1. **(a)** Write the line $3x+4y=5$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(b) Write the line $x-2y=3$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(c) Write the line $(x,y)=(2,-4)+t\begin{bmatrix}2\\6\end{bmatrix}$ in normal form $ax+by=c$.

(d) Write the line $(x,y)=(3,-1)+t\begin{bmatrix}-5\\2\end{bmatrix}$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(e) Write the line $\begin{cases}x=3-2t\\y=-1+4t\end{cases}$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(f) Write the line $\begin{cases}x=3-2t\\y=-1+4t\end{cases}$ in normal form $ax+by=c$.

(g) Write the line $y-3=7(x-5)$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(h) Write the line $y-3=7(x-5)$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

2. **(a)** Write the line $7x-2y=3$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(b) Write the line $3x+5y=-2$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(c) Write the line $(x,y)=(1,2)+t\begin{bmatrix}-3\\4\end{bmatrix}$ in normal form $ax+by=c$.

(d) Write the line $(x,y)=(5,-6)+t\begin{bmatrix}7\\8\end{bmatrix}$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(e) Write the line $\begin{cases}x=4-3t\\y=-1+5t\end{cases}$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(f) Write the line $\begin{cases}x=7-9t\\y=-8+5t\end{cases}$ in normal form $ax+by=c$.

(g) Write the line $y-4=3(x+2)$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(h) Write the line $y+8=3(x+4)$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

3. **(a)** Write the line $5x+7y=3$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(b) Write the line $7x-3y=5$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(c) Write the line $(x,y)=(1,-7)+t\begin{bmatrix}3\\-4\end{bmatrix}$ in normal form $ax+by=c$.

(d) Write the line $(x,y)=(2,5)+t\begin{bmatrix}7\\-3\end{bmatrix}$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(e) Write the line $\begin{cases}x=9-7t\\y=-3+5t\end{cases}$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(f) Write the line $\begin{cases}x=9+5t\\y=2-7t\end{cases}$ in normal form $ax+by=c$.

(g) Write the line $y-9=3(x-5)$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(h) Write the line $y-7=9(x-3)$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

4. **(a)** Write the line $5x-4y=7$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(b) Write the line $7x+4y=5$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(c) Write the line $(x,y)=(5,-3)+t\begin{bmatrix}7\\-4\end{bmatrix}$ in normal form $ax+by=c$.

(d) Write the line $(x,y)=(3,-2)+t\begin{bmatrix}-7\\5\end{bmatrix}$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

(e) Write the line $\begin{cases}x=5-7t\\y=-4+3t\end{cases}$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(f) Write the line $\begin{cases}x=3-7t\\y=-5+4t\end{cases}$ in normal form $ax+by=c$.

(g) Write the line $y-7=4(x-5)$ in vector form $(x,y)=(a,b)+t\begin{bmatrix}A\\B\end{bmatrix}$.

(h) Write the line $y+5=-4(x-7)$ in parametric form $\begin{cases}x=a+At\\y=b+Bt\end{cases}$.

5. Find a normal equation, a vector equation and parametric equations of the line passing through the points $P = (3, 2)$ and $Q = (-1, 5)$. Is the line perpendicular to the line $3x + 4y = 3$?

6. Find a normal equation, a vector equation and parametric equations of the line passing through the points $P = (-3, 2)$ and $Q = (-1, 5)$. Is the line perpendicular to the line $2x + 3y = 6$?

7. Find a normal equation, a vector equation and parametric equations of the line passing through the points $P = (9, -1)$ and $Q = (5, -3)$. Is the line perpendicular to the line $3x - y = 7$?

8. Find a normal equation, a vector equation and parametric equations of the line passing through the points $P = (-2, -1)$ and $Q = (6, -5)$. Is the line perpendicular to the line $2x - y = -3$?

9. Find the equation of the line through the point $P = (6, 3)$ perpendicular to the line segment $\overline{AB}$, where $A = (-5, 4)$ and $B = (1, -2)$.

10. Find the equation of the line through the point $Q = (3, -2)$ perpendicular to the line segment $\overline{AB}$, where $A = (8, -3)$ and $B = (-4, 2)$.

11. Find the equation of the line through the point $P = (-5, 4)$ perpendicular to the line segment $\overline{AB}$, where $A = (-7, 5)$ and $B = (2, -1)$.

12. Find the equation of the line through the point $P = (-3, 7)$ perpendicular to the line segment $\overline{AB}$, where $A = (-4, -3)$ and $B = (5, -5)$.

13. Find the equation of the line through the point $P = (6, 3)$ parallel to the line segment $\overline{AB}$, where $A = (-5, 4)$ and $B = (1, -2)$.

14. Find the equation of the line through the point $Q = (3, -2)$ parallel to the line segment $\overline{AB}$, where $A = (8, -3)$ and $B = (-4, 2)$.

15. Find the equation of the line through the point $P = (2, -1)$ parallel to the line segment $\overline{AB}$, where $A = (5, -7)$ and $B = (-1, 2)$.

16. Find the equation of the line through the point $P = (7, -3)$ parallel to the line segment $\overline{AB}$, where $A = (-3, -4)$ and $B = (3, 5)$.

17. Find the point P **on** the line $l: 3x-4y=7$ closest to the point $Q=(2,6)$, using a perpendicular line through Q.

18. Find the point P **on** the line $l: 2x-3y=4$ closest to the point $Q=(3,5)$, using a perpendicular line through Q.

19. Find the point P **on** the line $l: 6x-5y=4$ closest to the point $Q=(-13,5)$, using a perpendicular line through Q.

20. Find the point P **on** the line $l: 3x-2y=4$ closest to the point $Q=(-4,5)$, using a perpendicular line through Q.

21. Find the intersections of the following pairs of lines:

(a) $l: 3x+4y=5$ and $m: (x,y)=(2,-4)+t\begin{bmatrix}2\\6\end{bmatrix}$.

(b) $m: (x,y)=(2,-4)+t\begin{bmatrix}2\\6\end{bmatrix}$ and k the line through the points $P=(-2,-8)$ and $Q=(10,0)$.

(c) $l: 3x+4y=5$ and $n: x-2y=3$.

(d) $m: (x,y)=(2,-4)+t\begin{bmatrix}2\\6\end{bmatrix}$ and $p: (x,y)=(1,2)+s\begin{bmatrix}-1\\3\end{bmatrix}$.

(e) $m: (x,y)=(2,-4)+t\begin{bmatrix}2\\6\end{bmatrix}$ and the line q perpendicular to m through the point $R=(2,3)$.

(f) $a: y-3=7(x-5)$ and $m: (x,y)=(2,-4)+t\begin{bmatrix}2\\6\end{bmatrix}$.

(g) $a: y-3=7(x-5)$ and $b: \begin{cases}x=3-2t\\y=-1+4t\end{cases}$.

22. Find the intersections of the following pairs of lines:

(a) $l:\ x+2y=12$ and $m:\ (x,y)=(3,2)+t\begin{bmatrix}1\\2\end{bmatrix}$.

(b) $m:\ (x,y)=(3,2)+t\begin{bmatrix}1\\2\end{bmatrix}$ and k the line through the points $P=(6,3)$ and $Q=(0,1)$.

(c) $l:\ x+2y=12$ and $n:\ x-y=-1$.

(d) $m:\ (x,y)=(3,2)+t\begin{bmatrix}1\\2\end{bmatrix}$ and $p:\ (x,y)=(1,2)+s\begin{bmatrix}-1\\3\end{bmatrix}$.

(e) $m:\ (x,y)=(3,2)+t\begin{bmatrix}1\\2\end{bmatrix}$ and the line q perpendicular to m through the point $R=(10,6)$.

(f) $a:\ y+7=3(x-1)$ and $m:\ (x,y)=(3,2)+t\begin{bmatrix}1\\2\end{bmatrix}$.

(g) $a:\ y+7=3(x-1)$ and $b:\begin{cases}x=4+t\\y=4+5t\end{cases}$.

23. Find the intersections of the following pairs of lines:

(a) $\begin{cases}x=-3+t\\y=1-t\end{cases}$ and $(x,y)=(7,0)+s\begin{bmatrix}2\\1\end{bmatrix}$.

(b) $x+4y=13$ and the line through the points $(4,0)$ and $(5,-1)$.

(c) $(x,y)=(7,0)+s\begin{bmatrix}2\\1\end{bmatrix}$ and $(x,y)=(1,3)+t\begin{bmatrix}-1\\1\end{bmatrix}$.

(d) $x+4y=13$ and $(x,y)=(7,0)+s\begin{bmatrix}2\\1\end{bmatrix}$.

(e) $(x,y)=(9,1)+r\begin{bmatrix}4\\-1\end{bmatrix}$ and the line perpendicular to $x+4y=13$ through the point $(4,0)$.

(f) $y-1=-(x+3)$ and $\begin{cases}x=9+4r\\y=1-r\end{cases}$.

(g) $x+y=-2$ and $y+2=2(x-3)$.

24. Find the intersections of the following pairs of lines:

(a) $\begin{cases} x = 4+7t \\ y = 5+t \end{cases}$ and $(x,y) = (6,1) + s\begin{bmatrix} -3 \\ 1 \end{bmatrix}$.

(b) $3x - y = 7$ and the line through the points $(13, 2)$ and $(5, -2)$.

(c) $(x,y) = (4, 5) + s\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $(x,y) = (-2,-3) + t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(d) $3x - y = 7$ and $(x,y) = (-2,-3) + t\begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(e) $(x,y) = (9, 0) + r\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and the line perpendicular to $x - 2y = 4$ through the point $(3, 2)$.

(f) $y + 2 = -2(x-5)$ and $\begin{cases} x = 6-3r \\ y = 1+r \end{cases}$.

(g) $x - 2y = 9$ and $3(y-4) = -(x+3)$.

25. Determine the angles between the pairs of lines of problem **21 (a) - (g)**.

26. Determine the angles between the pairs of lines of problem **22 (a) - (g)**.

27. Determine the angles between the pairs of lines of problem **23 (a) - (g)**.

28. Determine the angles between the pairs of lines of problem **24 (a) - (g)**.

29. Find the distances of the point $A = (4, 2)$ to each of the 8 lines in problem **21**.

30. Find the distances of the point $A = (1, -2)$ to each of the 8 lines in problem **22**.

31. Find the distances of the point $A = (4, 0)$ to each of the 6 lines in problem **23**.

32. Find the distances of the point $A = (3, 2)$ to each of the 7 lines in problem **24**.

33. **(i)** Find the equations of the lines through $A = (\ 4, 2\ )$ parallel to the 8 lines defined in problem **21** .

(ii) Find the equations of the lines through $A = (\ 4, 2\ )$ perpendicular to the 8 lines defined in problem **21**.

34. **(i)** Find the equations of the lines through $A = (\ 1, -2\ )$ parallel to the 8 lines defined in problem **22**.

(ii) Find the equations of the lines through $A = (1, -2\ )$ perpendicular to the 8 lines defined in problem **22**.

35. **(i)** Find the equations of the lines through $A = (\ 4,\ 0\ )$ parallel to the 6 lines defined in problem **23**.

(ii) Find the equations of the lines through $A = (\ 5,\ -1\ )$ perpendicular to the 6 lines defined in problem **23**.

36. Find the equations of the lines through $A = (\ 6\ ,\ 1\ )$ parallel to the 7 lines defined in problem **24**.

(ii) Find the equations of the lines through $A = (\ 3,\ 2\ )$ perpendicular to the 6 lines defined in problem **24**.

3D lines

37. Find a vector equation of the line passing through the points $P = (\ 3, 2, 5\ )$ and $Q = (-1, 5, 7)$.

38. Find a vector equation of the line passing through the points $P = (\ 4,\ -1, 6\ )$ and $Q = (-2\ , 8, 3)$.

39. Find a vector equation of the line passing through the points $P = (\ 1,\ -2\ , 5\ )$ and $Q = (-3\ , 7, 4)$.

40. Find a vector equation of the line passing through the points $P = (\ 2, 3,\ -2\ )$ and $Q = (-2\ , 6,\ -3)$.

41. Determine if the points $P = (11, 2, 3)$, $Q = (-5, -6, 3)$ and $R = (-1, -4, 6)$ are on the line

$$k: \ (x, y, z) = (3, -2, 5) + t\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}.$$

42. Determine if the points $P = (3, 3, 3)$, $Q = (-5, 7, -3)$ and $R = (7, 1, 15)$ are on the line

$$l: \ (x, y, z) = (-1, 5, 3) + s\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$$

43. Determine if the points $P = (-3, 9, -4)$, $Q = (-1, 5, -4)$ and $R = (1, 1, -4)$ are on the line

$$l: \ (x, y, z) = (-2, 7, -1) + s\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$$

44. Determine if the points $P = (6, 5, -5)$, $Q = (-6, 1, -1)$ and $R = (-3, 2, -1)$ are on the line

$$l: \ (x, y, z) = (3, 4, -5) + s\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$$

45. Determine if the lines $k: \ (x, y, z) = (3, -2, 5) + t\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$

and $l: \ (x, y, z) = (-1, 5, 3) + s\begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}$ are parallel. If not, do they intersect?

46. Determine if the lines $k: \ (x, y, z) = (4, 2, 0) + t\begin{bmatrix} -0.5 \\ 1 \\ -2.5 \end{bmatrix}$

and $l: \ (x, y, z) = (3, 4, -5) + s\begin{bmatrix} 2 \\ -4 \\ 10 \end{bmatrix}$ are parallel. If not, do they intersect?

47. Determine if the lines $k:\ (x,y,z)=(1,-1,2)+t\begin{bmatrix}1\\2\\3\end{bmatrix}$

and $l:\ (x,y,z)=(0,3,1)+s\begin{bmatrix}1\\-1\\2\end{bmatrix}$ are parallel. If not, do they intersect?

48. Determine if the lines $k:\ (x,y,z)=(3,-2,5)+t\begin{bmatrix}-1\\2\\3\end{bmatrix}$

and $l:\ (x,y,z)=(-1,6,8)+s\begin{bmatrix}2\\-4\\3\end{bmatrix}$ are parallel. If not, do they intersect?

49. Write the line $\dfrac{x-1}{-3}=\dfrac{y+4}{5}=\dfrac{z-2}{6}$ in vector form $(x,y,z)=(a,b,c)+t\begin{bmatrix}A\\B\\C\end{bmatrix}$.

50. Write the line $\dfrac{x-5}{2}=\dfrac{y+3}{-4}=6-z$ in vector form $(x,y,z)=(a,b,c)+t\begin{bmatrix}A\\B\\C\end{bmatrix}$.

51. Write the line $\dfrac{x+4}{-7}=\dfrac{y-3}{2}=z-6$ in vector form $(x,y,z)=(a,b,c)+t\begin{bmatrix}A\\B\\C\end{bmatrix}$.

52. Write the line $\dfrac{x-4}{-5}=\dfrac{y-7}{3}=4-2z$ in vector form $(x,y,z)=(a,b,c)+t\begin{bmatrix}A\\B\\C\end{bmatrix}$.

53. Write the line $\begin{cases}x=3-7t\\y=-2+4t\\z=6+5t\end{cases}$ in vector form $(x,y,z)=(a,b,c)+t\begin{bmatrix}A\\B\\C\end{bmatrix}$.

54. Write the line $\begin{cases}x=7-2t\\y=5+3t\\z=-1+6t\end{cases}$ in vector form $(x,y,z)=(a,b,c)+t\begin{bmatrix}A\\B\\C\end{bmatrix}$.

55. Write the line $\begin{cases} x = 2+5t \\ y = -3+7t \\ z = 4-t \end{cases}$ in vector form $(x, y, z) = (a, b, c) + t\begin{bmatrix} A \\ B \\ C \end{bmatrix}$.

56. Write the line $\begin{cases} x = -1-6t \\ y = 2-3t \\ z = 5+4t \end{cases}$ in vector form $(x, y, z) = (a, b, c) + t\begin{bmatrix} A \\ B \\ C \end{bmatrix}$.

57. Line l goes through the points $P = (3, 2, 5)$ and $Q = (-1, 5, 7)$. Line k has equation

$k: (x, y, z) = (7, 3, 7) + s\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$. Are lines l and k parallel? If not, do they intersect?

58. Line l goes through the points $P = (6, -2, 1)$ and $Q = (4, 0, -1)$. Line k has equation

$k: (x, y, z) = (5, 2, 2) + s\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$. Are lines l and k parallel? If not, do they intersect?

59. Line l goes through the points $P = (1, -2, 3)$ and $Q = (25, -2, 21)$. Line k has equation

$k: (x, y, z) = (1, 2, 3) + s\begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix}$. Are lines l and k parallel? If not, do they intersect?

60. Line l goes through the points $P = (0, 4, 0)$ and $Q = (3, -2, 9)$. Line k has equation

$k: (x, y, z) = (1, 2, 3) + s\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$. Are lines l and k parallel? If not, do they intersect?

61. Find the points P on k and Q on l such that $\text{dist}(P,Q)$ is as small as possible, where

$$k:\ (x,y,z) = (1,-1,2) + t\begin{bmatrix}1\\2\\3\end{bmatrix} \quad \text{and} \quad l:\ (x,y,z) = (3,0,6) + s\begin{bmatrix}1\\-1\\2\end{bmatrix}.$$

62. Find the points P on k and Q on l such that $\text{dist}(P,Q)$ is as small as possible, where

$$k:\ (x,y,z) = (0,-1,0) + t\begin{bmatrix}1\\2\\2\end{bmatrix} \quad \text{and} \quad l:\ (x,y,z) = (0,2,1) + s\begin{bmatrix}1\\1\\1\end{bmatrix}.$$

63. Find the points P on k and Q on l such that $\text{dist}(P,Q)$ is as small as possible, where

$$k:\ (x,y,z) = (2,2,4) + t\begin{bmatrix}1\\0\\1\end{bmatrix} \quad \text{and} \quad l:\ (x,y,z) = (2,2,-5) + s\begin{bmatrix}3\\2\\4\end{bmatrix}.$$

64. Find the points P on k and Q on l such that $\text{dist}(P,Q)$ is as small as possible, where

$$k:\ (x,y,z) = (4,-1,9) + t\begin{bmatrix}2\\1\\0\end{bmatrix} \quad \text{and} \quad l:\ (x,y,z) = (2,4,5) + s\begin{bmatrix}0\\3\\2\end{bmatrix}.$$