Formal Methods and Functional Programming - Series 1

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Assignment 4

1. Parenthesizing Formulas

1. $(A \vee B) \to (C \to ((A \wedge C) \vee (B \wedge C)))$
2. $(A \to (B \to C)) \to ((A \wedge B) \to C)$

2. Proving Formulas

Proof for $(A \vee B) \to (C \to (A \wedge C) \vee (B \wedge C))$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A \vee B}\,\text{axiom} \quad \dfrac{\dfrac{\dfrac{}{\Gamma, A \vdash A}\,\text{axiom} \quad \dfrac{}{\Gamma, A \vdash C}\,\text{axiom}}{\Gamma, A \vdash A \wedge C}\,\wedge I}{\Gamma, A \vdash (A \wedge C) \vee (B \wedge C)}\,\vee IL \quad \dfrac{\dfrac{\dfrac{}{\Gamma, B \vdash B}\,\text{axiom} \quad \dfrac{}{\Gamma, B \vdash C}\,\text{axiom}}{\Gamma, B \vdash B \wedge C}\,\wedge I}{\Gamma, B \vdash (A \wedge C) \vee (B \wedge C)}\,\vee IL}{\Gamma \coloneqq A \vee B, C \vdash (A \wedge C) \vee (B \wedge C)}\,\vee E}{A \vee B \vdash C \to (A \wedge C) \vee (B \wedge C)}\,\to I}{(A \vee B) \to (C \to (A \wedge C) \vee (B \wedge C))}\,\to I$$

Proof for $(A \to (B \to C)) \to ((A \wedge B) \to C)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A \to (B \to C)}\,\text{axiom} \quad \dfrac{\dfrac{}{\Gamma \vdash A \wedge B}\,\text{axiom}}{\Gamma \vdash A}\,\wedge EL}{\Gamma \vdash B \to C}\,\to E \quad \dfrac{\dfrac{}{\Gamma \vdash A \wedge B}\,\text{axiom}}{\Gamma \vdash B}\,\wedge ER}{\Gamma \coloneqq A \to (B \to C), A \wedge B \vdash C}\,\to E}{A \to (B \to C) \vdash (A \wedge B) \to C}\,\to I}{(A \to (B \to C)) \to ((A \wedge B) \to C)}\,\to I$$

3. Elimination and Introduction Rules for $\leftrightarrow$

$$\dfrac{\Gamma \vdash A \to B \qquad \Gamma \vdash B \to A}{\Gamma \vdash A \leftrightarrow B}\,\leftrightarrow I$$

$$\dfrac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash A \to B}\,\leftrightarrow EL$$

$$\dfrac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash B \to A}\,\leftrightarrow ER$$