

Formal Methods and Functional Programming - Series 1

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Assignment 4

1. Parenthesizing Formulas

1. $(A \vee B) \rightarrow (C \rightarrow ((A \wedge C) \vee (B \wedge C)))$
2. $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)$

2. Proving Formulas

Proof for $(A \vee B) \rightarrow (C \rightarrow (A \wedge C) \vee (B \wedge C))$

$$\frac{\frac{\frac{\overline{\Gamma, A \vdash A} \text{ axiom} \quad \overline{\Gamma, A \vdash C} \text{ axiom}}{\Gamma, A \vdash A \wedge C} \wedge I \quad \frac{\overline{\Gamma, B \vdash B} \text{ axiom} \quad \overline{\Gamma, B \vdash C} \text{ axiom}}{\Gamma, B \vdash B \wedge C} \wedge I}{\frac{\overline{\Gamma, A \vdash (A \wedge C) \vee (B \wedge C)} \vee IR \quad \frac{\overline{\Gamma, B \vdash (A \wedge C) \vee (B \wedge C)} \vee IL}{\Gamma \vdash A \vee B} \vee E} \rightarrow I \quad \frac{\Gamma := A \vee B, C \vdash (A \wedge C) \vee (B \wedge C)}{A \vee B \vdash C \rightarrow (A \wedge C) \vee (B \wedge C)} \rightarrow I \quad \frac{A \vee B \vdash C \rightarrow (A \wedge C) \vee (B \wedge C)}{(A \vee B) \rightarrow (C \rightarrow (A \wedge C) \vee (B \wedge C))} \rightarrow I$$

Proof for $(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)$

$$\frac{\frac{\overline{\Gamma \vdash A \rightarrow (B \rightarrow C)} \text{ axiom} \quad \frac{\overline{\Gamma \vdash A \wedge B} \text{ axiom}}{\Gamma \vdash A} \wedge EL}{\Gamma \vdash B \rightarrow C} \rightarrow E \quad \frac{\overline{\Gamma \vdash A \wedge B} \text{ axiom}}{\Gamma \vdash B} \wedge ER}{\frac{\Gamma := A \rightarrow (B \rightarrow C), A \wedge B \vdash C}{A \rightarrow (B \rightarrow C) \vdash (A \wedge B) \rightarrow C} \rightarrow I} \rightarrow I \quad \frac{A \rightarrow (B \rightarrow C) \vdash (A \wedge B) \rightarrow C}{(A \rightarrow (B \rightarrow C)) \rightarrow ((A \wedge B) \rightarrow C)} \rightarrow I$$

3. Elimination and Introduction Rules for \leftrightarrow

$$\frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash B \rightarrow A}{\Gamma \vdash A \leftrightarrow B} \leftrightarrow I$$
$$\frac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash A \rightarrow B} \leftrightarrow EL$$
$$\frac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash B \rightarrow A} \leftrightarrow ER$$