Formal Methods and Functional Programming - Series 1

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Assignment 4

1. Parenthesizing Formulas

1. $(A \lor B) \to (C \to ((A \land C) \lor (B \land C)))$
2. $(A \to (B \to C)) \to ((A \land B) \to C)$

2. Proving Formulas

Proof for $(A \lor B) \to (C \to (A \land C) \lor (B \land C))$

$$
\cfrac{
\cfrac{\ }{\Gamma \vdash A \lor B}\text{ axiom}
\qquad
\cfrac{
\cfrac{
\cfrac{
\cfrac{\cfrac{\ }{\Gamma, A \vdash A}\text{ axiom} \quad \cfrac{\ }{\Gamma, A \vdash C}\text{ axiom}}{\Gamma, A \vdash A \land C}\land I
}{\Gamma, A \vdash (A \land C) \lor (B \land C)}\lor IL
}{\Gamma, A \vdash (A \land C) \lor (B \land C)}\lor IL
\qquad
\cfrac{
\cfrac{\cfrac{\ }{\Gamma, B \vdash B}\text{ axiom} \quad \cfrac{\ }{\Gamma, B \vdash C}\text{ axiom}}{\Gamma, B \vdash B \land C}\land I
}{\Gamma, B \vdash (A \land C) \lor (B \land C)}\lor IL
}{\Gamma := A \lor B, C \vdash (A \land C) \lor (B \land C)}\lor E
}{
\cfrac{
\cfrac{A \lor B \vdash C \to (A \land C) \lor (B \land C)}{(A \lor B) \to (C \to (A \land C) \lor (B \land C))}\to I
}{}\to I
}
$$

Proof for $(A \to (B \to C)) \to ((A \land B) \to C)$

$$
\cfrac{
\cfrac{
\cfrac{\cfrac{\ }{\Gamma \vdash A \to (B \to C)}\text{ axiom} \qquad \cfrac{\cfrac{\ }{\Gamma \vdash A \land B}\text{ axiom}}{\Gamma \vdash A}\land EL}{\Gamma \vdash B \to C}\to E \qquad \cfrac{\cfrac{\ }{\Gamma \vdash A \land B}\text{ axiom}}{\Gamma \vdash B}\land ER
}{\Gamma := A \to (B \to C), A \land B \vdash C}\to E
}{
\cfrac{A \to (B \to C) \vdash (A \land B) \to C}{(A \to (B \to C)) \to ((A \land B) \to C)}\to I
}\to I
$$

3. Elimination and Introduction Rules for $\leftrightarrow$

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\cfrac{\Gamma \vdash A \to B \qquad \Gamma \vdash B \to A}{\Gamma \vdash A \leftrightarrow B}\leftrightarrow I
$$

$$
\cfrac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash A \to B}\leftrightarrow EL
$$

$$
\cfrac{\Gamma \vdash A \leftrightarrow B}{\Gamma \vdash B \to A}\leftrightarrow ER
$$