3.2 polynomial functions of higher degree.

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0.$$

$a_n, a_{n-1}, \ldots, a_0$ Coefficients

if $a_n \neq 0$ $\deg f(x) = n$

Leading Coefficient

$f(x) = 3x^5 - 7x^3 + 8x - 2.$ Degree of $f = 5.$

$a_5 = 3$; $a_4 = 0$, $a_3 = -7$; $a_2 = 0$; $a_1 = 8$; $a_0 = -2.$

constant.

for any polynomial. Domain: $(-\infty, \infty)$.

Range. ?

$f(x) = x^2$ degree 2.

$D: (-\infty, \infty)$.

Range $[0, \infty)$.

parent function of Quadratic

$f(x) = x^2$ parent. function

Shift Right h units. $(x-h)^2$

$f(x) = a(x-h)^2 + K$

stretching Vertically. $a(x-h)^2$.

by $|a|$.

Shift up K units.

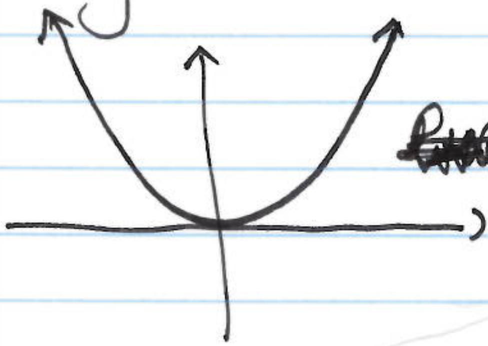
$a(x-h)^2 + K$.

End behavior. $y = x^2$.

as $x \to -\infty$ $y \to \infty$

as $x \to \infty$ $y \to \infty$

$y = x^2$



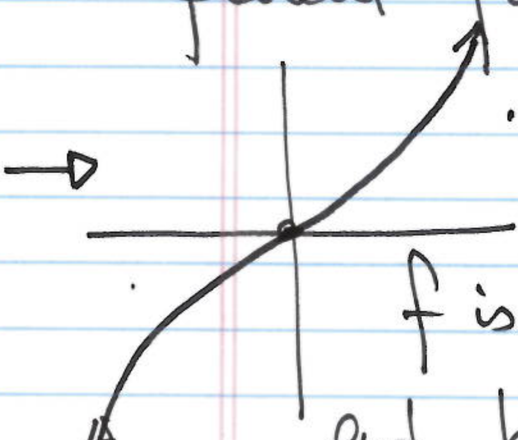
if $x < 0$ behavior.

f is decreasing.

if $x > 0$

f is increasing.

Parent function $y = x^3$ deg = 3.



· odd.

$D = (-\infty, \infty)$

$R = (-\infty, \infty)$.

f is increasing. for any x

end behavior. as $x \to -\infty$ $y \to -\infty$

as $x \to \infty$ $y \to \infty$

$f(x) = -x^3 + 4x$ D: $(-\infty, \infty)$ deg 3

Range $(-\infty, \infty)$.

$f(x) = 0$

$-x^3 + 4x = 0$.

$-x(x^2 - 4) = 0$

$-x(x-2)(x+2) = 0$

$x = 0, 2, -2$

end behavior.

$f(x) = -x^3 + 4x = -x^3\left(1 - \frac{4}{x^2}\right)$

as $x \to \infty$ $-(\infty)^3\left[1 - \frac{4}{(\infty)^2}\right]$

$-\infty(1 - 0) = \boxed{-\infty}$

End behavior $\quad y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

as $x \to \pm\infty \qquad y \sim a_n x^n$

$$y = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0).$$

$$= x^n \left[a_n + \frac{a_{n-1}}{x} + \frac{a_{n-2}}{x^2} + \dots + \frac{a_1}{x^{n-1}} + \frac{a_0}{x^n} \right]$$

$$f(x) = 7x^9 - 3x^5 + 100x^2 + 7000$$

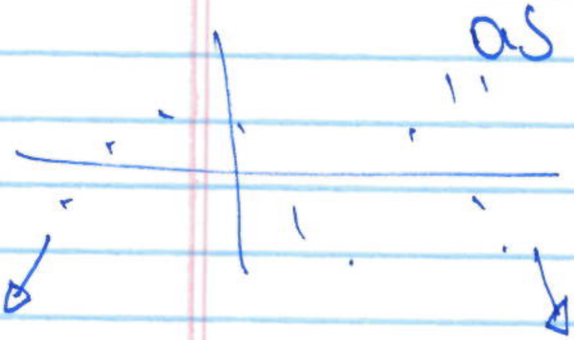
as $x \to \infty \qquad f(x) \to 7(\infty)^9 = \infty$

as $x \to -\infty \qquad y \to -\infty$

$$g(x) = -5x^6 + 1000x^5 - 3x^4 + 7$$

as $x \to \infty \qquad g(x) \to -\infty$

as $x \to -\infty \qquad g(x) \to -\infty$



repeated zeros. (multiplicity).

$$f(x) = x^2(x-1)^3(x+5)^2.$$

$$x^2\left[x^5 \text{- - - - - - -} +25\right]$$

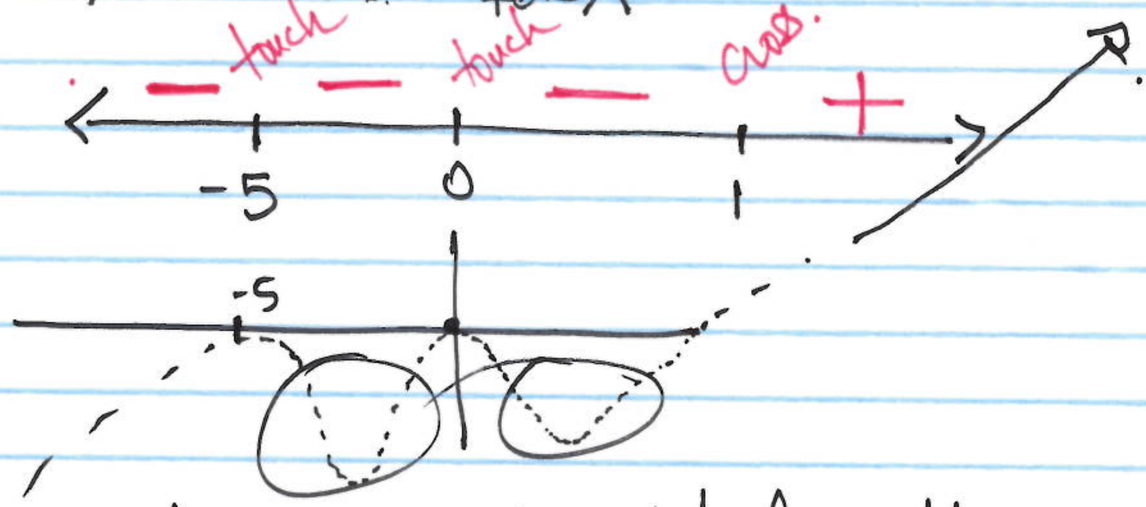
$\deg f = 7$

$f(x) \sim x^7$

Leading Coefficient $\underline{1}$

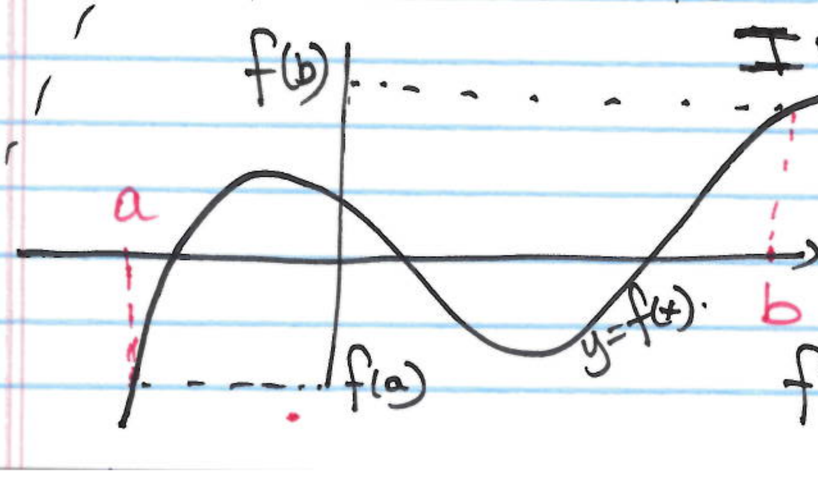
Constant 0

$$f(x) = x^7 \text{ - - - - - - } +25x^2$$



Intermediate Value theorem.

IVT



Given a polynomial function

$a \neq b$

$f(a) \neq f(b)$.

$f(a) < 0$ and $f(b) > 0$

Zeros of polynomial functions
X-intercepts.

Given $f(x)$ is a polynomial function and c is a real number.

Then all these statements are equivalent

1. $x = c$ is a zero of $f(x)$.
2. $f(c) = 0$ or $x = c$ is a solution to $f(x) = 0$
3. $(c, 0)$ is an x-intercept
4. $x - c$ is a factor of $f(x)$

Example: $f(x) = x^3 - x^2 - 2x$

$f(x) = 0$

$$x^3 - x^2 - 2x = 0$$

$$x(x^2 - x - 2) = 0.$$

$$x(x-2)(x+1) = 0$$

real zeros $x = 0$, $x = 2$, $x = -1$