6.4 Sum and Difference Formulas.

① $\sin(a+b) = \sin a \cos b + \sin b \cos a$

$\sin(\pi + x) = \cancel{\sin\pi}^{0} \cos x + \sin x \cos \pi = -\sin x$

$\sin(\tfrac{\pi}{2} + x) = \sin\tfrac{\pi}{2} \cos x + \sin x \cancel{\cos\tfrac{\pi}{2}}^{0} = \cos x$

② $\sin(a-b) = \sin a \cos b - \sin b \cos a$

True for any a & b

③ $\cos(a+b) = \cos a \cos b - \sin a \sin b.$

$\cos(\pi + x) = \cos\pi \cos x - \underset{0}{\sin\pi} \sin x$

$= -\cos x$ ✓

④ $\cos(a-b) = \cos a \cos b + \sin a \sin b.$

$\boxed{a=b}$

$$\boxed{\sin(2a) = 2\sin a \cos a}$$

$$\boxed{\cos(2a) = \cos^2 a - \sin^2 a}$$

$$\boxed{1 = \cos^2 a + \sin^2 a}$$

$$\boxed{\sin^2 a = \frac{1 - \cos 2a}{2}}$$

$\cos 2a = (1 - \sin^2 a) - \sin^2 a \longrightarrow \boxed{\cos 2a = 1 - 2\sin^2 a}$

$\cos 2a = \cos^2 a - (1 - \cos^2 a) \longrightarrow \boxed{\cos 2a = 2\cos^2 a - 1}$

$$\boxed{\cos^2 a = \frac{1 + \cos 2a}{2}}$$

#19 $\sin(\alpha+\beta+\alpha-\beta)+\sin(\alpha+\beta-\alpha+\beta) \cdot 2 \quad \sin(\alpha+\beta)\cos(\alpha-\beta)$

$\boxed{\sin 2\alpha+\sin 2\beta}$

$\sin(a+b)+\sin(a-b)=2\sin a\cos b.$

$$\boxed{\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$a+b=x$

$a-b=y \qquad 2b=x-y$

$2a=x+y \qquad b=\frac{x-y}{2}$

$a=\frac{x+y}{2}$

$\cos(a+b)+\cos(a-b)=2\cos a\cos b$

$$\boxed{\cos x+\cos y=2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$\cos x+\cos 3x=0$

$2\cos 2x\cos x=0$

$\cos 2x=0 \quad \text{or} \quad \cos x=0$

$\vdots$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

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$$\frac{\tan(5\pi/6) - \tan \pi/6}{1 + \tan\frac{5\pi}{6} \tan \pi/6} = \tan\left(\frac{5\pi}{6} - \pi/6\right) = \tan\left(\frac{4\pi}{6}\right).$$

$$\tan\left(\frac{2\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{3}\right).$$

$$= -\tan \pi/3 = \boxed{-\sqrt{3}}$$

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$$\frac{\tan 25 + \tan 110}{1 - \tan 25 \tan 110} = \tan(135) = \tan\left(\pi - \pi/4\right).$$

$$= -\tan \pi/4 = \boxed{-1}$$

6.4 #14 Find the exact Value.

a) $\sin\left(\frac{7\pi}{6} - \frac{\pi}{3}\right) = \sin\frac{7\pi}{6}\cos\frac{\pi}{3} - \sin\frac{\pi}{3}\cos\frac{7\pi}{6}$

$-\frac{1}{4} + \frac{3}{4} = \boxed{\frac{1}{2}} = \left(-\frac{1}{2}\right)\frac{1}{2} - \frac{\sqrt{3}}{2}\left(-\frac{\sqrt{3}}{2}\right)$

$\sin\frac{7\pi}{6} = \sin\left(\pi + \frac{\pi}{6}\right) = -\sin\frac{\pi}{6} = -\frac{1}{2}$

$\cos\frac{7\pi}{6} = \cos\left(\pi + \frac{\pi}{6}\right) = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

b) $\sin\frac{7\pi}{6} - \sin\frac{\pi}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2} = \boxed{-\frac{1+\sqrt{3}}{2}}$

(31) $\cos 60 \cos 10 - \sin 60 \sin 10 = \cos(60+10) = \boxed{\cos 70}$

(32) $\sin 110 \cos 80 + \sin 80 \cos 110 = \sin(110+80)$
$= \sin(190).$

(38) $\sin\frac{4\pi}{9}\cos\frac{\pi}{8} + \cos\frac{4\pi}{9}\sin\frac{\pi}{8}$

$\sin\left(\frac{4\pi}{9} + \frac{\pi}{8}\right) = \sin\left(\frac{41\pi}{72}\right)$

(40) $\cos\frac{\pi}{16}\cos\frac{3\pi}{16} - \sin\frac{\pi}{16}\sin\frac{3\pi}{16} = \cos\left(\frac{\pi}{16} + \frac{3\pi}{16}\right) = \cos\frac{\pi}{4} = \boxed{\frac{\sqrt{2}}{2}}$

6.5 Sum to product.

$$\sin a + \sin b = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$$\sin a - \sin b = 2\sin\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

$$\cos a + \cos b = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$

$b+\pi$

$$\cos a - \cos b = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$\cos(b+\pi) = -\cos b$

$$= 2\cos\left(\frac{a+b+\pi}{2}\right)\cos\left(\frac{a-b-\pi}{2}\right)$$

$$= 2\cos\left(\frac{\pi}{2} + \frac{a+b}{2}\right)\cdot\cos\left(\frac{-\pi}{2} + \frac{a-b}{2}\right)$$

$$= -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$

$$\cos 195 + \cos 105 = 2\cos(150)\cos 45$$

$$= 2\cos(180-30)\ \frac{\sqrt{2}}{2}$$

$$= 2(-\cos 30)\frac{\sqrt{2}}{2} = 2\left(-\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2} = -\frac{\sqrt{6}}{2}$$