Homotopy Type Theory Project report: Category Theory

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The aim of the project was to implement the foundational types and records occurring in basic category theory as outlined in chapter 9 of the Homotopy Type Theory book.

We defined the following notions:

a. category and precategory (in the file `Category.agda`)
b. isomorphisms in a category, the type $a \cong b$ of all isomorphic arrows from a to b (in `Category.agda`)
c. functors - composition of functors, unit laws and associativity (in `Functor.agda`)
d. natural transformations - composition, unit laws and associativity (in `Natural-transformation.agda`)
e. definition of adjoints (in `Adjoints.agda`)
f. left and right whiskering - composition of a functor with a natural transformation (in `Natural-transformation.agda`)
g. proof of Lemma 9.1.3 - for arbitrary arrow f in a category $\mathcal{C}$, `is-iso(f)` is a mere proposition and $a \cong b$ is a set (has homotopy level 0) (the proof is split in `Category.agda` and `Lemmas.ttt`)
h. lemma 9.1.4 - `idtoiso` - in `Category.agda`
i. lemma 9.1.8 - the type of objects in an arbitrary category has h-level 1 - and lemma 9.1.9 - in `Lemmas.agda`
j. construction of the precategory Set of all sets (of h-level 0) and functions between them - in (`Category.agda`)
k. construction of the functor precategory - the objects are functors between fixed precategories $\mathcal{A}$ and $\mathcal{B}$, the morphisms are natural transformations (in `Natural-transformation.agda`)
l. opposite and product (pre)categories - in `Categories.agda`
m. the hom functor $Hom_A : A^{op} \times A \to Set$, which maps a pair of objects (a, b) to $hom_A(a, b)$ and maps a pair of arrows $(f, f') : Hom_{A^{op} \times A}((a, b), (a', b'))$ to the function which maps g in $hom_A(a, b)$ to $f' \circ g \circ f$ - in `Functor.agda`

A natural extension of the current work would be implementation of lemmas 9.5.3 and 9.5.4 (Yoneda lemma).