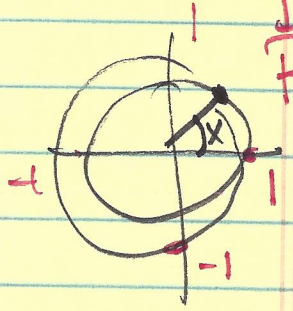
5.4 Graphs of Sine and Cosine functions

$y = \sin(x)$ is 2π periodic.



Domain $(-\infty, \infty)$.
Range $[-1, 1]$.

$\sin(x+2\pi) = \sin(x)$

$\sin(x+2n\pi) = \sin(x)$

$-1 \le \sin(x) \le 1$ $\quad n$ integer.

$\cos(x)$ is 2π periodic $\quad -1 \le \cos x \le 1$

Domain: $(-\infty, \infty)$.
Range $[-1, 1]$

$\cos(x+2n\pi) = \cos(x)$

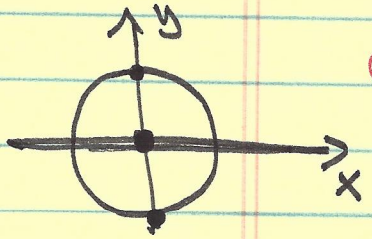
The function $\tan(x)$ is π periodic.

$$\tan(x+\pi) = \frac{\sin(x+\pi)}{\cos(x+\pi)} = \frac{-\sin(x)}{-\cos(x)} = \tan x$$

$\tan(x+\pi) = \tan(x)$.

$y = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$

Range $(-\infty, \infty)$



$\cos x = 0$

$x = \frac{\pi}{2} + n\pi$

$\cos(x) \neq 0$

Domain $\{x \mid x \neq (2n+1)\frac{\pi}{2}\}$

$x = \frac{\pi}{2} + n\pi = \frac{\pi}{2} + \frac{2n\pi}{2}$

$= \frac{\pi + 2n\pi}{2} = \frac{\pi}{2}(2n+1)$ ✓

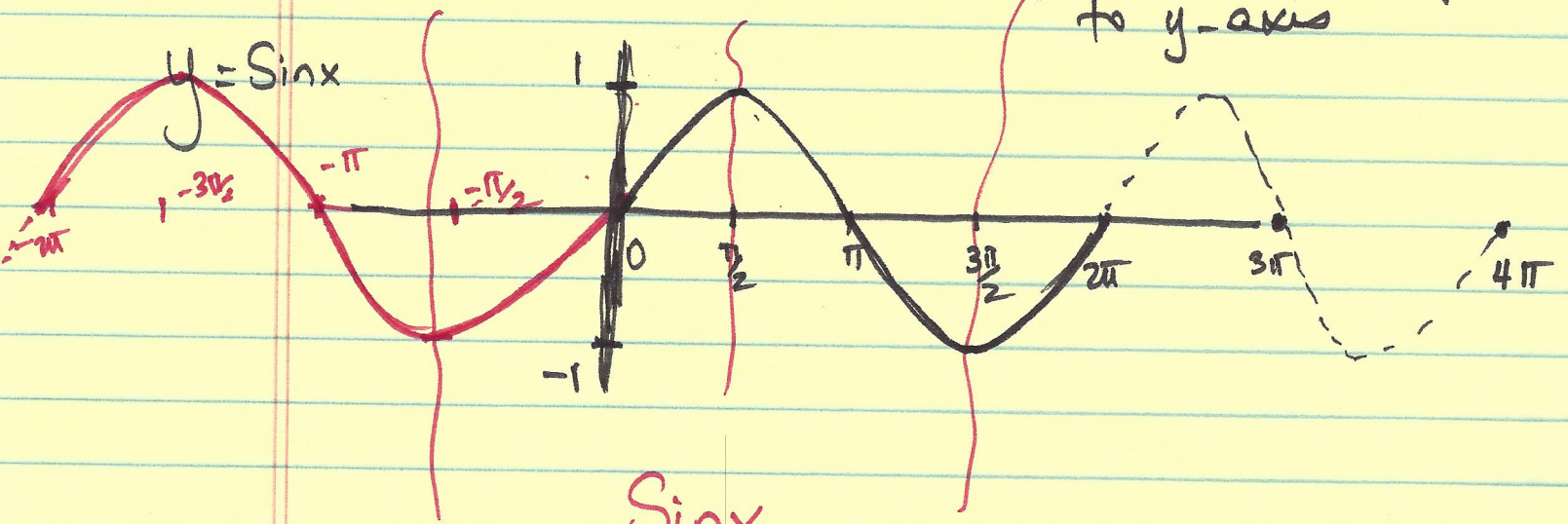
$y = \sin(x)$

$\sin(-x) = -\sin(x)$ odd function

Symmetry with respect to the origin

$\cos(-x) = \cos(x)$ even function

Symmetry with respect to y-axis



$\sin x$

$[-\pi/2, \pi/2] \longrightarrow [-1, 1]$ one-to-one.

$\sin(\arcsin x) = x$

$-1 \le x \le 1$

$\arcsin(x)$.

$y = \arcsin(x)$

$y = \sin^{-1}(x)$

Domain $[-1, 1]$.

Range $[-\pi/2, \pi/2]$.

QIV, QI

$\arcsin(\sin y) = y$

$\arcsin(\sin \pi/3) = \pi/3$

$\arcsin(\sin 3\pi/4) = \pi/4$

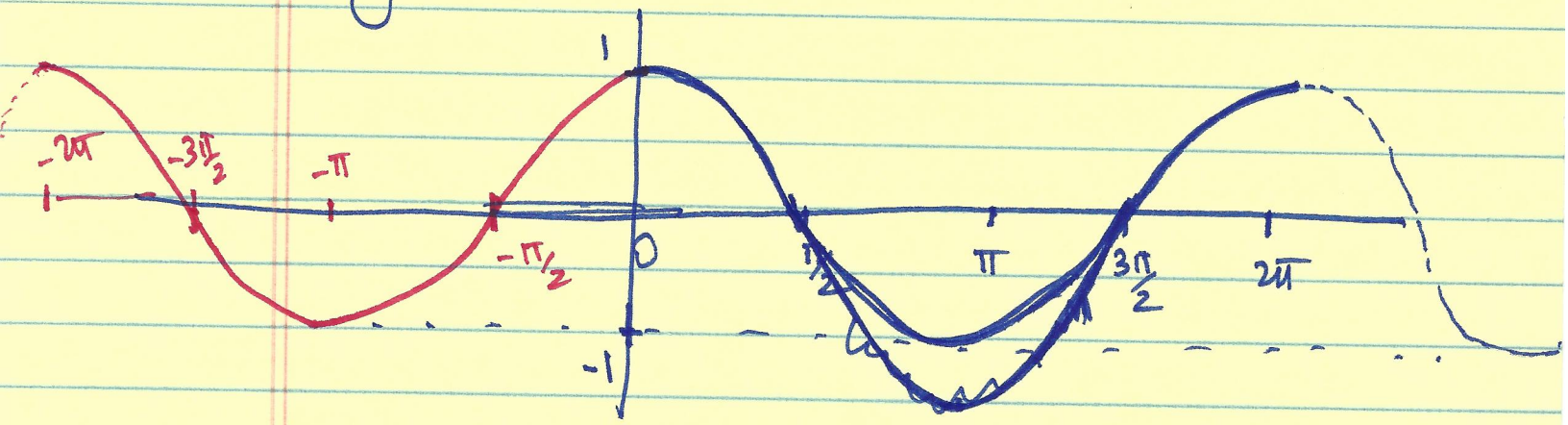
$-\pi/2 \le y \le \pi/2$

Examples

$\sin(\arcsin 1/2) = 1/2$

$\sin(\arcsin \sqrt{2}) =$ DNE

$y = \cos x$



$[0, \pi] \xrightarrow[\text{one-to-one}]{\cos x} [-1, 1]$.

$\arccos(x)$

Domain $[-1, 1]$

$y = \cos^{-1}(x)$. Range $[0, \pi]$.

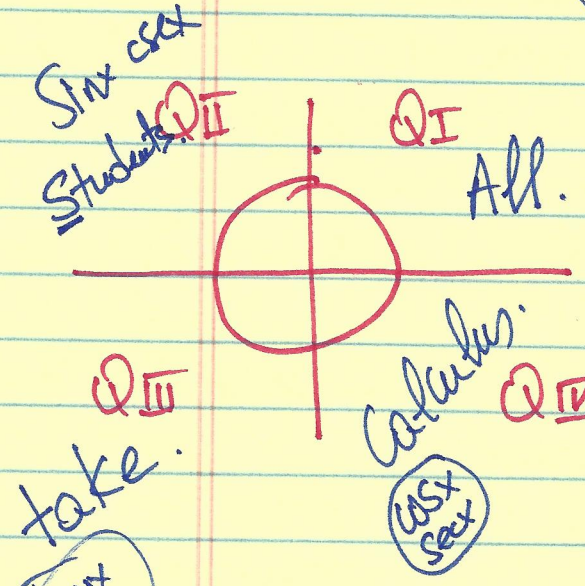
Q1, QII

$\cos(\arccos x) = x$

$-1 \le x \le 1$

$\arccos(\cos y) = y$.

$0 \le y \le \pi$



$\arccos(\cos \frac{\pi}{2}) = \pi/2$.

$\arccos(\cos \frac{3\pi}{4}) = \frac{3\pi}{4}$

$\cos(\arccos \frac{\sqrt{2}}{2}) = \frac{\sqrt{2}}{2}$

$\cos(\arccos -\frac{3}{5}) = -\frac{3}{5}$.

$\cos(\arccos \pi) =$ DNE

$$y = \sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\begin{cases} \sin\left(\frac{\pi}{2} - x\right) = \cos x \\ \cos\left(\frac{\pi}{2} - x\right) = \sin x \end{cases}$$

Graph of transformations of Sine and Cosine.

In general $y = a\sin(kx)$.

$y = a\sin k(x-b)$ | $y = a\cos k(x-b)$.

$|a|$ amplitude.

b phase shift

period $T = \frac{2\pi}{k}$

frequency $\omega = \frac{1}{T}$

appropriate interval.

on which to graph.

one complete period is.

$\left[b, b + \frac{2\pi}{k}\right]$

length.

#28 $y = 2\sin(x - \pi/3)$.

amplitude $|2| = 2$

period $T = \frac{2\pi}{1} = 2\pi$

phase shift $\pi/3$

Interval $\left[\frac{\pi}{3}, \frac{\pi}{3} + 2\pi\right] = \left[\frac{\pi}{3}, \frac{7\pi}{3}\right]$ ✓

#30 $y = 3\cos(x + \pi/4)$ $|a| = 3$

$T = \frac{2\pi}{1} = 2\pi$.

phase shift $-\pi/4$

interval $\left[-\frac{\pi}{4}, \frac{7\pi}{4}\right]$ ✓

#32 $y = \sin \frac{1}{2}(x + \pi/4)$. $|a| = 1$

$T = \frac{2\pi}{\frac{1}{2}} = 4\pi$.

phase shift $-\pi/4$

$\left[-\frac{\pi}{4}, \frac{15\pi}{4}\right]$.

$y = -3\sin(\pi x)$ $|a| = 3$

$T = \frac{2\pi}{\pi} = \boxed{2}$

$y = 3\sin(-5x)$ $|a| = 3$.

$= -3\sin(5x)$ $T = \frac{2\pi}{5}$ phase shift 0