Luminosity Distance - PHYS417 Project 2

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1 The Friedmann Equation

Show it can be written Predict behaviour (take limits)

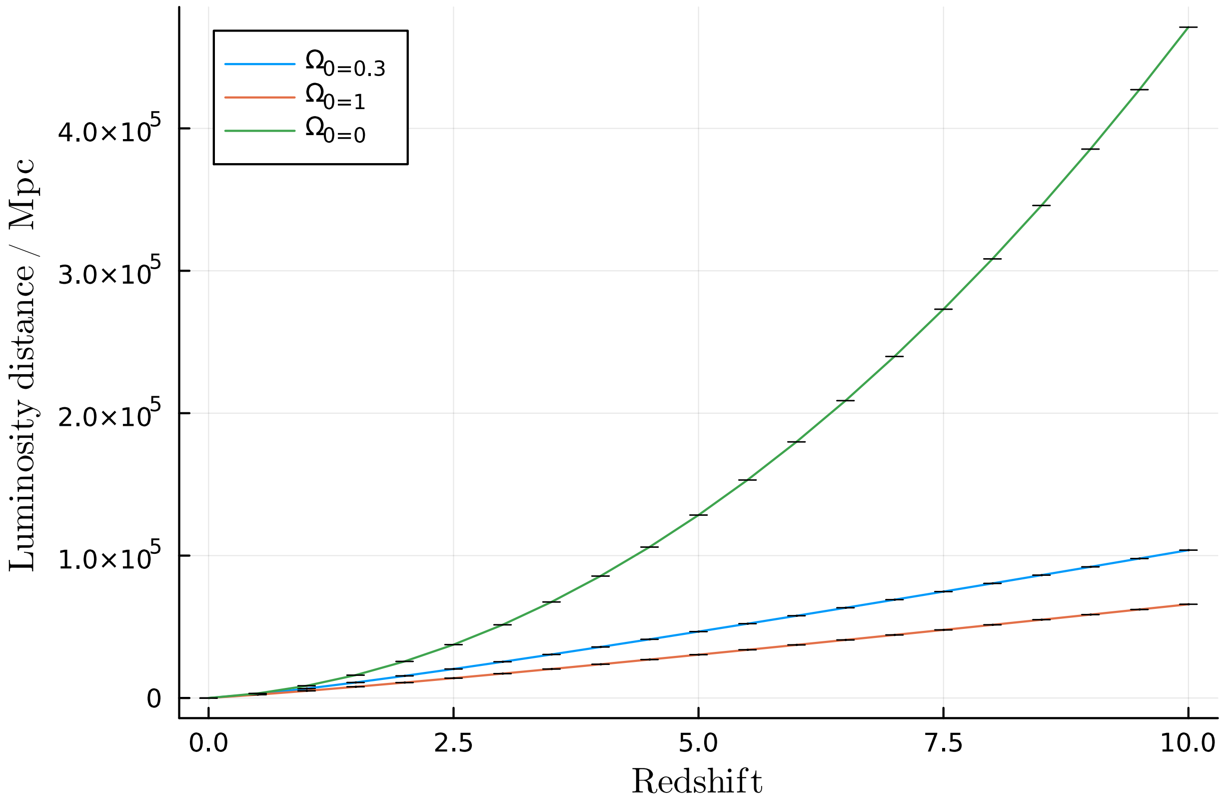
In a universe with both matter and dark energy we need to find d_p by numerical integration of

$$d_p(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{E(z')}$$

From here on out we'll assume a flat universe, unless stated otherwise.

```
using Unitful # Unit handling
using UnitfulAstro # Astronomical units
using PhysicalConstants.CODATA2018: c_0 # Speed of light from CODATA2018, with units
using QuadGK # Numerical integration
using Plots, Latexify, UnitfulLatexify, LaTeXStrings
using Measurements # Uncertainly handling

# Unitful doesn't export preferunits so we have to reference by package
Unitful.preferunits(u"Mpc",u"Msun")

# Define our cosmological parameters
Ω0::Real = 0.3
Ωk::Real = 0 # flat universe
ΩΛ::Real = 1 - Ω0
H0 = 70u"km/s/Mpc"

# This is a one line function definition
E(z::Real)::Number = sqrt(Ω0*(1+z)^3 + Ωk*(1+z)^2 + ΩΛ)

# Input type must be real and the output must be a length
# Unitful will determine and check the dimensions of the output
function dp(z::Real)::Unitful.Length
    """Calculate proper distance from redshift."""
    integral, err = quadgk(zz -> 1/E(zz), 0, z, rtol=1e-8)
    return c_0/H0 .* (integral ± err)
end

dl(z::Real) = dp(z) * (1+z)

z = 0:0.5:10
# dl.(z) vectorises dl so it acts elementwise on z
plot(z, upreferred.(dl.(z)), unitformat=latexify, label="\\Omega_0=$(Ω0)",
```

```
    legend=:topleft, xlabel=L"\mathrm{Redshift}", ylabel="\\mathrm{Luminosity\\
distance}")

# Matter dominated -> no dark energy
Ω0::Real = 1
ΩΛ::Real = 1 - Ω0
plot!(z, dl.(z), label="\\Omega_0=$(Ω0)") # plot!() updates last plot

# Dark energy dominated -> no matter
Ω0::Real = 0
ΩΛ::Real = 1 - Ω0
plot!(z, dl.(z), label="\\Omega_0=$(Ω0)")
```



Candle time.

We know that radial velocity is related to proper distance by

$$v_r = H_0 d_p$$

In the low redshift (and thus non-relativistic) limit

$$v_r \approx zc$$

so we can conclude

$$d_p \approx \frac{zc}{H_0}$$

Combining this with our luminosity distance equation and gives

$$d_l \approx \frac{zc}{H_0}(1+z) \approx \frac{zc}{H_0} \text{as } z \to 0$$

```
using CSV, DataFrames
using LsqFit

dl_from_m(dm::Number)::Unitful.Length = 10u"pc" * 10^(dm/5)

data = CSV.read("SCPUnion2.1_mu_vs_z.txt",
                DataFrame,
                header=["id", "z", "DMVal", "DMErr", "Prob"],
                skipto=6
        )

data.DM = data.DMVal .± data.DMErr

select!(data, :id, :z, :DM)  # Get rid of the other columns

data.dl = dl_from_m.(data.DM)

scatter(data.z, data.dl, unitformat=latexify, label="Candles",
    legend=:topleft, xlabel=L"\mathrm{Redshift}", ylabel="\\mathrm{Luminosity\\
distance}",
    markersize=3, markeralpha=0.75, xscale=:log10, yscale=:log10)

@. lowz_model(z, p) = z * c_0 / p[1] # z = redshift array, p = parameter array = [H0]

lowz_data = @view data[data.z .< 0.1, [2, 3]] # get z and DM for low redshift

lowz_fit = curve_fit(lowz_model, lowz_data.z, lowz_data.dl, [70u"km/s/Mpc"]) # last term
is starting parameter

Error: ArgumentError: dl not found
```