

Session #2 - Practice Solution

Jan. 30, 2025

1 Budget Constraint

Suppose you have \$60 to spend on either good x , with a price of \$10, or good y , with a price of \$6.

(a) The budget constraint is:

$$6Q_y + 10Q_x = 60 \quad \Rightarrow \quad Q_y = 10 - \frac{5}{3}Q_x$$

(b) If you receive an extra \$30, the new budget constraint is:

$$6Q_y + 10Q_x = 90 \quad \Rightarrow \quad Q_y = 15 - \frac{5}{3}Q_x$$

(c) If the extra \$30 is removed but the price of x decreases to \$5:

$$6Q_y + 5Q_x = 60 \quad \Rightarrow \quad Q_y = 10 - \frac{5}{6}Q_x$$

2 Utility Functions

Consider the following utility functions:

- (I) $u(x_1, x_2) = 4\sqrt{x_1x_2}$,
- (II) $u(x_1, x_2) = 4x_1 + x_2$,
- (III) $u(x_1, x_2) = \min\{4x_1, x_2\}$.

2.1 Marginal Utilities

- (I) $MU_1 = \frac{2\sqrt{x_2}}{\sqrt{x_1}}$, $MU_2 = \frac{2\sqrt{x_1}}{\sqrt{x_2}}$
- (II) $MU_1 = 4$, $MU_2 = 1$
- (III)

$$MU_1 = \begin{cases} 4, & \text{if } 4x_1 < x_2 \\ 0, & \text{if } 4x_1 \geq x_2 \end{cases}, \quad MU_2 = \begin{cases} 1, & \text{if } x_2 < 4x_1 \\ 0, & \text{if } x_2 \geq 4x_1 \end{cases}$$

2.2 Marginal Rate of Substitution (MRS)

The MRS of x_1 for x_2 is given by:

$$\text{MRS}_{x_1x_2} = \frac{MU_1}{MU_2}$$

3 Optimal Consumption Bundles

Given the budget constraint:

$$2x_1 + x_2 = 8$$

(I) Setting $\text{MRS} = \frac{P_1}{P_2}$, we get:

$$\frac{x_2}{x_1} = 2 \quad \Rightarrow \quad x_2 = 2x_1$$

Plugging into the budget constraint:

$$2x_1 + 2x_1 = 4x_1 = 8 \quad \Rightarrow \quad x_1 = 2, \quad x_2 = 4$$

(II) For perfect substitutes, the MRS is constant at 4:

$$x_1^* = 4, \quad x_2^* = 0$$

(III) For perfect complements, the consumption ratio is fixed at $4x_1 = x_2$:

$$(x_1^*, x_2^*) = \left(\frac{4}{3}, \frac{16}{3} \right)$$