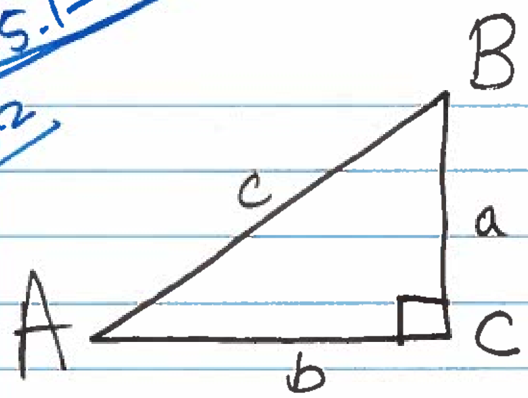
math 166
note 5.1–5.3
5.2




tell me everything you know

$90^\circ$

$\rightarrow a^2 + b^2 = c^2$ (Pythagorean theorem).

$\rightarrow m\angle A + m\angle B = 90^\circ$

$\sin\theta = \dfrac{opp}{hyp}$ $\qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{hyp}{opp}$

$\cos\theta = \dfrac{adj}{hyp}$ $\qquad \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{hyp}{adj}$

$\tan\theta = \dfrac{opp}{adj} = \dfrac{\sin\theta}{\cos\theta}$ $\qquad \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta} = \dfrac{a}{opp}$

Chap 5:

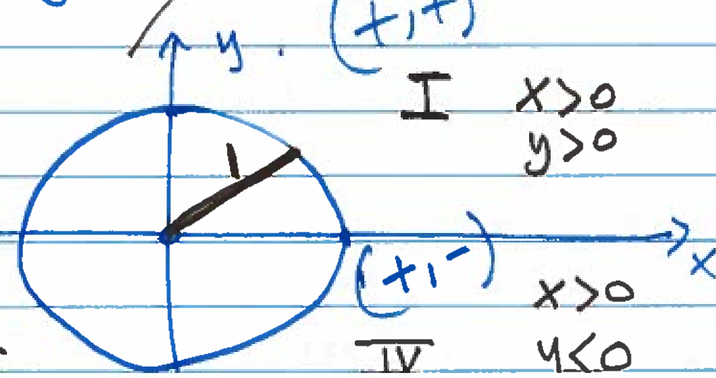
5.1 Unit Circle.
radius is 1, center, radius
Center (0,0).

equation of Circle.
$(x-h)^2 + (y-k)^2 = r^2$
center $(h,k)$
radius $= r$

Unit Circle: $\boxed{x^2 + y^2 = 1}$

$(-,+)$ $\quad x<0$, $y>0$ $\quad$ II

$(+,+)$ $\quad$ I $\quad x>0$, $y>0$




$(-,-)$ $\quad x<0$, $y<0$ $\quad$ III

$(+,-)$ $\quad x>0$, $y<0$ $\quad$ IV

~~Show that~~ Is $P\left(\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\right)$ on the Unit Circle

Unit Circle $\longrightarrow$ $x^2+y^2=1$ **No**

Is $\left(\frac{\sqrt{3}}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2 \stackrel{?}{=} 1$

$$\frac{3}{4}+\frac{3}{4}=\frac{6}{4}=\frac{3}{2}\neq 1.$$

Is $P\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ on the Unit Circle? yes.

$$\left(\frac{\sqrt{2}}{2}\right)^2+\left(-\frac{\sqrt{2}}{2}\right)^2=$$

$$\frac{2}{4}+\frac{2}{4}=\frac{4}{4}=1 \checkmark$$

$P\left(\frac{\sqrt{2}}{2}, y\right)$ is on the Unit Circle.

in Q IV $\quad y=-\frac{\sqrt{2}}{2}$

determine the value of $y$.

$$\left(\frac{\sqrt{2}}{2}\right)^2+y^2=1 \text{ and } y<0$$

$$\frac{1}{2}+y^2=1$$

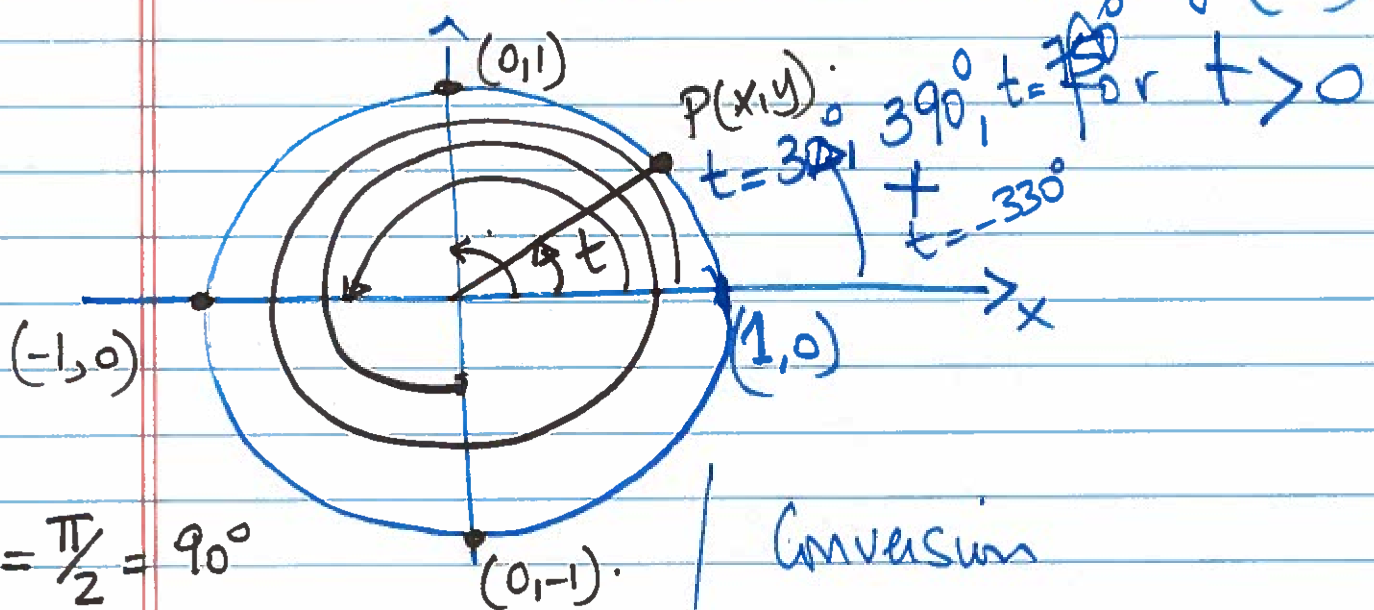
$$y^2=\frac{1}{2}\ ;\ |y|=\sqrt{\frac{1}{2}}=\frac{\sqrt{2}}{2}$$

$$y=\pm\frac{\sqrt{2}}{2}$$

terminal point on the Unit Circle.

Let $t > 0$ real number.

Let's mark off a distance $t$ along the Unit Circle starting $(1,0)$.




$t = \frac{\pi}{2} = 90°$

$t = \pi$ 180°

$t = \frac{3\pi}{2}$ 270°

$t = 2\pi$ 360°

Conversion

radian to degree.

$\frac{\pi}{6} \times \frac{180}{\pi} \longrightarrow \frac{180}{6} = \boxed{30°}$

if $\pi$ is 180°

then $\frac{\pi}{6}$ is ?

$\frac{\pi}{180} = \frac{\pi/6}{?}$

$\frac{\pi}{4} \times \frac{180}{\pi} \longrightarrow \frac{180}{4} = \boxed{45°}$

$\frac{\pi}{3} \times \frac{180}{\pi} \longrightarrow \frac{180}{3} = \boxed{60°}$

$\frac{\pi}{2} \times \frac{180}{\pi} \longrightarrow \frac{180}{2} = \boxed{90°}$

degree $\xrightarrow{\times \frac{\pi}{180}}$ radian.

$270° \times \frac{\pi}{180} \longrightarrow \frac{3}{2}\pi$

$300° \times \frac{\pi}{180} \longrightarrow \frac{5\pi}{3}$

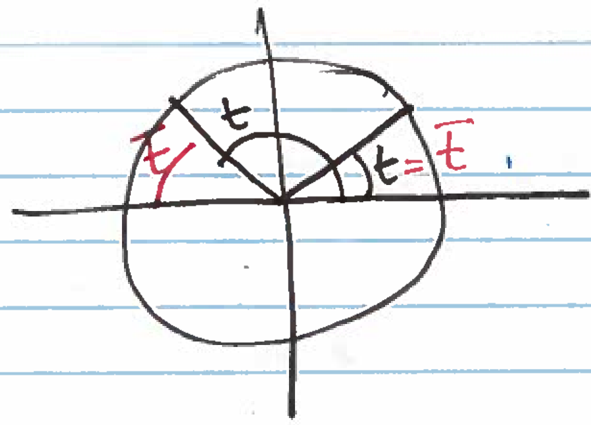
Q IV

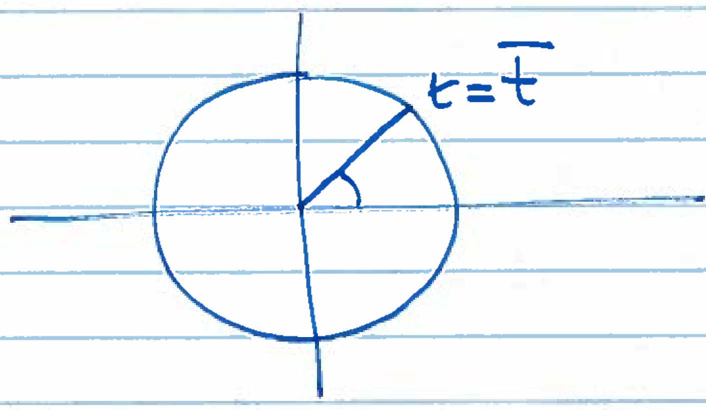
## 5.3 reference number.

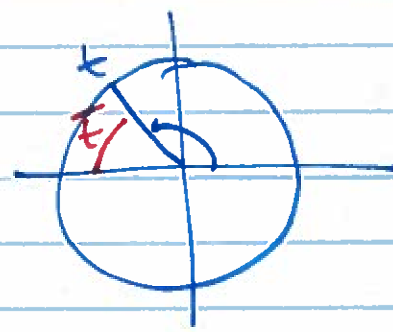
Let $t$ be a real number.

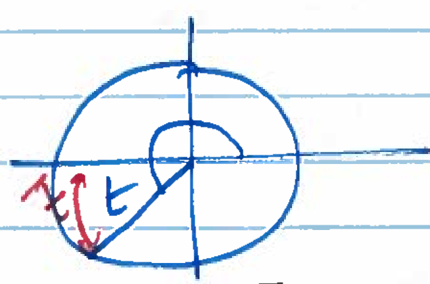
The reference number $\bar{t}$ associated with $t$ is the shortest distance along the unit circle with x-axis.

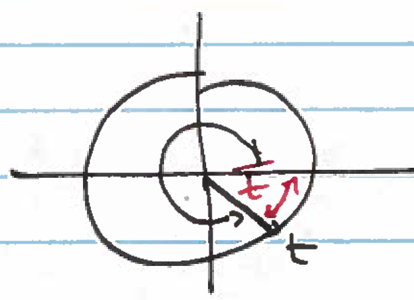
positive




$t = \frac{\pi}{8}$ ; $\bar{t} = \frac{\pi}{8}$

$t = \frac{5\pi}{6}$ $\therefore 150^\circ$ $\bar{t} = \frac{\pi}{6}$. $30^\circ$

$t = \frac{7\pi}{6}$ $\therefore 210^\circ$ $\bar{t} = \frac{\pi}{6}$ $30^\circ$

$t = \frac{7\pi}{4}$ ; $\bar{t} = (\frac{\pi}{4}$

$\therefore 315^\circ$ $45^\circ$

QIV

Find the reference number for each $t$.

$t = \frac{4\pi}{3}$ $Q_{III}$ ; $\bar{t} = \frac{4\pi}{3} - \pi = \boxed{\pi/3}$

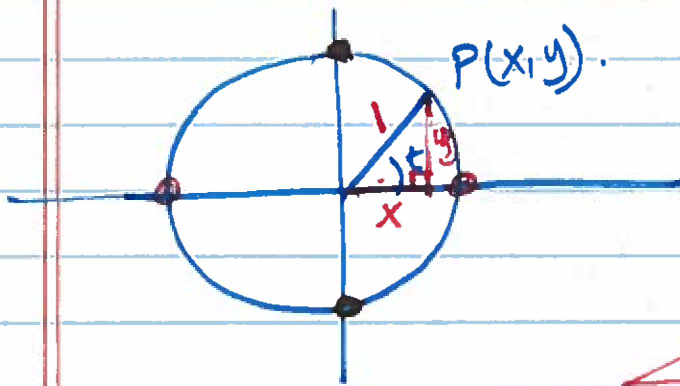
$t = 7\pi/3$ $Q_{I}$ ; $\bar{t} = \frac{7\pi}{3} - 2\pi = \frac{7\pi}{3} - \frac{6\pi}{3}$ [

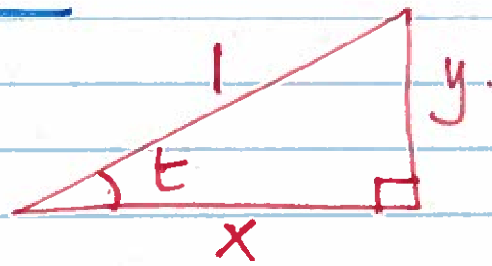
$t = 17\pi/6$. $Q_{II}$ ; $\bar{t} = \pi - \frac{5\pi}{6} = \boxed{\pi/6}$

$= \frac{12\pi + 5\pi}{6}$

$= 2\pi + \frac{5\pi}{6}$

# 5.3 Trigonometric functions of real numbers




$$\boxed{x^2+y^2=1}$$

$\sec(t) = \frac{1}{x}$ $\qquad \cos(t) = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1}$ $\qquad \boxed{\cos(t) = x}$

$\csc(t) = \frac{1}{y}$ $\qquad \sin(t) = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1}$ $\qquad \boxed{\sin(t) = y}$

$\cot(t) = \frac{x}{y}$ $\qquad \tan(t) = \frac{\text{opp}}{\text{adj}} = \frac{y}{x}$ $\qquad \boxed{\tan(t) = \frac{y}{x}}$

6 trigonometric functions

Domain : $\sin(x)$ $\quad \cos(x)$. $\quad (-\infty, \infty)$.

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ $\qquad \cos(x) \neq 0$

$\frac{\pi}{2}(\quad)$ $\qquad x \neq \frac{\pi}{2} + 2n\pi$

$\cos(x) = 0$. $\qquad$ or $\; x \neq \frac{3\pi}{2} + 2n\pi$

$\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \frac{13\pi}{2}$. $\qquad$ n integer.

$x = \frac{\pi}{2} + 2n\pi$ :.

$x = \frac{3\pi}{2} + 2n\pi$ :. $\frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \frac{15\pi}{2}, \ldots$

even numbers $2n$

$n$ integer.

odd numbers $(2n+1)$

$\cos(x)=0$

$x=(2n+1)\frac{\pi}{2}$

$n$ integer.

$f(x)=\tan(x)$

Domain $=\{x \mid x\neq(2n+1)\frac{\pi}{2}\}$.

$-\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}$ $\boxed{VA}$ $n$ integer.

$f(x)=\frac{1}{\cos(x)}=\sec(x)$. Domain: $\{x \mid x\neq(2n+1)$

$\cos(x)\neq 0 \therefore x\neq(2n+1)\frac{\pi}{2}$

$f(x)=\cot(x)=\frac{\cos(x)}{\sin(x)}$ ; $f(x)=\frac{1}{\sin(x)}=\csc(x)$

$f(x)=\frac{1}{x}$ Domain $\sin(x)\neq 0$. $x\neq n\pi$

$D_f=\{x \mid x\neq 0\}$ VA

$x=0$

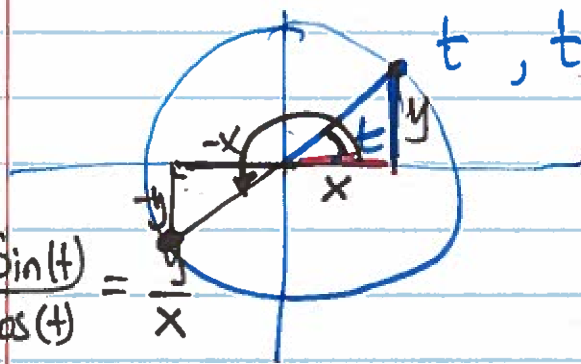
Domain $=\{x \mid x\neq n\pi\}$.

$n$ integer.

VA

$x=0, \pi, 2\pi$

# Periodic function




$t, t+2\pi, t+2n\pi$

$\cos(t)$

$\cos(t+2n\pi)$

$\tan(t) = \frac{\sin(t)}{\cos(t)} = \frac{y}{x}$

$\tan(t+\pi) = \frac{y}{x} = \frac{-y}{-x}$

$$\cos(t+2n\pi) = \cos(t)$$

$y = \cos(t)$ is $2\pi$ periodic

$y = \sec(t)$.

$$\sin(t+2n\pi) = \sin(t).$$

$y = \sin(t)$ is $2\pi$ periodic.

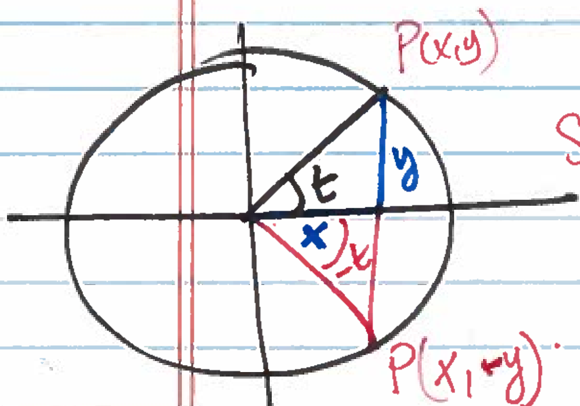
$y = \csc(t)$ ✓

$y = \tan(t)$ $\quad \tan(t+\pi) = \tan(t).$

$$\tan(t+n\pi) = \tan(t).$$

$\tan(t)$ is $\pi$ periodic

$\cot(t)$ is $\pi$ periodic.




$\cos(-t) = \cos(t)$ <u>even</u> y-axis

Sec(t) even

$\sin(-t) = -\sin(t)$ <u>odd</u> Reflection to origin

csc(t) odd

$\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin(t)}{\cos(t)}$

cot(t) odd reflection with respect to origin $= -\tan(t)$.

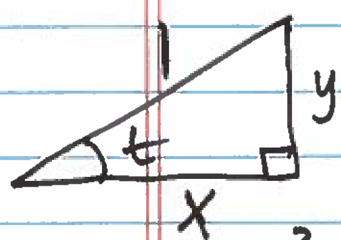
$$\sin\left(-\frac{\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\cos\left(-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}.$$

Special Values.

| $\theta$ | $0$ | $\pi/6$ | $\pi/4$ | $\pi/3$ | $\pi/2$ |
|---|---|---|---|---|---|
| $\sin(\theta)$ | $\sqrt{0}/2$ | $\sqrt{1}/2$ | $\sqrt{2}/2$ | $\sqrt{3}/2$ | $\sqrt{4}/2$ |
| $\cos(\theta)$ | $\sqrt{4}/2$ | $\sqrt{3}/2$ | $\sqrt{2}/2$ | $\sqrt{1}/2$ | $\sqrt{0}/2$ |
| $\tan(\theta)$ | $0$ | $\frac{1}{\sqrt{3}}$ | $1$ | $\sqrt{3}$ | und |
| $\cot(\theta)$ | und | $\sqrt{3}$ | $1$ | $1/\sqrt{3}$ | $0$ |

fundamental identities.




$\cos(t) = x \qquad \sin(t) = y.$

$\sin^2(t) = 1 - \cos^2(t)$

$$\boxed{\cos^2(t) = 1 - \sin^2(t)}$$

$x^2 + y^2 = 1$

$$\boxed{\cos^2(t) + \sin^2(t) = 1}$$ true for any $t$

$\cos^2\theta + \sin^2\theta = 1$

$\cos^2(t) = [\cos(t)]^2$

$\cos(t^2) \neq \cos^2(t).$

$$\frac{\cos^2(t)}{\cos^2(t)} + \frac{\sin^2(t)}{\cos^2(t)} = \frac{1}{\cos^2(t)}.$$

$$\boxed{1 + \tan^2(t) = \sec^2(t)}$$

$$\boxed{\sec^2(t) - \tan^2(t) = 1}$$

$$\boxed{\sec^2(t) - 1 = \tan^2(t)}$$

$$\frac{\cos^2(t)}{\sin^2(t)} + \frac{\sin^2(t)}{\sin^2(t)} = \frac{1}{\sin^2(t)}.$$

$$\boxed{1 + \cot^2(t) = \csc^2(t)}$$

HW

p406
5.1 : # 54, 60, 84, 98

416-417
5.2 # 12, 58, 64, 72, 80

429
5.3 # 72, 96, 118

Due 4/3/14
at 7pm