

# Formal Methods and Functional Programming - Series 1

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## Assignment 4

### 1. Parenthesizing Formulas

1.  $(A \vee B) \rightarrow (C \rightarrow ((A \wedge C) \vee (B \wedge C)))$
2.  $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)$

### 2. Proving Formulas

**Proof for**  $(A \vee B) \rightarrow (C \rightarrow (A \wedge C) \vee (B \wedge C))$

$$\frac{\frac{\frac{\frac{\frac{\Gamma, A \vdash A}{\text{axiom}}}{\Gamma, A \vdash A \wedge C}}{\Gamma, A \vdash (A \wedge C) \vee (B \wedge C)} \vee IL}{\Gamma, A \vdash (A \wedge C) \vee (B \wedge C)} \vee IL}{\Gamma \vdash A \vee B} \text{axiom} \quad \frac{\frac{\frac{\frac{\frac{\Gamma, B \vdash B}{\text{axiom}}}{\Gamma, B \vdash B \wedge C}}{\Gamma, B \vdash (A \wedge C) \vee (B \wedge C)} \vee IL}{\Gamma, B \vdash (A \wedge C) \vee (B \wedge C)} \vee IL}{\Gamma \vdash A \vee B, C \vdash (A \wedge C) \vee (B \wedge C)} \rightarrow I \quad \frac{\frac{\frac{\frac{\Gamma \vdash A \vee B, C \vdash (A \wedge C) \vee (B \wedge C)}{\rightarrow I}}{A \vee B \vdash C \rightarrow (A \wedge C) \vee (B \wedge C)} \rightarrow I}{(A \vee B) \rightarrow (C \rightarrow (A \wedge C) \vee (B \wedge C))} \rightarrow I$$

**Proof for**  $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)$

$$\frac{\frac{\frac{\frac{\frac{\Gamma \vdash A \rightarrow (B \rightarrow C)}{\text{axiom}}}{\Gamma \vdash B \rightarrow C}}{\Gamma \vdash B \rightarrow C} \rightarrow E \quad \frac{\frac{\frac{\frac{\Gamma \vdash A \wedge B}{\text{axiom}}}{\Gamma \vdash A} \wedge EL}{\Gamma \vdash B} \wedge ER}{\Gamma \vdash A \rightarrow (B \rightarrow C), A \wedge B \vdash C} \rightarrow I \quad \frac{\frac{\frac{\frac{\Gamma \vdash A \rightarrow (B \rightarrow C), A \wedge B \vdash C}{\rightarrow I}}{A \rightarrow (B \rightarrow C) \vdash (A \wedge B) \rightarrow C} \rightarrow I}{(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)} \rightarrow I$$

### 3. Elimination and Introduction Rules for $\leftrightarrow$

$$\frac{\frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash B \rightarrow A}{\Gamma \vdash A \leftrightarrow B} \leftrightarrow I}{\frac{\frac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash A \rightarrow B} \leftrightarrow EL}{\frac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash B \rightarrow A} \leftrightarrow ER}$$