2.5 Solving other types of Equations

p207 (5) $4x^4 - 16x^2 = 0$

factorization $4x^2(x^2 - 4) = 0$.

$$4x^2(x+2)(x-2) = 0$$

$$\boxed{x = 0, -2, 2}$$

(#10) $x^4 - 2x^3 = 16 + 8x - 4x^3$

$$(x^4 + 2x^3) - (8x + 16) = 0$$

"factor"

factor By Grouping

$$x^3(x+2) - 8(x+2) = 0$$

$$(x+2)(x^3 - 8) = 0 \qquad \text{Solu} \{-2, 2\}.$$

$$x + 2 = 0 \quad \text{or} \quad x^3 - 8 = 0$$

$$\boxed{x = -2} \qquad\qquad x^3 = 8$$

$$x = \sqrt[3]{8} = \boxed{2}$$

$$36t^4 + 29t^2 - 7 = 0$$

change the look

$$\text{let } x = t^2$$

$$36x^2 + 29x - 7 = 0 \qquad \boxed{1849}$$

$$x = \frac{-29 \pm \sqrt{(29)^2 - 4(36)(-7)}}{72}$$

$$x = \frac{-29 \pm 43}{72}$$

$$\frac{14}{72} = \boxed{\frac{7}{36}}$$

$$\frac{-72}{72} = \boxed{-1}$$

$$t^2 = \frac{7}{36} \qquad \text{or} \qquad t^2 = -1$$

$$\boxed{t = \pm \frac{\sqrt{7}}{6}} \qquad \boxed{t = \pm i}$$

(20) $3\sqrt{x} - 6 = 0$ Restriction $x \geq 0$

$3\sqrt{x} = 6$

Square → $\sqrt{x} = 2$

$\boxed{x = 4}$ ✓

(22) $\sqrt{2x+5} + 3 = 0$ $\left(\begin{array}{c}\text{Real} \\ \text{No Solution}\end{array}\right)$.

$\sqrt{2x+5} = -3$ ←→

$2x+5 = 9$
$2x = 4$
$x = 2.$

$\sqrt{i} = a + bi$

↓

$i = (a+bi)^2$

$0+i = a^2 - b^2 + 2abi$

$\begin{cases} a^2 - b^2 = 0 \\ 2ab = 1 \end{cases} \longrightarrow \begin{cases} a^2 = b^2 \\ ab = \frac{1}{2} \end{cases} \longrightarrow a = \pm b$

$\boxed{\begin{array}{c} a = b \\ -a = -b \end{array}}$

$ab = \frac{1}{2}$

↓

$a \cdot a = \frac{1}{2}$ $a^2 = \frac{1}{2}$

$a = \pm \frac{\sqrt{2}}{2}$

$\sqrt{i} \rightarrow \dfrac{\sqrt{2}}{2} + i\dfrac{\sqrt{2}}{2}$
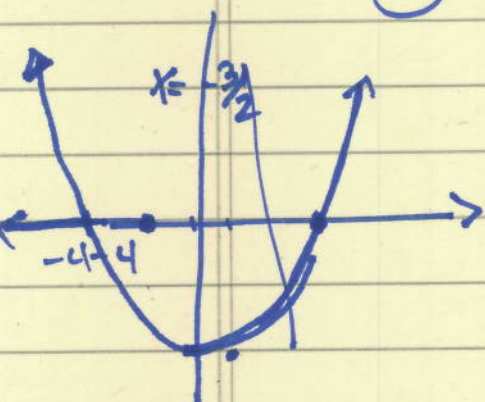
$\rightarrow -\dfrac{\sqrt{2}}{2} - i\dfrac{\sqrt{2}}{2}$

Hw2 p196-197

(#46) $y = x^2 + 3x - 4$ ← parabola opens up.

$a = 1 > 0$

Vert axis of symmetry

x-intercepts. (y=0)

$x^2 + 3x - 4 = 0$

$(x-1)(x+4) = 0$

$X = -\dfrac{b}{2a}$

$X = -\dfrac{3}{2(1)} = -\dfrac{3}{2}$

$\boxed{X = 1, -4}$

Vertex $\left(-\dfrac{3}{2}, -\dfrac{25}{4}\right)$

(#50)

$\dfrac{1}{3}x^2 - 5x + 25 = 0$

$\dfrac{9}{4} - \dfrac{9}{2} - 4$

$\dfrac{9 - 18 - 16}{4} = \dfrac{-25}{4}$

Discriminant

$b^2 - 4ac = 25 - 4\left(\dfrac{1}{3}\right)(25) < 0$

No real Solutions

$a = 9$

$b = -18$

$c = 9$

$b^2 - 4ac = 0$

$\dfrac{-(-18) \pm \sqrt{()}}{2(9)}$

$\boxed{\dfrac{18}{18} = 1}$

(#56) $9x^2 - 18x + 9 = 0$

$x^2 - 2x + 1 = 0$ $\therefore (x-1)^2 = 0$

$a = 1$

$b = -2$

$c = 1$
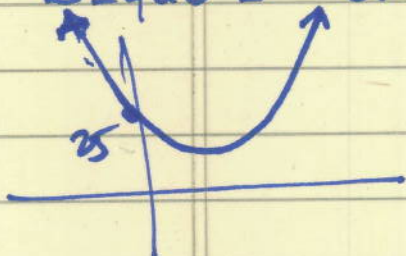
$X = \dfrac{-(-2) \pm \sqrt{(0)}}{2(1)} = \dfrac{2}{2} = \boxed{1}$ Double.

$b^2 - 4ac = 4 - 4(1)(1) = 0$

$(x-1)(x-1) = 0$

$x = 1 \quad x = 1$

(30) $\sqrt{x+5} - 2x = 3$ $\boxed{x \geq -\dfrac{3}{2}}$

Rest $\sqrt{x+5} = 2x+3.$

$x+5 \geq 0$ $x \geq -1.5$

$\boxed{x \geq -5}$ and $2x+3 \geq 0$

$-5 \quad -\dfrac{3}{2}$ $\boxed{x \geq -\dfrac{3}{2}}$
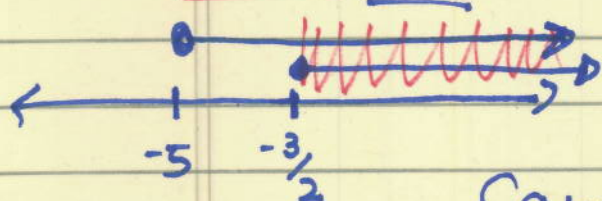
Square Both Sides.

$$x+5 = (2x+3)^2$$

$$x+5 = 4x^2 + 12x + 9$$

$$4x^2 + 11x + 4 = 0$$

$$X = \frac{-11 \pm \sqrt{121-64}}{8}$$ $\boxed{-0.43}$

$$X = \frac{-11 \pm \sqrt{57}}{8}$$ $\dfrac{-11+\sqrt{57}}{8}$

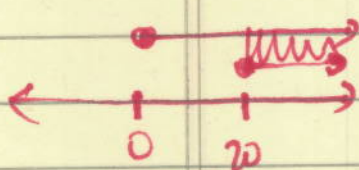$\dfrac{-11-\sqrt{57}}{8} \boxed{2.868}$

$$a^2 + 2ab + b^2{}_2 = 100$$
$$(a+b)^2 = (10)^2$$

(36) $\sqrt{x} + \sqrt{x-20} = 10$

$x \geq 0$ and $x \geq 20$

Square Both Side.

$\boxed{x \geq 20}$

$X + 2\sqrt{x}\sqrt{x-20} + X - 20 = 100$

$2\sqrt{x(x-20)} = 120 - 2x$

$\sqrt{x(x-20)} = 60 - X \qquad 60 - X \geq 0$

$\boxed{60 \geq X}$

Square Both Side.

$X(X-20) = 3600 - 120X + X^2 \qquad \boxed{20 \leq X \leq 60}$

$X^2 - 20X = 3600 - 120X + X^2 \qquad$ new

$100X - 3600 = 0.$

$X - 36 = 0$

$\boxed{X = 36} \checkmark$

(56) $\dfrac{4}{X+1} - \dfrac{3}{X+2} = \dfrac{1}{1}$

last step for fraction $\dfrac{A}{B} = \dfrac{C}{D}$

$\boxed{X \neq -1, -2}$

LHS. $\dfrac{4(X+2) - 3(X+1)}{(X+1)(X+2)} = \dfrac{1}{1}$

$\dfrac{X+5}{(X+1)(X+2)} = \dfrac{1}{1}$

Cross-multiply.

$(X+1)(X+2) = (X+5).$

$X^2 + 3x + 2 = X + 5$

$X^2 + 2x - 3 = 0$

$(X+3)(X-1) = 0$

$X = -3, 1 \checkmark$

$$\boxed{\#64} \quad 8\left(\frac{t}{t-1}\right)^2 - 2\left(\frac{t}{t-1}\right) - 3 = 0$$

$$\text{Let} \quad X = \frac{t}{t-1} \qquad \boxed{t \neq 1}$$

$$8X^2 - 2X - 3 = 0$$

$$X = \frac{2 \pm \sqrt{4 - 4(8)(-3)}}{2(8)} = \frac{2 \pm \sqrt{100}}{16}$$

$$X = \frac{2 \pm 10}{16} \quad \left\langle \begin{array}{l} \frac{12}{16} = \boxed{\frac{3}{4}} \\ \\ \frac{-8}{16} = \boxed{-\frac{1}{2}} \end{array} \right.$$

$$\frac{t}{t-1} = \frac{3}{4} \qquad \text{or} \qquad \frac{t}{t-1} = -\frac{1}{2}$$

cross-multi

$$4t = 3t - 3 \qquad\qquad\qquad 2t = 1 - t$$

$$\boxed{t = -3} \checkmark \qquad\qquad\qquad \boxed{t = \frac{1}{3}} \checkmark$$