot \frac{\cos x}{(\cos x+1)} \checkmark$$

(7) $$\frac{\tan x}{1+\sec x} = \frac{\frac{\sin x}{\cos x}}{1+\frac{1}{\cos x}} = \frac{\frac{\sin x}{\cos x}}{\frac{\cos x+1}{\cos x}}$$

$$= \frac{\sin x}{\cos x+1} \checkmark$$

(8) $$\frac{1+\cot x}{\csc x} = \frac{1+\frac{\cos x}{\sin x}}{\frac{1}{\sin x}} = \frac{\frac{\sin x+\cos x}{\sin x}}{\frac{1}{\sin x}}$$

$$= \boxed{\sin x+\cos x} \checkmark$$

Perform the addition or subtraction so that it is not a fractional form.

$$\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = \frac{\tan^2 x + (1+\sec x)^2}{(1+\sec x)\tan x}$$

$1+\tan^2 x = \sec^2 x$ key.

$$= \frac{\tan^2 x + 1 + 2\sec x + \sec^2 x}{(1+\sec x)\tan x}$$

$2\sec^2 x + 2\sec x$

$$= \frac{\sec^2 x + 2\sec x + \sec^2 x}{(1+\sec x)\tan x}$$

$$= \frac{2\sec x(\sec x + 1)}{(1+\sec x)\tan x}.$$

$$= \frac{2\sec x}{\tan x} = 2\csc x$$

$$= 2\frac{\frac{1}{\cos x}}{\frac{\sin x}{\cos x}} = \frac{2}{\sin x} = \boxed{2\csc x}$$

(10) $$\frac{3}{(\sec x - \tan x)} \cdot \frac{(\sec x + \tan x)}{(\sec x + \tan x)} = \frac{3(\sec x + \tan x)}{\sec^2 x - \tan^2 x}$$

$$= 3(\sec x + \tan x)$$

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#54 $(\sec^3 x - \sec^2 x) - (\sec x + 1)$

$\sec^2 x(\sec x - 1) - (\sec x - 1)$

$(\sec x - 1)(\sec^2 x - 1)$.

$(\sec x - 1)\tan^2 x$

Q2

Verify the identities

1) $\frac{\tan x}{\sec x} = \sin x$

LHS = RHS.

prove A = B

A = B → A − B → 0

exact. computed.

Error

C = C ✓

B = B ✓

$\frac{\tan x}{\sec x} = \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \frac{\sin x}{1} = \sin x$ ✓

$\frac{\cos\theta}{\sec\theta \sin\theta} = \csc\theta - \sin\theta$

$= \csc\theta \sin\theta$ ✓

LHS → $\frac{\cos\theta}{\frac{1}{\cos\theta} \cdot \sin\theta} = \frac{\cos^2\theta}{\sin\theta} = \frac{1 - \sin^2\theta}{\sin\theta} = \frac{1}{\sin\theta} - \sin\theta$