

Propositional Logic

$$\begin{array}{c}
\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge I \quad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash A} \wedge EL \quad \frac{\Gamma \vdash A \wedge B}{\Gamma \vdash B} \wedge ER \\
\frac{\Gamma, A \vdash B}{\Gamma \vdash A \rightarrow B} \rightarrow I \quad \frac{\Gamma \vdash A \rightarrow B \quad \Gamma \vdash A}{\Gamma \vdash B} \rightarrow E \\
\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee IL \quad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee IR \quad \frac{\Gamma \vdash A \vee B \quad \Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma \vdash C} \vee E \\
\frac{\Gamma \vdash \perp}{\Gamma \vdash A} \perp E \quad \frac{\Gamma \vdash \neg A \quad \Gamma \vdash A}{\Gamma \vdash B} \neg E
\end{array}$$

First-order Logic

Universal Quantification

$$\frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x.A(x)} \forall I^* \quad \frac{\Gamma \vdash \forall x.A(x)}{\Gamma \vdash A(t)} \forall E$$

* Side condition: x not free in any assumption in Γ .

Existential Quantification

$$\frac{\Gamma \vdash A(t)}{\Gamma \vdash \exists x.A(x)} \exists I \quad \frac{\Gamma \vdash \exists x.A(x) \quad \Gamma, A(x) \vdash B}{\Gamma \vdash B} \exists E^*$$

* Side condition: x is neither free in B nor free in Γ .

First-order Logic with Equality

$$\frac{}{\Gamma \vdash t = t} ref \quad \frac{\Gamma \vdash t = s}{\Gamma \vdash s = t} sym \quad \frac{\Gamma \vdash t = s \quad \Gamma \vdash s = r}{\Gamma \vdash t = r} trans$$

Congruence