Spectral entropy: a complementary index for rolling element bearing performance degradation assessment

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Abstract: Performance degradation assessment has been proposed to realize equipment's near-zero downtime and maximum productivity. Exploring effective indices is crucial for it. In this study, rolling element bearing has been taken as a research object, spectral entropy is proposed to be as a complementary index for its performance degradation assessment, and its accelerated life test has been performed to collect vibration data over a whole lifetime (normal-fault-failure). Results of both simulation and experiment show that spectral entropy is an effective complementary index.

Keywords: spectral entropy, performance degradation assessment, bearing accelerated life test

1 INTRODUCTION

In recent decades, machinery condition monitoring has received considerable attention, because its implementation can effectively prevent costly, even catastrophic, downtime [**1–3**]. Rolling element bearing is not only the most important but also a common failure unit in rotary machinery. Hence, an exact condition monitoring for it plays an important role in ensuring machinery's reliable running.

Abundant machinery fault information is included in its vibration signal [**4**], many correlative noteworthy researches have been carried out, mainly including vibration mathematical models [**5–8**], signal analysis, and intelligent fault diagnosis [**9–11**], thereinto, the research on signal analysis is extensive: statistical-based time-domain [**12–14**], fast Fourier transformation-based frequency-domain, high-frequency resonance [**15**], and advanced signal processing techniques such as wavelet (packet) transformation [**16**], empirical mode decomposition (EMD) [**17**], and cyclostationary analysis [**18**]. Most of these researches focused on fault diagnostics, they aimed to differentiate faults in different locations: inner race, outer race, and rolling element.

Performance degradation assessment [**19**] is a new research direction for prognostics which is much more efficient than diagnostics to achieve zero-downtime performance [**1**]. Compared with diagnostics, it focuses more on vibration trend over whole lifetime. Up to now, some research productions have been achieved [**19–25**], they mainly focused on the new intelligent assessment methods, such as a cerebellar model articulation controller, logistic regression, self-organizing map, and hidden Markov model. However, selecting the proper indices which is crucial to performance has not been paid attention to in previous researches. In fact, among existing signal analysis methods that mainly aim for fault classification, few have been researched on their trend with rolling element bearing performance degradation, except some time statistical indicators such as commonly used root mean square (RMS), crest factor, kurtosis, thereinto, RMS is the most commonly used one in engineering application, crest factor and kurtosis are pointed out that sensitive enough to impulse defect but reduce to normal-like levels as the damage grows [**25**]. Thus exploring new effective indices

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is a valuable work. Recently, some researches in this field have been published. Lybeck *et al.* [**26**] studied some indices' correlation with spall length, including RMS, kurtosis, signal peak amplitude, crest factor, and some higher-order statistics, and results showed that none of them are sensitive or consistent enough to be used as a sole indication of spall size. Yan and Gao [**27**] studied approximate entropy for assessing the bearing's defect size, and pointed out that it increases with defect size development. Tao *et al.* [**28**] proposed a new time-domain index based on Renyi entropy for condition monitoring of rolling element bearing, and compared with kurtosis and Honarvar third moment's trend with the defect development by simulation. These researches were either based on the artificial defect or simple mathematical model simulation which not only cannot depict bearing incipient defect but also cannot depict the complexity of actual performance degradation process. In this study, spectral entropy is proposed as a complementary index for rolling element bearing performance degradation assessment. Its trend with rolling element bearing performance degradation based on mathematical model and accelerated life test data has been studied. The consistency and difference between these results have been summarized.

This paper is organized as follows. In section 2, spectral entropy is educed and analysed after introducing the definition and properties of information entropy. In section 3, a vibration mathematical model of defective rolling element bearing is presented, then spectral entropy of a simulated periodic signal, Gaussian noise, and a defective rolling element bearing's vibration signal under different noise levels are computed and analysed. In section 4, experiment results are presented and analysed after introducing rolling element bearing accelerated life test. In section 5, conclusions made from these results are presented.

2 SPECTRAL ENTROPY

Spectral entropy is a generalization of information entropy. In this section, spectral entropy is educed and analysed after introducing the concept and properties of information entropy.

2.1 Concept and properties of information entropy

Information entropy was firstly proposed by Shannon [**29**] to measure information's uncertainty. For an absolutely continuous random variable X having probability density function $f(x)$, information entropy is defined as

$$H(X) = -\int f(x)\log_2(f(x))\,\mathrm{d}x \tag{1}$$

For discrete system, its brief formulation is as below

$$\begin{gathered} H = -\sum_{i=1}^{N} p_i \cdot \log_2(p_i) \\ \sum_{i=1}^{N} p_i = 1 \end{gathered} \tag{2}$$

where N is the number of symbols, p_i the probability of a symbol, and H is the uncertainty.

Information entropy holds the following properties [**30, 31**].

1. When the outcome of an event is certain, the entropy is zero.
2. When all the events are equiprobable, the entropy is maximal.
3. The increase of an events number results in entropy's increase.
4. The entropy is solely dependent on the probability distribution of the event occurrence.

The concept of entropy has been generalized in a number of different ways by different researchers [**32**] since the pioneering work of Shannon, ranging from communication, statistical mechanics, decision theory, and pattern recognition problems. In these disciplines, it is applied as a measure of disorder, unevenness of distribution, the degree of dependency, or complexity [**33**].

2.2 Spectral entropy

Spectral entropy has been proposed to measure the distribution of frequencies. $X(i), i = 1, 2, \ldots, N$ is the Fourier transformation of signal $x(i), i = 1, 2, \ldots, N$, it can be treated as a probability distribution for calculating entropy [**34**]. Then spectral entropy can be defined as below

$$\begin{gathered} SE = -\sum_{i=1}^{N} p_i \cdot \log_2(p_i) \\ p_i = \frac{X(i)}{\sum_{j=1}^{N} X(j)} \\ \sum_{i=1}^{N} p_i = 1 \end{gathered} \tag{3}$$

Spectral entropy can be normalized using equation (4) to avoid data length's influence [**33**], and this is used in this study

$$SE = \frac{-\left(\sum_{i=1}^{N} p_i \cdot \log_2(p_i)\right)}{\log_2(N)}$$

$$p_i = \frac{X(i)}{\sum_{j=1}^{N} X(j)}$$

$$\sum_{i=1}^{N} p_i = 1 \tag{4}$$

From the above definition of spectral entropy, it can be observed that: the number of frequency components is seen as the events number, p_i describes the ith frequency's percentage in whole spectrum, and $P = \{p_1, p_2, \ldots, p_N\}$ only depends on the distribution of X. Thus, spectral entropy is capable of evaluating the vibration signal's spectral structure. Spectral entropy yields larger value when amplitude distribution is flat, especially when amplitudes of each frequency component are equal, where it yields the largest value 1. Or it yields smaller value if amplitudes concentrate in few frequency components, especially when only one frequency component has non-zero amplitude, where it yields the smallest value 0. Thus, the range of spectral entropy defined in equation (4) is [0,1].

Spectral entropy has been used in fault diagnostics to differentiate different faults, such as normal valve, spring failure, and valve plate break [**33**, **35**, **36**]. In this study, its capacity of reflecting rolling element bearing performance degradation is studied.

3 SIMULATION BASED ON MATHEMATICAL MODEL

Rolling element bearing's defect deterioration is simulated by increasing impulse amplitude in a mathematical model. This section makes one, not only to know its trend with defect deterioration in theory (this supplies the base of analysis and explanation in following experiment validation), but also the influence of signal-noise-ratio (this supplies probable approaches for enhancing its capacity in further study).

3.1 Vibration mathematical model of rolling element bearing

Rolling element bearing is that class which the main load is transferred through elements in rolling contact than in sliding contact [**37**]. The sketch of the main fundamental elements and load zone can be seen in Appendix 2. The vibration signal of rolling element bearing is complex either under normal state or with defect, including components resulting from its structure (geometrical shape), tolerance, and surface deterioration such as spalling fatigue and abrasive wear [**6**]. McFadden [**5**] developed a model produced by a single point defect, which incorporates the effects of rolling element bearing geometry, shaft speed, load distribution, transfer function, and the exponential decay vibration. This model was extended to describe the vibration produced by multiple point defects [**7**]. The above models considered shocks as exactly periodicity. However, these shocks' periodicity is not exact, because the rolling elements experience some random slip (this is partly due to the fact that they have a faster spin rotation in the load zone than in the unloaded zone). In addition, their amplitudes are subjected to modulation by the rotations of the inner race (inner race fault), the outer race (outer race fault), or the cage (rolling element fault) [**38**]. Synthetically considering these factors, the mathematical model of defective rolling element bearing can be expressed as equation (5), which has been successfully used for describing faults in rolling element bearings [**39**, **40**].

$$\begin{aligned} x(t) &= \sum_{i=1}^{M} A_i \cdot \mathrm{s}(t - iT - \tau_i) + n(t) \\ A_i &= A_0 \cdot \cos(2\pi f_\mathrm{m} t + \varphi_\mathrm{A}) + C_\mathrm{A} \\ s(t) &= e^{-Bt} \cdot \cos(2\pi f_\mathrm{n} t + \varphi_\mathrm{w}) \end{aligned} \tag{5}$$

where A_i is the amplitude modulator with period $Q = 1/f_\mathrm{m}$, $s(t)$ the attenuated damping oscillation with mean impulse period T, τ_i the minor fluctuation around T, $n(t)$ the Gaussian noise with zero-mean, f_n the natural frequency related to bearing or system, A_0 the resonance intensity, C_A the arbitrary constant, and B is the coefficient of resonance damping, depending on system.

3.2 Simulation and analysis

As is well known, the vibration signal of a rolling element bearing in a normal state is Gaussian distribution [**41**], while it contains some periodic components in fact. Table 1 lists some correlative typical simulation signals' spectral entropy. It is observed that: (a) pure sine wave's spectral entropy is nearly zero (non-zero is resulted from discrete Fourier transformation's error of data with finite length), this is because pure sine wave's amplitudes concentrate on single frequency component; (b) Gaussian noise's spectral entropy is nearly to the largest value 1, this is because its amplitude distribution is flat; and (c) the spectral entropy of sine wave with Gaussian noise is large, but smaller than Gaussian noise's.

Inner race defect (IRD) is simulated using equation (5), bearing parameters and sampling frequency are the same as the following experiment. Figures 1 and

Table 1 Spectral entropy of some typical simulation signals

Type of signal	Spectral entropy
Pure sine wave	1.158e-013
Gaussian noise	0.98067
Sine wave + Gaussian noise (SNR* ≈ -5.7)	0.97079

*SNR $= 20 * \log 10(\sum_{i=1}^{N} x^2(i) / \sum_{i=1}^{N} n^2(i))$, $x(i)$ is the pure signal, $n(i)$ is the noise signal.

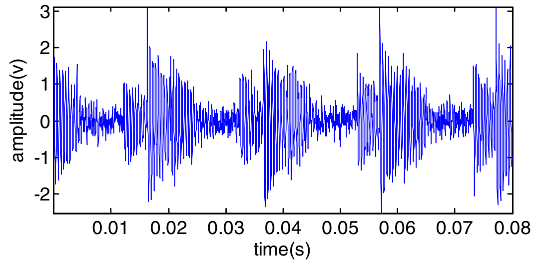


Fig. 1 Time wave of IRD

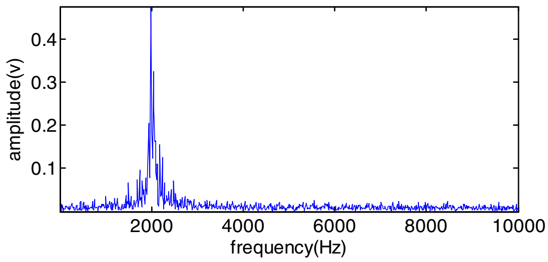


Fig. 2 Spectrum of IRD

2 are its time wave and spectrum, respectively. Defect development is simulated through increasing A_0 from 0.15 to 1.15 in equation (5). Three noise levels (corresponding standard deviations are $\sigma = 0.4$, $\sigma = 0.2$, and $\sigma = 0.1$, respectively) are studied to know about the influence of noise. Figure 3 shows spectral entropy's trend with defect development under different noise levels. It can be observed that: (a) spectral entropy decreases gradually with defect development, this is because spectrum amplitudes more concentrate on components round resonance frequency with the increase of resonance intensity; (b) the variation range of spectral entropy is different under the same increasing process of resonance intensity with different noise levels, it is about (0.98, 0.94) as $\sigma = 0.4$, about (0.97, 0.90) as $\sigma = 0.2$, about (0.96, 0.86) as $\sigma = 0.1$, that is, high noise level increases spectral entropy and reduces its variation range, low noise level decreases spectral entropy and increases its variation range. In fact, big variation range under the same defect development is favourable to detect degradation, so, denoising can improve spectral entropy's capacity of reflecting performance degradation process. Although only one condition, IRD, is presented, results of other conditions are similar.

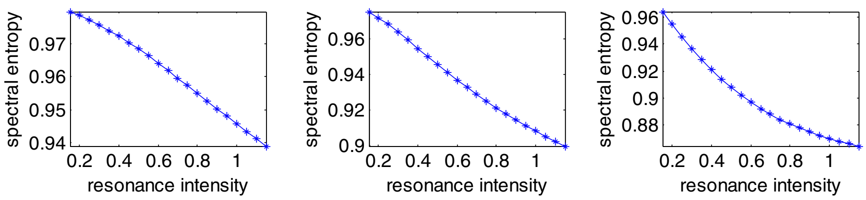
4 EXPERIMENT VALIDATION

From the above simulation, it can be observed that spectral entropy decreases with rolling element bearing's defect development, which shows that it is an effective index for reflecting rolling element bearing's performance degradation process in theory. In this section, its variation over rolling element bearing's actual degradation process is presented.

4.1 Experiment rig

Rolling element bearing accelerated life test [**42**] was performed to collect vibration data over a whole lifetime. Accelerated bearing life tester (ABLT-1A) was provided by Hangzhou Bearing Test & Research Center (HBRC) with the assistance of UNDP/UNIDO (ISO/IEC 17025 accreditation). It simultaneously hosts four rolling element bearings on one shaft driven by an AC motor and coupled by rub belts. A new one will be installed if one is failure, which improves test efficiency. Data acquisition system includes three acceleration sensors (installation sketch can be seen in Fig. 4), one signal switch instrument and DAQCard-6023E data acquisition card. Data acquisition software is programmed with National Instruments LabView. Experiment equipment can be seen in Fig. 5. One group (20 480 points) was collected per minute with 25.6 kHz sampling rate. Three channels' data was stored in one binary file. The tested rolling element bearing's type is 6307, and its corresponding parameters and operation condition can be seen in Table 2. Five characteristic frequencies can be seen in Table 3, corresponding computation can be seen in Appendix 2.

Two failure rolling element bearings, B1 and B2, are studied using 1062 (previous 900 points are cancelled because of instable running state) and 2469 groups, respectively. Because the amount of whole life's data is large and it exists difference more or less for each installation, only the part during which one rolling element bearing occurs failure is used, for example in



(a) $\sigma = 0.4$ (b) $\sigma = 0.1$ (c) $\sigma = 0.1$

Fig. 3 Spectral entropy's trend with defect development under different noise levels: (a) $\sigma = 0.4$, (b) $\sigma = 0.2$, and (c) $\sigma = 0.1$

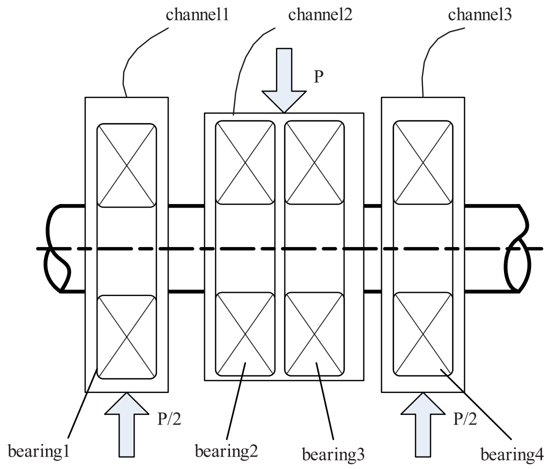


Fig. 4 The sketch of sensor installation

Fig. 5 Experiment equipment ABLT-1A

Table 2 Rolling element bearing's parameters and operation condition

Type	Ball number	Ball diameter (mm)	Pitch diameter (mm)	Contact angle	Motor speed (r/min)	Load (kN)
6307	8	13.494	58.5	0	3000	12.744

Table 3 Five characteristic frequencies of rolling element bearing 6307 (Hz)

f_r	f_c	f_b	f_i	f_o
50	19	102	246	153

Fig. 6, the degradation process of B1 is analysed using the data during $t1$. Figure 7 is one failure rolling element bearing, which shows that it is a complex process for rolling element bearing from normal to failure, and similar results can be found in other failure ones.

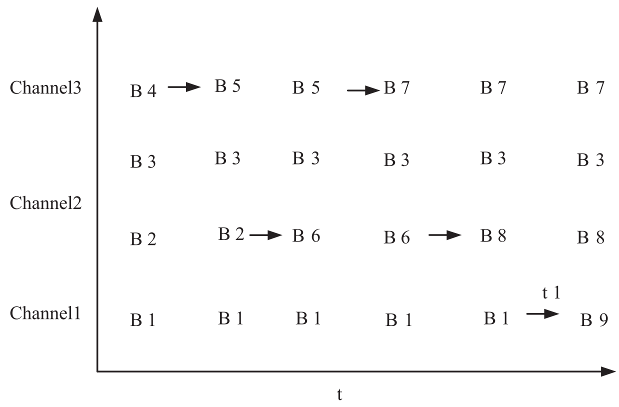


Fig. 6 The sketch of selecting analysed data

Fig. 7 One failure rolling element bearing

4.2 Results and analysis

Figures 8 and 9 show spectral entropy variation of B1 and B2, respectively. It is observed that there are two transit processes during each rolling element bearing's whole life time. The first one is that spectral entropy increases abruptly and then keeps stable during long time, the abrupt increase occurs at about 517 min for B1, at about 1297 min for B2. The second one is gradually decreasing up to final failure, which is resulted from resonance intensity's increase. It begins at about 978 min for B1, at about 2306 min for B2, and big instantaneous fluctuation can be observed for B1 and B2 during this process, at about 1045 and 2350 min, respectively.

B1 and B2's RMS are shown in Figs 10 and 11, respectively, to verify if spectral entropy can reflect rolling element bearing performance degradation. It is observed that there are also two transit processes during each rolling element bearing's whole life time. The first one is that RMS increases abruptly and keeps stable during long time. Its occurrence time is the same as spectral entropy's. However, this increase is so tiny that it is hardly to be observed. The second one is increasing gradually up to failure. Its occurrence time is the same as spectral entropy's for B1, and also occurs big instantaneous fluctuation at 1045 min, which may because big spall or crack is smoothed or rounded

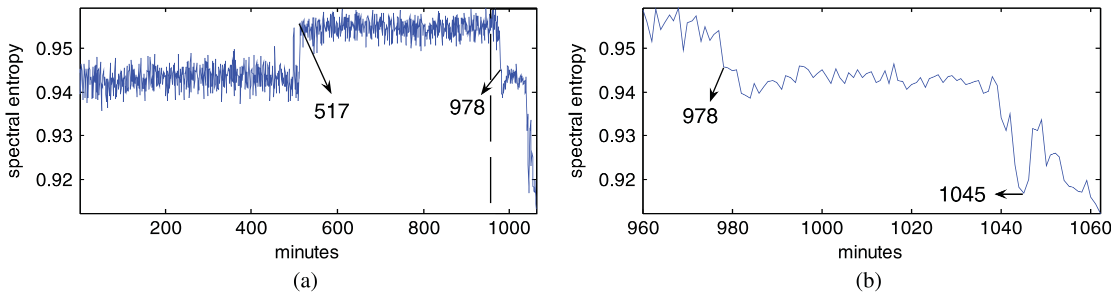


Fig. 8 B1's spectral entropy: (a) spectral entropy over whole life time and (b) local enlargement of (a)

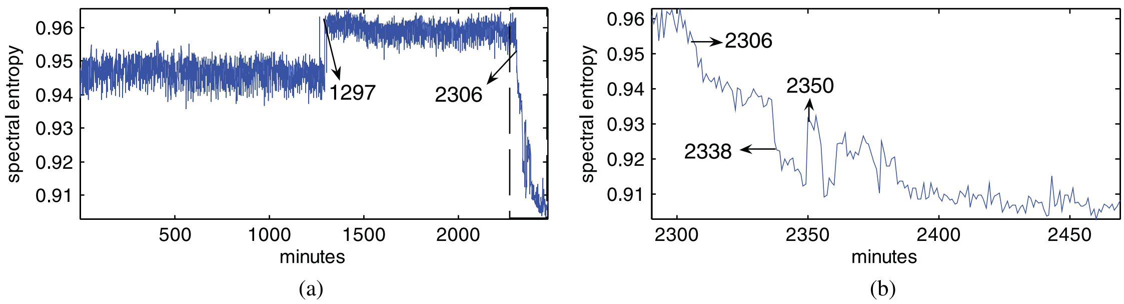


Fig. 9 B2's spectral entropy: (a) spectral entropy over whole life time and (b) local enlargement of (a)

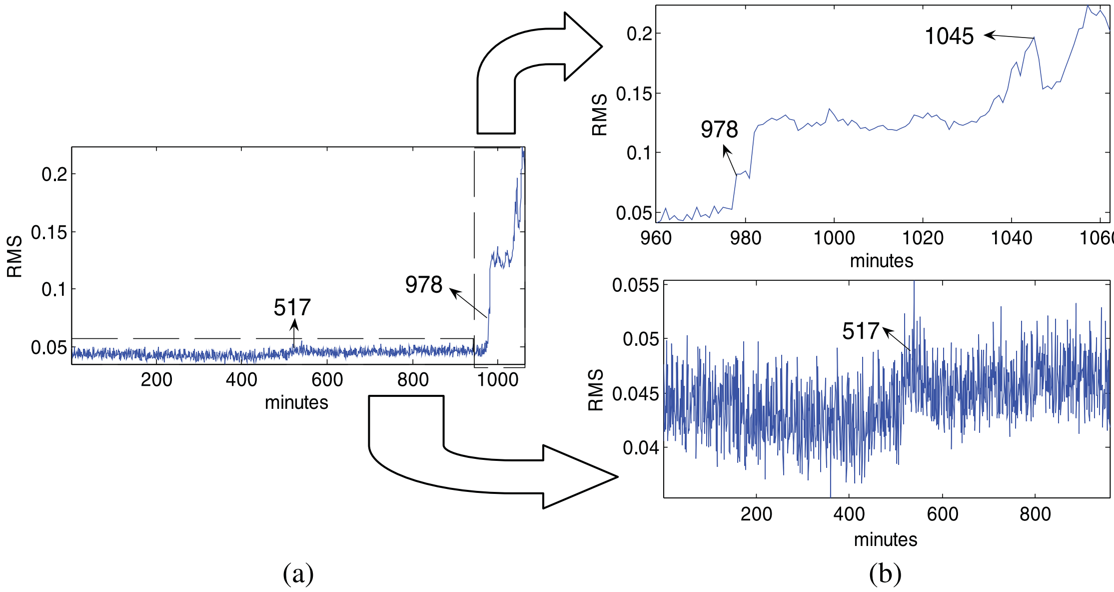
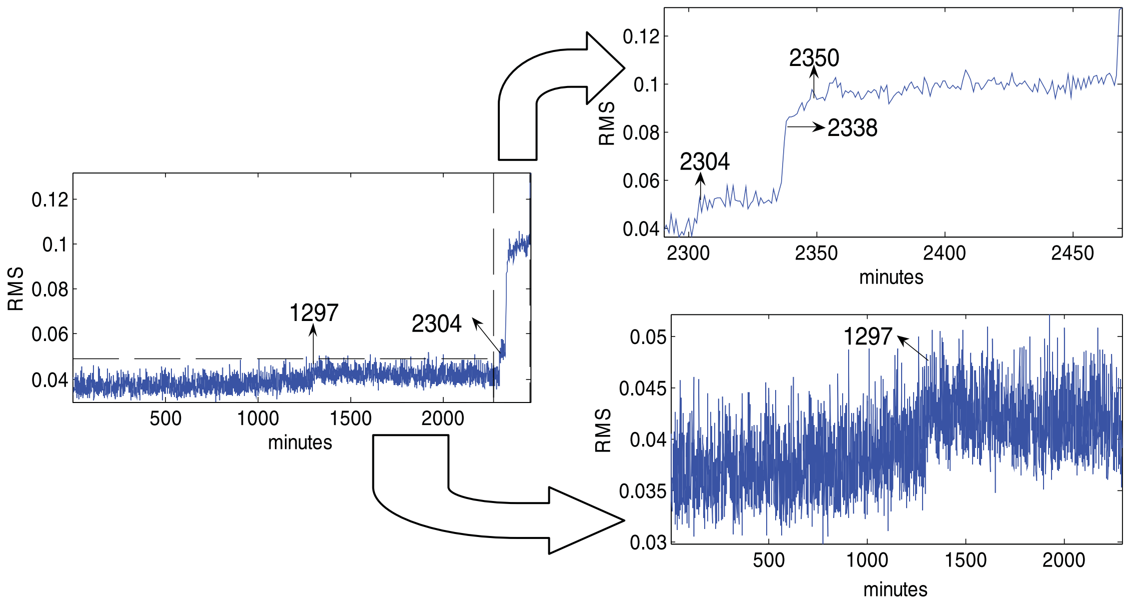


Fig. 10 B1's RMS: (a) RMS over whole life time and (b) local enlargement of (a)

resulting in renounce intensity falls rapidly [**43**]. Its occurrence time is a little earlier than spectral entropy for B2, at about 2304 min. Abrupt change occurs at 2338 min in Figs 9(b) and 11(b) during this process, which may because big spall's occurrence. And analysing further Figs 9(b) and 11(b), it can be found: (a) big instantaneous fluctuation occurs at 2350 min in Fig. 9(b), but this does not occur in Fig. 11(b); and (b) abrupt increase occurs as final failure in Fig. 11(b), but this does not occur in Fig. 9(b).

Only some of the above experimental phenomena can be explained properly, but proper explanations on



(a) (b)

Fig. 11 B2's RMS: (a) RMS over whole life time and (b) local enlargement of (a)

others, including (a) and (b) mentioned in the above paragraph and what changes of rolling element bearing can result in both energy and spectral entropy's increase abruptly, are worthy of further study, which mainly focuses on the influence of different defects on vibration signal.

From the above analysis, it is observed that spectral entropy can effectively reflect rolling element bearing's actual performance degradation process, and can more obviously detect incipient weak defect than RMS at least for these two rolling element bearings, which is very useful to realize predictive maintenance. In fact, one can find another phenomenon: B1 and B2's spectral entropy are very close, about 0.91, as final failure, while their RMS are so different, are more than 0.2 and about 0.12, respectively, this shows that spectral entropy is basically independent of vibration level for these two examples. In fact, this can be explained from the properties of spectral entropy. However, whether the frequencies structure are similar for different failure rolling element bearings should be studied further and summarized through abundant data.

5 CONCLUSIONS

In this study, spectral entropy is proposed to be used as a complementary index for rolling element bearing performance degradation assessment. A rolling element bearing accelerated life test has been performed to collect vibration data over its whole lifetime (normal-fault-failure). The results of both simulation and experiment show that it can effectively reflect the rolling element bearing's performance degradation process. Moreover, it holds an advantage over RMS, that is, it can more obviously detect an incipient weak defect at least for these two examples in the experiment. It can be observed that spectral entropy's values are close for different failure rolling element bearings, whether this is a common phenomenon should be studied further. However, it also has some disadvantages, such as its variation is not consistent enough with performance degradation, and it is not sensitive enough to final failure.

From simulation analysis, it can be found that low noise level is favourable to detect degradation, so its capacity can be enhanced either by selecting proper denoising methods or adopting better spectral analysis methods which can restrain noise.

Moreover, as shown in the previous research that different indices are sensitive to different faults and different degradation stages, spectral entropy is not an exception. Properly utilizing each effective index's advantages by some intelligent decision methods to realize rolling element bearing's comprehensive performance degradation assessment is one of the study directions.

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APPENDIX 1

Notation

A_i	amplitude modulator
A_0	resonance intensity
b	ball diameter
B	coefficient of resonance damping
C_A	arbitrary constant
D	pitch diameter
f_b	ball passing frequency
f_c	cage rotation frequency
f_i	inner race passing frequency
f_n	natural frequency related to bearing or system
f_o	outer race passing frequency
f_r	shaft rotation frequency
H	information entropy
$n(t)$	Gaussian noise with zero-mean
N	number of symbols
p_i	probability of a symbol
$s(t)$	attenuated damping oscillation
SE	spectral entropy
T	mean impulse period
z	ball number of rolling element
θ	contact angle
σ	standard deviation
τ_i	minor fluctuation around T

APPENDIX 2

In this appendix, some instructions on rolling element bearing are given. Figure 12 is a sketch of its structure and load distribution.

There are five characteristic frequencies for rolling element bearing: shaft rotation frequency f_r, cage rotation frequency f_c, ball passing frequency f_b, inner race passing frequency f_i, and outer race passing frequency f_o. For a rolling element bearing with stationary outer race, f_c, f_b, f_i, and f_o are computed as below [**4**]

$$f_c = \left(\frac{1 - d\cos\theta}{D}\right) \cdot \frac{f_r}{2} \tag{6}$$

$$f_b = \left[1 - \left(\frac{d}{D}\right)^2 \cos^2\theta\right] \cdot \frac{f_r}{(2 \cdot d \cdot D)} \tag{7}$$

$$f_i = \left(\frac{1 + d\cos\theta}{D}\right) \cdot z \cdot \frac{f_r}{2} \tag{8}$$

$$f_o = \left(\frac{1 - d\cos\theta}{D}\right) \cdot z \cdot \frac{f_r}{2} \tag{9}$$

where z is the number of balls.

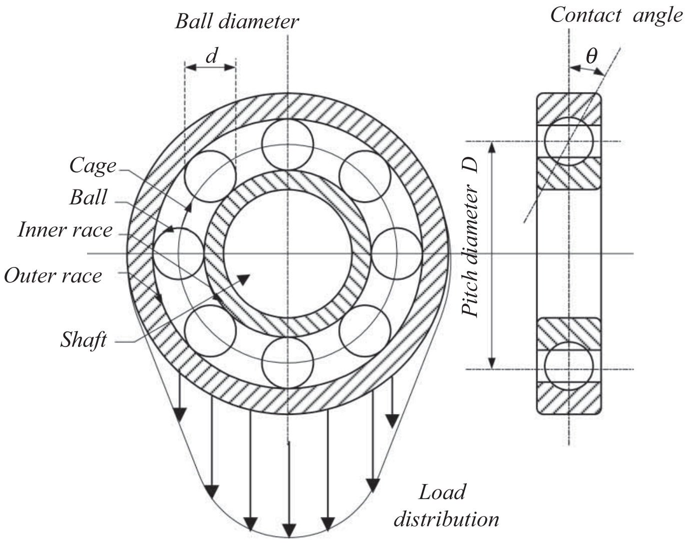

Fig. 12 Rolling element bearing's structure and load distribution